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Title page logo

Abstract Dummy Page.

"These violent delights have violent ends"
(Romeo and Juliet: Act 2, Scene 6, Line 9)

Преглед

Већим делом свога тока река Дрина протиче кроз тесне гудуре између стрмих планина или кроз дубоке кањоне окомито одсечених обала. Само на неколико места речног тока њене се обале проширују у отворене долине и стварају, било на једној било на обе стране реке, жупне, делимично равне, делимично таласасте пределе, подесне за обрађивање и насеља. Такво једно проширење настаје и овде, код Вишеграда, на месту где Дрина избија у наглом завоју из дубоког и уског теснаца који стварају Буткове Стијене и Узавничке планине. Заокрет који ту прави Дрина не-обично је оштар а планине са обе стране тако су стрме и толико ублизу да изгледају као затворен масив из којег река извире право, као из мрког зида. Али ту се планине одједном размичу у неправилан амфитеатар чији промер на најширем месту није већи од петнаестак километара ваздушне линије.

На том месту где Дрина избија целом тежином своје водене масе, зелене и запењене, из привидно затвореног склопа црних и стрмих планина, стоји велики и складно срезани мост од камена, са једанаест лукова широког распона. Од тог моста, као од основице, шири се лепезасто цела валовита долина, са вишеградском касабом и њеном околином, са засе-оцима полеглим у превоје брежуљака, прекривена њивама, испашама и шљивицима, изукрштана међама и плотовима и пошкропљена шумарцима и ретким скуповима црногорице. Тако, посматрано са дна видика, изгледа као да из широких лукова белог моста тече и разлива се не само зелена Дрина него и цео тај жупни и питоми простор, са свим што је на њему и јужним небом над њим.

На десној обали реке, почињући од самог моста, налази се главнина касабе, са чаршијом, делом у равници, а делом на обронцима брегова. На другој страни моста, дуж леве обале, протеже се Малухино поље, раштр-кано предграђе око друма који води пут Сарајева. Тако мост, састављајући два краја сарајевског друма, веже касабу са њеним предграђем.

Sammanfattning

De yngre tyckte inte att de hade levt den dagen, om det på kvällen när de somnade inte susade i deras öron och flimrade för deras ögon av allt som de hade hört och sett under dagen. De yngre tyckte inte att de hade levt den dagen, om det på kvällen när de somnade inte susade i deras öron och flimrade för deras ögon av allt som de hade hört och sett under dagen. De yngre tyckte inte att de hade levt den dagen, om det på kvällen när de somnade inte susade i deras öron och flimrade för deras ögon av allt som de hade hört och sett under dagen. De yngre tyckte inte att de hade levt den dagen, om det på kvällen när de somnade inte susade i deras öron och flimrade för deras ögon av allt som de hade hört och sett under dagen.

[illegible]

Acknowledgments

Woohoo!

List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

- I Radial Basis Function generated Finite Differences for Option Pricing Problems
- II BENCHOP — The BENCHmarking Project in Option Pricing
- III BENCHOP-SLV: The BENCHmarking project in Option Pricing — Stochastic and Local Volatility problems
- IV Pricing Derivatives under Multiple Stochastic Factors by Localized Radial Basis Function Methods
- V Pricing Financial Derivatives using Radial Basis Function generated Finite Differences with Polyharmonic Splines on Smoothly Varying Node Layouts
- VI SMOOTHING PAPER TITLE

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1. Introduction

The purpose of this thesis is to report on state of the art in Radial Basis Function generated Finite Difference (RBF-FD) methods for pricing of financial derivatives. Based on the six appended papers, this work provides a detailed overview of RBF-FD properties and challenges that arise when the RBF-FD method is used in financial applications. Furthermore, the manuscript aims to motivate further development of RBF-FD for finance.

Across the financial markets of the world, financial derivatives such as futures, options, and others are traded in substantial volumes. Therefore, knowing the prices of those financial instruments is of utmost importance at any given time. In order to make that possible in practice, it is very often required to employ a set of skills incorporating knowledge in financial theory, engineering methods, mathematical tools, and programming practice — which altogether constitute the field known as *financial engineering*.

Many of theoretical pricing models for financial derivatives can be represented using partial differential equations (PDEs). In many cases, those equations are time-dependent, of high spatial dimensions, and with challenging boundary conditions — which most often makes them analytically unsolvable. In those cases, numerical approximation as a mean of estimating their solution needs to be utilized. The field of *numerical analysis* is concerned with obtaining approximate solutions while maintaining reasonable bounds on errors. Unfortunately, there is no universal numerical method which could be used to solve all problems of this type efficiently. In fact, there are tremendously many numerical methods for solving different types of PDEs, and all those methods are featured with their own limitations in performance, stability, and accuracy — mostly depending on the problem details. Therefore, carefully selecting and developing numerical methods for particular applications has been the only way to build efficient PDE solvers in ongoing practice.

RBF-FD is a recent numerical method with potential to efficiently approximate solutions of PDEs in finance. Over the past years, besides the purely academic development and research of numerical properties, the method has been mainly applied for simulations of atmospheric phenomena. As its name suggests, the RBF-FD method is of a finite difference type, from the radial basis function family. As a finite difference method, RBF-FD approximates differential equations by linear systems of algebraic equations, known as difference equations. Radial basis functions are used as interpolants that enable local approximations of differential operators that are necessary for constructing the difference equations. Constructed like that, the method is featured with a sparse matrix of the linear system of difference equations, and it is relatively simple to implement like the standard finite difference methods. Moreover, the method is mesh-free, meaning that it does not require a structured discretization of the computational domain which makes it equally easy to use in spaces of different dimensions, and it is of a customizable order of accuracy — which are the features it inherits from the global radial basis function approximations. It is those properties that make the case for recognizing RBF-FD as a method with high potential for efficiently solving some analytically unfeasible pricing problems in finance.

Nevertheless, being a young method, RBF-FD is still under intense development and many challenges are faced when moving from simple theoretical cases toward more complex real-world applications. The core of this thesis deals with finding solutions for overcoming obstacles when financial derivatives are priced using RBF-FD to solve PDEs with several spatial dimensions. Thus, it represents a contribution to making the RBF-FD methods more reliable and efficient for use in financial applications.

The rest of this manuscript is organized as follows. Financial derivatives are introduced and defined in Chapter 2. An overview of some popular financial models and techniques for pricing of options are presented in Chapter 3. The properties of RBF-FD methods for solving PDEs in finance are presented in Chapter 4. Finally, some unsolved challenges with the suggestions for further development of the RBF-FD method for financial applications are shown in Chapter 5.

2. Financial Derivatives

Across the financial markets of the world, financial derivatives are traded in substantial volumes. According to the reports of BIS, the estimated total notional value of these financial instruments has been above half a quadrillion of USD during the current decade. That value is about the order of magnitude larger than the total world gross domestic product (GDP). The main reason for this astronomical market size is that there are numerous financial derivatives in existence, available for almost every type of investment asset.

- exchange-traded and over-the-counter markets

- types of traders: speculators, hedgers, arbitrageurs

- forwards & futures, options, and other derivatives

2.1 Forwards and Futures

2.2 Options

3. Option Pricing

Talk about stochastic and deterministic formulation.

3.1 Market Models

State, motivate and explain the models used in BENCHOP.

3.2 Pricing Methods

Talk about all the method groups from BENCHOP.

4. Radial Basis Function generated Finite Difference Methods

Similar to explanation in our papers, a bit of history and the method evolution throughout the papers.

4.1 Choosing Shape Parameters

4.2 Constructing Node Layouts

4.3 Smoothing Payoff Functions

5. Outlook and Further Development

Sumarize.

References

- [1] Peter Flynn. *Formatting information. A beginner's introduction to typesetting with L^AT_EX*. 3rd edition, 2005.
<http://www.ctan.org/tex-archive/info/beginlatex/beginlatex-3.6.pdf>.
- [2] Patrik W. Daly Helmut Kopka. *A Guide to L^AT_EX 2_ε*. Addison-Wesley Professional, 4th edition, 2003.
- [3] Frank Mittelbach, Michel Goossens with Johannes Braams, David Carlisle, and Chris Rowley. *The L^AT_EX 2_ε Companion*. Addison-Wesley Professional, 2nd edition, 2004.
- [4] Tobias Oetiker, Hubert Partl, Irene Hyna, and Elisabeth Schlegl. *The Not So Short Introduction to L^AT_EX 2_ε*. 4.14th edition, 2004.
<http://www.ctan.org/tex-archive/info/lshort/english/lshort.pdf>.
- [5] Paul W. Abrahams with Karl Berry and Kathryn A. Hargreaves. *T_EX for the Impatient*. Addison-Wesley Professional, 2003.
<http://www.tug.org/ftp/tex/impatient/book.pdf>.