Solitary Wave Propagation in a Novel Granular Chain Setup

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Abstract

Granular materials are ubiquitous in the modern world of engineering—from the gravel used in civil construction to the use of small particles as a shock-absorbing protective measure. The work presented here focuses on energy propagation within a one dimensional system of grains (a granular chain); the primary point of interest being the emergence of nondispersive pulses of energy, known as solitary waves (SW).

In this study, the SW was measured by a series of piezoelectric sensors as it propagated throughout a novel, experimental granular chain setup. Overall, this research contributes to the understanding of energy flow in granular media and provides a means to better control the energy propagation within such media.

Background

- **Granular Chain:** a discrete, one-dimensional system of macroscopic grains (spherical in shape); many implications in energy localization and shock absorption
- **Solitary Wave:** a nondispersive bundle of traveling energy that is known to form in granular chains; travels throughout the system at a constant speed that is proportional to the amount of energy in the wave; can be produced simply from an initial perturbation or strike to one end of the granular chain system
- **Hertz Law:** describes the potential energy interactions between adjacent grains in the granular chain when pressure is applied; the relationship between energy and grain overlap is nonlinear (for the spherical grains being used) and this allows for the formation of solitary waves

Purpose

1. To construct a macroscopic granular chain setup capable of supporting the propagation of solitary waves.
2. To measure the properties of the SW propagation in 5 different material types (silicon nitride, borosilicate, stainless steel 420, stainless steel 316, and brass).
3. To compare the experimental data to the data obtained previously through computer simulations.

Hypothesis

- The material softness of the experimental setup will be directly related to the velocity with which the solitary wave travels
- Computer simulations using similar material and physical properties will yield comparable results to that obtained through the experimental setup

Experimental Setup & Methods

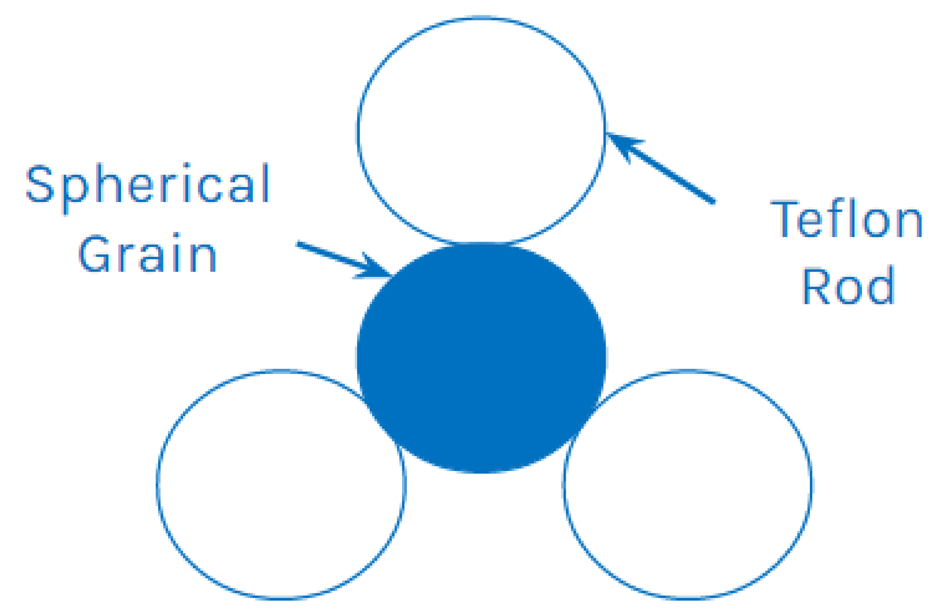

***Figure 1**: Cross sectional diagram of the experimental setup*

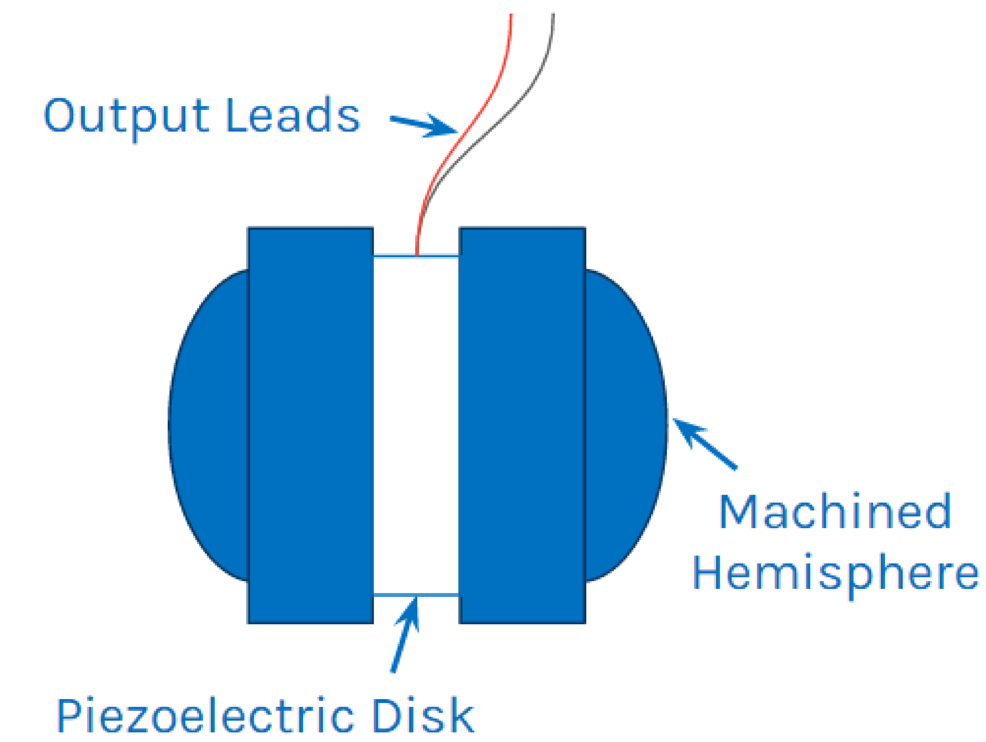

***Figure 2**: Diagram of the piezoelectric sensor insertion mechanism*

Main Apparatus

The granular chain setup itself is a track system composed of three Teflon rods and a hard metallic wall at each end. This track holds the spherical grains along the same central axis in order to properly support solitary wave propagation.

On the leftmost end of the system is a spherical cutout that allows the granular chain to be struck from the outside. A solenoid (simple push/pull mechanism) was used to strike the system, consistently forming solitary waves.

Piezoelectric Sensor

In order to measure the solitary wave propagation in the setup, piezoelectric disks were inserted into the granular chain. These sensors return a voltage signal that is proportional to the force applied to the ceramic disk surface.

A mechanism consisting of 2 hemi-spheres was designed to embed the sensor into the granular chain. This mechanism mimics the properties of an actual grain including the stiffness, mass, and dimensions.

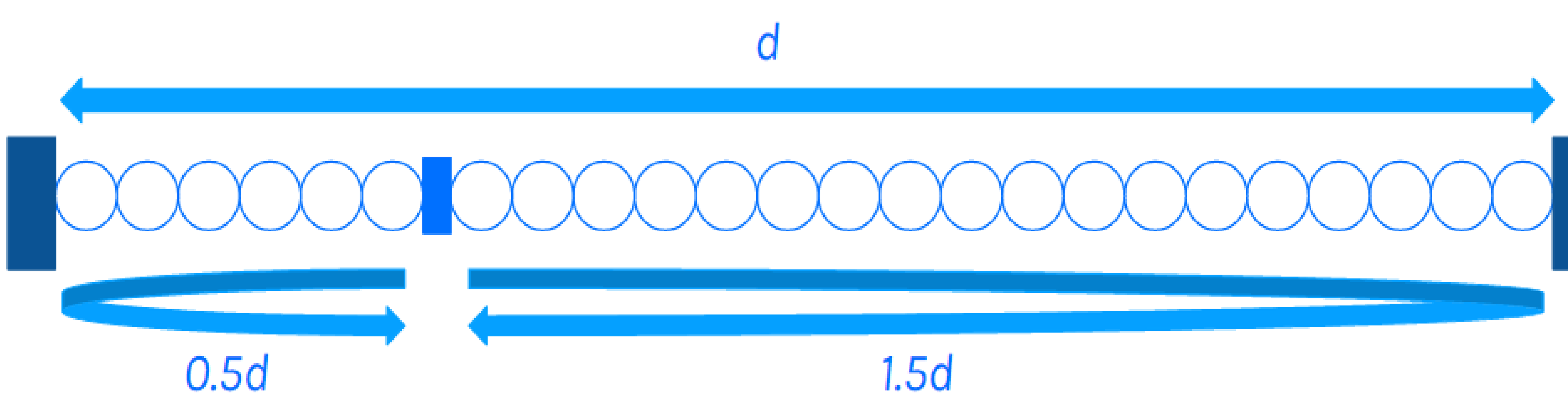

***Figure 3**: Diagram depicting the granular chain setup loaded with 24 grains and the piezoelectric sensor at quarter-way. The arrows represent the distance that the solitary wave travels, where 'd' is the length of the chain (0.288m)*

Data Collection

In order to collect data, the piezoelectric disk was inserted at four different positions in the setup (quarter-way, halfway, three-quarter way, and full length). During each trial, the force data was collected through the piezoelectric sensor and sent to an oscilloscope. Overall, at least 40 trials (10 at each of the 4 positions) were completed for each of the 5 material types.

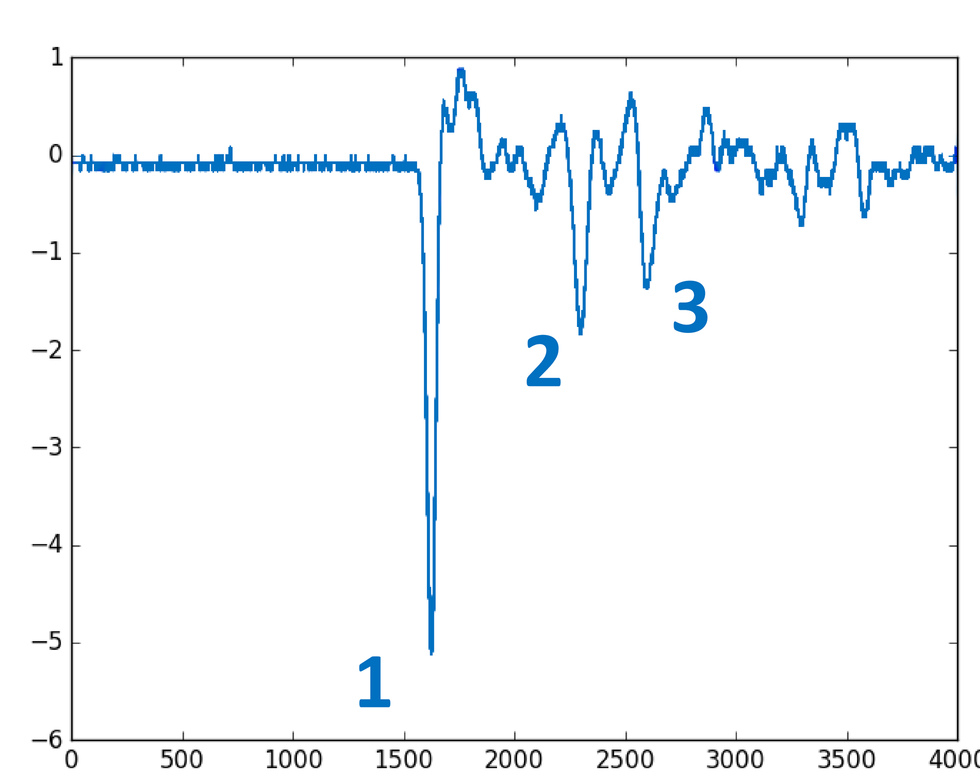

***Figure 4**: An example output signal from the piezoelectric sensor. 3 peaks are clearly defined, with each representing the time at which the SW passes the sensor in the granular chain*

Calculating Velocity

From the collected data, such as in ***Figure 4***, the time points at which each of the peaks occurred were compiled into a spreadsheet.

A formula was then developed to determine the velocity of the solitary wave that best fits the collected data. This model was deduced based on the fact that the wave velocity is constant and that the distance travelled can be calculated (***Figure 3***).

Data & Results

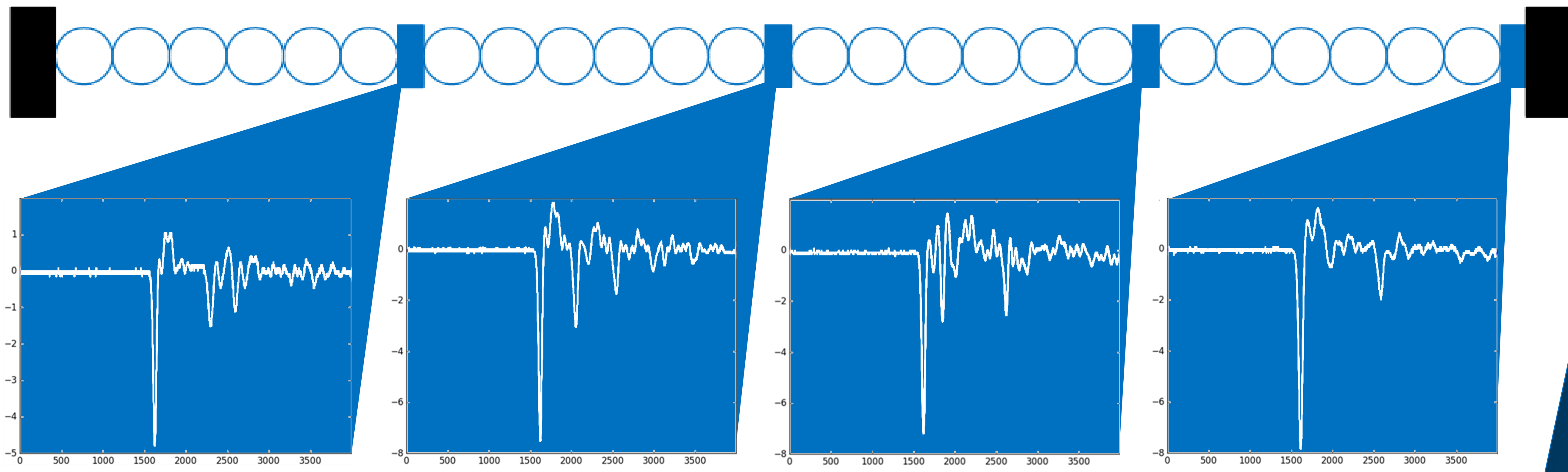

Figure 5: This diagram shows the results from four different experimental trials: with the piezoelectric sensor inserted at each of the four positions. The time points at which each of the peaks occur are then compiled in a spreadsheet and used to determine the solitary wave velocity.

Using the procedures described previously and the compiled experimental data (as in ***Figure 5***), the velocity of the solitary wave as it propagates through each of the five material types was determined. In order to examine the relationships between these velocities and the material properties, a means to quantify material softness was implemented (Softness was defined as $\rho \times D$, where ρ is the density of the material and D is related to the Young's Modulus and Poisson's Ratio of the material).

Furthermore, this definition of material softness was used to establish a reasonable means for comparing experimental data with computer simulation data. The data for each of the five experimental materials was compared directly with the data for a simulation material that had a similar softness value.

Experimental Material	Solitary Wave Velocity (m/s)	Softness ($\rho \times D$)	Comparable Simulation Material	Solitary Wave Velocity (m/s)	Softness ($\rho \times D$)
Stainless steel 316	439.7371742	0.0559569	Stainless steel	414.5454545	0.0551
Stainless steel 420	441.9787207	0.0533814			
Brass	351.7432348	0.0990308	Red oak wood	359.0551181	0.0786
Borosilicate	602.5844841	0.0503775	Pyrex	438.4615385	0.0479
Silicon nitride	806.4340483	0.0144044	Boron carbide	894.1176471	0.00805

Table 1: Table *comparing the solitary wave velocities and softness values of the experimental materials used to the corresponding values of comparable simulation materials*

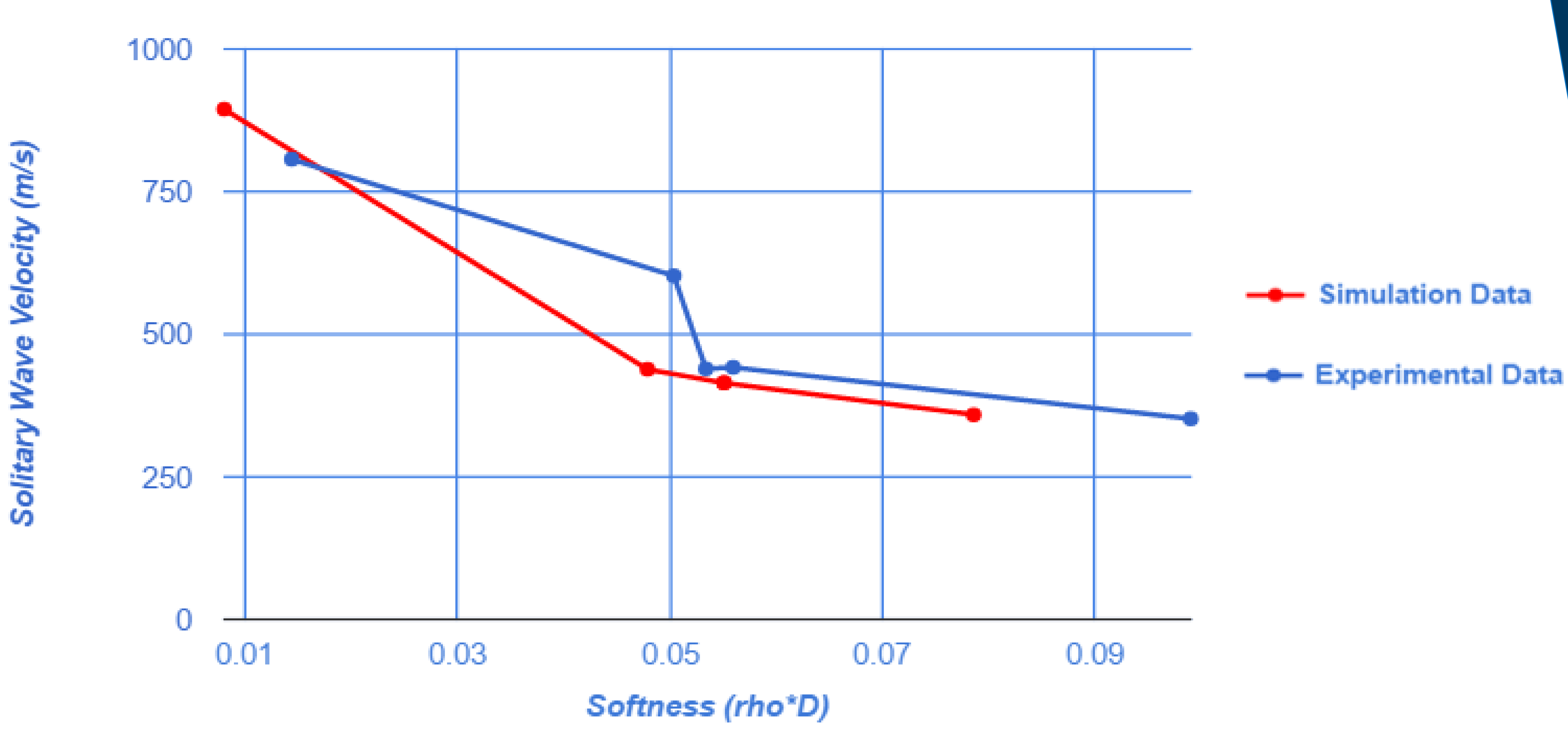

*Figure 6: This plot displays the data in **Table 1**. The experimental data is plotted with the simulation data on the same axes, where softness ($\rho \times D$) is represented on the x-axis and solitary wave velocity (m/s) is represented on the y-axis. Note that larger values of $\rho \times D$ indicate softer materials.*

Conclusions

In examining the experimental results (***Figure 6***), the major finding was the inverse relationship between solitary wave velocity and grain material softness, where the velocity decreases as the material becomes softer.

Furthermore, the experimental data obtained is well supported by the comparable simulation data; this can be observed in the similarity between the two datasets in ***Figure 6***. Such evidence confirms the validity of the novel experimental setup in its ability to support solitary waves and provides confidence in the use of a macroscopic setup for the study of energy propagation within granular media.

Applications

The findings in this study regarding the effects of material properties on solitary wave velocity can be exploited in a variety of granular systems as a practical method to better control mechanical energy transfer.

The success of the novel experimental setup suggests that similar macroscopic models can be utilized to study the behaviour of various granular media in a controlled, adjustable, and easy-to-observe environment.

Future Work

- The setup constructed in this work can be modified to study the energy localization behaviour and discrete breathers present within granular chains.
- This can be done by alternating softer material types in the center of the system, allowing for the bulk of the energy to be trapped in the softer materials.
- The granular chain setup can also be improved by the implementation of accelerometers as a means to collect acceleration data on the solitary wave propagation. Such a setup can ultimately be used to study the grain to grain interactions within multi-dimensional systems.

Acknowledgements

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References

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