Math 61 Introduction to Discrete Structures **Summer Session A 2024**
Homework #1

Instructions: Solve the following problems and be sure to show all of your work.

1. **Math Induction.** Using Math Induction, prove that for every positive integer n,

$$1(1!) + 2(2!) + \cdots + n(n!) = (n + 1)! - 1$$

2. **Math Induction.** Using Math Induction, prove that for every positive integer $n \geq 3$,

$$2n + 1 \leq 2^n$$

3. **Math Induction.** Using Math Induction, prove that

$$11^n - 6 \text{ is divisible by } 5, \text{ for all } n \geq 1$$

4. **Sets.** Let the universe be the set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{2, 4, 6, 8\}$. List the elements of each set.
   (a) $B \cap C$
   (b) $\overline{A} - B$
   (c) $A \cup B - (C - B)$
5. **Sets.** Let $A = \{x \big| x^2 - 4x + 4 = 1\}$ and $B = \{1, 3\}$. Prove that $A = B$.
6. **Sets.** Let $X = \{1, 2\}$ and $Y = \{a, b, c\}$. List the elements in each set.
   (a) $X \times Y$
   (b) $Y \times X$
   (c) $X \times X \times X$
7. **Sets.** Suppose there is a group of 191 students, of which 10 are taking French, business, and music; 36 are taking French and business; 20 are taking French and music; 18 are taking business and music; 65 are taking French; 76 are taking business; and 63 are taking music.
   (a) How many are taking none of the three subjects?
   (b) Draw a Venn Diagram to illustrate the universal set U of students, set F those students taking French, set B for those students taking business, and set M for those students taking music. Write the number of students belonging each region depicted in the diagram.
8. **Functions.** Determine whether

$$f = \{(1, c), (2, a), (3, b), (4, c), (2, d)\}$$

   is a function from $X = \{1, 2, 3, 4\}$ to $Y = \{a, b, c, d\}$. If it is a function,
   (a) find its domain and range
   (b) draw its arrow diagram
   (c) determine if it is one-to-one, onto, or both. If it is both, give the description of the inverse function as a set of ordered pairs, draw its arrow diagram, and give the domain and range of the inverse function.

9. **Functions.** Determine whether

$$g = \{(1, c), (2, d), (3, a), (4, b)\}$$

is a function from $X = \{1, 2, 3, 4\}$ to $Y = \{a, b, c, d\}$. If it is a function,

(a) find its domain and range

(b) draw its arrow diagram

(c) determine if it is one-to-one, onto, or both. If it is both, give the description of the inverse function as a set of ordered pairs, draw its arrow diagram, and give the domain and range of the inverse function.

10. **Functions.** Given

$$g = \{(1, b), (2, c), (3, a)\}$$

a function from $X = \{1, 2, 3\}$ to $Y = \{a, b, c, d\}$ and

$$f = \{(a, x), (b, x), (c, z), (d, w)\}$$

a function from Y to $Z = \{w, x, y, z\}$, write $f \circ g$ as a set of ordered pairs and draw the arrow diagram of $f \circ g$.

11. **Sequences and Strings.** Let t be the sequence defined by

$$t_n = 2n - 1, \qquad n \geq 1$$

(a) Find $\sum_{i=1}^{3} t_i$.

(b) Find $\sum_{i=3}^{7} t_i$.

(c) Find $\prod_{i=1}^{3} t_i$.

(d) Find $\prod_{i=3}^{6} t_i$.

(e) Find a formula that represents this sequence as a sequence whose lower index is 0.

(f) Is t decreasing?

(g) Is t increasing?

(h) Is t nondecreasing?

(i) Is t nonincreasing?

12. **Sequences and Strings.** Compute the given quantity using the strings

$$\alpha = baab, \qquad \beta = caaba, \qquad \gamma = bbab$$

(a) $\alpha\beta$

(b) $\beta\alpha$

(c) $\lambda\alpha$

(d) $\beta\lambda$

(e) $|\alpha\beta|$

(f) $|\alpha\alpha|$

13. **Sequences and Strings.** Find all substrings of the string $aabaabb$.

14. **Relations.** Draw the digraph of the relation $R = \{(1,2),(2,1),(3,3),(1,1),(2,2)\}$ on $X = \{1,2,3\}$.

15. **Relations.** Determine whether the relation

$$(x,y) \in R \text{ if } x \geq y$$

defined of the set of positive integers is reflexive, symmetric, antisymmetric, transitive, and/or a partial order.