Relativity Quiz - 1

John Rötter

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Problem 1:

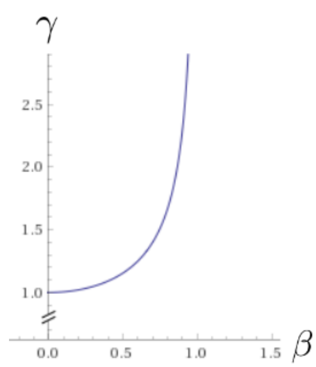
a.) Consider a particle's energy is twice it's rest mass energy. Then $E = \gamma mc^2$ with $E = 2mc^2$ gives that $\gamma = 2$. Therefore the Lorentz factor is,

$$\gamma = 2.$$

b.) Since $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ then we can rearrange using algebra and find that $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$. Plugging in $\gamma = 2$ gives us

$$\beta = \frac{\sqrt{3}}{2}.$$

c.) The allowed values of β and γ are that $\beta \in [0, 1)$ and $\gamma \in [1, \infty)$
d.) Below is the graph of $\gamma(\beta)$.



Problem 2:

a.) On average, the muon will live for $2.2\mu s$ in it's rest frame before it will decay.
b.) If the muon has an energy of 60 GeV then using $E = \gamma mc^2$ we can find that

$$\begin{aligned}\gamma &= \frac{E}{mc^2} \\ &= \frac{60}{.105} \\ &= 571\end{aligned}$$

Therefore, using $\Delta t = \gamma \Delta \tau$ we can see that $\Delta t = (571)(2.2\mu s) = 1256\mu s$. Therefore the muon will live for $0.001256s$ in the lab frame.
c.) Since we know $\gamma = 571$, we can find that $\beta = 0.999$ so the muon will travel for 376423m before decaying so it is safe to say we do not need to worry about the muon decaying before it reaches the detector.
d.)As found before, $\gamma = 571$ and $v = .999c$ for our muon. Additionally, the muon mass and charge are known to be 105 MeV/c and $-1e$ respectively. Therefore we can use the formula for radius of curvature,

$$\begin{aligned}r &= \frac{\gamma mv}{|q|B} \\ &= \frac{198 * .999}{|-1|(3.8T)} \\ &= 52m\end{aligned}$$

Therefore, the radius of curvature is 52 meters.
e.) The μ^+ will decay using the weak interaction, and the W boson will decay further into a positron. Each step along the way a neutrino must also be created. Thus, the full interaction must be,

$$\mu^+ \rightarrow \bar{\nu_\mu} + W^+ \rightarrow \bar{\nu_\mu} + e^+ + \nu_e. \tag{1}$$

f.) Similar to the μ^+ decay, the μ^- decay just has the charge and lepton number reversed giving,

$$\mu^- \rightarrow \nu_\mu + W^- \rightarrow \nu_\mu + e^- + \bar{\nu_e}. \tag{2}$$

Problem 3:

a.) The Pythagorean equation for E, m, and $|\vec{p}| = p$ is

$$E^2 = m^2c^4 + p^2c^2. \tag{3}$$

b.) The relation from Problem 1, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, gives that $\gamma^2 = \frac{1}{1-\beta^2}$. This can be rearranged to give,

$$1 = \beta^2 + \frac{1}{\gamma^2}. \tag{4}$$