Artificial Intelligence II

Part 2: Lecture 4

Brent Davis

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Slides are adapted from Olga Vesker (UW), Steve Seitz (UW), David Jacobs (UMD), Rober Collins (Penn State), Yalda Mohsenadeh (Western)

Computer Vision

Motion

Outline

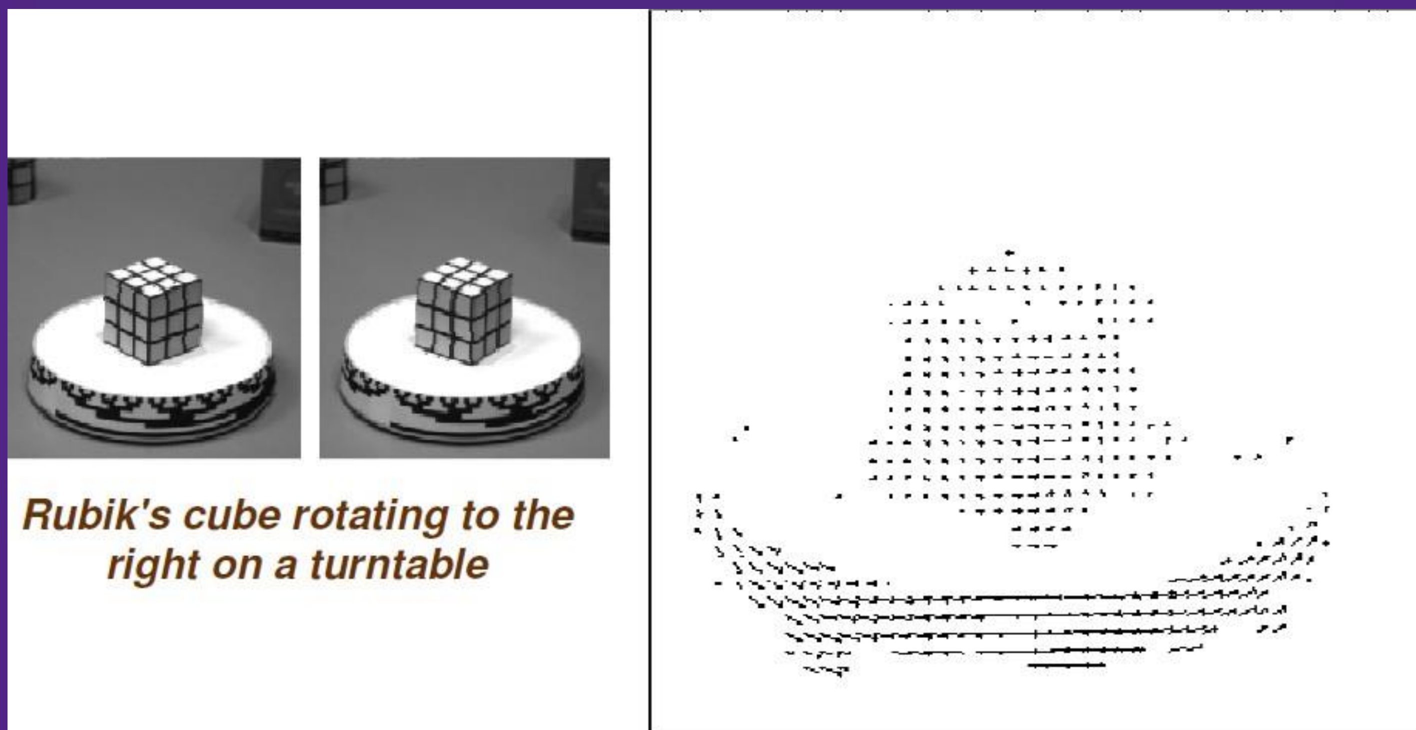
- Motion Estimation
  - Motion field
  - Optical flow field
- Methods for optical flow estimation
  - Discrete Search
  - Lucas-Kanade approach to optical flow
- Motion Tracking
  - Harris corner detection

Why estimate motion?

- Many applications
  - Track objects
  - Correct for camera jitters
  - Align images
  - Special effects

Optical flow and Motionfield

- Optical flow is the apparent motion of brightness patterns between 2 (or several) frames in an image sequence
  - Usually represent optical flow by a 2 dimensional vector (u, v)



Rubik's cube rotating to the right on a turntable

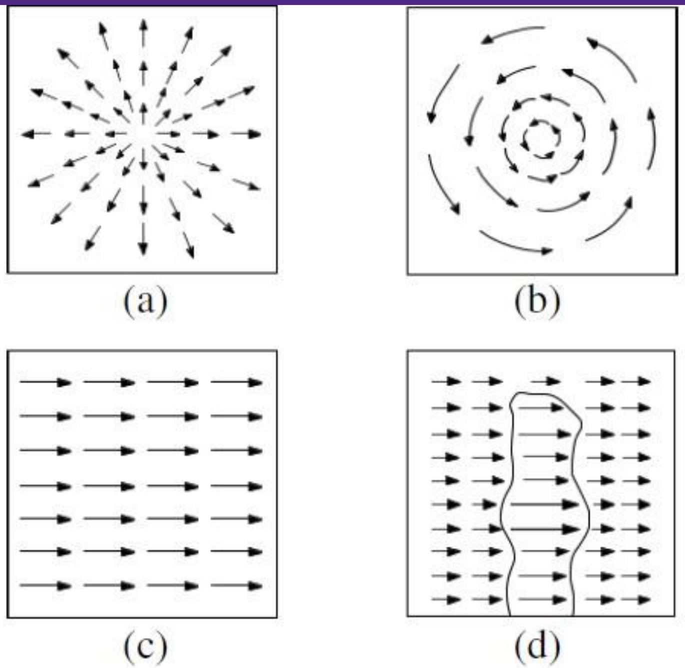
Optical flow and motion field

- Optical flow is the apparent motion of brightness patterns between 2 (or several) frames in an image sequence
- Why does brightness changes between frames?
- Assuming that the illumination does not change:
  - Changes are due to relative motion between the scene and the camera
  - There are three possibilities
    - Camera still, moving scene
    - Moving camera, still scene
    - Moving camera, moving scene
    - Camera still, still scene (no change)
- Optical flow is what we can estimate from image sequence

Motion Field (MF)

- The actual relative motion between 3D scene and the camera is 3 dimensional
  - motion will have horizontal (x), vertical (y), and depth (z) components, in general
- We can project these 3D motions onto the image plane
- What we get is a 2 dimensional motion field
- Motion field is the *projection* of the actual 3D motion in the scene onto the image plane
- Motion Field is what we actually **need** to estimate for applications

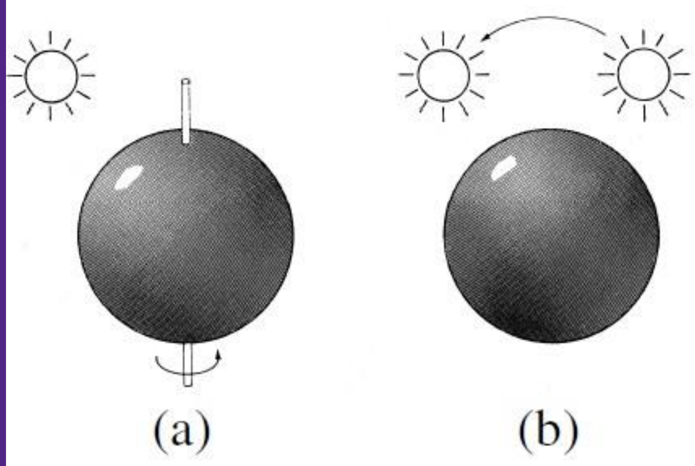
Examples of Motion field



(a) Translation perpendicular to a surface. (b) Rotation about axis perpendicular to image plane. (c) Translation parallel to a surface at a constant distance. (d) Translation parallel to an obstacle in front of a more distant background.

Optical Flow vs. Motion Field

- Optical flow is the apparent motion of brightness patterns
- We equate optical flow field with motion field
- Frequently works, but not always

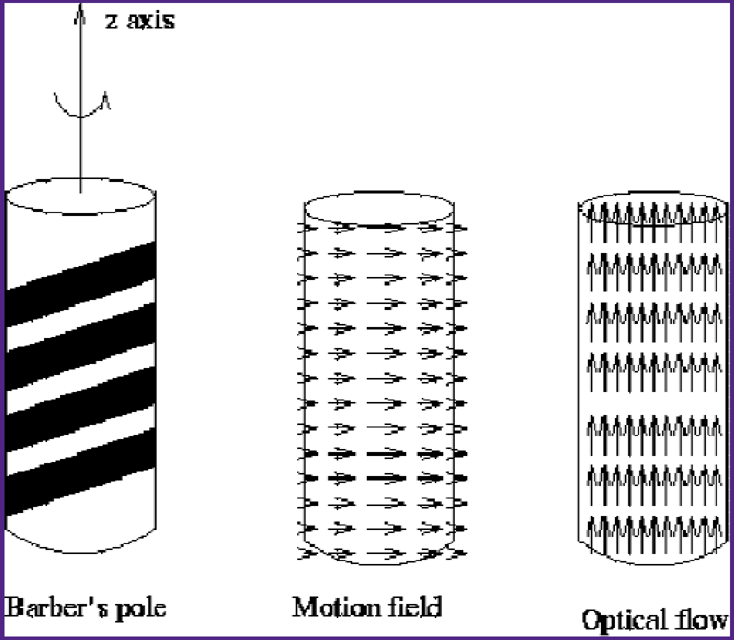


(a) A smooth sphere is rotating under constant illumination. Thus the optical flow field is zero, but the motion field is not

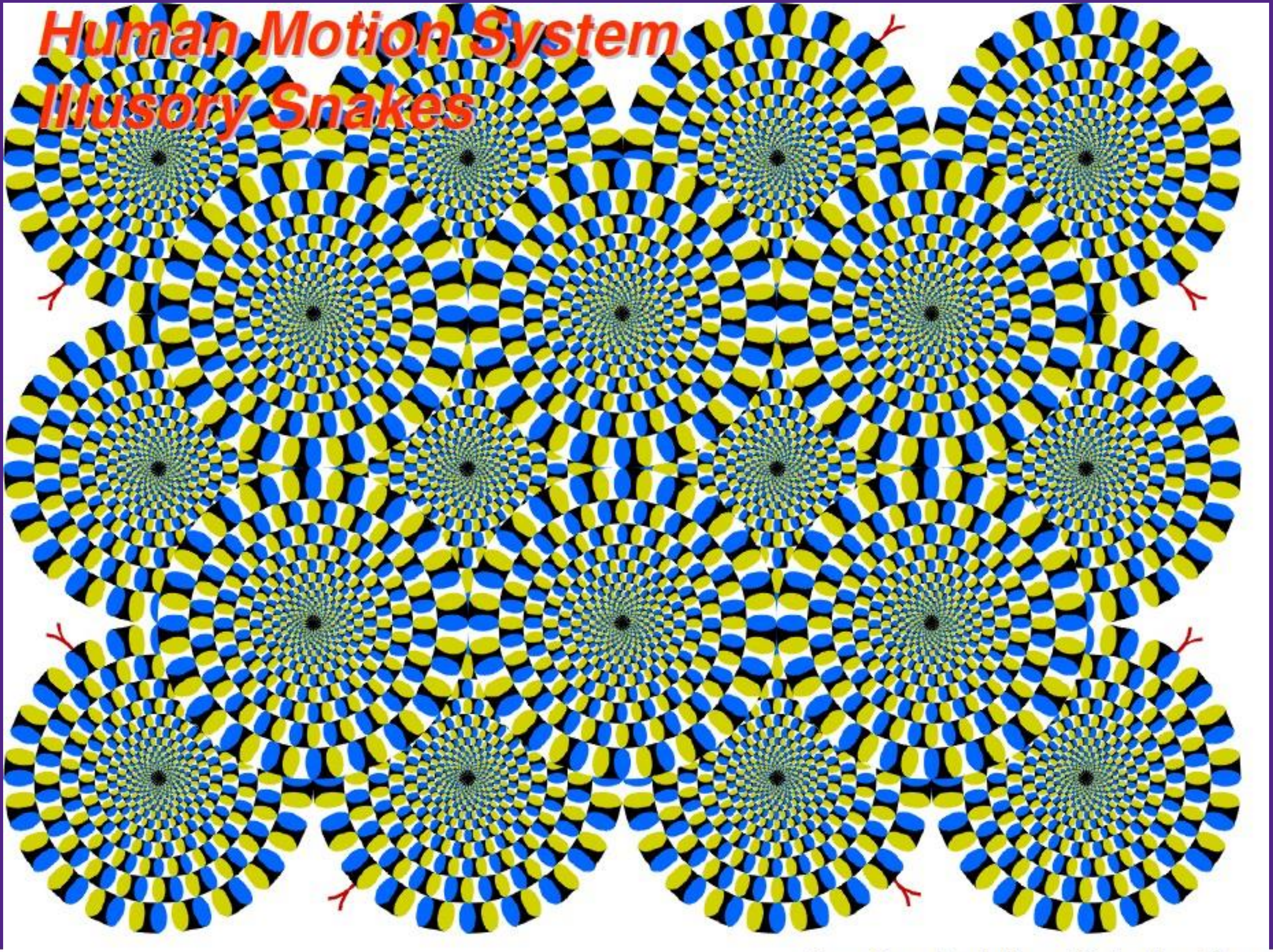
(b) A fixed sphere is illuminated by a moving source—the shading of the image changes. Thus the motion field is zero, but the optical flow field is not

Optical Flow vs. Motion Field

- Motion field and optical flow are very different

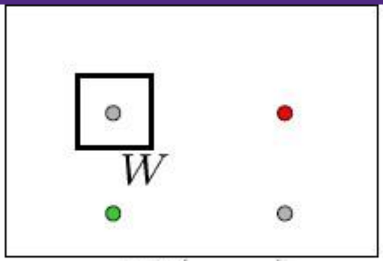


Human Motion System
Illusory Snakes

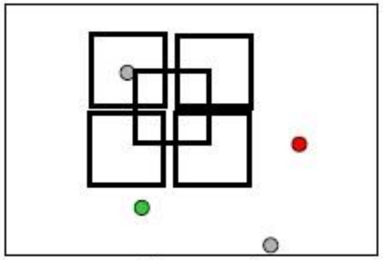


from Gary Bradski and Sebastian Thrun

Discrete search for optical flow



$H(x, y)$



$I(x, y)$

- Given window W in ***H***, find best matching window in ***I***
- Minimize SSD (sum squared difference) or SAD (sum of absolute differences) of pixels in window

$$min_{(u,v)} \left\{ \sum_{(x,y) \in W} |I(x+u, y+v) - H(x,y)|^2 \right\}$$

- search over a specified range of (u,v) values
  - this (u,v) range defines the **search range**
- can use integral image technique for fast search

Computing Optical Flow: Brightness Constancy Equation

- Can we estimate optical flow without search over all possible locations?
  - Yes! If the motion is small …
- Let P be a moving point in 3D
  - At time t, P has coordinates (X(t), Y(t), Z(t))
  - Let P =(X(t), y(t)) be the coordinates of its image at time t
  - Let I(X(t), Y(t), t) be the brightness at P at time t.
- Brightness Constance Assumption:
  - As P moves over time, I(X(t), Y(t), t) remains constant

Computing Optical Flow: Brightness Constancy Equation

$$I[x(t), y(t), t] = \text{constant}$$

Taking derivative with respect to time:

$$\frac{d\,I[x(t), y(t), t]}{dt} = 0$$

$$\Downarrow$$

$$\frac{\partial I}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial I}{\partial t} = 0$$

Computing Optical Flow: Brightness Constancy Equation

1 equation with 2 unknowns

$$\frac{\partial I}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial I}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial I}{\partial t}=0$$

Let

$$\nabla I=\begin{bmatrix}\frac{\partial I}{\partial x}\\ \frac{\partial I}{\partial y}\end{bmatrix}$$ (Frame spatial gradient)

$$\begin{bmatrix}u\\ v\end{bmatrix}=\begin{bmatrix}\frac{\partial x}{\partial t}\\ \frac{\partial y}{\partial t}\end{bmatrix}$$ (optical flow)

$$I_t=\frac{\partial I}{\partial t}$$ (derivative across frames)

Computing Optical Flow: Brightness Constancy Equation

$$\frac{\partial I}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial I}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial I}{\partial t}=0$$

- Written using dot product notation:

$$\begin{bmatrix} I_x \\ I_y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t = 0$$

  - Where I have used more compact notation:

$$\frac{\partial I}{\partial x} = I_x \qquad \frac{\partial I}{\partial y} = I_y$$

Computing Optical Flow: Brightness Constancy Equation

1 equation with 2 unknowns: $\begin{bmatrix} I_x \\ I_y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t = 0$

- Intuitively, what does this constraint mean?
  - The component of the flow in the gradient direction is determined
  - Recall that gradient points in the direction perpendicular to the edge
  - The component of the flow parallel to an edge is unknown

Computing Optical Flow: Brightness Constancy Equation

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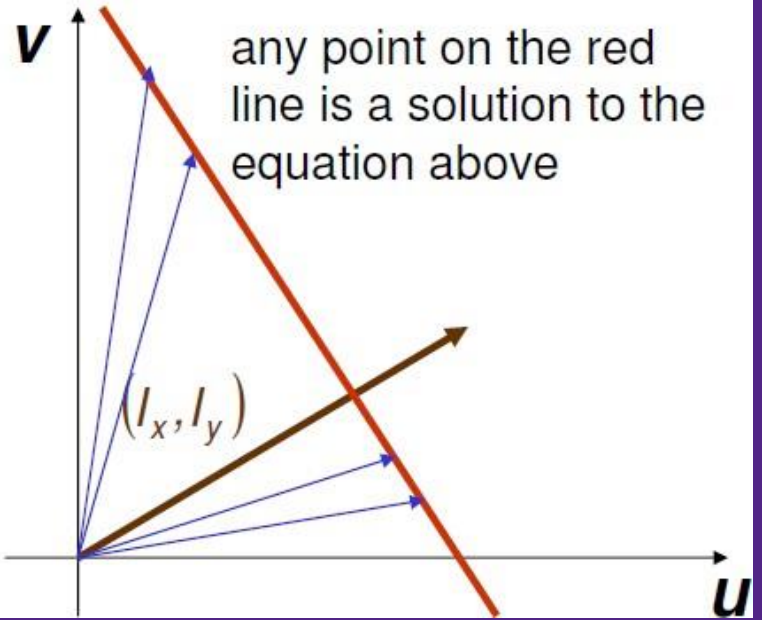


Computing Optical Flow: Brightness Constancy Equation

- How to get more equations for a pixel?
  - Basic idea: impose additional constraints
    - most common is to assume that the flow field is smooth locally
    - one method: pretend the pixel's neighbors have the same (u,v)
      - If we use a 5x5 window, that gives us 25 equations per pixel!

$$I_t(p_i) + \nabla I(p_i) \cdot \begin{bmatrix} u \\ v \end{bmatrix} = 0$$

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_{25}) \end{bmatrix}$$

matrix $\boldsymbol{A}$ 25x2 | vector $\boldsymbol{d}$ 2x1 | vector $\boldsymbol{b}$ 25x1

Computing Optical Flow: Brightness Constancy Equation

- I_x and I_y are computed just as before (recall lectures on filtering)
  - For example, can use Sobel operator

$$s_x = \frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad s_y = \frac{1}{8}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

  - Note that 1/8 factor is now mandatory, unlike in edge detection, since we want the actual gradient value

Computing Optical Flow: Brightness Constancy Equation

- I_t is the derivative between the frames

121	121	122	123	122	123
121	121	122	123	122	123
122	123	124	123	124	123
120	122	122	123	122	123
121	121	124	123	124	123
125	120	124	123	124	123

I^5: frame at time = 5

121	121	122	123	20	20
121	121	122	123	22	22
122	123	124	123	24	21
120	122	122	123	22	22
121	121	124	123	24	23
125	120	124	123	24	24

I^6: frame at time = 5

- Simplest approximation to $I_t(p) = I^{t+1}(p) - I^t(p)$
- For example for pixel with coordinates (4,3) above

$$I_t(4,3) = 22 - 122 = -100$$

Lukas-Kanade Flow

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_{25}) \end{bmatrix}$$

matrix ***A*** 25x2 | vector ***d*** 2x1 | vector ***b*** 25x1

- Problem: now we have more equations than unknowns
- Can't find the exact solution d, but can solve Least Squares Problem:

$$\underset{25\text{x}2}{A}\ \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Lukas-Kanade Flow

$$\underset{25\times 2}{A}\ \underset{2\times 1}{d} = \underset{25\times 1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

- Solution: solve least squares problem
  - minimum least squares solution given by solution (in ***d***) of:

$$\underset{2\times 2}{(A^T A)}\ \underset{2\times 1}{d} = \underset{2\times 1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = -\underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

  - The summations are over all pixels in the K x K window
  - This technique was first proposed by Lucas & Kanade (1981)
  - Note: solution is at sub-pixel precision, that is you can get answer like u= 0.7 and v = -0.33
    - Contrast this with discrete search: to find answer at sub-pixel precision, you have to search at sub-pixel precision (usually)

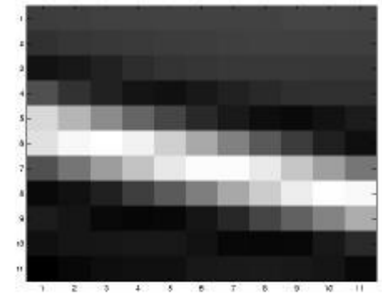
Conditions for solvability

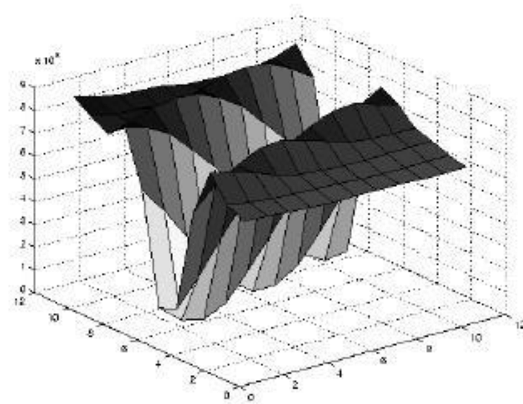
- Optimal (u, v) satisfies Lucas-Kanade equation

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = -\underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- When is this solvable?
  - $\mathbf{A^TA}$ should be invertible
  - $\mathbf{A^TA}$ entries should not be too small (noise)
  - $\mathbf{A^TA}$ should be well-conditioned
    - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)
    - The eigenvectors of A^TA relate to edge direction and magnitude

Edge



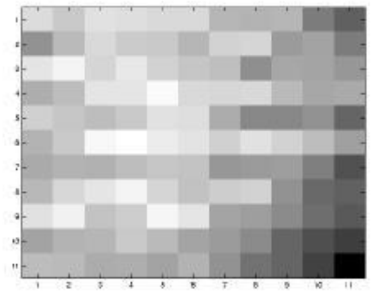


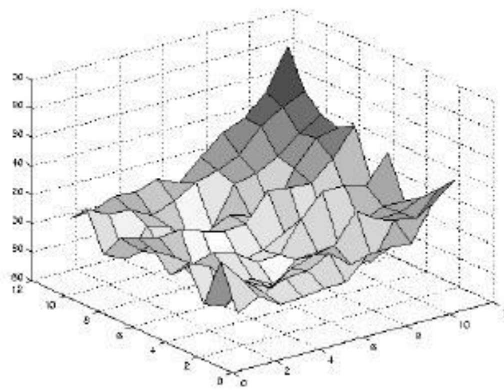
$$\sum \nabla I (\nabla I)^T$$

– gradients very large or very small

– large λ_1, small λ_2

Low texture regions



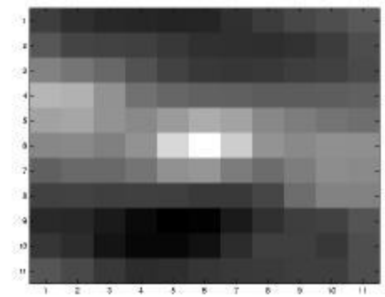


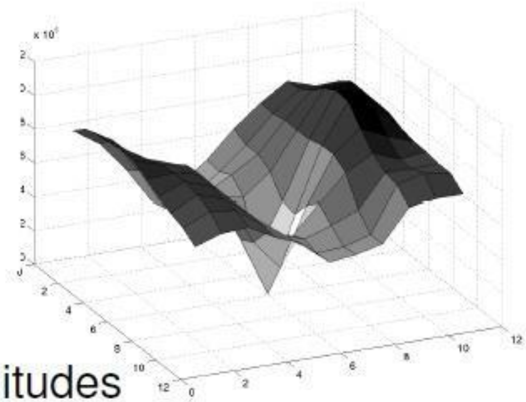
$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

High textured regions





$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

Observations

- This is a two image problem BUT
  - Can measure sensitivity by just looking at one of the images!
  - This tells us which pixels are easy to track, which are hard
    - very useful for feature tracking

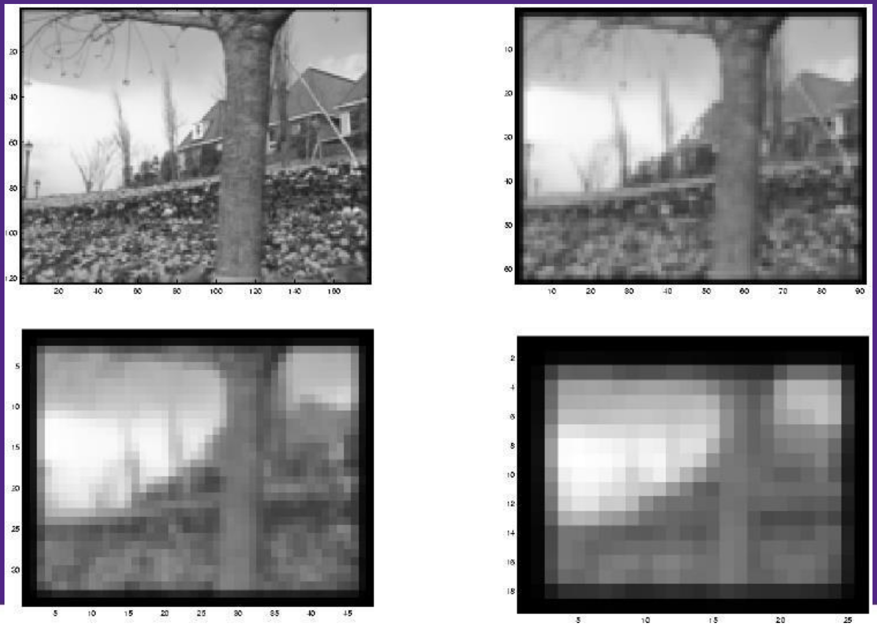
Errors in Lucas-Kanade

- What are the potential causes of errors in this procedure?
  - Suppose A^TA is easily invertible
  - Suppose there is not much noise in the image

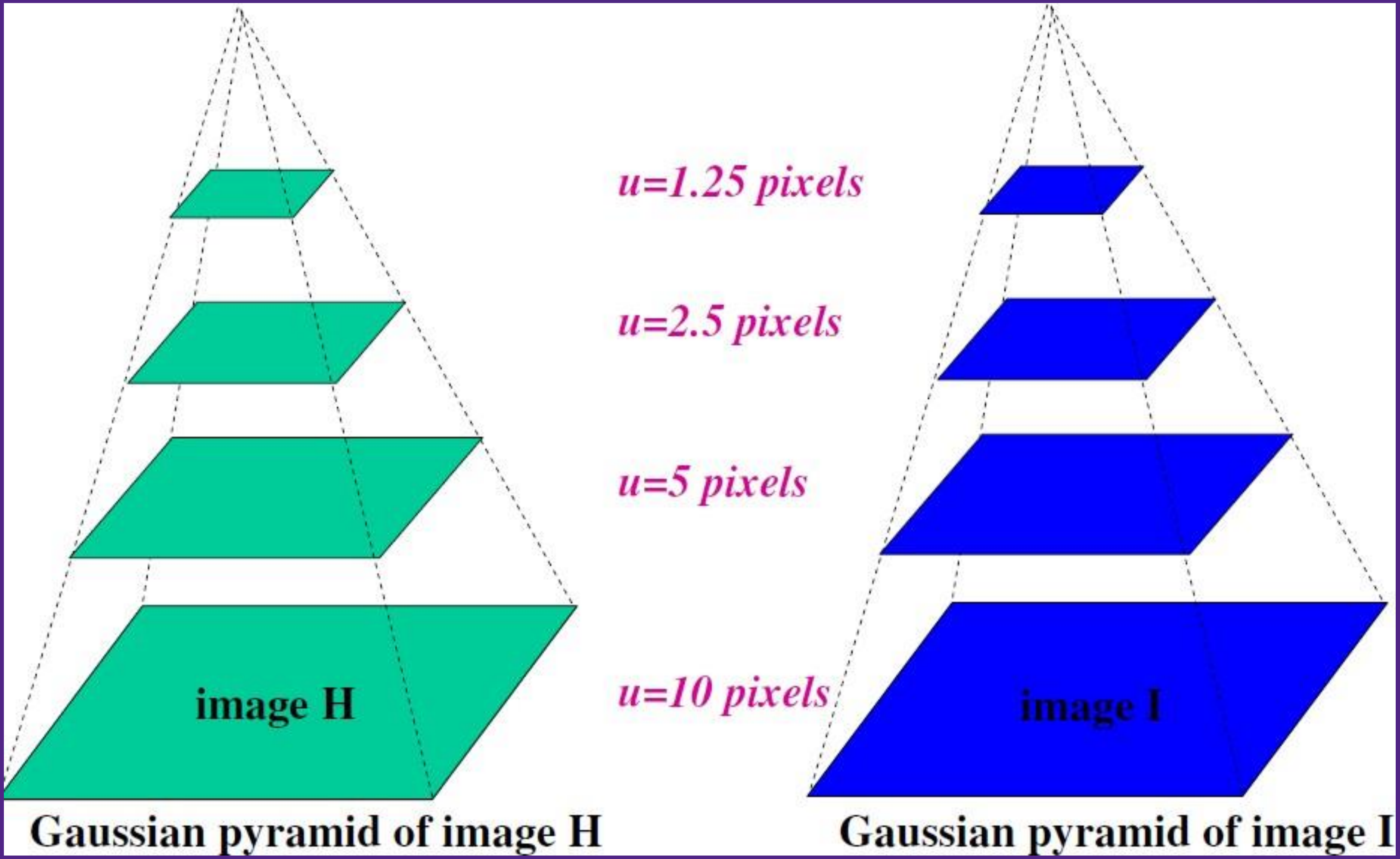
- When our assumptions are violated
  - Brightness constancy is **not** satisfied
  - The motion is **not** small
  - A point does **not** move like its neighbors
    - window size is too large
    - what is the ideal window size?

Revisiting the Small Motion Assumption

- What if the motion is not small enough? How can we solve this problem?
  - Reduce resolution



Coarse to fine optical flow estimation



Coarse to fine optical flow estimation

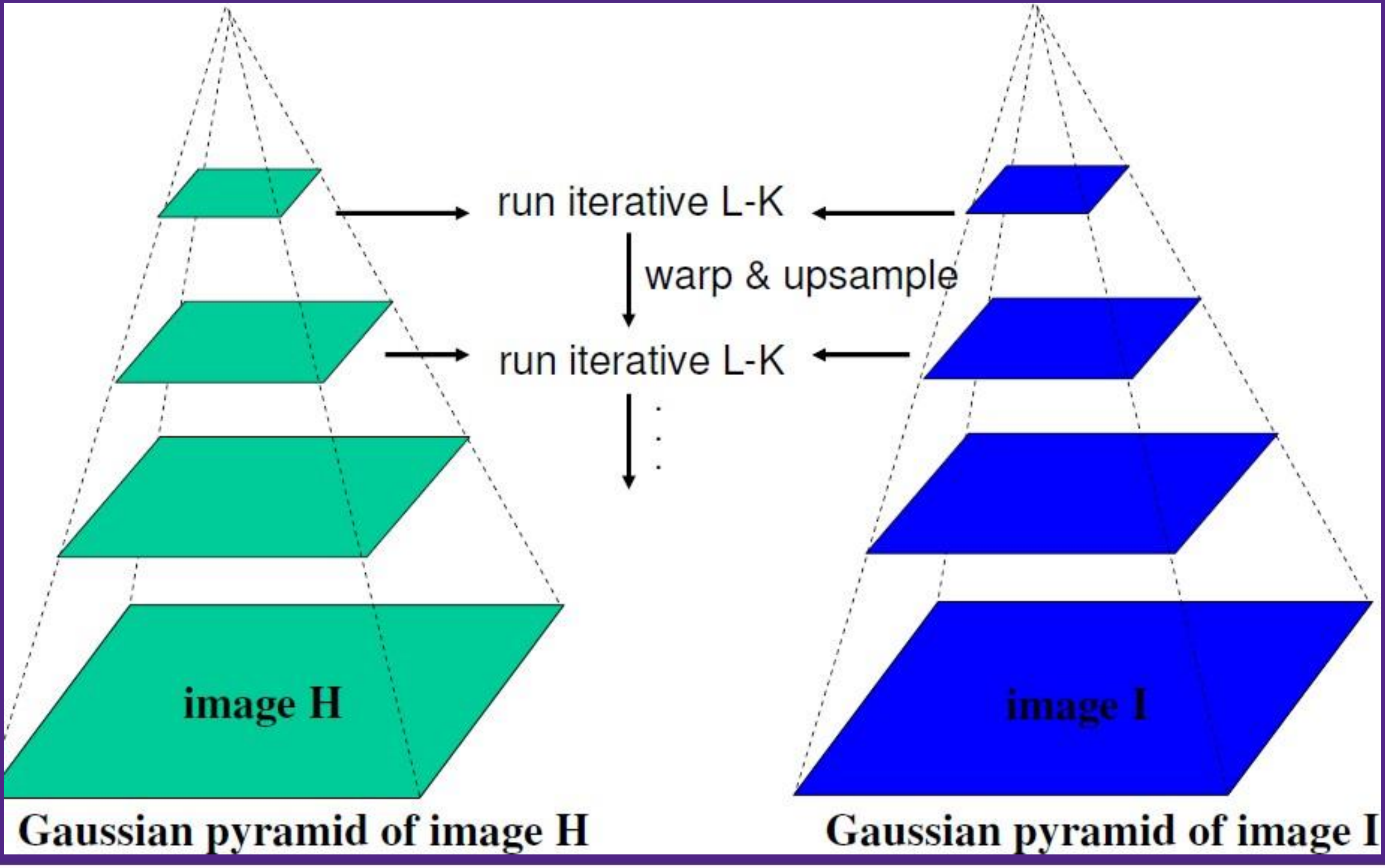
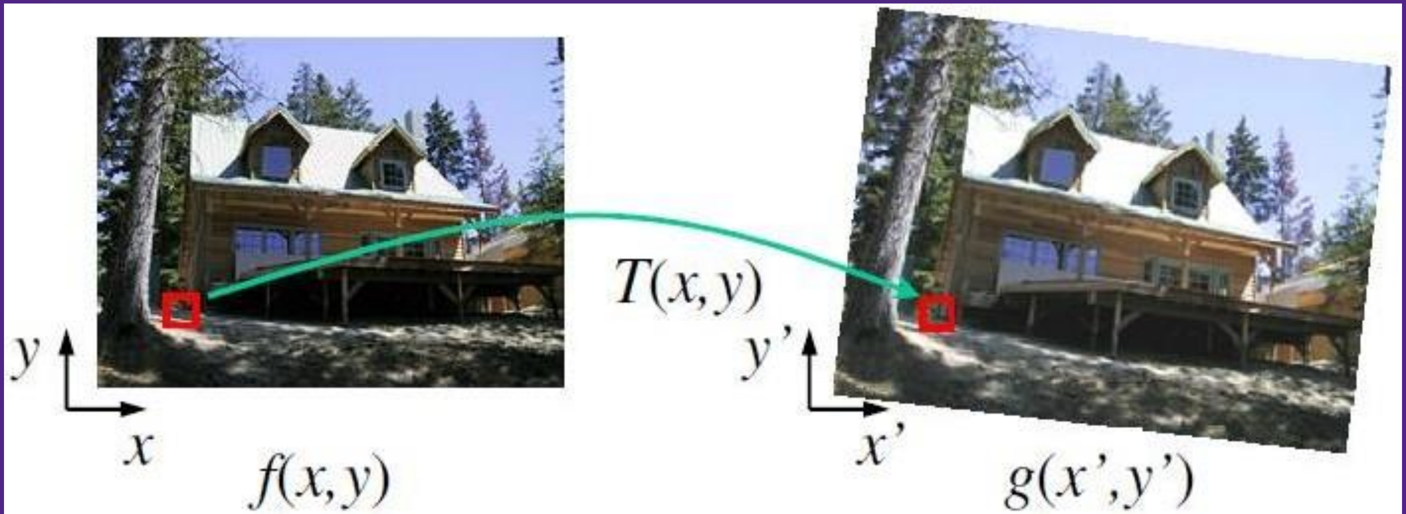


Image Warping

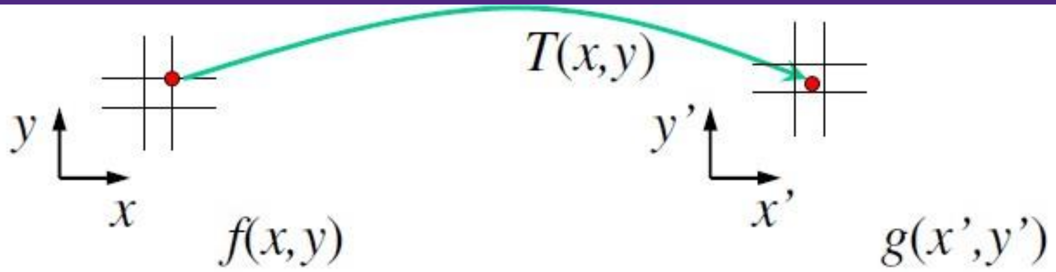


- Send each pixel $f(x,y)$ to its corresponding location

  $(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

Forward Warping



- Send each pixel $f(x,y)$ to its corresponding location

  $(x',y') = T(x,y)$ in the second image

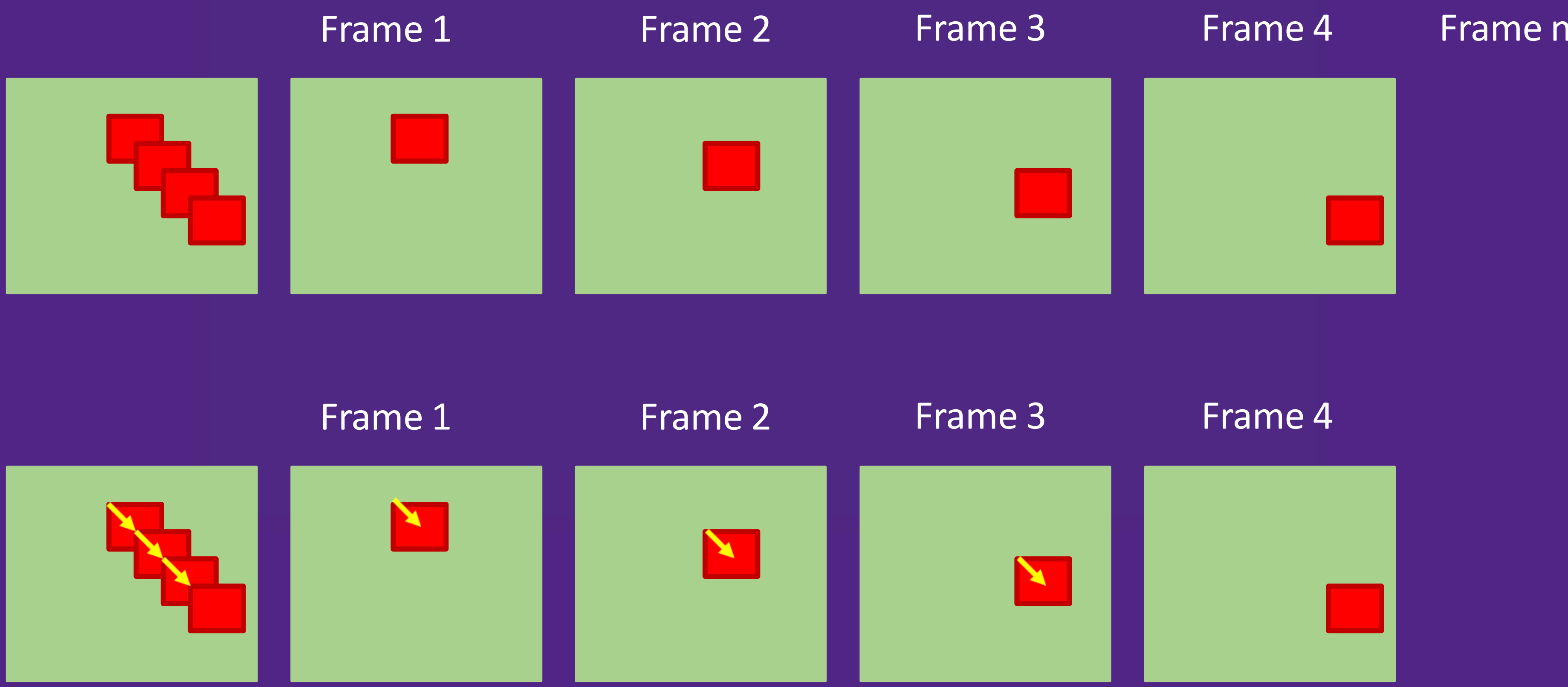
Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

– Known as "splatting"

Western Science

Motion Tracking

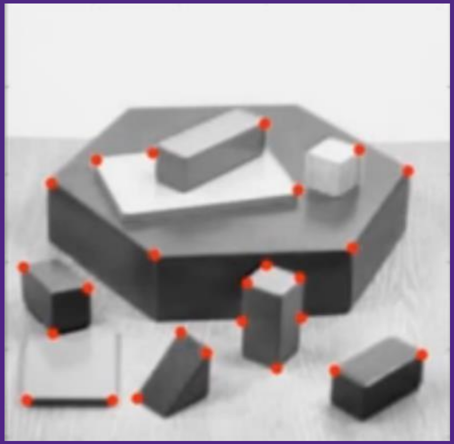


Motion Tracking

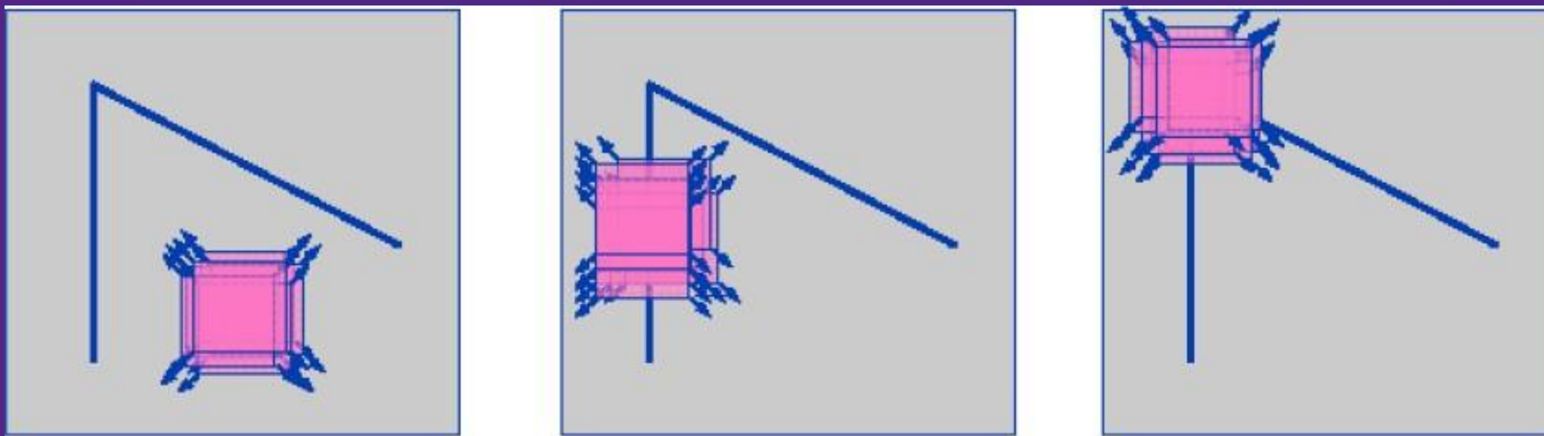
- Suppose we have more than two images
- How to track a point through all of the images?
  - In principle, we could estimate motion between each pair of consecutive frames
  - Given point in first frame, follow arrows to trace out it's path
  - Problem: DRIFT
  - small errors will tend to grow and grow over time—the point will drift way off course
- Feature Tracking
  - Choose only the points ("features") that are easily tracked
  - How to find these features?
    - windows where $\sum \nabla I (\nabla I)^T$ has two large eigenvalues
  - Called the Harris Corner Detector

Harris corner detector

- Corner is an intersection of two edges
- They are good features to track/match
- Harris corner gives a mathematical representation for this concept.



Harris Corner Detector: BasicIdea



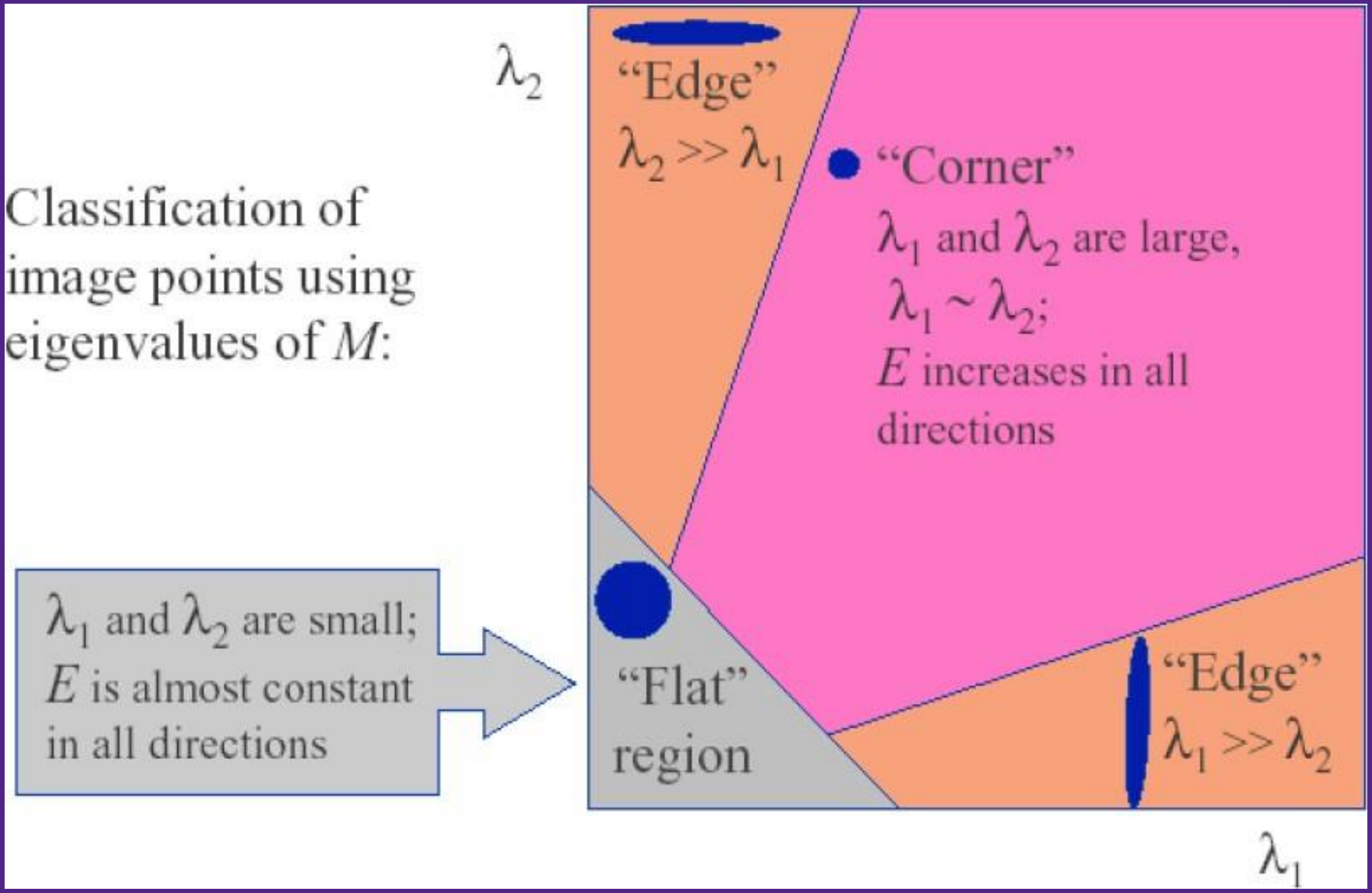
"flat" region:
no change in
all directions

"edge":
no change along
the edge direction

"corner":
significant change
in all directions

Harris corner detector gives a mathematical approach for determining which case holds.

Classification via Eigenvalues

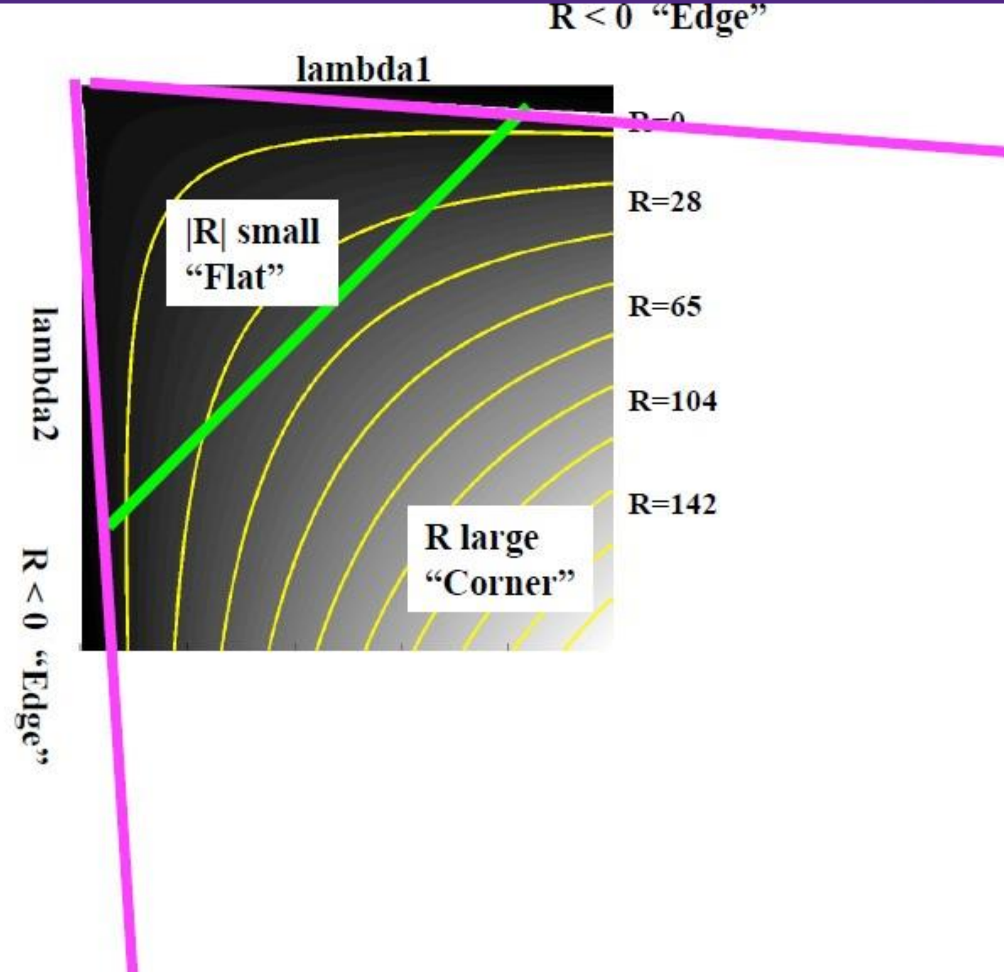


Harris corner detector

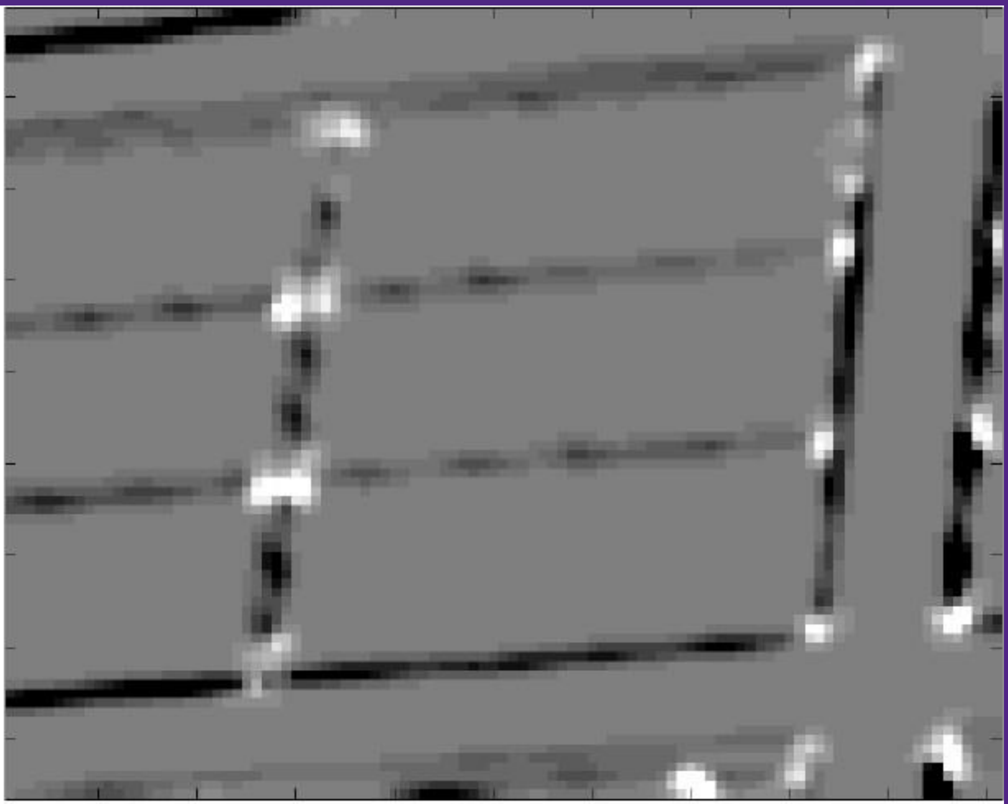
- $M(x,y) \quad \sum_{x,y \in w} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I^2 \end{bmatrix}$
- Measure of corner response:
  - $R = \det M - k\,(trace\,M)^2$
  - $\det M = \lambda_1 \lambda_2$
  - $trace\,M = \lambda_1 + \lambda_2$
  - K is an empirically determined constant; k =0.04 – 0.06
- Response value greater than a threshold is a corner

Corner response map

- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



Example



Harris R score.

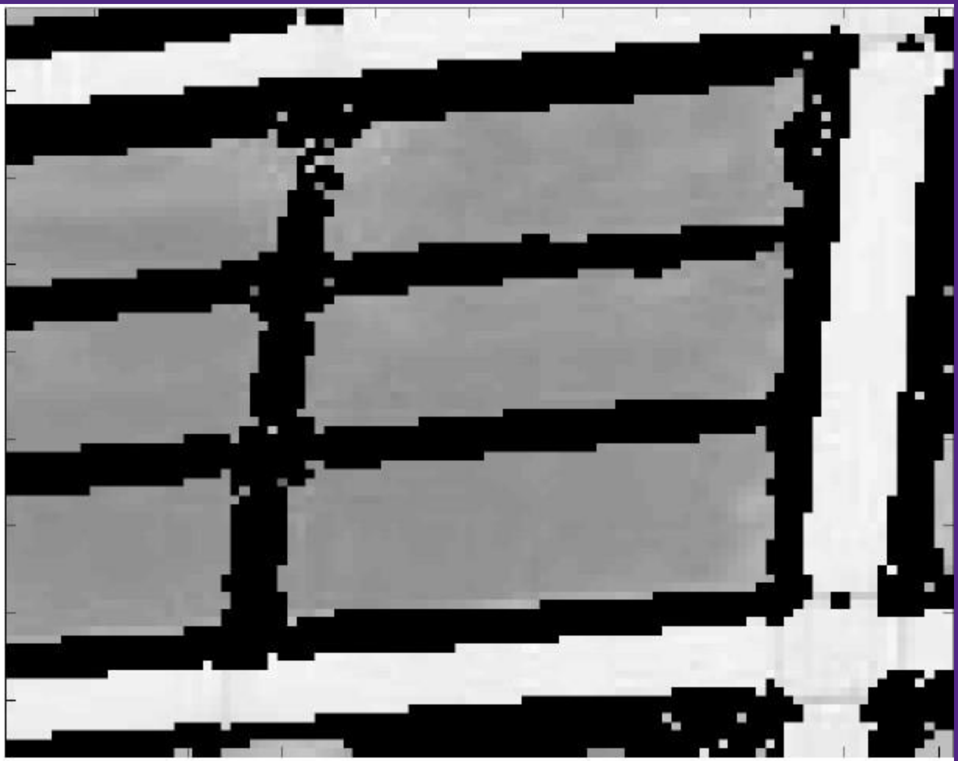
Ix, Iy computed using Sobel operator

Windowing function w = Gaussian, sigma=1

Example

Threshold: > 10000
(corners)

Example



Threshold: -10000 < R < 10000
(neither edges nor corners)

Harris Corner DetectionAlgorithm

1. Color to grayscale
2. Spatial derivative calculation
3. Structure tensor setup (M)
4. Corner response calculation
5. Non-maximum suppression

Tracking Features

- Feature tracking
  - Compute optical flow for that feature for each consecutive H, I
- When will this go wrong?
  - Occlusions—feature may disappear
    - need mechanism for deleting, adding new features
  - Changes in shape, orientation
    - allow the feature to deform
  - Changes in color
  - Large motions

Tracking over many features

- Feature tracking with m frames
  1. Select features in first frame
  2. Given feature in frame i, compute position in i+1
  3. Select more features if needed
  4. i = i + 1
  5. If i < m, go to step 2

- Issues
  - Discrete search vs. Lucas Kanade?
    - depends on expected magnitude of motion
    - discrete search is more flexible
  - Compare feature in frame i to i+1 or frame 1 to i+1?
    - affects tendency to drift..
  - How big should search window be?
    - too small: lost features. Too large: slow