a)

$f_1 = \log(n^n)$ ✓

$\log(n^n) = n \log n$

$\underline{O(n \log n)}$

$f_2 = (\log n)^n$

$O((\log n)^n)$

$f_3 = \log(n^{6006})$ ✓

$O(\log n)$

$f_4 = (\log n)^{6006}$

$O((\log n)^{6006})$

$f_5 = \log\log(6006n)$ ✓

$O(\log(\log(n)))$

$\{f_5, f_3, f_1, f_4, f_2\}$

$\{f_5, f_3, f_4, f_1, f_2\}$

$O((\log n)^{6006}) < O(n \log n)$

(b)

$f_1 = 2^n \quad O(2^n) \quad O(f_1) < O(f_2)$

$f_2 = 6006^n \quad O(6006^n)$

$f_3 = 2^{6006^n} \quad O(f_4) < O(f_3)$

$f_4 = 6006^{2^n} \quad O(f_4) > O(f_5)$

$f_5 = 6006^{n^2}$

$\{f_1, f_2, f_5, f_4, f_3\}$ ✓

(c)

$f_1 = n^n \quad O(n^n)$

$f_2 = \binom{n}{n-6} \sim n^6 \quad O(n^6)$

$f_3 = (6n!) \quad O(n^n)$

$f_4 = \binom{n}{n/6} = \frac{n!}{\left(\frac{n}{6}\right)!\left(n-\frac{n}{6}\right)!} =$

$= \frac{n!}{\left(\frac{n}{6}\right)!\left(\frac{5}{6}n\right)!} \sim \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\sqrt{2\pi\frac{n}{6}}\left(\frac{n}{6e}\right)^{\frac{n}{6}}}$

$\sqrt{2\pi\frac{5}{6}n}\left(\frac{5n}{6e}\right)^{\frac{5}{6}n}$

$$\frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\sqrt{\pi^2 \frac{5n^2}{9}}\left(\frac{5n}{6e}\right)^{\frac{5}{6}n}\left(\frac{n}{6e}\right)^{\frac{n}{6}}} = \frac{\sqrt{2\pi n}\left(\frac{n}{\cancel{e}}\right)^n}{\frac{\pi n}{3}\sqrt{5}\,(5)^{\frac{5}{6}n}\cdot\left(\frac{n}{\cancel{6e}}\right)^{\cancel{n}}} \quad \frac{\sqrt{2\pi n}}{\frac{\pi n}{3}\sqrt{5}\,(5)^{\frac{5n}{6}}\left(\frac{1}{6}\right)^n}$$

$$\frac{n!}{\left(\frac{n}{6}\right)!\left(\frac{5}{6}n\right)!} \sim \frac{n^{\frac{1}{6}n}}{\left(\frac{n}{6}\right)!} \sim \frac{\cancel{n^{\frac{1}{6}}}n}{\sqrt{2\pi\frac{n}{6}}\left(\frac{n}{6e}\right)^{\frac{n}{6}}} = \frac{\cancel{n^{\frac{1}{6}n}}}{\sqrt{\frac{\pi n}{3}}\left(\frac{1}{6e}\right)^{\frac{n}{6}}\cdot\cancel{(n)^{\frac{n}{6}}}} =$$

$$\frac{(6e)^{\frac{n}{6}}}{\sqrt{\frac{n\pi}{3}}} \sim \frac{6^{\frac{n}{6}}}{\ }$$

$$\frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\sqrt{2\pi\frac{n}{6}}\left(\frac{n}{6e}\right)^{\frac{n}{6}}\cdot\sqrt{2\pi\frac{5n}{6}}\left(\frac{5n}{6e}\right)^{\frac{5n}{6}}} \simeq \frac{n^n\, e^{-n}}{\left(\frac{n}{6}\right)^{\frac{n}{6}}\cancel{e^{-n/6}}\left(\frac{5n}{6}\right)^{\frac{5n}{6}}\cancel{e^{5n/6}}} =$$

$$\frac{\cancel{n^n}}{\cancel{n^{n/6}}\cdot\cancel{n^{5n/6}}\cdot 1/6^{n/6}\cdot(5n)^{5n/6}\cdot 1/6^{5n/6}} = \left(\frac{6}{5^{5/6}}\right)^n$$

$$O\left(\left(6/5^{5/6}\right)^n\right)$$

$$f_5 = n^6 \qquad O(n^6)$$

$$\{\{f_2, f_5\}\ f_4,\ f_1,\ f_3\}$$

(d) $f_1 = n^{n+4} + 4! \sim O(n^{n+4})$

✓ $f_2 = n^{7\sqrt{n}} = e^{\ln n^{7\sqrt{n}}} = e^{7\sqrt{n}\,\ln(n)} \sim O(\sqrt{n}\,\ln(n))$

$f_3 = 4^{3n \log n} \sim 4^{n \log n}$

✓ $f_4 = 7^{n^2}$ $\quad O(7^{n^2})$

✓ $f_5 = n^{12 + 1/n} = n^{\frac{12n+1}{n}} = n^{\frac{12n}{n} + \frac{1}{n}} = n^{12} \cdot \sqrt[n]{n} \sim n^{12}$ $\quad O(n^{12})$

$\{f_2, f_5, f_1, f_3, f_4\}$