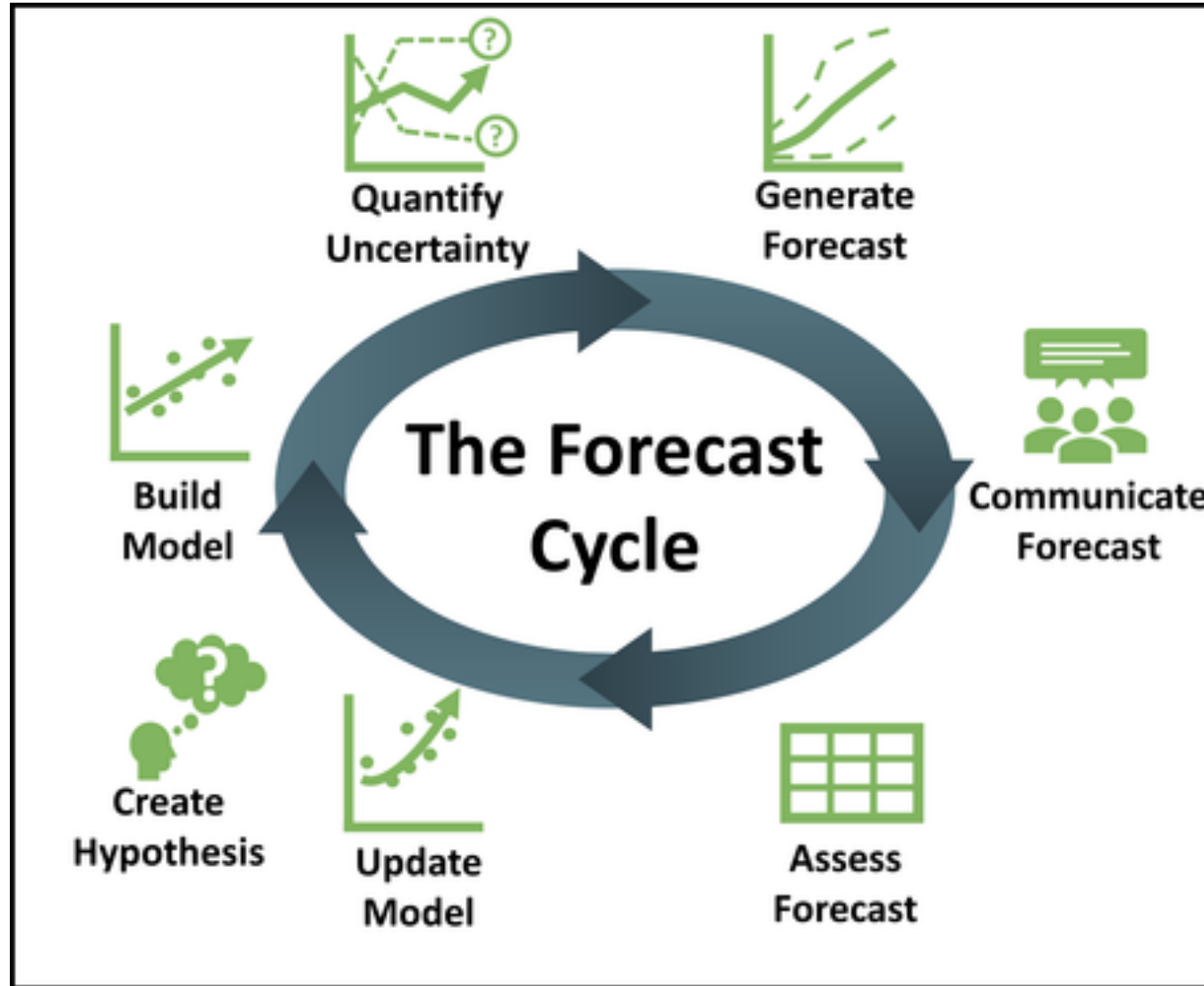


Autoregressive models

Flexible workhorse of time series forecasting



Moore, T.N., C.C. Carey, and R.Q. Thomas.

Autoregressive models

The current state is a linear combination of past states.
In other words a regression against itself.

$$\text{AR(P)} \quad Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3} \dots + \beta_p Y_{t-p} + \varepsilon_t$$

Autoregressive models

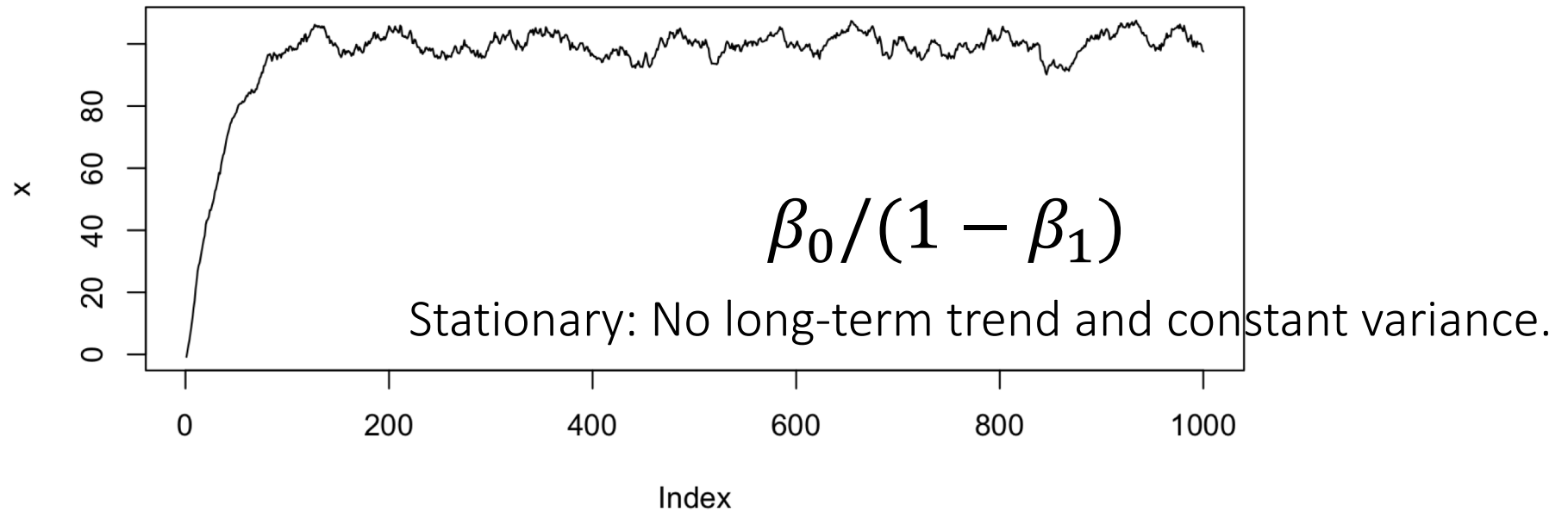
The current state is a linear combination of past states.
In other words a regression against itself.

AR(1)

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t$$

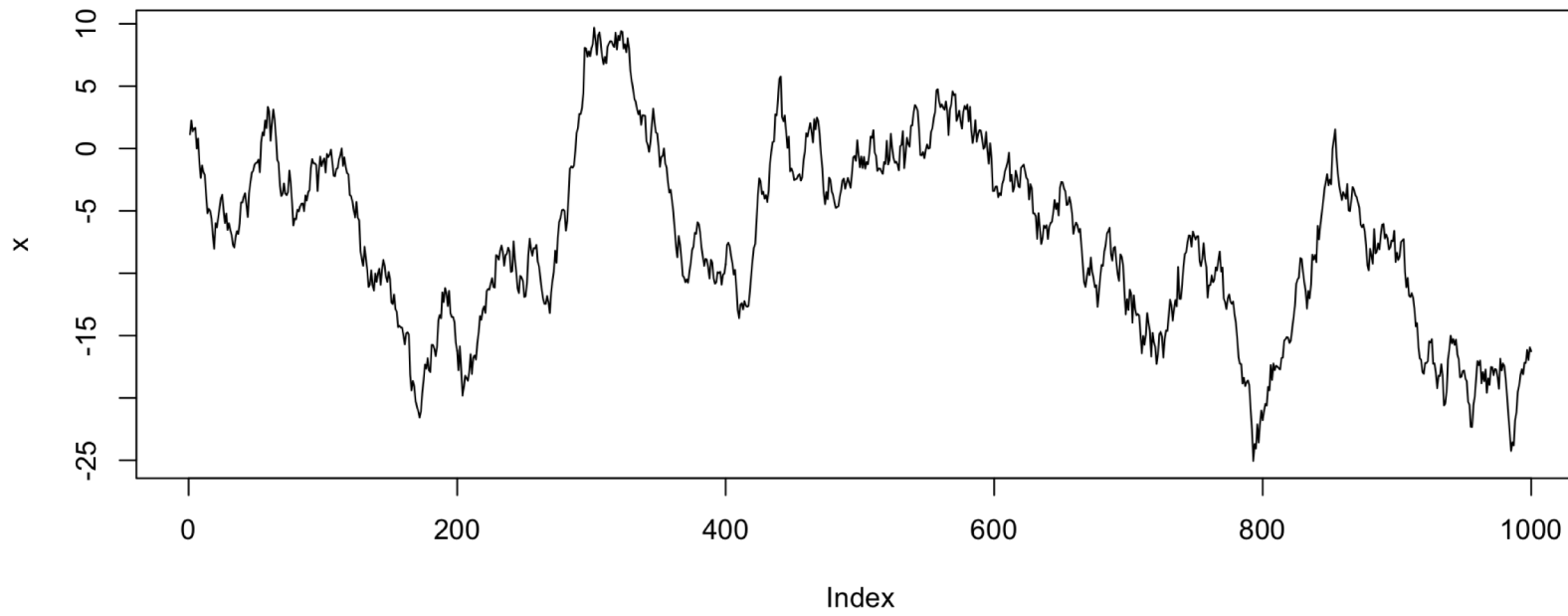
Autoregressive models

AR(1)
$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t$$



Random walk

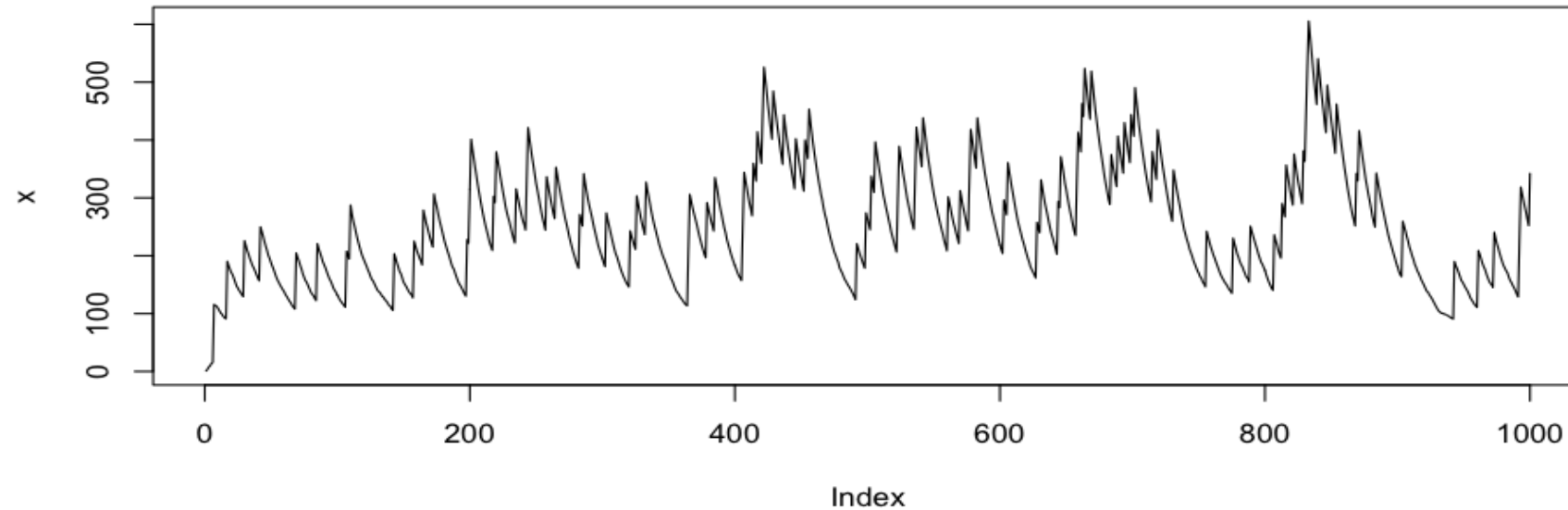
$$Y_t = 0 + 1 * Y_{t-1} + \varepsilon_t$$



What makes AR models so useful?

Very little data required, but can be made more complex

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 P_t + \varepsilon_t$$



Simple model for soil moisture dynamics

What makes AR models so useful?

Can also represent mechanistic models, e.g. Gompertz model

$$n_t = n_{t-1} e^{a+(b-1)\ln(n_{t-1})}$$

What makes AR models so useful?

$$n_t = n_{t-1} e^{a + (b-1)\ln(n_{t-1})}$$

$$\ln(n_t) = \ln(n_{t-1}) + a + (b - 1)\ln(n_{t-1})$$

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$$\ln(n_t) = \ln(n_{t-1}) + a + b * \ln(n_{t-1}) - \ln(n_{t-1})$$

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$$\ln(n_t) = \ln(n_{t-1}) + a + b * \ln(n_{t-1}) - \ln(n_{t-1})$$

$$\ln(n_t) = a + b * \ln(n_{t-1})$$

a = low density growth rate

$a/(1 - b)$ = carrying capacity

Inverse and forward modeling

In today's lab, we will use both inverse and forward modeling

- 1) Inverse modeling to fit AR(1) model to NDVI data
- 2) Forward model to forecast NDVI given fit AR(1), where initial condition is last NDVI measurement