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CSC560
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Analysis of MinMax2 Algorithms

Analysis of Linear Algorithm:

$C_b(n) = n + 1$ (when *n* integers are sorted from greatest to least)

$C_w(n) = 4(n-4) + 5$ (when *n* integers are sorted so that the two smallest integers and larges integer are found in the first four elements of the array)

Analysis of Divide and Conquer Algorithm:

Recurrence:

$$C(n) = \begin{cases} 5 & \text{if } n = 4 \\ 2\,C(\frac{n}{2}) + 4 & \text{if } n > 4 \end{cases}$$

Solving top-down:

$$C(n) = 2\,C(\frac{n}{2}) + 4$$

$$C(\frac{n}{2}) = 2\,C(\frac{n}{4}) + 4$$

$$C(n) = 4\,C(\frac{n}{4}) + 3 \cdot 4$$

$$C(\frac{n}{4}) = 2\,C(\frac{n}{8}) + 4$$

$$C(n) = 8\,C(\frac{n}{8}) + 7 \cdot 4$$

Closed form:

$$C(n) = 5(\frac{n}{4}) + 4(2^{\log_2 n - 2} - 1)$$

Induction proof:

Base case:

$$C(4) = 5(\frac{4}{4}) + 4(2^{\log_2 4 - 2} - 1) = 5$$

Induction case:

Assume: $C(2^k) = 5(2^{k-2}) + 4(2^{k-2} - 1)$ for all $k \leq K$

Show: $C(2^{K+1}) = 5(2^{K-1}) + 4(2^{K-1} - 1)$

$$C(2^{K+1}) = 2C(\frac{2^{K+1}}{2}) + 4$$

$$C(2^{K+1}) = 2C(2^K) + 4$$

$$C(2^{K+1}) = 2[5(2^{K-2}) + 4(2^{K-2} - 1)] + 4$$

$$C(2^{K+1}) = 10(2^{K-2}) + 8(2^{K-2} - 1) + 4$$

$$C(2^{K+1}) = 5(2^{K-1}) + 8(2^{K-2}) - 8 + 4$$

$$C(2^{K+1}) = 5(2^{K-1}) + 4[2(2^{K-2}) - 2 + 1]$$

$$C(2^{K+1}) = 5(2^{K-1}) + 4(2^{K-1} - 1)$$

Analysis of Runs

For an input of 64 using the linear algorithm...

$$C_b(64) = 64 + 1 = 65$$

$$C_w(64) = 4(64-4) + 5 = 245$$

For an input of 64 using the divide and conquer algorithm...

$$C(64) = 5(\frac{64}{4}) + 4(2^{\log_2 64 - 2} - 1) = 5(16) + 4(15) = 140$$

Comparing the ranges found for the numbers of comparisons for both algorithms when running the program with the results found from the mathematical analysis showed consistent results. The number of comparisons made during the runs using the linear algorithm were all between the best and worst outcomes predicted by the analysis. The number of comparisons made during every run using the divide and conquer algorithm matched the number of comparisons predicted by the analysis exactly.