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A simple model for the structure constant of temperature fluctuations in the lower atmosphere

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Abstract. A model for C_T, the structure constant of temperature fluctuations, for the lowest 50–500 m of the atmosphere over land has been derived from similarity expressions and simple surface flux parametrisations. Only very few data are required by the model: standard weather data and estimates of ground characteristics. Universal flux profiles for the nocturnal boundary layer lead to C_T profiles which increase in value near the top of the layer. Validation of model predictions by slant path laser scintillometer and differential thermometer measurements over flat terrain gives very good agreement for daytime and good agreement for night-time. Slant path scintillometer measurements show that even in extremely mountainous terrain the daytime model gives reasonable results.

1. Introduction

Optical techniques for imaging, ranging, communication or active remote sensing are often limited by atmospheric turbulent refractive index fluctuations. In the visible and IR these refractive index fluctuations over land are caused mostly by temperature fluctuations. The spectrum of these fluctuations is proportional to the structure constant of the temperature fluctuations, given in the inertial subrange by

$$C_T^2 = \frac{\langle(\Theta(r_0) - \Theta(r_0 + r))^2\rangle}{r^{2/3}} \tag{1}$$

where r_0 is a position vector, r is a separation vector (within the inertial subrange) and Θ is the potential temperature. The brackets $\langle\ \rangle$ indicate a time average. In the visible C_T is related to the structure constant of the refractive index fluctuations C_n by (Lawrence *et al* 1970)

$$C_n = 79 \times 10^{-6}(P/T^2)C_T \tag{2}$$

with the air pressure P in hPa and the temperature T in K. The highest values of C_T normally occur near the ground in the atmospheric surface layer. They vary with time and height over some orders of magnitude. We derive a C_T model for about the lowest 100 m of the atmosphere in terms of standard weather data and simple surface parameters. We finally compare model predictions with platinum wire differential thermometer and laser scintillometer measurements in flat and mountainous terrain.

2. Model

The C_T model is based on the similarity expressions with asymptotic scaling of Wyngaard *et al* (1971):

$$\frac{C_T^2 z^{2/3}}{T_*^2} = 4.9(1 - 7z/L)^{-2/3} \tag{3}$$

for the unstable case (daytime) and

$$\frac{C_T^2 z^{2/3}}{T_*^2} = 4.9[1 + 2.4(z/L)^{2/3}] \tag{4}$$

for the stable case (night-time). Here z is the height above the ground; T_* is $-Q_0/u_*$, where $Q_0 = \langle(T - \langle T\rangle)(w - \langle w\rangle)\rangle$ is the temperature flux with w the vertical velocity component and u_* is the friction velocity; and L is the Monin–Oboukhov length defined as $L = -(u_*^3 T)/(kgQ_0)$ with the von Karman constant $k = 0.35$ and the acceleration of gravity $g = 9.8\ \mathrm{m\ s^{-2}}$.

In the daytime the surface layer, where the fluxes are assumed to be constant, extends to a height of typically 100 m (on sunny days often up to 500 m), whereas it often does not comprise more than 10 m during the night. In order to have about the same range of applicability for night-time as for daytime, we will extend equation (4) to the whole nocturnal boundary layer, which normally covers the lowest 50–500 m. Now the fluxes can no longer be taken to be constant; we have to use local, height-dependent variables of the heat flux Q, the friction velocity u_1, and the Monin–

Oboukhov length L_1. We rewrite equation (4) as

$$C_T^2 = 4.9\frac{T_*^2}{z^{2/3}} + 11.8\frac{Q^2}{u_1^2 L_1^{2/3}}. \quad (5)$$

The second term dominates at greater heights, in the so-called z-less stratification layer. The lower, z-dependent layer is here assumed to be identical to the constant-flux surface layer. Nieuwstadt (1984) gives the following general profiles of u_1 and Q:

$$u_1/u_* = (1 - z/h)^{3/4} \quad (6)$$

$$Q/Q_0 = 1 - z/h \quad (7)$$

where h is the height of the turbulent nocturnal boundary layer. These relations should be valid for stationary conditions over flat terrain. This finally provides the boundary layer model equation

$$C_T^2 = 4.9\frac{T_*^2}{z^{2/3}} + 11.8\frac{(Q_0^2)^{4/3} k^{2/3} g^{2/3}}{u_*^4 (1 - z/h)^{1/3} T^{2/3}}. \quad (8)$$

Now modelling is reduced to a parametrisation of the surface fluxes and knowledge of h.

u_* is provided by the flux profile relationship (Businger 1973)

$$\frac{u}{u_*} = \frac{1}{k}\left(\ln\frac{z_u}{z_0} - \psi\right) \quad (9)$$

where

$$\psi = 2\ln[(1+y)/2] + \ln[(1+y^2)/2] - 2\tan^{-1} y + \pi/2$$

$$y = (1 - 15 z_u/L)^{1/4} \qquad \text{for } z_u/L < 0 \text{ (unstable)}$$

$$\psi = -4.7\, z_u/L \qquad \text{for } z_u/L > 0 \text{ (stable)}.$$

In equation (9) u is the wind speed measured at the height z_u and z_0 is the roughness length of the surface. It can be seen from equation (4) that u_* becomes less important as $-z/L$ exceeds 0.14. On the other hand, from $-z/L \leqslant 0.14$ and $z \geqslant z_u$ it follows that $\psi \leqslant 0.34$, which is typically less than 10% of $\ln(z_u/z_0)$ in equation (9). Thus it is acceptable to use the neutral case approximation $\psi = 0$ under all unstable conditions for $z \geqslant z_u$.

For the daytime heat flux a very simple expression was given by Holtslag and Van Ulden (1983). This is, slightly modified,

$$Q_0 = \frac{\eta}{c_p \rho}\frac{1 - \alpha + \gamma/s}{1 + \gamma/s}(1 - A)R - \beta \quad (10)$$

where c_p is the specific heat of air at constant pressure, ρ is the density of air, γ is c_p/r_w with r_w the latent heat of water vaporisation and s is $\partial q_s/\partial T$ with q_s the saturation specific humidity. Both γ and s are functions only of temperature. The surface humidity parameter α is equal to 1 over wet grass and is 0 over dry surfaces. Corresponding to the humidity of the ground and its vegetation coverage, intermediate values must be chosen. R is the solar irradiation, which can be estimated from astronomical parameters and the cloud coverage, and A is the surface albedo. The constants η and β are about 0.9 and 20 W m^{-2} respectively.

Estimation of night-time Q_0 is much more difficult because infrared cooling, ground heat conduction, sensible and latent heat fluxes have the same order of magnitude. For modelling all these processes more information than usually available is needed, e.g. ground humidity and temperature profile. We used another approach: for low wind speeds we take the maximum possible downwards heat flux for which equation (9) can be solved, i.e.

$$Q_0 = -\frac{4}{27}\frac{k^2 u^3 T}{4.7 z_u g[\ln(z_u/z_0)]^2}. \quad (11)$$

Because under stronger wind Q_0 becomes saturated, we limited Q_0 to $H_{max}/(c_p\rho)$, where we chose $H_{max} = 5$ W m^{-2} within the first 4 h after sunset (the upper soil layer is still heated from the day) and 10 W m^{-2} elsewhere, in agreement with our experimental results. Generally these values have to be adjusted to the respective ground and weather conditions. Low heat conductivity of the ground tends to increase these values, and cloud cover or high humidity tends to decrease them.

There are no reliable parametrisations of h from surface data. Because h becomes very critical as z/h approaches 1, h should be measured directly, or modelling is limited to $z/h \simeq 0.5$. For rough estimation only we can use the expression of Zilitinkevich (1972):

$$h = 0.4(u_* L/f)^{1/2} \quad (12)$$

where f is the Coriolis parameter.

3. Experimental verification

Model predictions have been tested by two experiments. One of them was carried out over homogeneous, flat terrain. Point measurements of C_T were taken by a platinum wire differential thermometer at a height of 2.2 m. Additionally, two laser scintillometers, one using a He–Ne laser (0.63 μm) and the other using a CO_2 laser (10.6 μm), measured path-averaged C_n. The laser sources were on a small hill at a height of 30 m compared to the flat environment. The path length was chosen to be 1600 m, making turbulent inner scale effects on the scintillation negligible. For model input we measured the solar radiation and the wind speed. The ground was moderately dry, about 50% irregularly covered with grass. We chose the parameters $z_0 = 0.05$ m and $\alpha = 0.5$. A was measured to be 0.3. We did not measure h, but we expect that h should be significantly higher than 30 m. Thus we used the approximation $z/h = 0$ for night-time.

Neglecting the influence of humidity fluctuations for both wavelengths, C_n was computed from equation (2). The log-intensity variance for a spherical wave is (Ishimaru 1978)

$$\sigma_{\ln I}^2 = 2.25 k_*^{7/6} \int_0^X C_n^2(z(x))(x/X)^{5/6}(X - x)^{5/6}\,dx \quad (13)$$

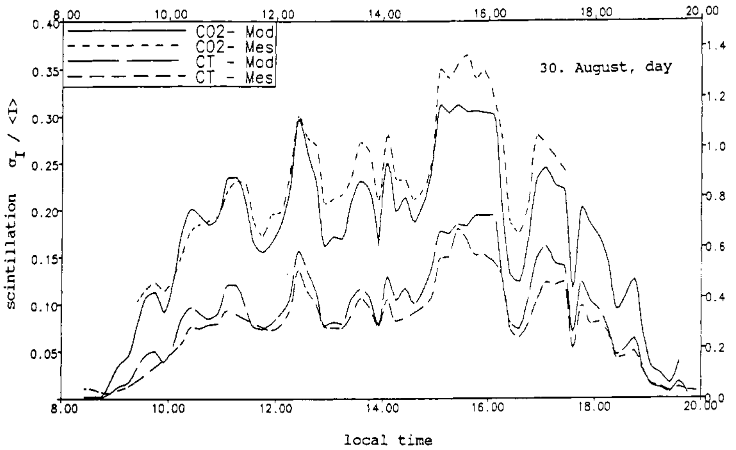

Figure 1. Comparison of model prediction of C_T and CO_2 laser scintillation (λ = 10.6 μm) with measurements for one day.

where k_* is the optical wavenumber, X is the total path length and x is the length coordinate along the path. We related the measured σ_I to $\sigma_{\ln I}$ by

$$\sigma_I^2/\langle I\rangle^2 = \exp(\sigma_{\ln I}^2) - 1 \qquad (14)$$

which is valid for a log-normal distribution of I. For daytime we used the CO_2 system because the He–Ne scintillation became saturated. At night we operated only the He–Ne system.

Figure 1 gives a comparison for daytime of both the single-level model results with the point C_T measurements and the slant path integrated model predictions with the scintillation values. Conditions were broken clouds and wind speed between 1 and 4 m s^{-1}. A similar comparison is given in figure 2 for night-time. There is very good agreement, although the short-term fluctuations of the night-time model exceed those observed. Model runs of other days not shown here provide comparable agreement.

Another experiment was carried out in the Bavarian Alps on sunny summer days. The He–Ne scintillometer path between a mountain top and a valley can be seen in figure 3. For application of the daytime surface layer model the z values were chosen to be the respective vertical distances to the ground (ground profile idealised to a polygon), which had a maximum value of 300 m. Wind and radiation values have been averaged from measurements on both sides of the scintillometer path. For the different surface types (grass, forest, rocks) we chose typical values of z_0, α and A.

In figure 4 measurements (symbols 10 min averages) and model predictions (curves) of scintillation are plotted against the solar irradiation. The wind speed was taken as a parameter. The overall agreement is good. Only for lower irradiation values did the observed scintillation exceed the model predictions significantly. On the other hand, the predicted wind speed dependence was not observed. One has to take into account that in the complicated dynamic system of a mountain valley (slope winds, etc) the application of the similarity expressions must be considered carefully. Also the two local wind measurements cannot be considered as representative of the whole path.

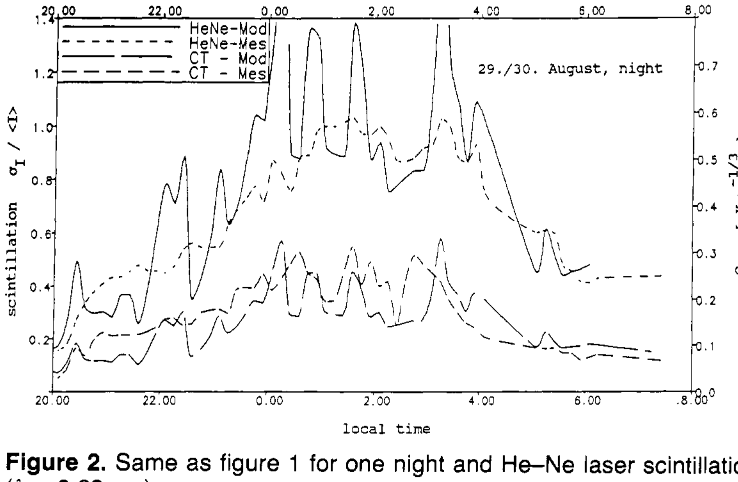

Figure 2. Same as figure 1 for one night and He–Ne laser scintillation (λ = 0.63 μm).

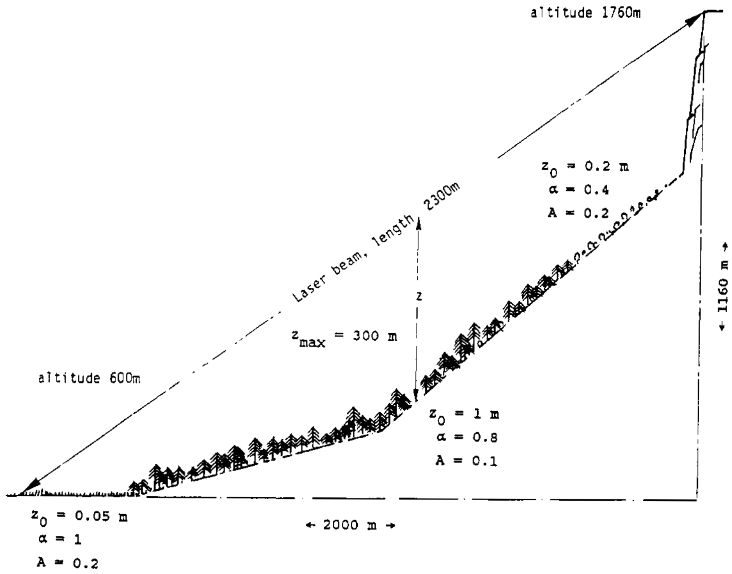

Figure 3. Cross section of the scintillation experiment site in the Bavarian Alps. The sketch shows the simplified geometry and parameters used for the calculations.

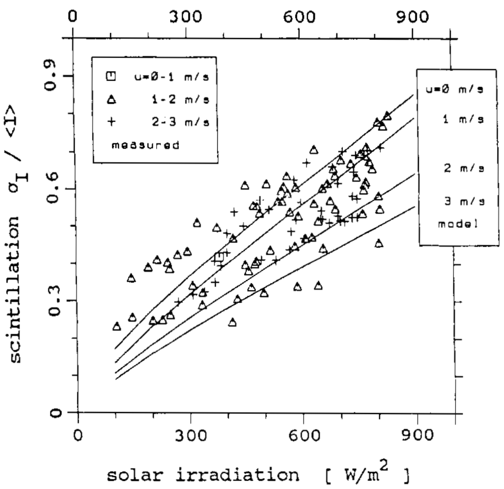

Figure 4. Comparison of measured laser scintillation (symbols) with model prediction (curves) for the Bavarian Alps experiment. Wind speed is taken as a parameter.

4. Discussion

A simple way to estimate the C_T profile near the ground has been demonstrated. Only knowledge of u, R, T, z_0, α and A for the daytime model or u, H_{max}, z_0 and h for the night-time model is required. The experimental results indicate high accuracy and applicability even under very extreme topographic conditions for daytime. The surface layer night-time accuracy suffers from uncertainties in Q_0 determination.

A verification of the night-time boundary layer equation to greater heights over flat terrain would be of great interest. There are observations (e.g. Tsvang 1969) which show an increase of C_T above about 100 m, supporting the model prediction. On the other hand, our model contradicts the empirical model of Kunkel and Walters (1983). They expect a rapid decrease of C_T^2 near h with $[1 + 100(z/h)^2]^{-1}$. Different definitions of h (Mahrt *et al* 1982) together with the occurrence of an accumulation layer (André and Mahrt 1982, Wetzel 1982) could give an explanation of this discrepancy.

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