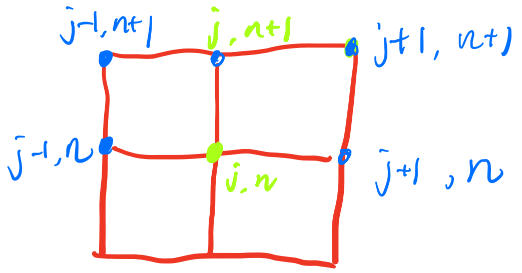

$$u_j^{n+1} - u_j^n = \frac{1}{2}\cdot r\left(u_{j+1}^{n+1} + u_{j+1}^n - 2u_j^{n+1} - 2u_j^n + u_{j-1}^{n+1} + u_{j-1}^n\right) + \overset{\text{tao}}{\frac{1}{2}}(\sin t_{n+1} + \sin t_n)$$

$$-\frac{1}{2} r u_{j-1}^{n+1} + (1+r) u_j^{n+1} - \frac{1}{2} r u_{j+1}^{n+1} = \frac{r}{2} u_{j-1}^n + (1-r) u_j^n + \frac{r}{2} u_{j+1}^n + \overset{\text{tao}}{\frac{1}{2}}(\sin t_{n+1} + \sin t_n)$$

$J = 1$

$$\begin{bmatrix} -\frac{r}{2} & 1+r & -\frac{r}{2} & \cdots & & \\ 0 & -\frac{r}{2} & 1+r & -\frac{r}{2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\frac{r}{2} & 1+r & -\frac{r}{2} \end{bmatrix} \begin{bmatrix} u_0^{n+1} \\ u_1^{n+1} \\ \vdots \\ u_J^{n+1} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & 1-r & \frac{r}{2} & & & \\ \frac{r}{2} & 1-r & \frac{r}{2} & & & \\ & \ddots & \ddots & \ddots & & \\ & & & \frac{r}{2} & 1-r & \frac{r}{2} \end{bmatrix} \begin{bmatrix} u_0^n \\ u_1^n \\ \vdots \\ u_J^n \end{bmatrix} + \overset{\text{tao}}{\frac{1}{2}} \begin{bmatrix} \sin t_{n+1} + \sin t_n \\ \sin t_{n+1} + \sin t_n \\ \vdots \\ \sin t_{n+1} + \sin t_n \end{bmatrix}$$

$J-1 \times J+1 \quad (J+1)\times 1$

$j = 0$

$$-\frac{r}{2} u_{-1}^{n+1} + (1+r) u_0^{n+1} - \frac{1}{2} r u_1^{n+1} = \frac{r}{2} u_{-1}^n + (1-r) u_0^n + \frac{r}{2} u_1^n + \overset{\text{tao}}{\frac{1}{2}}(\sin t_{n+1} + \sin t_n)$$

$$\frac{r}{2}\left(u_{-1}^n + u_{-1}^{n+1}\right) = \frac{r}{2}\left(u_1^n + u_1^{n+1}\right)$$

$$\underset{1+r}{\left(1+\frac{r}{2}\right)} u_0^{n+1} \underset{-r}{-\frac{r}{2}} u_1^{n+1} = (1-r) u_0^n + r u_1^n + \overset{\text{tao}}{\frac{1}{2}}(\sin t_{n+1} + \sin t_n)$$

$j = J$

$$-\frac{1}{2} r u_{J-1}^{n+1} + (1+r) u_J^{n+1} - \frac{1}{2} r u_{J+1}^{n+1} = \frac{r}{2} u_{J-1}^n + (1-r) u_J^n + \frac{r}{2} u_{J+1}^n + \overset{\text{tao}}{\frac{1}{2}}(\sin t_{n+1} + \sin t_n)$$

$$\frac{r}{2}\left(u_{J+1}^{n} + u_{J+1}^{n+1}\right) = \frac{r}{2}\left(u_{J-1}^{n} + u_{J-1}^{n+1}\right)$$

ta0

$$-r\,u_{J-1}^{n+1} + (1+r)u_{J}^{n+1} = r\,u_{J-1}^{n} + (1-r)u_{J}^{n} + \frac{1}{2}(\sin t_{n+1} + \sin t_n)$$

$j = 0$

ta0

$$\left(1+\frac{r}{2}\right)u_{0}^{n+1} - \frac{r}{2}u_{1}^{n+1} = (1-r)u_0^n + r\,u_1^n + \frac{1}{2}(\sin t_{n+1} + \sin t_n)$$

$1+r$ 0 r 1

$j = 1 \sim J-1$

ta0

$$-\frac{1}{2}r\,u_{j-1}^{n+1} + (1+r)u_j^{n+1} - \frac{1}{2}r\,u_{j+1}^{n+1} = \frac{r}{2}u_{j-1}^{n} + (1-r)u_j^{n} + \frac{r}{2}u_{j+1}^{n} + \frac{1}{2}(\sin t_{n+1} + \sin t_n)$$

$j = J$

ta0

$$-r\,u_{J-1}^{n+1} + (1+r)u_J^{n+1} = r\,u_{J-1}^{n} + (1-r)u_J^{n} + \frac{1}{2}(\sin t_{n+1} + \sin t_n)$$

ta0

$$\begin{array}{ccccc} 1+r & -r & 0 & & \\ \end{array}$$

$$\begin{bmatrix} 0 & \cancel{1+\frac{r}{2}} & \cancel{-\frac{r}{2}} & & & \\ -\frac{r}{2} & 1+\frac{r}{2} & -\frac{r}{2} & & & \\ & -\frac{r}{2} & 1+\frac{r}{2} & -\frac{r}{2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\frac{r}{2} & 1+\frac{r}{2} & -\frac{r}{2} \\ & & & & -r & 1+r \end{bmatrix} \begin{bmatrix} u_0^{n+1} \\ u_1^{n+1} \\ u_2^{n+1} \\ \vdots \\ u_{J-1}^{n+1} \\ u_J^{n+1} \end{bmatrix} = \begin{bmatrix} 1-r & r & & & & \\ \frac{r}{2} & 1-r & \frac{r}{2} & & & \\ & \frac{r}{2} & 1-r & \frac{r}{2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & \frac{r}{2} & 1-r & \frac{r}{2} \\ & & & & r & 1-r \end{bmatrix} \begin{bmatrix} u_0^{n} \\ u_1^{n} \\ u_2^{n} \\ \vdots \\ u_{J-1}^{n} \\ u_J^{n} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \sin t_{n+1} + \sin t_n \\ \vdots \\ \sin t_{n+1} + \sin t_n \end{bmatrix}$$

$$u[0, :] = \cos \pi x$$