SYPT Week 6 - Collision

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1 Kinematics

$$\omega = \dot{\phi}$$

$$\alpha = \ddot{\phi}$$

Centre of Mass:

$$\begin{cases} \ddot{y}_\mathsf{c} = \ddot{y}_\mathsf{p} + \ddot{\phi}\,\ell\sin\phi - \dot{\phi}^2\ell\cos\phi \\ \ddot{x}_\mathsf{c} = \ddot{x}_\mathsf{p} + \ddot{\phi}\,\ell\cos\phi + \dot{\phi}^2\ell\sin\phi \end{cases}$$

2 Dynamics

2.1 Spring-Mass-Damper

$$N - mg = m\ddot{y}_\mathsf{c}$$

$$N - mg = m(\boxed{\ddot{y}_\mathsf{p}} + \boxed{\ddot{\phi}}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi)$$

Finding $\ddot{y}_\mathsf{p}$:

$$\boxed{\begin{aligned} m\ddot{y}_\mathsf{p} &= -mg - kx^{\frac{3}{2}} - c\dot{y}_\mathsf{p} \\ \boxed{\ddot{y}_\mathsf{p}} &= -g - \frac{k}{m}x^{\frac{3}{2}} - \frac{c}{m}\boxed{\dot{y}_\mathsf{p}} \end{aligned}}$$

$\ddot{\phi}$ depends on whether the friction is static or kinetic.

2.2 Friction

2.2.1 Kinetic friction

$$F_\mathsf{fric} = m\ddot{x}_\mathsf{c}$$

$$F_\mathsf{fric} = m(\boxed{\ddot{x}_\mathsf{p}} + \boxed{\ddot{\phi}}\ell\cos\phi + \dot{\phi}^2\ell\sin\phi)$$

$$\mu_k N = F_\mathsf{fric}$$

$$\boxed{F_\mathsf{fric} = \mu_k m(\ddot{y}_\mathsf{p} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi + g)}$$

Finding $\ddot{\phi}$ using method 1:

$$I\ddot{\phi} = N\ell\sin\phi - F_\mathsf{friction}\ell\cos\phi$$

$$I\ddot{\phi} = N\ell \sin\phi - \mu_k N\ell\cos\phi$$

$$\ddot{\phi} = \frac{N\ell(\sin\phi - \mu_k\cos\phi)}{I}$$

$$\boxed{\ddot{\phi} = \frac{m\ell(\ddot{y}_{\mathsf{p}} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi + g)(\sin\phi - \mu_k\cos\phi)}{I}}$$

Finding $\ddot{\phi}$ using method ②:

$$\begin{cases} N - mg = m(\ddot{y}_{\mathsf{p}} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi) \\ F_{\mathsf{fric}} = \mu_k m(\ddot{y}_{\mathsf{p}} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi + g) \\ I\ddot{\phi} = N\ell\sin\phi - F_{\mathsf{friction}}\ell\cos\phi \end{cases}$$

Solving the equation simultaneously for $\ddot{\phi}$ and rearranging the terms,

$$\boxed{m\ell\sin\theta(g + \ddot{y}_p) - m\ell\cos\theta\,\ddot{x}_p = I\ddot{\phi}}$$

Solving for $\ddot{\phi}$, the same solution will be obtained.

2.2.2 Static friction

$$\ddot{x}_p = 0$$

$$F_{\mathsf{friction}} = m(\boxed{\ddot{\phi}}\ell\cos\phi + \dot{\phi}^2\ell\sin\phi)$$

Non-slip condition:

$$m(\ddot{\phi}\ell\cos\phi + \dot{\phi}^2\ell\sin\phi) \le \mu_s N$$

$$m(\ddot{\phi}\ell\cos\phi + \dot{\phi}^2\ell\sin\phi) \le \mu_s m(\ddot{y}_{\mathsf{p}} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi + g)$$

$$\boxed{\mu_s \ge \frac{(\ddot{\phi}\ell\cos\phi + \dot{\phi}^2\ell\sin\phi)}{(\ddot{y}_{\mathsf{p}} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi + g)}}$$

Finding $\ddot{\phi}$:

$$\boxed{m\ell\sin\theta(g + \ddot{y}_p) - m\ell\cos\theta\,\ddot{x}_p = I\ddot{\phi}}$$

$$\ddot{x}_p = 0$$

Moment of inertia about $I_{\mathsf{p}} = I_{\mathsf{c}} + m\ell^2$

$$m(g + \ddot{y}_p)\ell\sin\phi = I_p\ddot{\phi}$$

$$\ddot{\phi} = \frac{m(g + \ddot{y}_p)\ell\cos\phi}{I}$$

3 Time of collision

Capsule will leave the ground when normal force becomes 0:

$$0 = m(\ddot{y}_{\mathsf{p}} + \ddot{\phi}\ell\sin\phi - \dot{\phi}^2\ell\cos\phi + g)$$

4 Some other things

ℓ is a function of θ:

$$\ell(\theta) = R\sqrt{2(1 + \sin\theta)}$$

ϕ is also a function of θ:

$$\phi(\theta) = -\frac{\pi - 2\theta}{4}$$

ℓ is therefore a function of ϕ:

$$\ell(\phi) = R\sqrt{2(1 + \sin\frac{4\phi + \pi}{2})}$$