Independence

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Lecture 11

Readings: RN 13.4. PM 8.2.

Outline

Learning Goals

By the end of the lecture, you should be able to

- Given a probabilistic model,
  determine if two variables are unconditionally independent.
- Given a probabilistic model,
  determine if two variables are conditionally independent given a third variable.
- Give examples of deriving a compact representation of a joint distribution by using independence and/or conditional independence assumptions.

Learning Goals

Unconditional and Conditional Independence

Revisiting the Learning goals

(Unconditional) Independence

$$P(X \wedge Y) = \underbrace{P(X)\,P(Y|X)} = P(Y)\,P(X)$$

Definition ((unconditional) independence)

X and Y are (unconditionally) independent iff

$$P(X|Y) = P(X)$$

$$P(Y|X) = P(Y)$$

$$\rightarrow \quad P(X \wedge Y) = P(X)P(Y)$$

Learning Y does NOT influence your belief about X.

Conditional Independence

Definition (conditional independence)

X and Y are conditionally independent given Z if

$$P(X|Y \wedge Z) = P(X|Z).$$

$$P(Y|X \wedge Z) = P(Y|Z).$$

$$P(Y \wedge X|Z) = P(Y|Z)P(X|Z).$$

Learning Y does NOT influence your belief about X if you already know Z.

CQ: Deriving a compact representation

n random variables, $2^n - 1$ probabilities.

Boolean

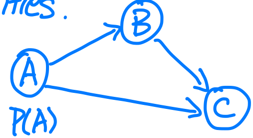
CQ: Consider a model with three random variables, A, B, C.

1. What is the minimum number of probabilities required to specify the joint distribution? 7 probabilities.

$P(A \wedge B \wedge C)$ $\quad 2^3 = 8$ probabilities.

$= P(A) * P(B|A) * P(C|A \wedge B)$

$P(B|A)$
$P(B|\neg A)$



$P(A)$

$P(C|A \wedge B)$
$P(C|A \wedge \neg B)$
$P(C|\neg A \wedge B)$
$P(C|\neg A \wedge \neg B)$

$1 + 2 + 4 = 7$

2. Assume that A, B, and C are independent. What is the minimum number of probabilities required to specify the joint distribution? 3 probabilities.

$P(A \wedge B \wedge C) = P(A) * P(B) * P(C)$

$1 + 1 + 1 = 3$

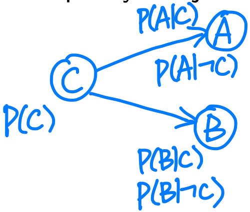
P(A) P(B) P(C)

CQ: Deriving a compact representation

CQ: Consider a model with three random variables, A, B, C.

1. What is the minimum number of probabilities required to specify the joint distribution? 7 probabilities

2. Assume that A and B are conditionally independent given C. What is the minimum number of probabilities required to specify the joint distribution? 5 probabilities



$$P(A \land B \land C)$$
$$= P(C) * P(A|C) * P(B|C \land A)$$
$$= P(C) * P(A|C) * P(B|C)$$
$$1 + 2 + 2 = 5$$

Revisiting the Learning Goals

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