Annotated Solutions for 1996 Statistics Waiver Exam

(1) C

The response is on a log scale, so the slope measures the percentage change in Y per unit change in time (here year). To see the details, write down the fit for two years. The difference on the log scale is .1385,

$$\begin{aligned} 0.1385 &= \log(\text{estimated } VW_t) - \log(\text{estimated } VW_{t-1}) \\ &= \log(VW_t/VW_{t-1}) \\ &= \log(1 + (VW_t - VW_{t-1})/ VW_{t-1}) \\ &\approx (VW_t - VW_{t-1})/ VW_{t-1} \end{aligned}$$

which is roughly the percentage change (divided by 100). Thus, in this example, for each change in year, the index rose by about 13.85%.

(2) C

Prediction intervals from the model are found on the scale of the response. Thus, the endpoints of the 95% interval (assume 95% unless otherwise indicated) are

$$-269.3 + 0.1385(1995) \pm 2\ (0.097)$$

on the log scale. Considering the upper endpoint, converted to the scale of the VW index we have

$$\exp(-269.3 + 0.1385(1995) + 2\ (0.097)) = \exp(7.202) = 1341.$$

(3) A

The plot of the residuals shows that the model is over-predicting the residuals near the end of the series. This error will be compounded when the predictions are put back on the VW scale (as in #2 above).

(4) C

This is a time series, so we should always check for autocorrelation. It is quite apparent in the sequence plot of the residuals.

(5) B

Remember, the fitted model is on the scale of the response, so pay attention to the units.

(6) D

A confidence interval for the slope in this fit is $0.01385 \pm 2\ 0.001$. The slope for the other period is far outside this interval. Assuming a comparable fit in the earlier peroid (its RMSE is similar), the width of the interval in the earlier period would be similar. The slopes are far apart.

(7) B

With financial returns, one should always be concerned about fat tails in the distribution of the data. There is some evidence of such behavior in this example, though the effect is slight.

(8) C

The point is leveraged as well as inconsistent with the rest of the data. It is influential. Removing it would cause the slope to increase and the intercept to decrease. (Draw the lines to see the effect on the intercept.)

(9) C

This fit lacks an interaction, so the fits are parallel. The slope implies a change of 0.01 seconds/KB.

(10) A

Is the interaction significant? No, so use the model with parallel fits. Program A is consistently about 0.30 seconds faster for all file sizes. (Had there been interaction, one program might have been better for small files, with the other better for larger files.)

(11) E

The fits are quite comparable (from looking at the plots). They only differ in the intercept terms, as shown in Model 1.

(12) D

An important assumption is the assumption of unequal error variation. We could see this best in a plot of the residuals for each group.

(13) C (with the responses ordered correctly as a,b,c,d,e)

The constrained fit is in fact Model 1. The difference in fits is *twice* the shown term for the dummy variable (which indicates the difference of each from the baseline model).

(14) C

The smallest possible variance inflation factor is one. Had the file sizes not been comparable, we might have found some deviation from this lower limit. Remember, when you have a balanced experiment such as this, collinearity will be small.

(15) D

You need to use the full model with an interaction term to compare the two separate fits to just one. Differences from a single model are represented in the *two* dummy variable terms in Model 2.

(16) D

Intercepts are predictions when the predictor is zero, but they are also often extrapolations from the data. Here, the intercept is not significantly different from zero.

(17) B

This point helps us see that the variability has increased for large orders. Simply ignoring this point would hide the lack of constant variance. Use outliers to gain insights if you can.

(18) A

The number sold is 80% (roughly, since the intercept is near zero) of the number requested. So, the professors are requesting 25% more than they need.

(19) E

The partial F for Department is not large, so departments do not differ in their behavior – assuming no interaction, of course.

(20) D

I'd like to also look for interaction, but that's not a choice. With grouping, why should the variances be the same? Check that as well. Since we can see that the residuals appear to lack constant variation from the plots, we can fix that by doing a weighted analysis (i.e., give more weight to the more precise observations).

(21) B

You can see the interaction term in the profile plot. Recall the scheduling example. Marginal analysis often conceal interactions.

(22) D

The interactions are very significant (F=27143), so the effects of the promotions vary by region.

(23) B OR C

In general, it is unwise to interpret so-called main effects when the interaction is significant. Again, refer to the scheduling example. Nonetheless, the profile plot suggests differences among regions (e.g., south looks high compared to mountain).

(24) A

Looking at the mountain–promotion term (estimate –7.79 with a t of –1.44) shows a small difference between the programs in this region. Sure a one-sample, multiple comparison would be nicer, but the effect is not significant here – so it would not be significant there either.

(25) C

Look at the profile plot. The south region with gift program is highest.

(26) E

There is one outlier, but with this much data, the results will not be overly distorted due to this one observation. The variances differ slightly, perhaps, but not enough to have much effect.

(27) E

You can read the right answer from the plot.

(28) D

The fit gives the probability for *not* purchasing a shirt (labeled “no” at the right side of the plot). The effect is significant.

(29) C

This term is not significantly different from zero. The p-value is much larger than 0.05, and the implied t-statistic is about 0.7.

(30) C

They have to cut the price to obtain this effect. You can come close just by looking at the plot. In detail, note that the slope for price implies that the odds for *no* purchase get multiplied by exp(0.031)=1.031 for each $1 increase. To *increase* the odds by 20%, we would have to cut the price by about $7 (i.e., 20/3). Alternatively, just find the odds at each of the shown prices.

(31) C

Just as in regular regression, the use of a categorical term with no interaction implies that the effect of the slope (here, the impact on purchase probability) is the same in both groups.

(32) C

The key word is “significant.” Look at the overall F (F=82).

(33) D

R^2 is a squared correlation, so it does not change. T-stats are measures of significant, and also do not change. The slopes do.

(34) A

This is a partial coefficient, and must be interpreted “holding the other factors fixed.” One of the other factors is the size of the house in square feet. The only way to add a bathroom without increasing the size is to convert the space.

(35) E

The RMSE is huge ($40,000) and can only predict the price to within about 80 thousand dollars.

(36) B

Bonferroni implies that we must use a threshold of 0.05/5 = 0.01 for this comparison.

(37) B

These are partial coefficients. It matters what type of space is added.

(38) D

Collinearity is present as shown in the VIFs. It is not large, but may be enough.

(39) B

There is not enough data at the right to see variation for the expensive houses, but you can certainly see that a few are quite expensive.

(40) A

Collinearity reduces the horizontal range of the points shown in the leverage plot.

(41) D

The DW is less than one, indicating autocorrelation, but what is the source of this autocorrelation? A look at the fit shows that the analysis also missed an important trend. Getting the equation correct is always the first step of the analysis.

(42) E

You cannot compare the RMSEs here since the models are fit to different response variables. Similarly, the R^2 statistics are not directly comparable either since they measure how much variation in transformed response variables has been captured. You are not "explaining" the same variation in the two cases.

(43) B

Again, the word "significant" is key. The t-statistic implies that the addition of $Week^2$ has significantly improved the fit.

(44) C

Collinearity is not the same as interaction.

(45) D

The others are valid statements about interaction. Redundant predictors often occur when there is collinearity.