Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875

   B. 0.2676

   C. 0.5

   D. 0.6987

Ans: B

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.

   A. More employees at the processing center are older than 44 than between 38 and 44.

   B. A training program for employees under the age of 30 at the centre would be expected to attract about 36 employees.

ANS :

A) False, as the mean is around 38, more of the employees are in the age group between 38 and 44.

B) False, as more number of people lies between 38 and 44. Only 36 people will get attracted

3. If X_1 ~ $N(\mu, \sigma^2)$ and X_2 ~ $N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS : lets consider 2X1 then the mean and variance will be $(2\mu, 4\sigma^2)$

   Both are normal random variables 2X1 will have a wider spread when compared to X1 + X2. Now consider X1 + x2 then the mean and variance will be $(2\mu, 2\sigma^2)$.

4. Let X ~ $N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9

   B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

ANS : D

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

   B. Specify the 5th percentile of profit (in Rupees) for the company

   C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS :

A) The Rupee range (centred on the mean) such that it contains 95% probability for the annual profit of the company is (126, 954).

B) 5th percentile of profit in Rupees = 2.775 * 45 = 124.88 = 125

C) both divisions have a similar probability of making a loss in a given year, but Profit1 has a slightly lower probability.