



$$u_j^{n+1} - u_j^n = \frac{1}{2} \cdot r \left(\underbrace{u_{j+1}^{n+1}} + \underbrace{u_{j+1}^n} - \underbrace{2u_j^{n+1}} - \underbrace{2u_j^n} + \underbrace{u_{j-1}^{n+1}} + \underbrace{u_{j-1}^n} \right) + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$-\frac{1}{2}r u_{j-1}^{n+1} + (1+r) u_j^{n+1} - \frac{1}{2}r u_{j+1}^{n+1} = \frac{r}{2} u_{j-1}^n + (1-r) u_j^n + \frac{r}{2} u_{j+1}^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$j=1$$

$$\begin{bmatrix} -\frac{r}{2} & 1+r & -\frac{r}{2} & \dots & 0 \\ 0 & -\frac{r}{2} & 1+r & -\frac{r}{2} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{r}{2} & 1+r & -\frac{r}{2} & \dots & 0 \end{bmatrix} \begin{bmatrix} u_0^{n+1} \\ u_1^{n+1} \\ \vdots \\ u_J^{n+1} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & 1-r & \frac{r}{2} & \dots & 0 \\ 0 & -\frac{r}{2} & 1-r & \frac{r}{2} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -\frac{r}{2} & 1-r & \frac{r}{2} & \dots \end{bmatrix} \begin{bmatrix} u_0^n \\ u_1^n \\ \vdots \\ u_J^n \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \sin t_{n+1} + \sin t_n \\ \sin t_{n+1} + \sin t_n \\ \vdots \\ \sin t_{n+1} + \sin t_n \end{bmatrix}$$

$J-1 \times J+1 \quad (J+1) \times 1$

$$j=0$$

$$-\frac{r}{2} u_{-1}^{n+1} + (1+r) u_0^{n+1} - \frac{1}{2} r u_1^{n+1} = \frac{r}{2} u_{-1}^n + (1-r) u_0^n + \frac{r}{2} u_1^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$\frac{r}{2} (u_{-1}^n + u_1^{n+1}) = \frac{r}{2} (u_1^n + u_1^{n+1})$$

$$(1 + \frac{r}{2}) u_1^{n+1} - \frac{r}{2} u_2^{n+1} = (1-r) u_0^n + r u_1^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$j=J$$

$$-\frac{1}{2}r u_J^{n+1} + (1+r) u_J^{n+1} - \frac{1}{2}r u_{J+1}^{n+1} = \frac{r}{2} u_{J-1}^n + (1-r) u_J^n + \frac{r}{2} u_{J+1}^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$\frac{r}{2} (u_{j+1}^n + u_{j+1}^{n+1}) = \frac{r}{2} (u_{j-1}^n + u_{j-1}^{n+1})$$

$$-r u_{j-1}^{n+1} + (1+r) u_j^{n+1} = r u_{j-1}^n + (1-r) u_j^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$j=0$$

$$(1+\frac{r}{2}) u_1^{n+1} - \frac{r}{2} u_2^{n+1} = (1-r) u_0^n + r u_1^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$j=1 \sim j-1$$

$$-\frac{1}{2} r u_{j-1}^{n+1} + (1+r) u_j^{n+1} - \frac{1}{2} r u_{j+1}^{n+1} = \frac{r}{2} u_{j-1}^n + (1-r) u_j^n + \frac{r}{2} u_{j+1}^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$j=j$$

$$-r u_{j-1}^{n+1} + (1+r) u_j^{n+1} = r u_{j-1}^n + (1-r) u_j^n + \frac{1}{2} (\sin t_{n+1} + \sin t_n)$$

$$\begin{bmatrix} 0 & 1+\frac{r}{2} & -\frac{r}{2} & & & \\ \frac{r}{2} & 1+\frac{r}{2} & -\frac{r}{2} & & & \\ & -\frac{r}{2} & 1+\frac{r}{2} & -\frac{r}{2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\frac{r}{2} & 1+\frac{r}{2} & -\frac{r}{2} \\ & & & & -r & 1+r \end{bmatrix} \begin{bmatrix} u_0^{n+1} \\ u_1^{n+1} \\ u_2^{n+1} \\ \vdots \\ u_{j-1}^{n+1} \\ u_j^{n+1} \end{bmatrix} = \begin{bmatrix} 1-r & r & & & & \\ \frac{r}{2} & 1-r & \frac{r}{2} & & & \\ & \frac{r}{2} & 1-r & \frac{r}{2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\frac{r}{2} & 1-r & \frac{r}{2} \\ & & & & r & 1-r \end{bmatrix} \begin{bmatrix} u_0^n \\ u_1^n \\ u_2^n \\ \vdots \\ u_{j-1}^n \\ u_j^n \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \sin t_{n+1} + \sin t_n \\ \sin t_{n+1} + \sin t_n \\ \sin t_{n+1} + \sin t_n \\ \vdots \\ \sin t_{n+1} + \sin t_n \end{bmatrix}$$

$$u[0, :] = \cos \eta x$$