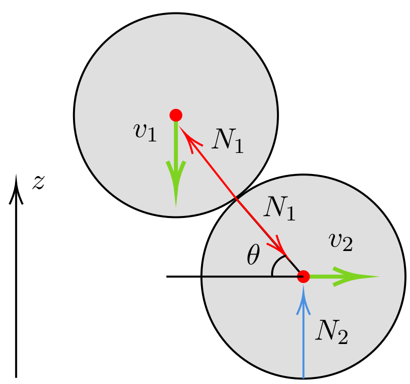
POTD 538

bob the legend

November 15, 2021

1. First possible method



Relating velocity of top and bottom cylinders

$$v_2 = v_1 \tan\theta \tag{1}$$

Relative velocity $v_{2,1}$ where $v_2 - v_{2,1} = v_1$

$$v_{2,1} = v_1\sqrt{\tan^2\theta - 1} \tag{2}$$

So to find rate of change of θ, just let $v_{2,1} = r\frac{d\theta}{dt}$ since $v_{2,1}$ always points in the tangential direction and rate of change of θ is essentially just angular velocity of the bottom cylinders around the top one? so we get:

$$\omega = \frac{v_1\sqrt{\tan^2\theta - 1}}{r} \tag{3}$$

differentiating both sides to get $\frac{d^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\sqrt{\tan^2\theta - 1}}{r}\frac{dv_1}{dt} + v_1\frac{d(\sqrt{\tan^2\theta - 1})}{dt} \tag{4}$$

where $\frac{d\theta}{dt} = \omega$

so how to find dv_1/dt? (just a_1 where a refers to acceleration) Write out $F = ma$ for both the top and bottom

$$mg - 3N\sin\theta = ma_1 \quad (5) \qquad N\cos\theta = ma_1\tan\theta \quad (6)$$

Solving for a_1 gives us

$$a = \frac{g}{1 + 3\tan^2\theta} \tag{7}$$

Final solution for $\frac{d^2\theta}{dt^2}$

$$\boxed{\frac{d^2\theta}{dt^2} = \frac{g}{1 + 3\tan^2\theta}\frac{\sqrt{\tan^2\theta - 1}}{r} + \frac{1}{2}(\tan^2\theta - 1)^{-\frac{1}{2}}2\sec^2\theta\tan\theta r\omega^2} \tag{8}$$