Math 61 Introduction to Discrete Structures | **Summer Session A 2022**
Homework #2

Instructions: Solve the following problems and be sure to show all of your work.

1. **Relations.** Let R_1 and R_2 be relations on $\{1, 2, 3, 4\}$ given by

$$R_1 = \{(1, 1), (1, 2), (3, 4), (4, 2)\}$$
$$R_2 = \{(1, 1), (2, 1), (3, 1), (4, 4), (2, 2)\}$$

   List the elements of $R_2 \circ R_1$ and $R_1 \circ R_2$.

2. **Equivalence Relations.** Determine whether the given relation is an equivalence relation on $X = \{1, 2, 3, 4, 5\}$. If the relation is an equivalence relation, list the equivalence classes.

$$R = \{(x, y) \in X \times X \mid 3 \text{ divides } (x - y)\}$$

3. **Equivalence Relations.** List the ordered pairs of the equivalence relation R on the set $X = \{1, 2, 3, 4\}$ defined by the partition $\mathscr{F} = \{\{1, 4\}, \{2, 3\}\}$. Also, find the equivalence classes [1], [2], [3], and [4].

4. **Equivalence Relations.** Let f be a function from X to Y. Define a relation R on X by

$$xRy \text{ if and only if } f(x) = f(y)$$

   Prove that R is an equivalence relation on X.

5. **Matrices of Relations.** Suppose $X = \{1, 2, 3\}$ and $Y = \{x, y\}$, and $Z = \{a, b, c\}$ are ordered sets. If R_1 is the relation from X to Y given by

$$R_1 = \{(1, x), (1, y), (2, x), (3, x)\}$$

   and R_2 is the relation from Y to Z given by

$$R_2 = \{(x, b), (y, b), (y, a), (y, c)\}$$

   (a) Find the matrix A_1 of the relation R_1 (relative to the given orderings).
   (b) Find the matrix A_2 of the relation R_2 (relative to the given orderings).
   (c) Find the matrix product $A_1 A_2$.
   (d) Use the result of part (c) to find the matrix of the relation $R_2 \circ R_1$.
   (e) Use the result of part (d) to find the relation $R_2 \circ R_1$ (as a set of ordered pairs).

6. **Matrices of Relations.** Suppose the matrix A of a relation R on an ordered set $X = \{w, x, y, z\}$ is given by

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Use A to determine if R is reflexive.
   (b) Use A to determine if R is symmetric.
   (c) Use A to determine if R is anti-symmetric.
   (d) Use A to determine if R is transitive.
   (e) Find R as a set of ordered pairs.
   (f) Find R as a digraph.

7. **Basic Principles.** How many different car license plates can be constructed if the licenses contain three letters followed by two digits

   (a) if repetitions are allowed.
   (b) if repetitions are not allowed.

8. **Basic Principles.** How many strings of length 5 formed using the letters ABCDEFG without repetitions

   (a) begin with AC or DB in that order?
   (b) contain letters B and D consecutively in either order (i.e., BD or DB)?

9. **Basic Principles.** A **bit** is a binary digit (a digit that is 0 or 1). How many eight-bit strings either start with a 1 or end with a 1 or both?

10. **Permutations and Combinations.** In how many ways can five distinct Martians and eight distinct Jovians wait in line if no two Martians stand together.

11. **Permutations and Combinations.** Let $X = \{a, b, c, d\}$.

   (a) Compute the number of 3-combinations of X.
   (b) List the 3-combinations of X.
   (c) Compute the number of 3-permutations of X.
   (d) List the 3-permutations of X.

12. **Permutations and Combinations.** Show that the number of n-bit strings having exactly k 0's with no two 0's consecutive, is $C(n-k+1, k)$.

13. **Generalized Permutations and Combinations.** Determine the number of strings that can be formed by ordering the letters given.

MESSINESS

14. **Generalized Permutations and Combinations.** Suppose there are piles of identical red, blue, and green balls where each pile contains at least 10 balls.

   (a) In how many ways can 10 balls be selected?
   (b) In how many ways can 10 balls be selected if at least one red ball must be selected?
   (c) In how many ways can 10 balls be selected if at most one red ball must be selected?