CMS Office Hours

Quiz 1 Solutions: *Relatively* Easy **2020-Jun-30**

$* * *$ Use natural units. $\hbar = c \equiv 1$ $* * *$

A **Toolkit Equation** is one which you should add to your Physics Toolkit.

(...once you discover it of course!)

1. (a) Given that $E \geq 2m$ and using the **Toolkit Equation**: $E = \gamma m$ (for a massive particle), we see that $\gamma \geq 2$.

   (b) By definition,

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}, \text{ where } \beta \equiv v/c \tag{1}$$
$$\implies \beta = \sqrt{1 - \frac{1}{\gamma^2}}.$$

   Using $\gamma = 2$ from before gives $\beta = 0.866$. Therefore a particle becomes relativistic when it is going 86.6% the speed of light!

   (c) Using $\beta = v/c$ and knowing that $0 \leq v \leq c$, it must be the case that

$$0 \leq \beta \leq 1.$$

   From Equation 1 above, we see that

$$1 \leq \gamma < \inf .$$

   (d)

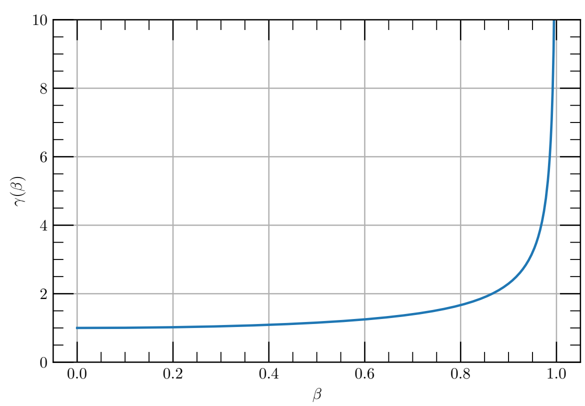

Figure 1: Plot showing γ as a function of β.

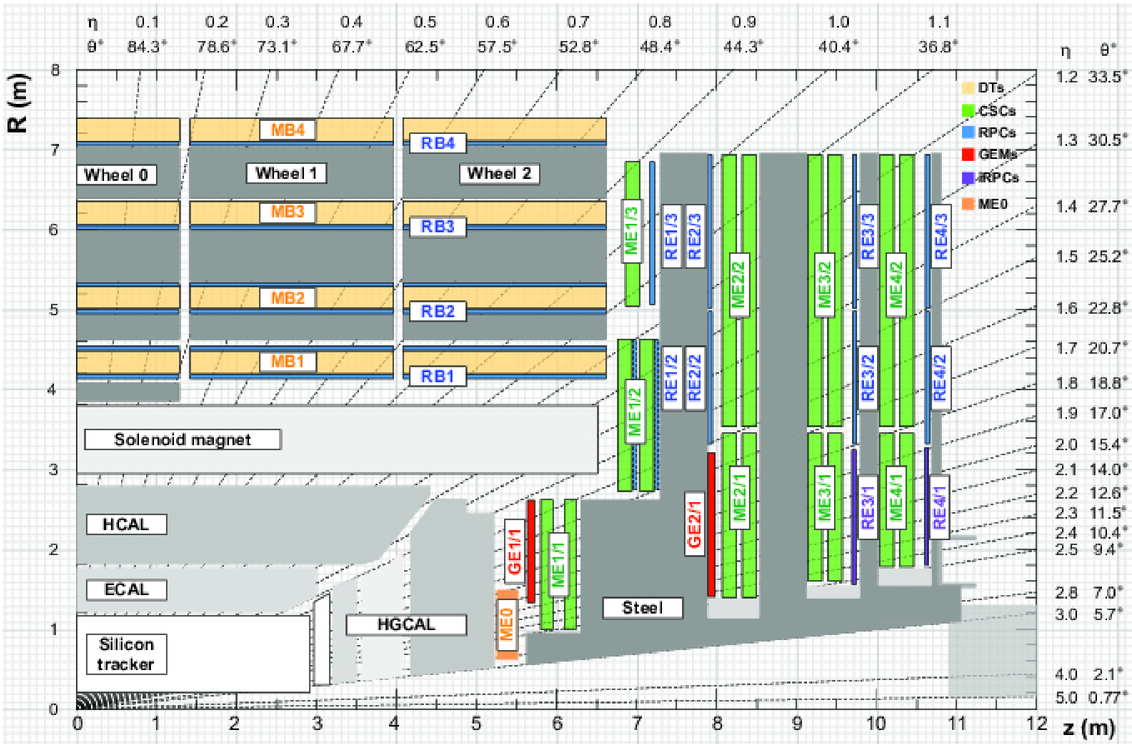

Figure 2: A quadrant view of the CMS detector.

2. (a) *Look it up!* The lifetime of a muon (τ_μ) is about **2.2 μs**. This is how long it takes for the muon to decay in its rest frame.

   (b) The physicists in the lab will 'see' the muon moving very fast; its lifetime *in the lab frame* will be **dilated**:

$$t_{\text{lab}} = \gamma\tau$$

$$\gamma = \frac{E}{m} = \frac{60 \text{ GeV}}{0.1057 \text{ GeV}} = 567.6 \quad (\textit{ultra relativistic!})$$

$$\implies t_{\text{lab}} = (567.6)(2.2\ \mu\text{s}) = 1.25 \text{ ms}$$

   Comment: In this way, we can "make particles live longer" by speeding them up! The muon lives 500 times longer in the lab frame compared to its rest frame.

   (c) In the lab frame the muon lives for 1.25 ms while travelling at nearly the speed of light:

$$d_{\text{lab}} = v_{\text{lab}} t_{\text{lab}} \approx c t_{\text{lab}} = 375 \text{ Km}$$

   The distance from the interaction point (i.e., the point where the protons collide) to the end of the Muon System is about 7.5 m along the $\eta = 0$

direction, about 11.0 m along the $\eta = 2.5$ direction, and therefore a maximum of about 13.3 m along the $\eta = 1.2$ direction (convince yourself of this using Figure 2).

Answer: *Nope!* Physicists won't have to lose any sleep over muons decaying before the muon system.

(d) We need to relate the transverse momentum ($\vec{p}_T$) to the charge of the particle (q), magnetic field strength ($\vec{B}$), and the radius of curvature (R):

$$|\vec{p}_T| = q|\vec{B}|R. \quad \textbf{(Toolkit Equation)} \tag{2}$$

You can convince yourself of this equation by using elementary classical mechanics of centripetal forces (see Fig. 3). Since $\gamma \approx 568 \gg 2$ and the motion of the muon is perpendicular to the $\vec{B}$ field, we can say that

$$E = 60 \text{ GeV} \approx \vec{p} = \vec{p}_T.$$

Since natural units take some getting used to, let's use SI units in Eqn. 2 keeping in mind that $e = 1.602 \times 10^{-19}$ C:

$$R = \frac{|\vec{p}_T|}{q|\vec{B}|} = \frac{60 \text{ GeV}/c}{e(3.8\text{T})} \times \left(\frac{10^9 \text{ eV}}{1 \text{ GeV}}\right)\left(\frac{e \text{ J/C}}{1 \text{ eV}}\right),$$

where the last conversion factor uses the fact that 1.602×10^{-19} J $= 1$ eV. Once the dust settles, we have:

$$R = \frac{60 \times 10^9 \text{ J}}{3 \times 10^8 \text{ m/s} \cdot 3.8 \text{ T}} = 52.6 \text{ m}.$$

Comment: In practice, this problem is worked in the *reverse* way: we measure the radius of curvature (technically the **sagitta**) to figure out what the p_T was.

(e)

$$\mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e$$

Comment: The positron is the only particle with a mass *lighter* than the muon and which also conserves its electric charge (+1). The 2 neutrinos are there to conserve **muon lepton number** (−1 on both sides) and **electron lepton number** (0 on both sides). *The neutrinos are also necessary to conserve momentum!*

(f)

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$$

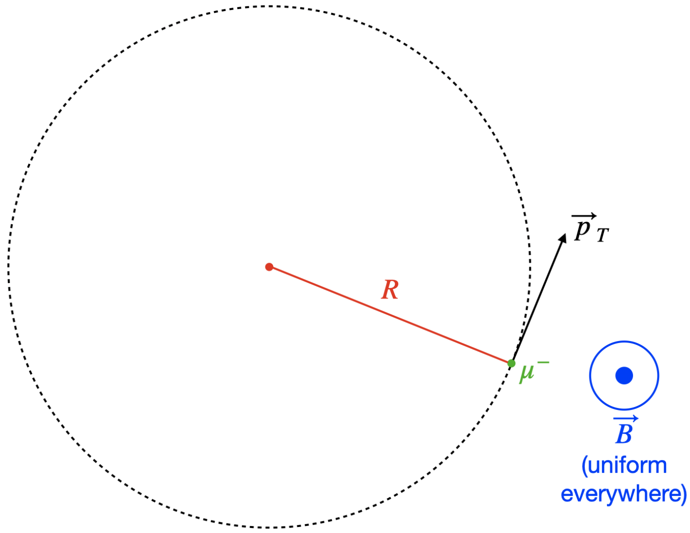

Figure 3: A muon has a circular trajectory in a magnetic field.

3. In High-Energy Particle Physics, there are a couple useful *Pythagorean equations* (equations of the form: $c^2 = a^2 + b^2$).

   (a)
   $$E^2 = m^2 + \vec{p}^{\,2} \quad \textbf{(Toolkit Equation)}$$

   (b) Starting from Eqn. 1:
   $$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$
   Do some algebra to get:
   $$1^2 = \beta^2 + \frac{1}{\gamma^2} \quad \textbf{(Toolkit Equation)}$$
   Comment: Often during special relativity calculations, you will come across factors like: $1/\gamma^2, \gamma^2\beta^2$, etc. so it is useful to have this equation handy to convert easily between them.