Decision Networks

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Lecture 17

Readings: RN 16.5 - 16.6. PM 9.3 - 9.4.

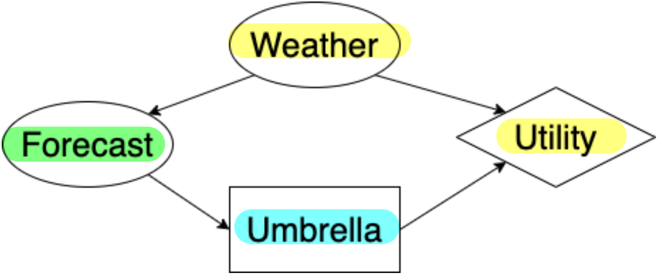
Outline

Learning Goals

By the end of the lecture, you should be able to

- Model a one-off decision problem by constructing a decision network containing nodes, arcs, conditional probability distributions, and a utility function.
- Choose the best action by evaluating a decision network.

Should I take my umbrella or not?



Should I take my umbrella or not?

$P(\text{Weather} = \text{rain}) = 0.3$

Weather	Forecast	P(Forecast | Weather)
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

The utility function

Weather	Umbrella	u(Weather, Umbrella)
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

Should I take my umbrella or not?

P(Weather = rain) = 0.3

Weather	Forecast	P(Forecast | Weather)
norain	sunny	0.7
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The utility function

Weather	Umbrella	u(Weather, Umbrella)
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rain	takeit	70
rain	leaveit	0

A policy

- A policy specifies what the agent should do under all contingencies.
- For each decision variable, a policy specifies a value for the decision variable for each assignment of values to its parents.

For the weather decision network, how many possible policies are there?

Forecast has 3 values. For each value of Forecast, there are 2 possible decisions.

$2^3 = 8$ possible policies.

Solving the weather problem

Two approaches

- Compute the expected utility of each policy, and choose the policy that maximizes the expected utility.
- Use the variable elimination algorithm.

The Expected Utility of a Policy

Consider the policy π_1 below.

- take the umbrella if the forecast is cloudy, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_1?

$$
\begin{aligned}
EU(\pi_1) = {} & P(norain) * P(sunny \mid norain) * u(norain, leave\ it) \\
& + P(norain) * P(cloudy \mid norain) * u(norain, take\ it) \\
& + P(norain) * P(rainy \mid norain) * u(norain, leave\ it) \\
& + P(rain) * P(sunny \mid rain) * u(rain, leave\ it) \\
& + P(rain) * P(cloudy \mid rain) * u(rain, take\ it) \\
& + P(rain) * P(rainy \mid rain) * u(rain, leave\ it)
\end{aligned}
$$

$$
\begin{aligned}
EU(\pi_1) = {} & P(norain) * P(sunny \mid norain) * u(norain, leave\ it) \\
& + P(norain) * P(cloudy \mid norain) * u(norain, take\ it) \\
& + P(norain) * P(rainy \mid norain) * u(norain, leave\ it) \\
& + P(rain) * P(sunny \mid rain) * u(rain, leave\ it) \\
& + P(rain) * P(cloudy \mid rain) * u(rain, take\ it) \\
& + P(rain) * P(rainy \mid rain) * u(rain, leave\ it)
\end{aligned}
$$

$$
\begin{aligned}
= {} & 0.7 * 0.7 * 100 \\
& + 0.7 * 0.2 * 20 \\
& + 0.7 * 0.1 * 100 \\
& + 0.3 * 0.15 * 0 \\
& + 0.3 * 0.25 * 70 \\
& + 0.3 * 0.60 * 0
\end{aligned}
$$

$$= 49 + 2.8 + 7 + 0 + 5.25 + 0$$

$$= \underline{64.05}$$

CQ: The Expected Utility of a Policy

CQ: Consider the policy π_2 below:

- take the umbrella if the forecast is rainy, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_2?

$$
\begin{aligned}
EU(\pi_2) = {} & P(norain) * P(sunny \mid norain) * u(norain, leave\ it) \\
& + P(norain) * P(cloudy \mid norain) * u(norain, leave\ it) \\
& + P(norain) * P(rainy \mid norain) * u(norain, take\ it) \\
& + P(rain) * P(sunny \mid rain) * u(rain, leave\ it) \\
& + P(rain) * P(cloudy \mid rain) * u(rain, leave\ it) \\
& + P(rain) * P(rainy \mid rain) * u(rain, take\ it)
\end{aligned}
$$

$$
\begin{aligned}
= {} & 0.7 * 0.7 * 100 \\
& + 0.7 * 0.2 * 100 \\
& + 0.7 * 0.1 * 20 \\
& + 0.3 * 0.15 * 0 \\
& + 0.3 * 0.25 * 0 \\
& + 0.3 * 0.60 * 70
\end{aligned}
$$

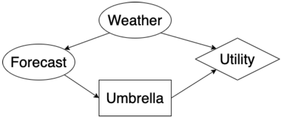
$$
= 49 + 14 + 1.4 + 0 + 0 + 12.6
$$

$$
= \boxed{77}
$$

Variable elimination algorithm

1. Remove all variables that are not ancestors of the utility node.
2. Create factors.
3. While there are decision nodes remaining
   3.1 Sum out each random variable that is not a parent of a decision node.
   3.2 Find the optimal policy for the last decision.
4. Return the optimal policies.
5. Sum out all remaining random variables to get the expected utility of the optimal policy.

Applying VEA



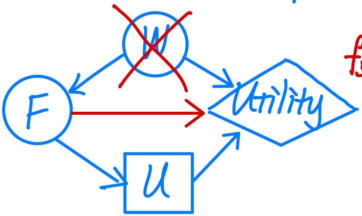
Step 1: Nothing to be done.

Step 2: Define three factors.
f1(Weather), f2(Forecast, Weather), f3(Weather, Umbrella).

Step 3.1: Weather is not a parent of any decision node. Sum out Weather.

$f1 * f2 * f3 = f4$ (Weather, Forecast, Umbrella).

sum out Weather from f4 to produce f5.



f_5(Forecast, Umbrella)

Create factors

$f_1(\text{Weather})$.

Weather	val
rain	0.3
norain	0.7

$f_2(\text{Weather, Forecast})$.

Weather	Forecast	P(Forecast | Weather)
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

The utility function

$f_3(\text{Weather, Umbrella})$.

Weather	Umbrella	u(Weather, Umbrella)
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

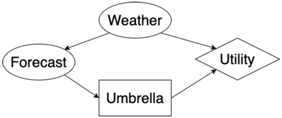
f_4(Weather, Forecast, Umbrella)

Weather	Forecast	Umbrella	val
norain	sunny	take it	9.8
norain	sunny	leave it	49
norain	cloudy	take it	2.8
norain	cloudy	leave it	14
norain	rainy	take it	1.4
norain	rainy	leave it	7
rain	sunny	take it	3.15
rain	sunny	leave it	0
rain	cloudy	take it	5.25
rain	cloudy	leave it	0
rain	rainy	take it	12.6
rain	rainy	leave it	0

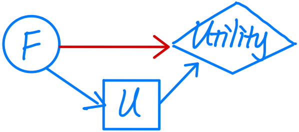
f_5(Forecast, Umbrella)

Forecast	Umbrella	val
sunny	take it	12.95
sunny	leave it	49
cloudy	take it	8.05
cloudy	leave it	14
rainy	take it	14
rainy	leave it	7

Applying VEA



Step 3.2: Find the optimal policy for Umbrella.



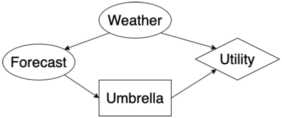
f_5(Forecast, Umbrella)

Forecast	Umbrella	val
sunny	take it	12.95
sunny	leave it	49
cloudy	take it	8.05
cloudy	leave it	14
rainy	take it	14
rainy	leave it	7

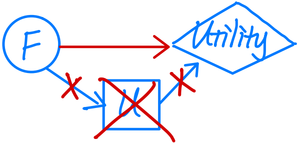
The optimal policy for Umbrella:

Forecast	Umbrella
sunny	leave it
cloudy	leave it
rainy	take it

Applying VEA



Step 3.2: Find the optimal policy for Umbrella.



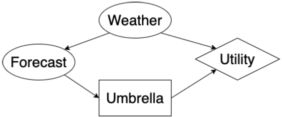
f_5(Forecast, Umbrella)

Forecast	Umbrella	val
sunny	take it	12.95
sunny	leave it	49
cloudy	take it	8.05
cloudy	leave it	14
rainy	take it	14
rainy	leave it	7

f_6(Forecast)

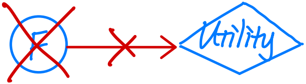
Forecast	val
sunny	49
cloudy	14
rainy	14

The Expected Utility of the Optimal Policy



$f_6(\text{Forecast})$

Forecast	val
sunny	49
cloudy	14
rainy	14

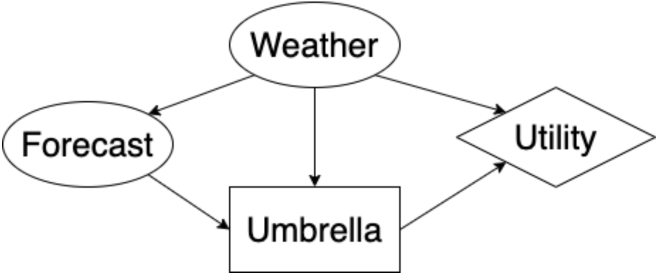


Sum out Forecast from f_6 to produce $f_7()$.

The expected utility of the optimal policy is

$$49 + 14 + 14 = 77$$

What if I can observe the Weather directly?



Applying VEA

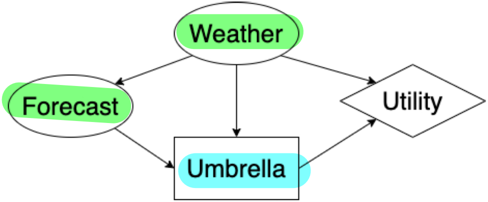
Step 1: Nothing to be done.

Step 2: Define 3 factors.

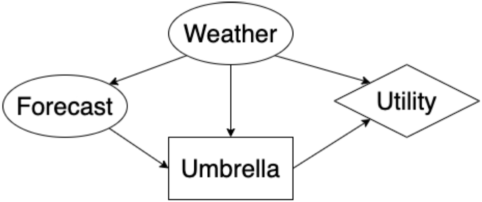
$f_1(\text{Weather})$, $f_2(\text{Forecast}, \text{Weather})$, $f_3(\text{Weather}, \text{Umbrella})$.

Step 3.1: Nothing to be done.

Step 3.2: Find the optimal policy for Umbrella. f_3



Applying VEA



$f_3(\text{Weather}, \text{Umbrella})$

Weather	Umbrella	u(Weather, Umbrella)
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

$f_4(\text{Weather})$

Weather	value
norain	100
rain	70

The optimal policy:

Weather = norain, leaveit.

Weather = rain, take it.

Sum out all the remaining variables.

f_1(Weather).

Weather	val
rain	0.3
norain	0.7

f_4(Weather)

Weather	value
norain	100
rain	70

f_2(Weather, Forecast)

Weather	Forecast	P(Forecast | Weather)
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

① Sum out Forecast from f_2 to get f_5.

f_5(Weather).

Weather	val
rain	1
norain	1

② Multiply all the factors together and sum out Weather.

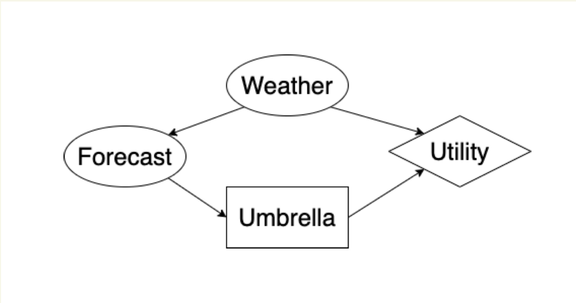
f_6(Weather)

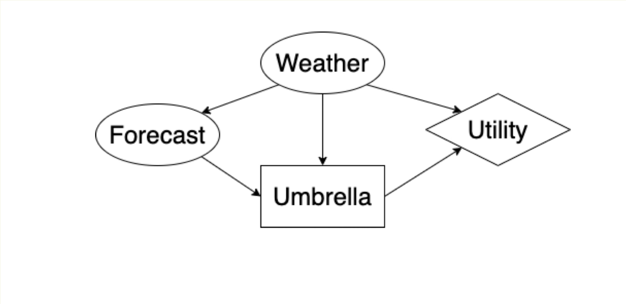
Weather	val
rain	21
norain	70

f_7()

val
91

The expected utility of the optimal policy.





The expected utility of the optimal policy:

77

91

The value of getting perfect information on Weather

$= 91 - 77 = 14$

Revisiting the Learning Goals

By the end of the lecture, you should be able to

- Model a one-off decision problem by constructing a decision network containing nodes, arcs, conditional probability distributions, and a utility function.
- Choose the best action by evaluating a decision network.