8. $Var(\hat{f}(x_0))$ is the reproducible error
ε is the irreducible error

9. use that $\hat{f}(x_0) = \mathbb{E}[Y \mid X = x_0]$
that $Var(\hat{f}(x_0)) + [Bias(\hat{f}(x_0))]^2$ goes to 0
that we get the left one term is ε

10

$$\mathbb{E}\left[(y_0 - \hat{f}(x_0))^2\right] = \mathbb{E}\left[(y_0 - \mathbb{E}(\hat{f}(x_0)) + \mathbb{E}[\hat{f}(x_0)] - \hat{f}(x_0))^2\right]$$

$$= \mathbb{E}\left[(y_0 - \mathbb{E}(\hat{f}(x_0))^2\right] + 2\mathbb{E}\left[(\hat{f}(x_0) - \mathbb{E}[\hat{f}(x_0)]) \cdot (\mathbb{E}[\hat{f}(x_0)] - y_0)\right]$$

$$+ \mathbb{E}\left[(\mathbb{E}[\hat{f}(x_0)] - \hat{f}(x_0))^2\right]$$

the middle term is:

$$\mathbb{E}\left[(\hat{f}(x_0) - \mathbb{E}(\hat{f}(x_0))) \cdot (\mathbb{E}(\hat{f}(x_0)) - y_0)\right]$$

since $\mathbb{E}(\hat{f}(x_0))$ and y_0 are constants we can take $\mathbb{E}$ inside that is

$$\underbrace{(\mathbb{E}(\hat{f}(x_0)) - \mathbb{E}(\hat{f}(x_0)))}_{\text{is } 0} \cdot (\mathbb{E}(\hat{f}(x_0)) - y_0)$$

so the term comes out as 0

then the third term is Variance$(\hat{f}(x_0))$

so $\mathbb{E}[(y_0 - \hat{f}(x_0))^2] = \text{Variance} + \mathbb{E}[(y_0 - \mathbb{E}[\hat{f}(x_0)])^2]$

and $\mathbb{E}[(y_0 - \mathbb{E}[\hat{f}(x_0)])^2] = \mathbb{E}[(f(x_0) + \varepsilon - \mathbb{E}\hat{f}(x_0))^2]$

$$= \mathbb{E}[\varepsilon^2] + \mathbb{E}[(f(x_0) - \mathbb{E}\hat{f}(x_0))^2] - 2(f(x_0) - \mathbb{E}\hat{f}(x_0)) \cdot \mathbb{E}(\varepsilon)$$

since $f(x_0)$ and $\mathbb{E}\hat{f}(x_0)$ are constants, and $\mathbb{E}(\varepsilon) = 0$

$$= \text{Var}(\varepsilon) + \underbrace{(f(x_0) - \mathbb{E}\hat{f}(x_0))^2}_{(\text{bias})^2}$$

So that $\mathbb{E}[(y_0 - \hat{f}(x_0))^2] = \text{Variance} + (\text{bias})^2 + \text{Var}(\varepsilon)$

QED