The Phenotypic Signature of Host-Parasite Coevolution in Continuous Space

A Preprint

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Abstract

Here we identify the phenotypic signature of host-parasite coevolution in continuous space

Keywords blah · blee · bloo · these are optional and can be removed

1 Introduction

The host is denoted H and the parasite P. Spatial location is x ($\in \mathbb{R}^1$ or $\mathbb{R}^2$) and individual trait value of a host/parasite is $z_H/z_P \in \mathbb{R}^1$. Assuming a trait-matching model, where host fitness is minimized and parasite fitness is maximized when $z_H = z_P$, we have the following two fitness functions:

$$W_H \propto \exp\left(\frac{B_H}{2}(z_H - z_P)^2\right), \tag{1a}$$

$$W_P \propto \exp\left(-\frac{B_P}{2}(z_H - z_P)^2\right), \tag{1b}$$

where $B_H, B_P > 0$ capture the strengths of biotic selection experienced by each species. Using a standard approach, we derive the following non-spatial coevolutionary model:

$$\frac{d\bar{z}_H}{dt} = -G_H B_H(\bar{z}_P - \bar{z}_H), \tag{2a}$$

$$\frac{d\bar{z}_P}{dt} = G_P B_P(\bar{z}_H - \bar{z}_P), \tag{2b}$$

where $G_H, G_P > 0$ denote the additive genetic variances of each species. If we assume interactions occur locally (individuals interact only when they 'collide'), then we obtain the following continuous space model:

$$\frac{\partial\bar{z}_H}{\partial t} = -G_H B_H(\bar{z}_P - \bar{z}_H) + \frac{D_H}{2}\Delta\bar{z}_H, \tag{3a}$$

$$\frac{\partial\bar{z}_P}{\partial t} = G_P B_P(\bar{z}_H - \bar{z}_P) + \frac{D_P}{2}\Delta\bar{z}_P, \tag{3b}$$

where $D_H, D_P > 0$ are dispersal rates for each species and $\Delta = \frac{\partial^2}{\partial x_1^2}$ in one-dimensional space and $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$ in two-dimensional space.

2 Headings: first level

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2.1 Headings: second level

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j|y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \tag{4}$$

2.1.1 Headings: third level

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Paragraph Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

3 Examples of citations, figures, tables, references

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The documentation for `natbib` may be found at

`http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf`

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated...

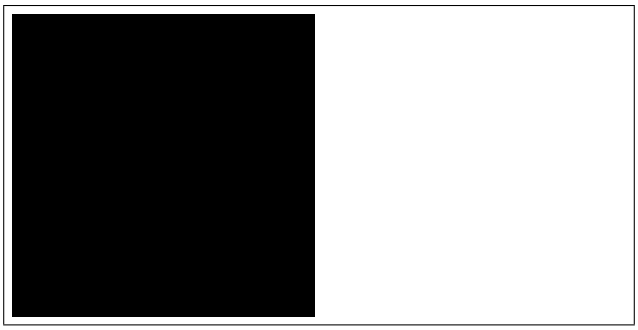

Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

`https://www.ctan.org/pkg/booktabs`

3.1 Figures

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See Figure 1. Here is how you add footnotes. [^Sample of the first footnote.]

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3.2 Tables

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See awesome Table~1.

3.3 Lists

- Lorem ipsum dolor sit amet
- consectetur adipiscing elit.
- Aliquam dignissim blandit est, in dictum tortor gravida eget. In ac rutrum magna.

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