Excitation of Electromagnetic Waves in Pre-bunched CFEL

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Abstract
The growth rate of the CFEL is found to increase with the modulation index and reaches maximum when the modulation index approaching unity. The growth rate is calculated at experimentally known CFEL parameters and it is seen that beam pre-bunching on Cerenkov free electron laser (CFEL) offers considerable enhancement in gain when the phase of the pre-bunching electron beam is $-\pi/2$, i.e., when the pre-bunched beam is in the retarding zone.

Index Terms - Free electron laser, Cerenkov free electron laser, Pre-bunched electron beam.

I. Introduction

Cerenkov free electron laser (CFEL) is the widely used source of broad- band, high power microwave generation at short wavelengths. A Cerenkov free electron laser generally employs two kinds of slow wave structures: (i) A dielectric whose dielectric constant is $|\varepsilon|>1$ reduces the phase velocity of the radiation below c. A moderately relativistic electron beam can excite the electromagnetic radiation by cerenkov emission, (ii) A plasma lining have a dielectric constant $\varepsilon=1-\frac{\omega_p^2}{\omega^2}$ can act as a slowing down medium for $\omega_p \gg \omega$ so that $|\varepsilon| \gg 1$(where ω_p is the electron plasma frequency and ω is the radiation frequency).

Recently, a lot of research work has been carried out in studying the free electron laser [6-15] by pre-bunched electron beams. Simulation of enhanced pre-bunching in free electron lasers for the generation of high gain radiation at high frequencies has been demonstrated by Freund *et al.* [13]. It is demonstrated that free electron laser with the pre-bunched beam combines the best characteristics of amplifiers and oscillators.
A theoretical model for gain and efficiency enhancement in a FEL using pre-bunched electron beam has been developed and studied by Beniwal *et al.* [14]. It is seen that growth rate increases with the increase in the modulation index. Sharma and Bhasin have studied the gain and efficiency enhancement in a slow wave FEL using pre-bunched electron beam in a dielectric loaded waveguide [15]. They have found that the growth rate and gain of a slow wave FEL increase with the modulation index and is maximum when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave.
The organization of the paper is as follows: We follow perturbation techniques to obtain the growth rate and efficiency of CFEL. We have calculated the increase in growth rate and efficiency with the increase in modulation index.

II. Instability Analysis

Consider a dielectric loaded waveguide of effective permittivity ε_1. A pre-bunched relativistic electron beam of density n_{b0}, velocity $v_b \hat{z}$, relativistic gamma factor $\gamma = 1 + \frac{eV_b}{mc^2}(1+\Delta \sin \omega_0 \tau) \approx \gamma_0 (1 + \Delta \sin \omega_0 \tau)$ [where Δ is the modulation index (its value lie from 0 to 1), mc^2 is the rest mass energy of the electrons, e is the electronic charge, $\omega_0 \left(\approx k_{z0} v_b\right)$ and k_{z0} are the modulation frequency and wave number of the pre-bunched electron beam], respectively propagates through the waveguide (cf. Fig. 1).

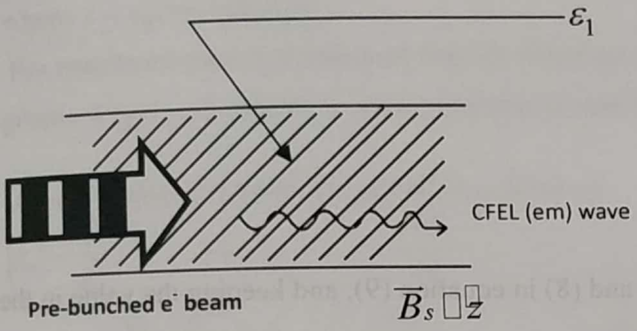

Fig.1. A schematic diagram of the Cerenkov free electron laser

An electromagnetic signal E_1 is also present in the interaction region.

$$\overrightarrow{E}_1 = \overrightarrow{E}_0 e^{-i\left(\omega_1 t - \overrightarrow{k}_1 . \overrightarrow{x}\right)}, \qquad (1)$$

$$\overrightarrow{B}_1 = \frac{c}{\omega_1} \overrightarrow{k}_1 \times \overrightarrow{E}_1, \qquad (2)$$

where $\overrightarrow{E}_0$ and $\overrightarrow{k}_1$ lie in the x-z plane, i.e., $\frac{\partial}{\partial y} = ik_y = 0$. The response of the beam electrons to the signal is governed by the relativistic equation of motion

$$\frac{\partial}{\partial t}(\gamma \vec{v}) + \vec{v}.\nabla(\gamma \vec{v}) = -\frac{e}{m}(\overrightarrow{E} + \frac{1}{c}\vec{v} \times \overrightarrow{B}). \qquad (3)$$

Expanding

$$\vec{v} = v_b \hat{z} + \vec{v}_1, \quad \gamma' = \gamma + \gamma^3 \frac{\overrightarrow{vb}.\overrightarrow{v1}}{c^2}$$

and linearizing equation (3), we get

$$\gamma \overline{v_1} + \gamma^3 \frac{v_b^2}{c^2} v_{z1}\hat{z} = \frac{e}{im(\omega_1 - k_z v_b)}\left[\overline{E_1}\left(1 - \frac{k_z v_b}{\omega_1}\right) + \overline{k}\frac{v_b E_z}{\omega_1}\right]. \qquad (4)$$

Velocity components in the x and z directions are given by

$$v_{x1} = \frac{e}{im\gamma(\omega_1 - k_z v_b)}\left[E_{x1} - \frac{k_z v_b E_{x1}}{\omega_1} + \frac{k_{x1} v_b E_{z1}}{\omega_1}\right]. \qquad (5)$$

$$v_{z1} = \frac{eE_{z1}}{im(\omega_1 - k_z v_b)\gamma^3}. \qquad (6)$$

On linearizing and solving equation of continuity, we obtain density perturbation

$$n_1 = n_{b0}\frac{\overline{k1}.\overline{v_1}}{(\omega_1 - k_z v_b)}. \qquad (7)$$

Using the value of v_{x1} and v_{z1} from equations (5) and (6) in equation (7), we get

$$n_1 = \frac{en_{b0}}{im(\omega_1 - k_z v_b)^2}\left[\frac{E_{x1} k_{x1}}{\gamma}\left(1 - \frac{k_z v_b}{\omega_1}\right) + \frac{k_{x1}^2 v_b E_{z1}}{\gamma\omega_1} + \frac{k_{z1} E_{z1}}{\gamma^3}\right]. \qquad (8)$$

The perturbed current density is given by

$$\overrightarrow{J1} = -n_{b0} e\overrightarrow{v_1} - n_1 e\overrightarrow{v_b}. \qquad (9)$$

Substituting the values of $\overline{v_1}$ and n_1 from equations (5), (6) and (8) in equation (9), and keeping the value in the wave equation, we obtain

$$k_1^2 \overline{E_1} - \overline{k_1}\left(\overline{k_1 . E_1}\right) - \frac{\omega_1^2}{c^2}\varepsilon\overline{E_1} = \frac{4\pi i\omega_1 \overline{J_1}}{c^2} \qquad (10)$$

and writing x and z components of the latter, we obtain

$$\left(k_{z1}^2 - \frac{\omega_1^2}{c^2}\varepsilon + \frac{\omega_{pb}^2}{\gamma c^2}\right)E_{x1} = \left(k_{x1}k_z - \frac{\omega_{pb}^2}{\gamma c^2}\frac{k_{x1}v_b}{(\omega_1 - k_z v_b)}\right)E_{z1}, \qquad (11)$$

where $\omega_{pb}^2 = \frac{4\pi n_{bo}}{m}e^2$.

Equation (11) gives the dispersion relation and can be further rearranged by taking $\omega_{pb}{}^2$ terms to the right hand side and retaining only those terms which have a resonance denominator $(\omega_1 - k_z v_b)^2$, we get

$$\left(\omega_1^2 - \frac{k_1^2 c^2}{\varepsilon}\right)(\omega_1 - k_z v_b)^2 = \frac{\omega_{pb}^2}{\gamma^3\varepsilon}(\omega_1^2 + k_{x1}^2 v_b^2 \gamma^2). \qquad (12)$$

The two factors on the left–hand side of equation (12) when equated to zero $\omega_1 - \frac{k_1 c}{\sqrt{\varepsilon}} = 0, \omega_1 - k_z v_b = 0$, give radiation and beam modes, respectively. To determine the growth rate of the CFEL instability, we use the first order perturbation techniques .In the presence of the right hand side terms $\left(i.e., n_{b0} \neq 0\right)$, we assume that the eigen functions are not modified but their eigen value are. We expand ω_1 as

$$\omega_1 = \omega_{1r} + \delta = k_z v_b + \delta = k_{z0} v_b + \delta,$$

where δ is the small frequency mismatch and $\omega_{1r} = \frac{k_1 c}{\sqrt{\varepsilon}}$.

On further solving equation (12) we obtain

$$\delta = \left[\frac{\omega_{pb}^2(\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma^2)}{2\omega_{1r}\gamma^3 \varepsilon}\right]^{1/3} e^{i\frac{2n\pi}{3}}, n = 0,1,2,3........ \quad (13)$$

Hence the growth rate, i.e., the imaginary part of δ is given as

$$\Gamma = \left[\frac{\omega_{pb}^2(\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma_0^2)}{2\omega_{1r}\gamma^3 \varepsilon}\right]^{1/3} \frac{\sqrt{3}}{2} \quad (14)$$

where $\gamma = \gamma_0(1 + \Delta \sin \omega_0 \tau)$.

For maximum gain it is assumed that all electrons are bunched in the decelerating zone, i.e., $\omega_0 \tau = -\pi/2$. This gives $\gamma = \gamma_0(1 - \Delta)$, where Δ is the modulation index, its value lies between 0 to 1 and $\Delta \neq 1$.

Using Equation (13) the real part of δ is given as

$$|\delta_r| = \frac{\Gamma}{\sqrt{3}}, \qquad \text{i.e., } \omega_1 = k_z v_b - \frac{\Gamma}{\sqrt{3}}$$

or $v_b = \frac{\omega_1}{k_z} + \frac{\Gamma}{\sqrt{3}} k_z$ i.e., $v_b > \frac{\omega_1}{k_z}$.

This is the necessary condition for electron bunching and net energy transfer from beam electrons to the radiation wave.

III. Results and Discussions

In the numerical calculations we have used typical parameters of Cerenkov free electron laser (CFEL). For the comparative study we have also used parameters of free electron maser experiment with a pre-bunched electron beam [1], (e.g., beam energy = 0.07 MeV and beam current I_b = 1.0A) and other parameters are same as CFEL.

In Fig. 2, we have plotted the variation of the growth Γ (in rad /sec) as a function of modulation index Δ when the phase of the pre-bunched beam is $-\pi/2$, i.e., when the electron beam is in the decelerating zone, (a) plot for CFEL parameters and (b) plot for FEL parameters. From Fig. 2, it can be seen that the growth rate increases with the modulation index (in both cases), when $\Delta \approx 0.80$ and beyond this value of modulation index, i.e., when Δ increases from 0.80 to 0.98, the growth rate increases by a factor of 9 for CFEL parameters and by10 for FEL parameters. For modulation index Δ=0, i.e., without modulated beam, the value of the growth is found to be 2.696×10^{10} rad/sec for CFEL parameters and 2.438×10^{10} rad/sec for FEL parameters.

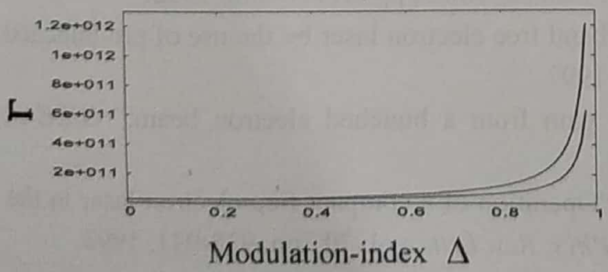

Fig.2. Growth rate Γ (in rad /sec) as a function of modulation index Δ for (a) CFEL parameters with $E_b = 1.35$ MeV, $I_b = 1.0$ KA and for $\sin\omega_0\tau = -1$, (b) FEL parameters with $E_b = 0.07$MeV, $I_b = 1.0$A and for $\sin\omega_0\tau = -1$.

Increase in growth rate is more for case (b) i.e., with FEL parameters.

If we introduce plasma in the interaction region of CFEL then further reducing the requirements on beam energy for generating shorter wavelength radiation.

The growth rate of the pre-bunched CFEL increases with the modulation index and consequently the gain and efficiency of the device also increases with the modulation index.

IV. Conclusion

In conclusion, we can say that by increasing the modulation index the growth rate, gain, and efficiency of the pre-bunched CFEL increases.

The growth rate of electromagnetic wave is more in FEL than in CFEL, as the modulation index increases from 0.85 to 0.98. In addition to this it is seen that by using pre-bunched electron beams, requirement for beam energy can be reduced drastically for generating high frequency radiations, for both CFEL and FEL.

The scheme seems to work well at millimeter and sub-millimeter wavelengths.

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