Introduction to Bayesian Networks

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Lecture 11b

Readings: RN 13.4. PM 8.3.

Outline

Learning Goals

By the end of the lecture, you should be able to

- Describe components of a Bayesian network.
- Compute a joint probability given a Bayesian network.
- Given a Bayesian network, determine if two variables are independent or conditionally independent given a third variable.
- Given a joint probability distribution and an order of the variables, construct a Bayesian network that correctly represents the independent relationships among the variables in the distribution.

Learning Goals

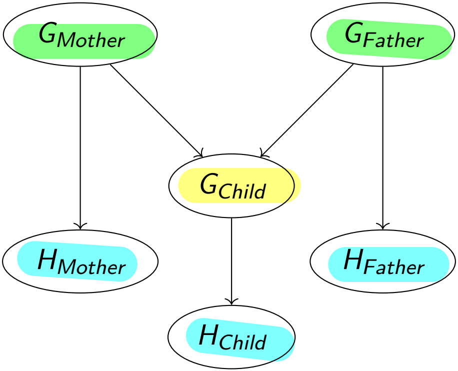
Examples of Bayesian Networks

Semantics of Bayes Net

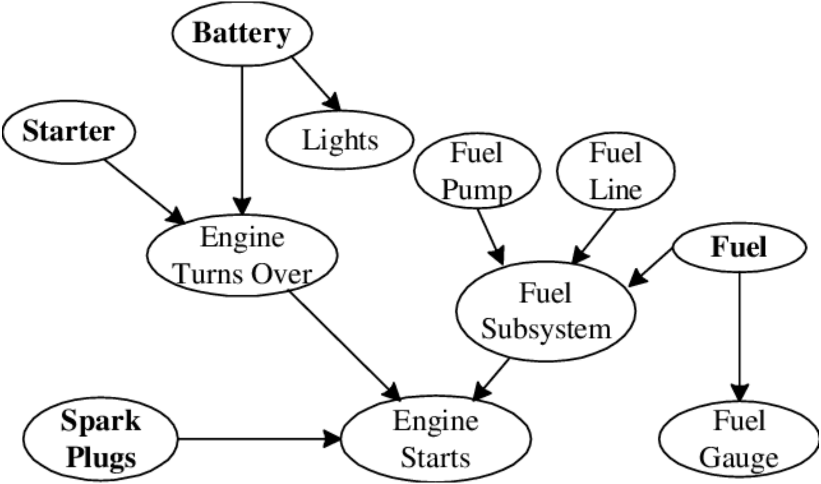
Constructing Bayes Nets

Revisiting the Learning goals

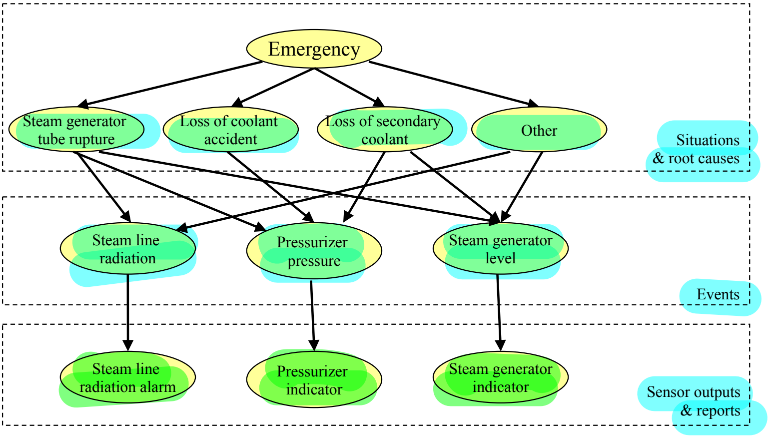
Inheritance of Handedness



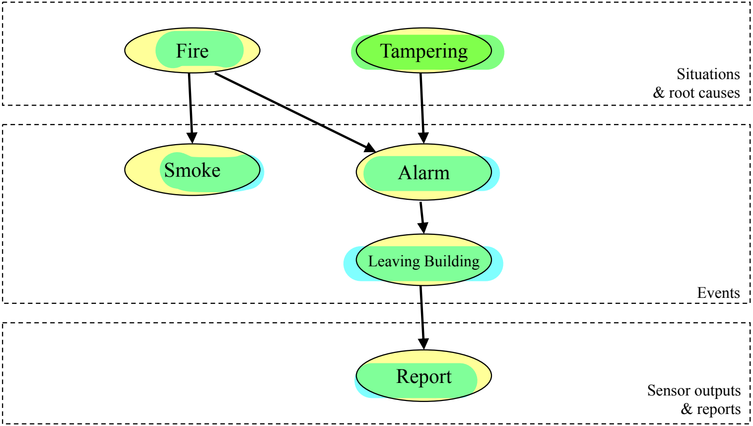
Car Diagnostic Network



Example: Nuclear power plant operations

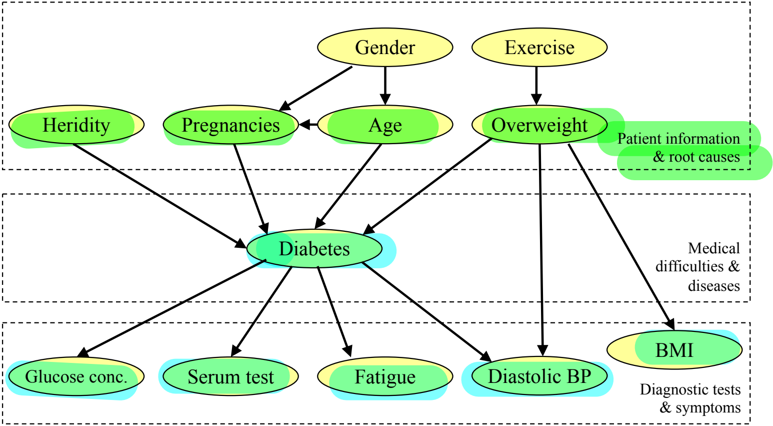


Example: Fire alarms



Report: "report of people leaving building because a fire alarm went off"

Example: Medical diagnosis of diabetes



Why Bayesian Networks?

A probabilistic model of the Holmes scenario:

- The random variables:
  Earthquake, Radio, Burglary, Alarm, Watson, and Gibbon.
- # of probabilities in the joint distribution: $2^6 = 64$.
- For example,
  $P(E \land R \land B \land A \land W \land G) =?$
  $P(E \land R \land B \land A \land W \land \neg G) =?$
  ... etc ...

We can compute any probability using the joint distribution, but

- Quickly become intractable as the number of variables grows.
- Unnatural and tedious to specify all the probabilities.

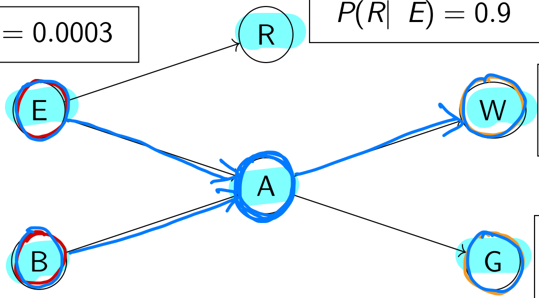
Why Bayesian Networks?

A Bayesian Network

- is a compact version of the joint distribution, and
- takes advantage of the unconditional and conditional independence among the variables.

A Bayesian Network for the Holmes Scenario

$2^6 - 1 = 63 \quad 1+1+2+4+2+2 = 12$



$P(E) = 0.0003$

$P(R|\neg E) = 0.0002$
$P(R|\ E) = 0.9$

$P(W|\neg A) = 0.4$
$P(W|\ A) = 0.8$

$P(G|\neg A) = 0.04$
$P(G|\ A) = 0.4$

$\boxed{P(B) = 0.0001}$

$P(A|\neg B \wedge \neg E) = 0.01$
$P(A|\neg B \wedge\ E) = 0.2$
$P(A|\ B \wedge \neg E) = 0.95$
$P(A|\ B \wedge\ E) = 0.96$

Bayesian Network

A Bayesian Network is a directed acyclic graph.

- Each node corresponds to a random variable.
- X is a parent of Y if there is an arrow from node X to node Y.
- Each node X_i has a conditional probability distribution $P(X_i|Parents(X_i))$ that quantifies the effect of the parents on the node.

The Semantics of Bayesian Networks

Two ways to understand Bayesian Networks:

- A representation of the joint probability distribution
- An encoding of the conditional independence assumptions

Learning Goals

Examples of Bayesian Networks

Semantics of Bayes Net

Representing the Joint Distribution

Encoding the Conditional Independence Relationships

Constructing Bayes Nets

Revisiting the Learning goals

Representing the joint distribution

We can compute each joint probability using the following formula.

$$P(X_n \wedge \cdots \wedge X_1) = \prod_{i=1}^{n} P(X_i | Parents(X_i))$$

Representing the joint distribution

Example: What is the probability that

- The alarm has sounded,
- Neither a burglary nor an earthquake has occurred,
- Both Watson and Gibbon call and say they hear the alarm, and
- There is no radio report of an earthquake?

$$P(\neg B \land \neg E \land A \land \neg R \land W \land G)$$

$$= P(\neg B) * P(\neg E) * P(A \mid \neg B \land \neg E) * P(\neg R \mid \neg E) * P(W \mid A) * P(G \mid A)$$

$$= (1 - 0.0001)(1 - 0.0003)(0.01)(1 - 0.0002)(0.4)(0.8)$$

$$= 0.0032$$

CQ: Calculating the joint probability

CQ: What is the probability that

- NEITHER a burglary NOR an earthquake has occurred,
- The alarm has NOT sounded,
- NEITHER of Watson and Gibbon is calling, and
- There is NO radio report of an earthquake?

(A) 0.5699
(B) 0.6699
(C) 0.7699
(D) 0.8699
(E) 0.9699

$(1-0.0001)*(1-0.0003)*(1-0.01)*(1-0.4)$
$*(1-0.04)*(1-0.0002) = 0.5699.$

Nothing "exciting" is happening in the world.
"No news is good news!"

Burglary, Alarm and Watson

CQ Unconditional Independence

CQ: Are Burglary and Watson independent?

$$\text{Burglary} \longrightarrow \text{Alarm} \longrightarrow \text{Watson}$$

(A) Yes

(B) No

If we learned B, would our belief of W change?

If Burglary is happening, then Alarm is likely going off, and Watson is likely calling Holmes.

CQ: Conditional Independence

CQ: Are Burglary and Watson conditionally independent given Alarm?

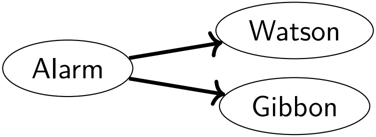
(A) Yes

(B) No

Watson does not observe Burglary directly. Burglary could only influence Watson through Alarm.

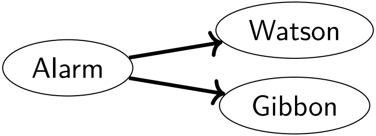
If Alarm is known, we have "cut the chain" in the middle. Burglary and Watson cannot affect each other anymore.

Alarm, Watson and Gibbon



CQ Unconditional Independence

CQ: Are Watson and Gibbon independent?



(A) Yes

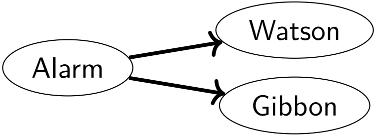
(B) No

If we learned the value of Watson, does this influence our belief about Gibbon?

If Watson is calling, then it's more likely that the Alarm is going off, which means that it is more likely that Gibbon is calling.

CQ Conditional Independence

CQ: Are Watson and Gibbon conditionally independent given Alarm?

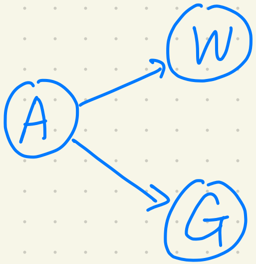


(A) Yes

(B) No

Alarm as an event.
Watson and Gibbon are noisy sensors for the event. The value of each sensor depends entirely on the event.

If we know whether the event is happening or not, the two sensors can no longer affect each other.



$P(A) = 0.1$

$P(W|A) = 0.8$ $\quad$ $P(G|A) = 0.4$

$P(W|\neg A) = 0.4$ $\quad$ $P(G|\neg A) = 0.1$

$$P(W|A \wedge G) = P(W|A \wedge \neg G) = P(W|A)$$

$$P(W|\neg A \wedge G) = P(W|\neg A \wedge \neg G) = P(W|\neg A)$$

$$P(W \wedge A \wedge G) = P(A)\,P(W|A)\,P(G|A) = 0.1 * 0.8 * 0.4 = 0.032$$

$$P(\neg W \wedge A \wedge G) = P(A)\,P(\neg W|A)\,P(G|A) = 0.1 * 0.2 * 0.4 = 0.008$$

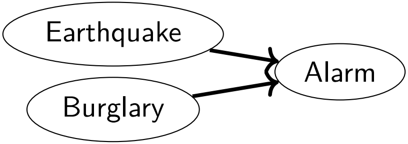
$$P(W|A \wedge G) = \frac{0.032}{0.032 + 0.008} = 0.8 \qquad P(W|A) = \frac{0.08}{0.08 + 0.02} = 0.8$$

$$P(W \wedge A) = P(W \wedge A \wedge G) + P(W \wedge A \wedge \neg G) = 0.032 + 0.048 = 0.08$$

$$P(W \wedge A \wedge \neg G) = P(A)\,P(W|A)\,P(\neg G|A) = 0.1 * 0.8 * 0.6 = 0.048$$

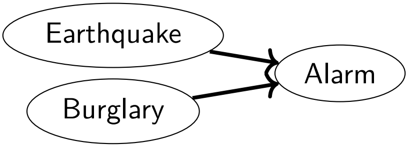
$$P(\neg W \wedge A) = P(\neg W \wedge A \wedge G) + P(\neg W \wedge A \wedge \neg G) = 0.008 + 0.012 = 0.02$$

Earthquake, Burglary, and Alarm



CQ Unconditional Independence

CQ: Are Earthquake and Burglary independent?



(A) Yes

(B) No

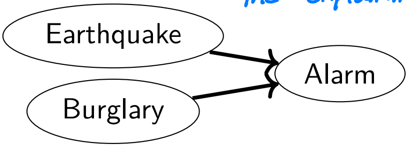
If we learned whether an earthquake is happening or not, this does NOT change our belief about Burglary, and vice versa.

Let's assume that looting is not more frequent during an earthquake. $\Downarrow$

CQ: Conditional Independence

CQ: Are Earthquake and Burglary conditionally independent given Alarm?

the "explaining away" effect



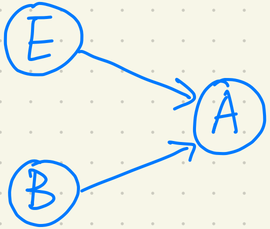
Suppose that the Alarm is going off.

(A) Yes

(B) No

If an earthquake is happening, then it is less likely that the Alarm is caused by a Burglary.

If an earthquake is not happening, then it is more likely that a Burglary is happening and causing the alarm to go off.



$P(E) = 0.1$ $\quad$ $P(A \mid E \wedge B) = 0.9$ $\quad$ $P(A \mid \neg E \wedge B) = 0.7$

$P(B) = 0.2$ $\quad$ $P(A \mid E \wedge \neg B) = 0.5$ $\quad$ $P(A \mid \neg E \wedge \neg B) = 0.1$

Prove that $P(E \mid A \wedge B) \neq P(E \mid A)$.

$P(E)\ P(B \mid E)\ P(A \mid E \wedge B)$

$$P(E \wedge A \wedge B) = P(E)\,P(B)\,P(A \mid E \wedge B) = 0.1 * 0.2 * 0.9 = 0.018.$$

$$P(\neg E \wedge A \wedge B) = P(\neg E)\,P(B)\,P(A \mid \neg E \wedge B) = 0.9 * 0.2 * 0.7 = 0.126.$$

$$\frac{P(E \wedge A \wedge B)}{P(A \wedge B)} = P(E \mid A \wedge B) = \frac{0.018}{0.018 + 0.126} = 0.125$$

$$P(E \wedge A \wedge \neg B) = P(E)\,P(\neg B)\,P(A \mid E \wedge \neg B) = 0.1 * 0.8 * 0.5 = 0.04$$

$$P(E \wedge A) = P(E \wedge A \wedge B) + P(E \wedge A \wedge \neg B) = 0.018 + 0.04 = 0.058$$

$$P(\neg E \wedge A \wedge \neg B) = P(\neg E)\,P(\neg B)\,P(A \mid \neg E \wedge \neg B) = 0.9 * 0.8 * 0.1 = 0.072$$

$$P(\neg E \wedge A) = P(\neg E \wedge A \wedge B) + P(\neg E \wedge A \wedge \neg B) = 0.126 + 0.072 = 0.198$$

$$P(E \mid A) = \frac{0.058}{0.058 + 0.198} = 0.227$$

Revisiting the Learning Goals

By the end of the lecture, you should be able to

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- Compute a joint probability given a Bayesian network.
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