Math 61 Introduction to Discrete Structures **Summer Session A 2024**
Homework #3

Instructions: Solve the following problems and be sure to show all of your work.

1. **Binomial Coefficients and Combinatorial Identities.** Expand $(2c - 3d)^5$ using the Binomial Theorem.

2. **Binomial Coefficients and Combinatorial Identities.** Find the coefficient of the term x^4y^7 when expanding the expression $(x + y)^{11}$.

3. **Binomial Coefficients and Combinatorial Identities.** Use the Binomial Theorem to show that

$$0 = \sum_{k=0}^{n} (-1)^k C(n, k)$$

4. **The Pigeonhole Principle.** An inventory consists of a list of 100 items, each marked "available" or "unavailable." There are 55 available items. Show that there are at least two available items in the list exactly nine items apart.

5. **The Pigeonhole Principle.** Professor Euclid is paid every other week on Friday. Show that in some month, she is paid three times.

6. **Recurrence Relations.** Find a recurrence relation and initial conditions that begins with the given terms.

$$1, 1, 2, 4, 16, 128, 4096$$

7. **Recurrence Relations.** Assume that a person invests \$2000 at 14% interest compounded annually. Let A_n represent the amount at the end of n years.
   (a) Find a recurrence relation for the sequence $\{A_n\}_{n=0}^{\infty}$. Justify your answer.
   (b) Find an initial condition for the sequence $\{A_n\}_{n=0}^{\infty}$.
   (c) Find A_1, A_2, and A_3.
   (d) Find an explicit formula for A_n.

8. **Recurrence Relations.**
   Let S_n denote the number of n-bit strings that do not contain the pattern 00.
   (a) Find a recurrence relation and initial conditions for the sequence $\{S_n\}$. Justify your answer.
   (b) Show that for all integers $n \geq 1$, $S_n = f_{n+2}$, where f denotes the Fibonacci sequence.

9. **Solving Recurrence Relations.** Solve the given recurrence relation for the initial conditions given.

$$a_0 = 1$$
$$a_n = 2^n a_{n-1} \text{ if } n \geq 1$$

10. **Solving Recurrence Relations.** Solve the given recurrence relation for the initial conditions given.

$$a_0 = 4$$
$$a_1 = 10$$
$$a_n = 2a_{n-1} + 8a_{n-2} \text{ if } n \geq 1$$

11. **Solving Recurrence Relations.** Solve the given recurrence relation for the initial conditions given.

$$
\begin{aligned}
a_0 &= 1 \\
a_1 &= 1 \\
a_n &= 6a_{n-1} - 9a_{n-2} \text{ if } n \geq 2
\end{aligned}
$$