Jackalope population dynamics

Fitting models to simulated data can be a helpful exercise for understanding how models behave under a wide range of parameter values and under what conditions model fitting algorithms converge or break down. This exercise creates a simulated time-series of population dynamics and then fits a population model to these data.

The (mythical) Jackalope is a species of hare native to the high plains of Wyoming and nearby states. It's population dynamics are controlled by the availability of forage (which is highly variable and driven by climate) and coyote predation. We will simulate jackalope dynamics using a density-dependent Gompertz population model.

The Gompertz model is

$$n_t = n_{t-1}e^{a+(b-1)log(n_{t-1})}$$

Where n_t is population density at time t, a is the intrinsic rate of population increase, and b is a parameter describing the strength of density dependence. While this model may look complicated at first, if we take transform the model to the log scale the equation simplifies to a very simple and easy to fit linear equation (see Ives et al. 2003 for further detail),

$$log(n_t) = a + b * log(n_{t-1})$$

While this model includes some of the processes that drive Jackalope density, it is intentionally a simplification. Thus, we will add process error to our model to account for any factor that might dive variability in density, but are omitted from our model.

$$log(n_t) = a + b * log(n_{t-1}) + \epsilon_t$$
$$\epsilon_t \sim Normal(0, \sigma^2)$$

Here is ϵ_t is the deviation of our Gompertz from the observed density, which we would expect to be normally distributed since we are modeling on the log scale (i.e. population densities on the log scale can vary from $-\infty$ to ∞).

Given this data model to

1. Generate a simulated data set with known parameter values for Jackalope density over time (R code provided)
2. Program a JAGS model to fit the above model to the simulated data which will require two steps 1) Specifying a likelihood and 2) specifying a prior.
3. Obtain posterior summary statistics for a, b, and sigma. How do they compare with the values used to simulate the data?
4. Play with differing parameter values in the simualtion data. How do estimates of parameters change when sigma is increased?

Extra Credit. Our current model assumes that Jackalope are perfectly observed. Add a "data" model to the simulation and JAGS code using $y_t \sim Poisson(n_t)$ where y_t is the oberserved count and n_t is a latent, true jackalope density.