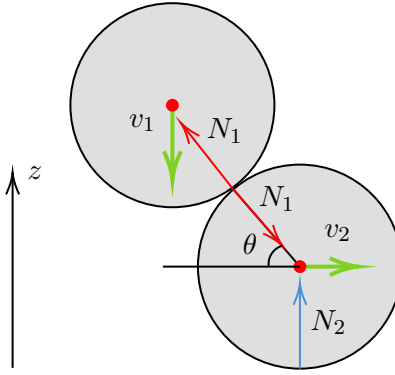


# POTD 538

bob the legend

March 21, 2022

## 1. First possible method



Relating velocity of top and bottom cylinders

$$v_2 = v_1 \tan \theta \quad (1)$$

Relative velocity  $v_{2,1}$  where  $v_2 - v_{2,1} = v_1$

$$v_{2,1} = v_1 \sqrt{\tan^2 \theta - 1} \quad (2)$$

So to find rate of change of  $\theta$ , just let  $v_{2,1} = r \frac{d\theta}{dt}$  since  $v_{2,1}$  always points in the tangential direction and rate of change of  $\theta$  is essentially just angular velocity of the bottom cylinders around the top one? so we get:

$$\omega = \frac{v_1 \sqrt{\tan^2 \theta - 1}}{r} \quad (3)$$

differentiating both sides to get  $\frac{d^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\sqrt{\tan^2 \theta - 1}}{r} \frac{dv_1}{dt} + v_1 \frac{d(\sqrt{\tan^2 \theta - 1})}{dt} \quad (4)$$

where  $\frac{d\theta}{dt} = \omega$

so how to find  $dv_1/dt$ ? (just  $a_1$  where  $a$  refers to acceleration) Write out  $F = ma$  for both the top and bottom

$$mg - 3N \sin \theta = ma_1 \quad (5) \qquad N \cos \theta = ma_1 \tan \theta \quad (6)$$

Solving for  $a_1$  gives us

$$a = \frac{g}{1 + 3 \tan^2 \theta} \quad (7)$$

Final solution for  $\frac{d^2\theta}{dt^2}$

$$\boxed{\frac{d^2\theta}{dt^2} = \frac{g}{1 + 3 \tan^2 \theta} \frac{\sqrt{\tan^2 \theta - 1}}{r} + \frac{1}{2}(\tan^2 \theta - 1)^{-\frac{1}{2}} 2 \sec^2 \theta \tan \theta r \omega^2} \quad (8)$$