Summary of Equations in Aerodynamics Lecture Notes

By Kelvin Le

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Contents

1 Fundamental Aerodynamics Equations

1.1 Equation of State for a Gas (Ideal Gas Law)

$$p = \rho R T \tag{1}$$

where:

- p = Pressure (Pa or N/m^2)
- ρ = Density (kg/m^3)
- R = Specific Gas Constant (J/kg·K)
- T = Temperature (K)

1.2 Specific Volume

$$v = \frac{1}{\rho} \tag{2}$$

where v is the specific volume (m^3/kg).

1.3 Kinetic Energy and Temperature Relation

$$K_e = \frac{3}{2} k T \tag{3}$$

where:

- K_e = Average kinetic energy of molecules (J)
- k = Boltzmann constant (1.38×10^{-23} J/K)
- T = Temperature (K)

1.4 Shear Stress Due to Viscosity (Newton's Law of Friction)

$$\tau = \mu \frac{du}{dy} \tag{4}$$

where:

- τ = Shear stress (Pa or N/m^2)
- μ = Dynamic viscosity (Pa·s or N·s/m^2)
- $\frac{du}{dy}$ = Velocity gradient (s^{-1})

1.5 Reynolds Number (Flow Characterization)

$$Re = \frac{\rho V l}{\mu} = \frac{V l}{\nu} \tag{5}$$

where:

- Re = Reynolds number (dimensionless)
- V = Flow velocity (m/s)
- l = Characteristic length (m)
- μ = Dynamic viscosity (Pa·s or N·s/m^2)
- ν = Kinematic viscosity (m^2/s)

1.6 Mach Number (Compressibility Effects)

$$M = \frac{V}{a} \tag{6}$$

where:

- M = Mach number (dimensionless)
- V = Object speed (m/s)
- a = Speed of sound in the medium (m/s)

2 Example Calculations

2.1 Finding Temperature Using Ideal Gas Law

$$T = \frac{p}{\rho R} \tag{7}$$

Example:

$$T = \frac{8.9876 \times 10^4}{(1.1117)(287)} = 281K \tag{8}$$

2.2 Air Weight in a Room

First, use the equation of state to find ρ:

$$\rho = \frac{p}{RT} \tag{9}$$

Then, find mass:

$$m = \rho V \tag{10}$$

where m is mass (kg) and V is volume (m^3).

Finally, weight:

$$W = mg \tag{11}$$

where g is gravitational acceleration (9.81 m/s^2), and W is weight (N).

2.3 Percentage Change in Air Weight Due to Temperature Drop

Since the temperature changes, we recalculate density:

$$\rho = \frac{2116}{(1716)(460-10)} = 0.00274\frac{\text{slug}}{\text{ft}^3} \tag{12}$$

Compare densities:

$$\Delta\rho = 0.00274 - 0.00237 = 0.00037\frac{\text{slug}}{\text{ft}^3} \tag{13}$$

Percentage change:

$$\%\text{change} = \frac{\Delta\rho}{\rho} \times 100 = \frac{0.00037}{0.00237} \times 100 = 15.6\% \text{ increase} \tag{14}$$

Alternative solution:

$$\Delta W = \Delta m \cdot g = 0.888 \times 32.2 = 28.5936 \text{ lb} \tag{15}$$

$$\%\text{change} = \frac{\Delta W}{W_1} \times 100 = \frac{28.5936}{183} \times 100 = 15.6\% \tag{16}$$