# Approaches to Modeling, Types of uncertainty

Ecological Forecasting, Jan 20th

First, checking in on lab from Tuesday.

Any questions, or issues?

# Revisiting logistic growth

$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

Low density growth rate

Carrying Capacity

# Forward Modeling

$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

Estimated

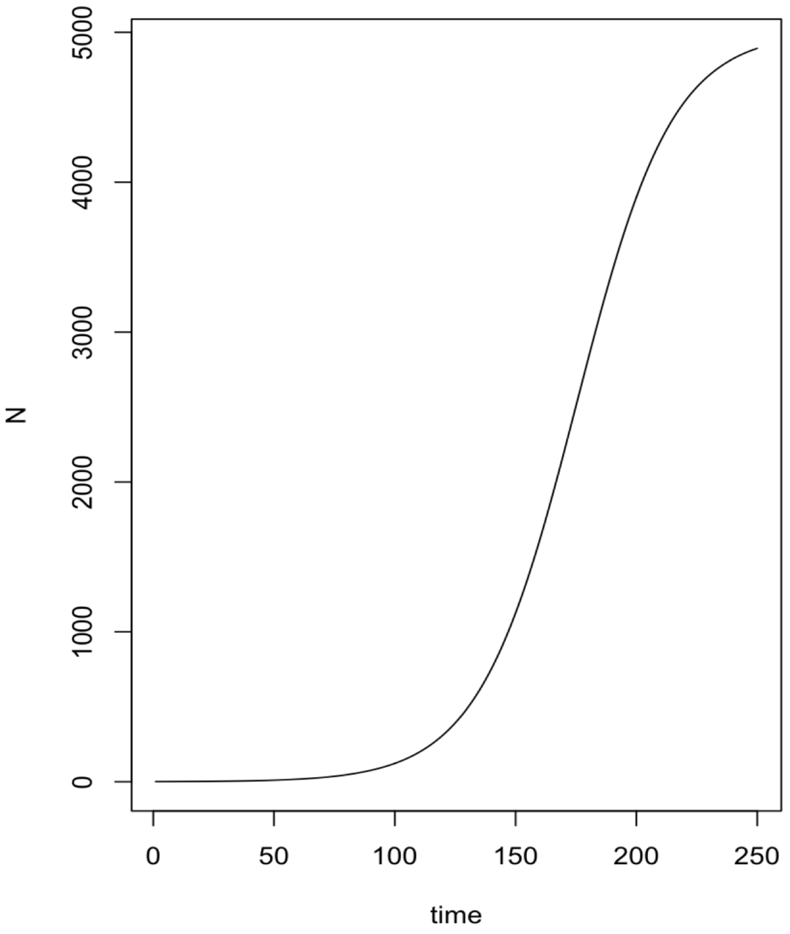
Data

Data

# Forward Modeling

$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

Model is purely deterministic,
stochasticity can be added in some forms



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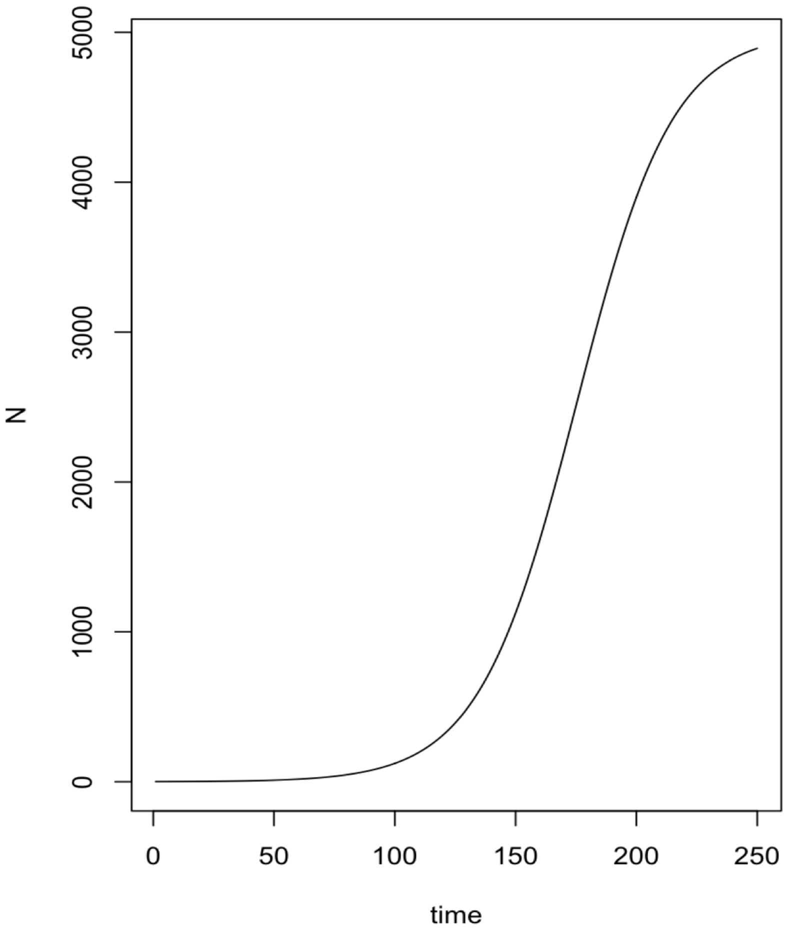

# Forward Modeling

Anyone know of any applications of forward modeling?

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| Parameter | Quantity | Units |
|---|---|---|
| *Aboveground state variables* | | |
| $n(\mathbf{z}, \mathbf{x}, a, t)$ | density† of size $\mathbf{z}$ type $\mathbf{x}$ plants in gaps of age $a$ at time $t$ | $m^{-2}$ |
| $p(a, t)$ | distribution of gap ages $a$ at time $t$ | dimensionless |
| *State dimensions* | | |
| $\mathbf{z}$ | plant size $\mathbf{z} = [z_s, z_a] = [B_s, B_a]$ | kg C, kg C |
| $\mathbf{x}$ | plant type $[x_1, x_2] \cdot x_1 = 0$ if $C_3$, 1 if $C_4$, $x_2$ = leaf longevity | dimensionless, yr |
| $a$ | gap age | yr |
| $t$ | time | yr |
| *Plant resource environment* | | |
| $\mathbf{r}$ | resource vector $\mathbf{r} = [\phi, W, N]$ | $J \cdot m^{-2} \cdot s^{-1}$, $m^3$ $H_2O/m^2$, kg $N/m^2$ |
| $\phi$ | photosynthetically active radiation (PAR) | $J \cdot m^{-2} \cdot s^{-1}$ |
| $W$ | soil water content | $m^3$ $H_2O/m^2$ |
| $N$ | plant available soil nitrogen content | kg $N/m^2$ |
| *Transition rates* | | |
| $g_s$ | structural biomass growth rate | kg C/yr |
| $g_a$ | living biomass tissue growth rate | kg C/yr |
| $\mu$ | mortality rate | $yr^{-1}$ |
| $f$ | fecundity | $yr^{-1}$ |
| $\lambda$ | total disturbance rate $\lambda = \lambda_F + \lambda_{DI}$ | $yr^{-1}$ |
| $\lambda_F$ | fire frequency | $yr^{-1}$ |
| $\lambda_{DI}$ | rate of canopy gap formation | $yr^{-1}$ |
| $s$ | survivorship of plants following disturbance | dimensionless |
| *Plant size characteristics* | | |
| $h$ | height | m |
| $B_a$ | living biomass $(B_l + B_r + B_{sw})$ | kg C |
| $B_s$ | structural stem biomass | kg C |
| $B_l$ | leaf biomass | kg C |
| $B_r$ | root biomass | kg C |
| $B_{sw}$ | sapwood biomass | kg C |
| *Leaf-level carbon and water fluxes* | | |
| $A_n$ | net rate of carbon gain per unit leaf area | $\mu mol$ $C \cdot m^{-2} \cdot s^{-1}$ |
| $\Psi$ | evapotranspiration rate per unit leaf area | $\mu mol$ $H_2O \cdot m^{-2} \cdot yr^{-1}$ |
| *Decomposition model state variables and coefficients* | | |
| $C_1$ | fast soil carbon pool | kg $C/m^2$ |
| $C_2$ | structural soil carbon pool | kg $C/m^2$ |
| $N_1$ | fast soil nitrogen pool | kg $N/m^2$ |
| $N_2$ | structural soil nitrogen pool | kg $N/m^2$ |
| *Miscellaneous* | | |
| $n_0(\mathbf{z}, \mathbf{x}, a)$ | initial plant density $n(\mathbf{z}, \mathbf{x}, a, 0)$† | $m^{-2}$ |
| $p_0(a)$ | initial gap age distribution | dimensionless |
| $\mathbf{z}_0$ | seedling size | kg C, kg C |
| $y$ | integer gap number $(1 \ldots Q)$ | dimensionless |
| $h^*$ | height above which mortality is treated as disturbance | m |
| $c_s$ | stomatal conductance | $\mu mol$ $H_2O \cdot m^{-2} \cdot s^{-1}$ |
| $T_L$ | leaf temperature | °C |
| $T_A$ | atmospheric air temperature | °C |
| $C_i$ | interstitial concentration of $CO_2$ | mol/mol |
| $C_a$ | atmospheric concentration of $CO_2$ | mol/mol |

† $n(\mathbf{z}, \mathbf{x}, a, t)$ is technically a density distribution where $n(\mathbf{z}, \mathbf{x}, a, t)dz_s dz_a da$ is the per $m^{-2}$ density of type $\mathbf{x}$ plants between size $z_s$ and $z_s + dz_s$, and size $z_a$ and $z_a + dz_a$ in gaps aged between $a$ and $a + da$ at time $t$.

Paul Moorecroft, Ecosystem Demography model

# Forward Modeling

## Anyone know of any applications of forward modeling?

Forward modeling historically has been of the most common modeling approaches. Big push to make more complicated forward models came in the 1970s, 1980s with increases in computing power.

# Inverse modeling

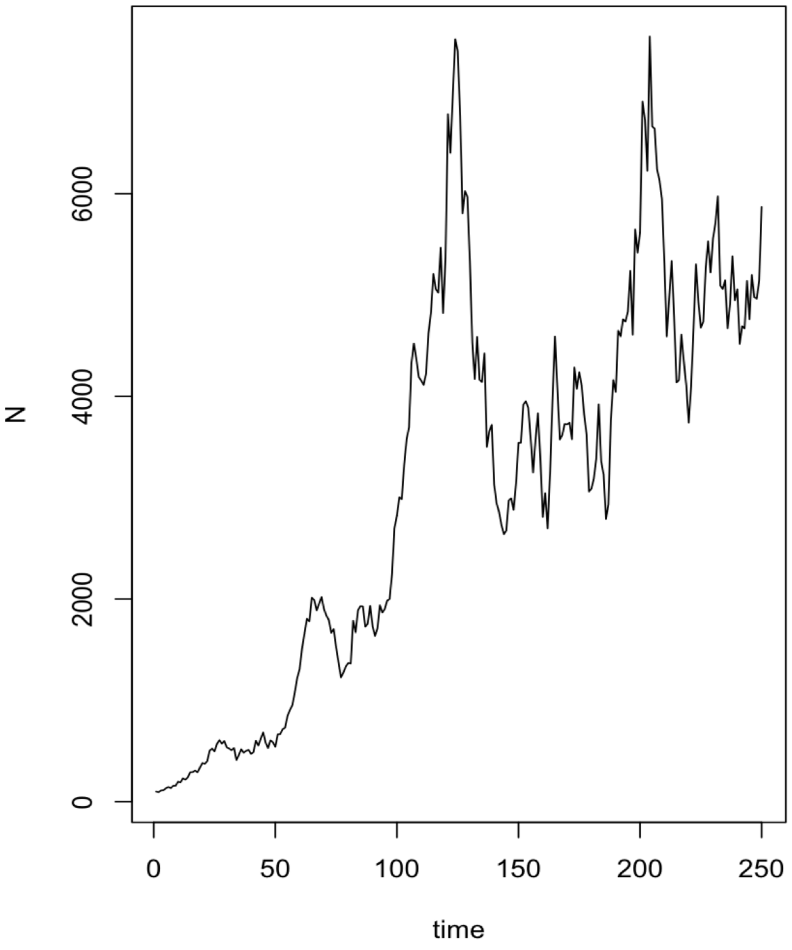
$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

Data

Estimated

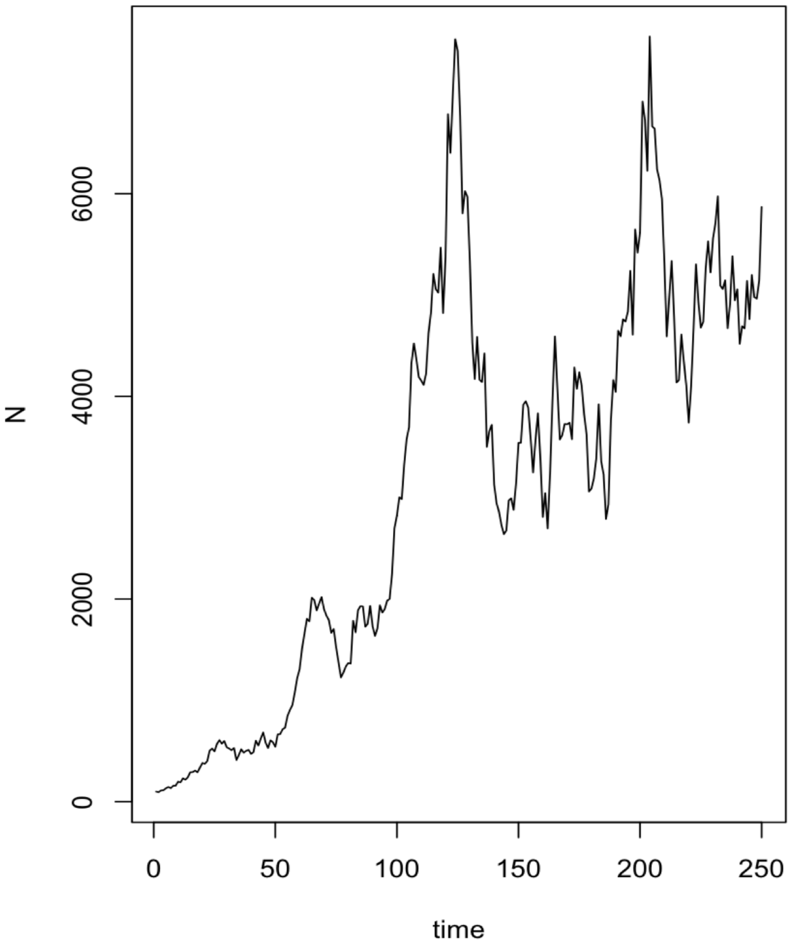
Estimated

# Inverse Modeling




$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

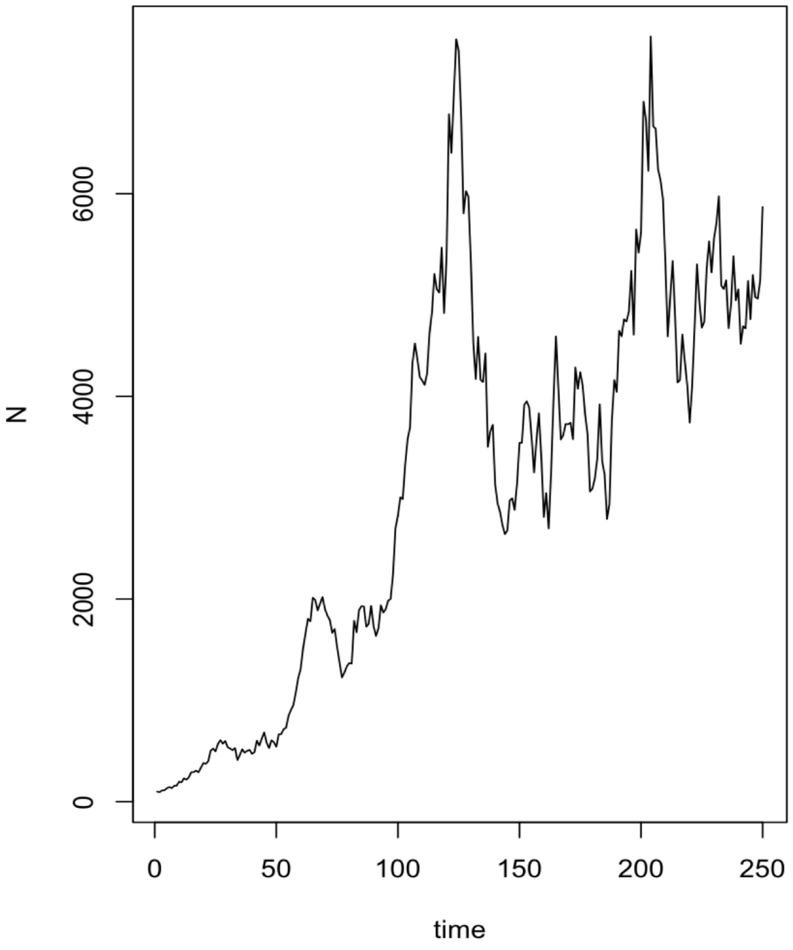
# Inverse Modeling




Probability distributions describes connection between data and model

$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

# Forward and Inverse don’t match

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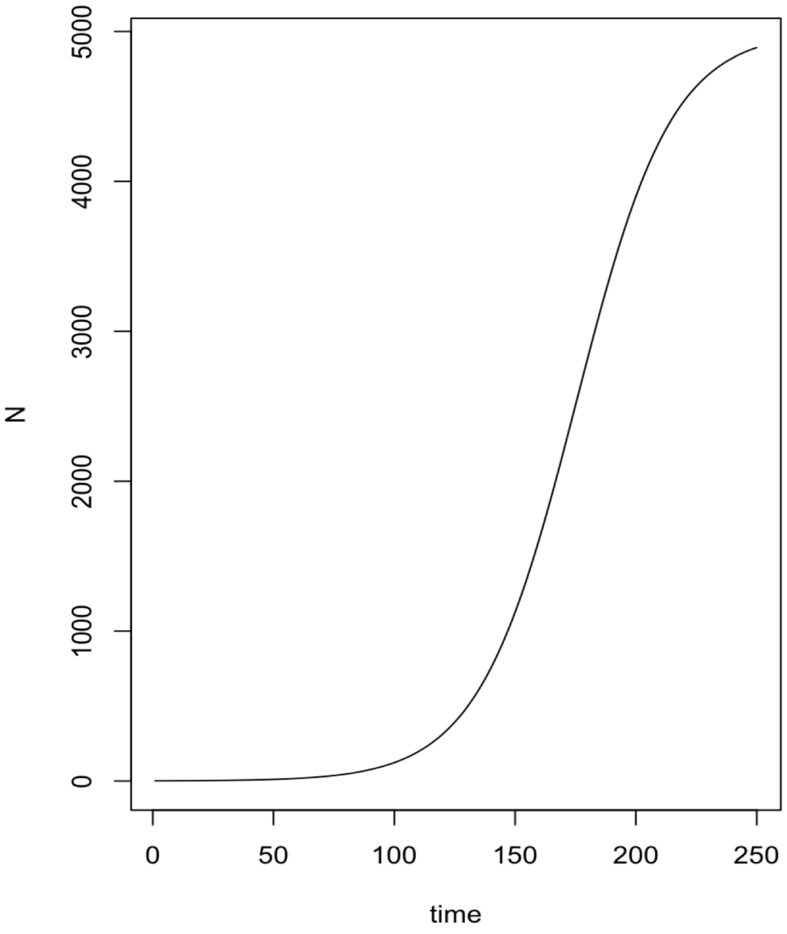
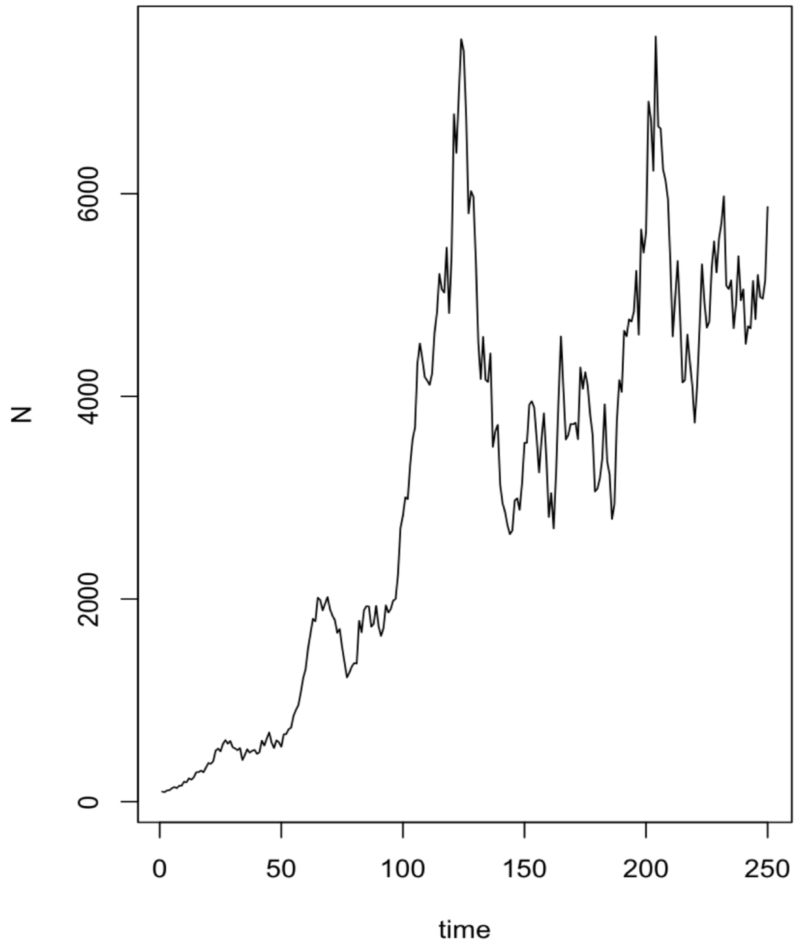


# What accounts for this?

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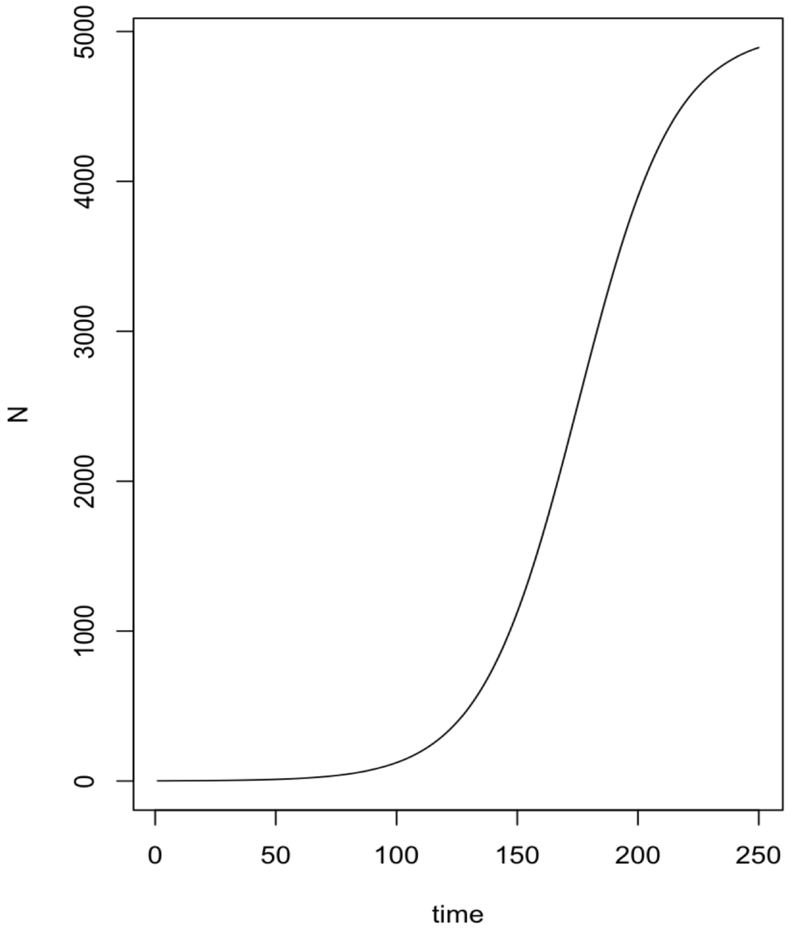


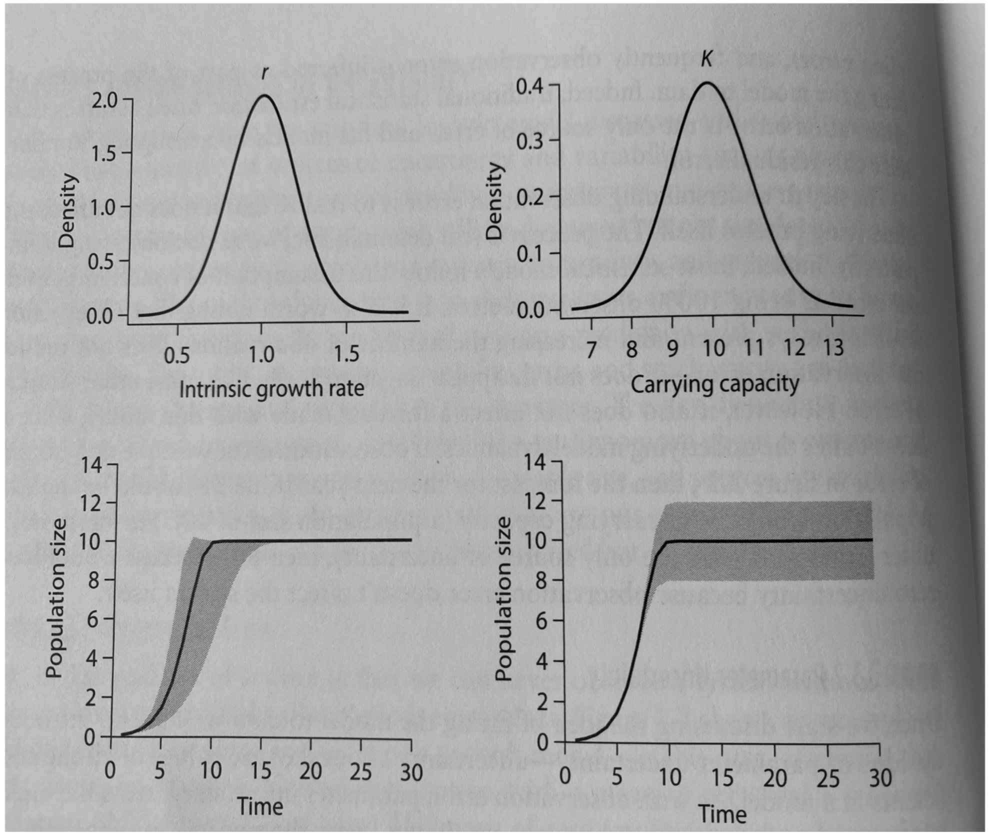
# What accounts for this?

Variability and uncertainty

# Types of variability and uncertainty

Parameter uncertainty: The true values of the parameter in the models are unknown

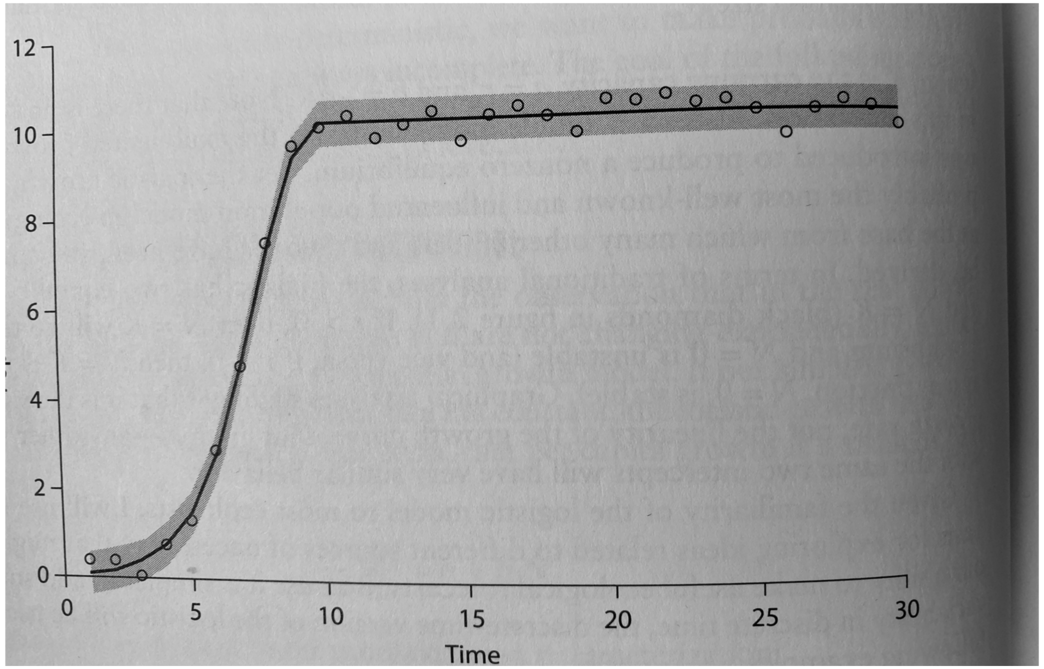
This type of uncertainty can be accounted for by both forward and inverse modeling approaches



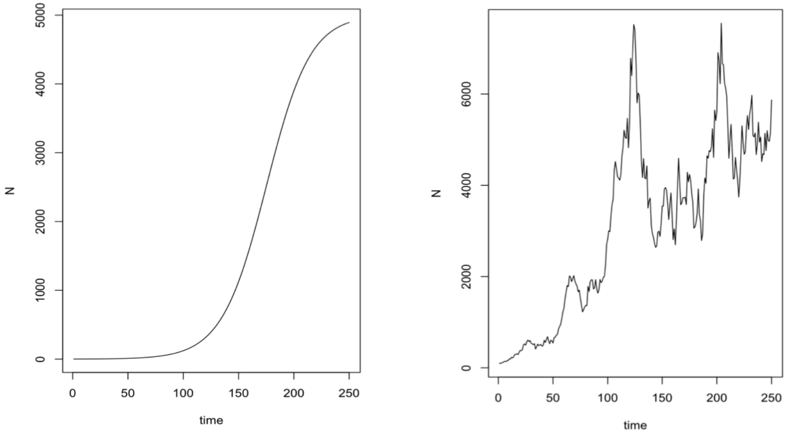
# Types of variability and uncertainty

## Observation uncertainty: The true state of the system can not be observed without error.

If observation error is the only source of error then the underlying forecast is deterministic.



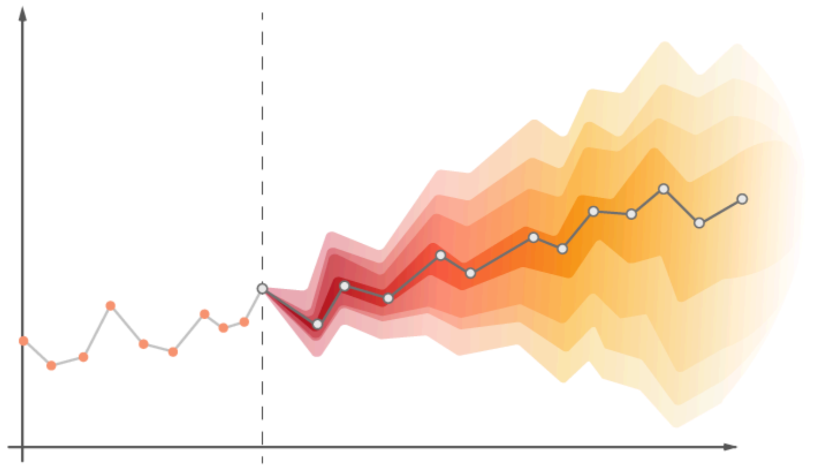
# Types of variability and uncertainty




## Process error/variability: Our models do not include all of the relevant processes.

This source of error is very common in ecology and environmental sciences, but hard (impossible?) to include with forward modeling approaches.

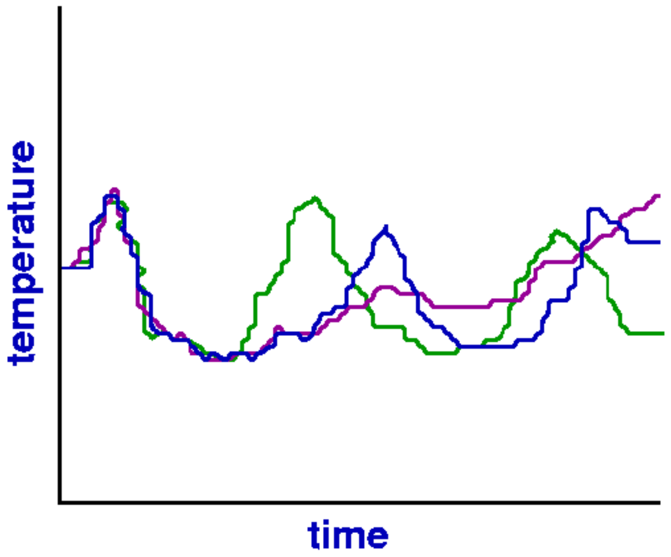
Because process error is uncertainty about the true state of a system, it propagates out in forecasts, often leading to greater uncertainty as time/space goes on.



# Types of variability and uncertainty

Initial condition uncertainty: Not knowing the current state of the system.

This is most important in chaotic systems like weather.




# Grab bag of other uncertainties

**Driver uncertainty:** If forecast includes "drivers" like temperature, than uncertainty in the forecast of that driver propagate through to forecasts. One of the reasons why the most ecologically realistic model may not be the best for forecasting.

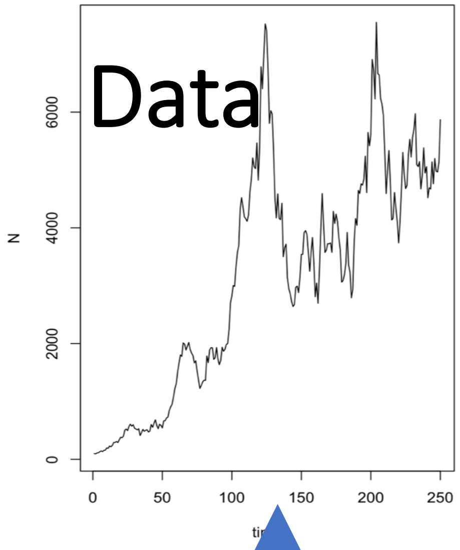
**Numerical approximation:** Computers make approximation errors. This rarely matters for most of our applications, but good to be aware of.

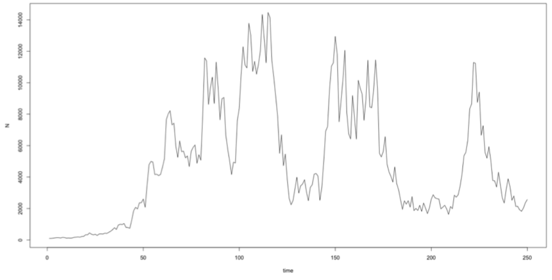
# So how do we build models and forecasts in the face of all of these sources of uncertainty?

# Models as scaffolds

We use models to describe the relationship (or covariance) between different sources of data, and data yet to be observed.

# Models as scaffolds



Forecast

$$N_{t+1} = N_t + rN_t(1 - N_t/K)$$

Data

# Soapbox

Models are sometimes described as process-based, mechanistic, statistical, phenomenological.

What does that mean?

# Soapbox

Any (or at least most) useful models contain some element of all of these.