Problem 1 - Magnetized ball

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Since magnetisation is uniform, the bound current density within the sphere is:

$$J = \nabla \times M = 0 \tag{1}$$

On the other hand, the bound surface current density is

$$K = M \cdot n \tag{2}$$

where n is the surface normal vector. We can find the vector potential of the uniformly magnetised sphere:

$$A = \frac{\mu_0}{4\pi} \int \frac{K}{r} R^2 \sin\theta d\theta d\phi \tag{3}$$

And find the B field using $B = \nabla \times A$

Finally, we obtain the magnetic field inside a uniformly magnetised sphere as

$$B = \frac{2}{3}\mu_0 M \tag{4}$$

The total magnetic dipole of this sphere would then be:

$$m = M\frac{4}{3}\pi R^3 \tag{5}$$

$$m = \frac{2B\pi R^3}{\mu_0} \tag{6}$$

The initial and final magnetic field inside the sphere can be found based on the graph given (the y intercepts):

$$\Delta m = \frac{2(B_{20} - B_{120})\pi R^3}{\mu_0} = \boxed{0.85 Am^2} (to 2 s.f) \tag{7}$$

$(B_{20} - B_{120} \approx 0.17T)$