The Phenotypic Signature of Host-Parasite Coevolution in Continuous Space

A Preprint

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Abstract

Here we identify the phenotypic signature of host-parasite coevolution in continuous space

Keywords blah · blee · bloo · these are optional and can be removed

1 Introduction

The host is denoted H and the parasite P. Spatial location is x ($\in \mathbb{R}^1$ or $\mathbb{R}^2$) and individual trait value of a host/parasite is $z_H/z_P \in \mathbb{R}^1$. Assuming a trait-matching model, where host fitness is minimized and parasite fitness is maximized when $z_H = z_P$, we have the following two fitness functions:

$$w_H \propto \exp\left(\frac{B_H}{2}(z_H - z_P)^2\right), \tag{1a}$$

$$w_P \propto \exp\left(-\frac{B_P}{2}(z_H - z_P)^2\right), \tag{1b}$$

where $B_H, B_P > 0$ capture the strengths of biotic selection experienced by each species. Using a standard approach, we derive the following non-spatial coevolutionary model:

$$\frac{d\bar{z}_H}{dt} = -G_H B_H(\bar{z}_P - \bar{z}_H), \tag{2a}$$

$$\frac{d\bar{z}_P}{dt} = G_P B_P(\bar{z}_H - \bar{z}_P), \tag{2b}$$

where $G_H, G_P > 0$ denote the additive genetic variances of each species. If we assume interactions occur locally (individuals interact only when they 'collide') and abundance densities are spatially homogeneous, then we can obtain the following continuous space model:

$$\frac{\partial\bar{z}_H}{\partial t} = -G_H B_H(\bar{z}_P - \bar{z}_H) + \frac{D_H}{2}\Delta\bar{z}_H, \tag{3a}$$

$$\frac{\partial\bar{z}_P}{\partial t} = G_P B_P(\bar{z}_H - \bar{z}_P) + \frac{D_P}{2}\Delta\bar{z}_P, \tag{3b}$$

where $D_H, D_P > 0$ are dispersal rates for each species and $\Delta = \frac{\partial^2}{\partial x_1^2}$ in one-dimensional space and $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$ in two-dimensional space. If we want to incorporate the effects of random genetic drift, we may be able to justify

$$\frac{\partial \bar{z}_H}{\partial t} = -G_H B_H(\bar{z}_P - \bar{z}_H) + \frac{D_H}{2}\Delta \bar{z}_H + \sqrt{\frac{G_H}{N_H}}\dot{W}_H, \tag{4a}$$

$$\frac{\partial \bar{z}_P}{\partial t} = G_P B_P(\bar{z}_H - \bar{z}_P) + \frac{D_P}{2}\Delta \bar{z}_P + \sqrt{\frac{G_P}{N_P}}\dot{W}_P, \tag{4b}$$

where N_P, N_H are the local abundance densities (here assumed to be constant in space and time) and $\dot{W}_H, \dot{W}_P$ are space-time white noise processes. This SPDE model may possess some major benefits over more traditional continuous-space models of population genetics. In particular, it is a simple extension of the stochastic heat equation, which is known to admit solutions in dimensions one, two and three.

2 Headings: first level

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2.1 Headings: second level

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \tag{5}$$

2.1.1 Headings: third level

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

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3 Examples of citations, figures, tables, references

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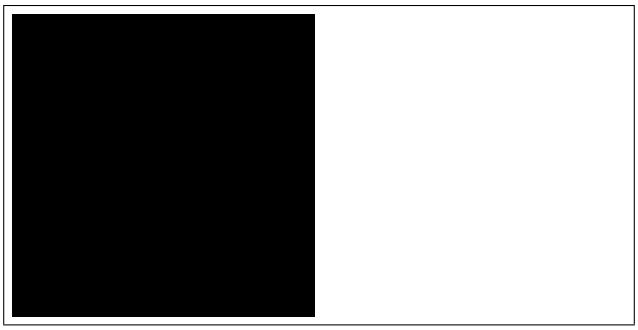

Figure 1: Sample figure caption.

Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui. some text (Kour and Saabne 2014b, 2014a) and see Hadash et al. (2018).

The documentation for `natbib` may be found at

`http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf`

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

```
\citet{hasselmo} investigated\dots
```

produces

> Hasselmo, et al. (1995) investigated...

`https://www.ctan.org/pkg/booktabs`

3.1 Figures

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See Figure 1. Here is how you add footnotes. [^Sample of the first footnote.]

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3.2 Tables

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See awesome Table~1.

Table 1: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

3.3 Lists

- Lorem ipsum dolor sit amet
- consectetur adipiscing elit.
- Aliquam dignissim blandit est, in dictum tortor gravida eget. In ac rutrum magna.

Hadash, Guy, Einat Kermany, Boaz Carmeli, Ofer Lavi, George Kour, and Alon Jacovi. 2018. "Estimate and Replace: A Novel Approach to Integrating Deep Neural Networks with Existing Applications." *arXiv Preprint arXiv:1804.09028*.

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