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# THE PHENOTYPIC SIGNATURE OF HOST-PARASITE COEVOLUTION IN CONTINUOUS SPACE

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A PREPRINT

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## Abstract

Here we identify the phenotypic signature of host-parasite coevolution in continuous space

**Keywords** blah · bleee · bloo · these are optional and can be removed

## 1 Introduction

The host is denoted  $H$  and the parasite  $P$ . Spatial location is  $x$  ( $\in \mathbb{R}^1$  or  $\mathbb{R}^2$ ) and individual trait value of a host/parasite is  $z_H/z_P \in \mathbb{R}^1$ . Assuming a trait-matching model, where host fitness is minimized and parasite fitness is maximized when  $z_H = z_P$ , we have the following two fitness functions:

$$W_H \propto \exp\left(\frac{B_H}{2}(z_H - z_P)^2\right), \quad (1a)$$

$$W_P \propto \exp\left(-\frac{B_P}{2}(z_H - z_P)^2\right), \quad (1b)$$

where  $B_H, B_P > 0$  capture the strengths of biotic selection experienced by each species. Using a standard approach, we derive the following non-spatial coevolutionary model:

$$\frac{d\bar{z}_H}{dt} = -G_H B_H (\bar{z}_P - \bar{z}_H), \quad (2a)$$

$$\frac{d\bar{z}_P}{dt} = G_P B_P (\bar{z}_H - \bar{z}_P), \quad (2b)$$

where  $G_H, G_P > 0$  denote the additive genetic variances of each species. If we assume interactions occur locally (individuals interact only when they ‘collide’), then we obtain the following continuous space model:

$$\frac{\partial \bar{z}_H}{\partial t} = -G_H B_H (\bar{z}_P - \bar{z}_H) + \frac{D_H}{2} \Delta \bar{z}_H, \quad (3a)$$

$$\frac{\partial \bar{z}_P}{\partial t} = G_P B_P (\bar{z}_H - \bar{z}_P) + \frac{D_P}{2} \Delta \bar{z}_P, \quad (3b)$$

where  $D_H, D_P > 0$  are dispersal rates for each species and  $\Delta = \frac{\partial^2}{\partial x_1^2}$  in one-dimensional space and  $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$  in two-dimensional space.

## 2 Headings: first level

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### 2.1 Headings: second level

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^N \sum_{j=1}^N \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \quad (4)$$

#### 2.1.1 Headings: third level

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## 3 Examples of citations, figures, tables, references

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The documentation for `natbib` may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

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\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated...

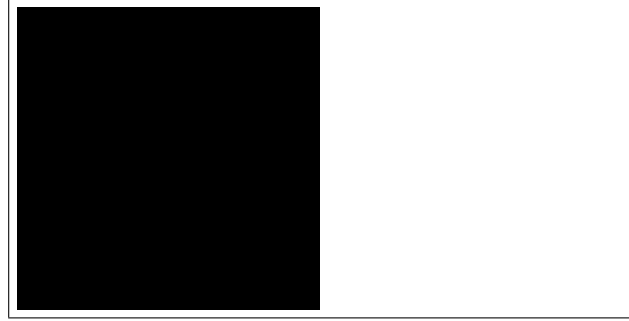


Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

<https://www.ctan.org/pkg/booktabs>

### 3.1 Figures

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See Figure 1. Here is how you add footnotes. [^Sample of the first footnote.]

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### 3.2 Tables

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See awesome Table~1.

### 3.3 Lists

- Lorem ipsum dolor sit amet
- consectetur adipiscing elit.
- Aliquam dignissim blandit est, in dictum tortor gravida eget. In ac rutrum magna.

Hadash, Guy, Einat Kermany, Boaz Carmeli, Ofer Lavi, George Kour, and Alon Jacovi. 2018. “Estimate and Replace: A Novel Approach to Integrating Deep Neural Networks with Existing Applications.” *arXiv Preprint arXiv:1804.09028*.

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