Optimizing Retirement Portfolios for Future Cash Flow Requirements using Grid-Based, Monte Carlo Optimal Control Solvers

by Daniel Rogers

Overview

Much research has been done into calculating sustainable withdrawal rates from retirement portfolios and looking to minimize the probability-of-ruin (PoR) for an investor self-annuitizing their retirement. Most of these studies examine portfolios with constant asset allocations since this makes the problem much more manageable than allowing for dynamic portfolio behavior. However, this also limits the applicability of their results and goes against observed behavior where investors are counseled to invest aggressively earlier in life and become more conservative as they approach retirement. This paper will look at one method for allowing dynamic asset allocation over the lifetime of an investor and analyze the impact this has on sustainable withdrawal rates and PoR. It will also look at investment decisions that can be made before retirement in order to fully fund a retirement account and minimize the probability-of-ruin later in life. The approach involves using a grid-based, Monte Carlo approximation of an optimal control function that seeks to find the optimal asset allocation at points of time in an investor's life. We will find that the results obtained agree closely with conventional wisdom and that investors are well-advised to 1) invest aggressively early in life and slowly become more conservative, 2) begin saving early for retirement, and 3) limit withdrawal rates in retirement to 4% of your initial retirement wealth.

Problem Description

We begin by assuming a Markowitz market model where single-period asset returns are drawn from a multivariate normal distribution:

$$\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

An investor's problem is to determine the allocation of assets in a portfolio at any given point in time. If the percentage of wealth invested in each asset is given by the vector $\boldsymbol{\pi}_t \in \mathbb{R}^n$ then the investor's wealth will evolve according to:

$$W_{t+1} = (W_t + C_t)\,(\mathbf{x} \cdot \boldsymbol{\pi}_t)$$

W_t is the value of the portfolio at time t, and C_t represents cash flows going into or coming out of the portfolio. With the cash flows known before-hand, we will find that an investor's decisions depend only on time and his current wealth level. He is thus seeking to find the optimal control function:

$$\boldsymbol{\pi}_t = \boldsymbol{\pi}(t, W_t)$$

so that the system is completely described by:

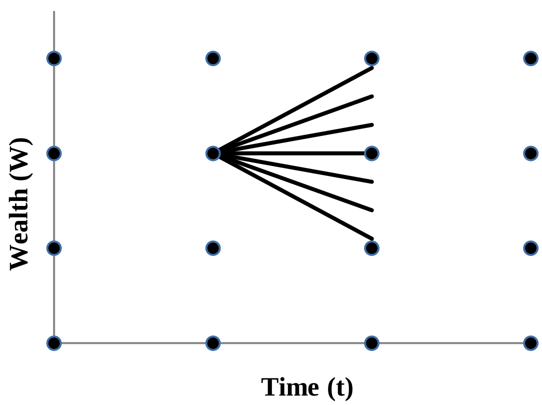
$$f(t, W_t, C_t, \boldsymbol{\pi}_t)$$

If we assume that an investor's cash flows are positive from time $t = 0$ up to the time of retirement, $t = t_R$, and then turn negative until the simulation end time, $t = T$, then we will notice that a non-negative terminal wealth ($W_T > 0$) indicates that the investor made it to the end of retirement without running out of money. The utility function we are seeking to maximize is the probability of surviving (PoS) retirement without running out of money:

$$U(\boldsymbol{\pi}_t) = \Pr(W_T > 0)$$

The optimal control problem is to find the function $\boldsymbol{\pi}_t = \boldsymbol{\pi}(t, W_t)$ that maximizes this utility.

A Grid-Based, Monte Carlo Approximation



A graphical depiction of the grid-based, Monte Carlo approximation. We define a grid of points (t, W) and, taking any point on that grid, can simulate random asset returns and resulting values of wealth W_{t+1}. If the expected utility of wealth at point $(t+1, W_{t+1})$ is known or can be estimated, we c an determine the optimal portfolio allocation at (t, W) to maximize the expected utility.

In general, the solution to optimal control problems can be difficult to find. However, in our case we are lucky that the utility function only depends on our final state and not on any of the intervening states. In this case the utility at time T can be found directly, and the utility at time $T - 1$ can be estimated using Monte Carlo techniques. The problem at time $T - 1$ is to find the portfolio for a given point $(T - 1, W_{T-1})$ which maximizes the expected utility at time T. A Monte Carlo simulation of asset returns can be used to find the optimal portfolio allocation without difficulty. However, when we step back to time $T - 2$, we will not be able to so easily calculate the expected utility at every possible future point $(T - 1, W_{T-1})$. Doing so would quickly cause the problem to explode in size and dimension to the point where it becomes intractable.

To avoid this problem, we limit ourselves to calculating the optimal portfolio allocations and expected utilities along the line $T - 1$ to a limited number of points. Then, when we step back to any point at time $T - 2$ we can again sample a random set of asset returns and calculate the expected utility of any portfolio by interpolating between the estimated utilities at time $T - 1$. This process can be repeated as we move backward through time to calculated optimal portfolios and expected utilities at a series of grid points over the assumed problem space.

If we define a grid:

$$\mathcal{A}_{grid} = \left\{(t, W_t) \mid t = \{0,1,2,3,\dots,T\}, W = \left\{W_{MIN} + k\,\frac{W_{MAX} - W_{MIN}}{K}\right\}_{k=\{0,1,2,\dots,K\}}\right\}$$

the result of the above process will be a set of approximations to the optimal control function at each point on this grid:

$$\hat{\pi}(t, W_t) \qquad (t, W_t) \in \mathcal{A}_{grid}$$

From this we can interpolate the optimal control function $\hat{\boldsymbol{\pi}}_{\boldsymbol{t}}$ for any point $\{(t, W_t) \mid 0 \le t \le T$, $W_{MIN} \le W \le W_{MAX}\}$. We will also be able to save the expected utilities at each point on the grid so that we can estimate values of:

$$f_U(t, W_t) = E(U(\boldsymbol{\pi}_{\boldsymbol{t}}) \mid t, W_t)$$

that is, the expected utility of the optimal control function given the starting point (t, W_t). However, in practice we will also be able to estimate the value of $f_U(t, W_t)$ using Monte Carlo simulations once we have our estimate of the optimal control function. This will provide expected utilities that can be made as accurate as necessary by increasing the number of runs in the Monte Carlo simulation.

A Note on Problem Size and Number of Assets

There are three factors that determine the size of the computational problem needed to estimate the optimal control function:

1. The size of the grid (the number of years simulated and the range and precision of the wealth points sampled at each point in time)
2. The number of random samples (random asset returns) used in the Monte Carlo optimization at each point.
3. The number of assets

While the problem scales linearly as we add new grid points or increase the number of random samples in the Monte Carlo optimization, adding new assets causes the problem to explode much faster. This is primarily because adding a new asset requires us to increase the number of random samples needed to capture the dynamics of the asset returns. If portfolios only include one risky asset and a risk-free asset with a deterministic return, we might find that an arbitrarily chosen number of R samples is enough to represent the possible returns on the risky asset. However, if we include two risky assets we will likely need R^2 samples of returns to provide the same precision in our simulation. Even a problem involving 2 risky assets can quickly exceed the computing power of a standard desktop computer.

This problem can be solved if we restrict ourselves to the Markowitz model and remember that portfolio return and variance can be calculated as:

$$r = \boldsymbol{\pi} \cdot \boldsymbol{\mu}$$

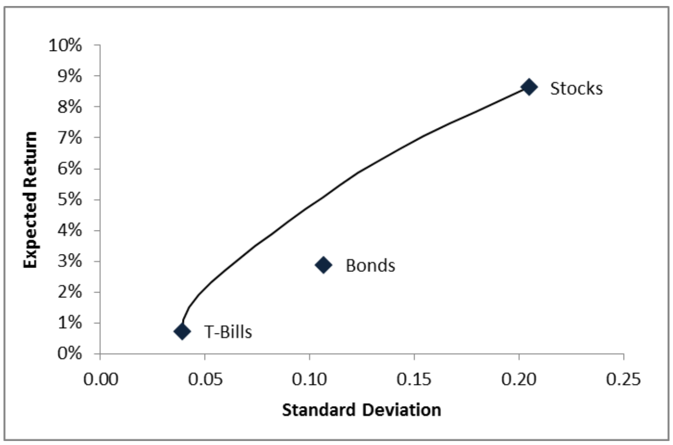
$$\sigma^2 = \boldsymbol{\pi}^{\mathbf{T}} \boldsymbol{\Sigma} \, \boldsymbol{\pi}$$

This means that we can simulate the returns of this portfolio by taking sample returns (x) from a normal distribution and transform them to the desired scale with:

$$x' = r + x\sigma$$

To save time we can even sample from the normal distribution once and re-use the same sample set for all points on the grid, applying the transformation above for each possible portfolio.

We can also simplify the problem by limiting ourselves to portfolios that lie on the efficient frontier. In this case we are assuming that these portfolios (which have superior risk and return profiles in a Markowitz market) will always dominate other portfolios in our optimization. The efficient frontier is unchanging and can be pre-calculated before performing the optimization. It can also be represented as a parameter curve with a single parameter varying from 0 to 1 as we traverse the curve. Portfolio optimization at each point in the grid-based solver then becomes an optimization involving only a single parameter instead of N (or $N - 1$) parameters. This offers substantial performance increases even for markets with as little as three assets.



The efficient frontier for a market with three assets. Instead of performing an optimization of an n-asset portfolio (which would require having n-1 free variables), we can calculate the efficient frontier and form a parametric line representing it. The optimization then involves only a single, bounded variable.

Results

In this section we will demonstrate the results of this approach by analyzing the problems of both sustainable withdrawal rates and required contribution rates for retirement portfolios. In both cases we will show that the optimal control approach provides improved performance for an investor saving for retirement. We will also provide guidelines and benchmarks that investors can use to manage their retirement portfolios both before and during retirement. We will make use of the following simple market models, defined by their expected returns and covariance matrixes and created from the Ibbotson data set using inflation-adjusted, annual returns:

Market A: Two Assets
(Bonds & Stocks)

$$\boldsymbol{\mu} = [.0288 \quad .0864]^T$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} .011423 & .002334 \\ .002334 & .042067 \end{bmatrix}$$

Market B: Three Assets
(T-Bills, Bonds, & Stocks)

$$\boldsymbol{\mu} = [.0072 \quad .0288 \quad .0864]^T$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} .001533 & .002333 & .000783 \\ .002333 & .011423 & .002334 \\ .000783 & .002334 & .042067 \end{bmatrix}$$

The assets included these models are:

- T-Bills – short-term (30 day) Treasury Bills
- Bonds – long-term (20 years) Treasury Bonds
- Stocks – The S&P 500

The first model was chosen to mimic the results of the Trinity studies for sustainable portfolio withdrawal rates. It consists of long-term bonds and an equity portfolio. The second model attempts to add a risk-free asset that better represents an investor's investment choices, and offers a low-risk investment alternative. It should be noted that the T-Bills do not represent a risk-free investment, because the returns on all assets are being adjusted for inflation. The inclusion of TIPS would be the only truly risk-free asset under this model, but these are still too new to consider using in our model.

Sustainable Withdrawal Rates

We begin by determining a sustainable withdrawal rate from a retirement portfolio. In this case, we assume that the investor has reached retirement with a portfolio that has a given initial value. He is then going to withdrawal a fixed percentage of that initial portfolio value during the course of his retirement while investing the remainder of his wealth. The goal is to make it through retirement without running out of money.

The Trinity papers approach this problem by examining the historical performance of portfolios that invest a fixed percentage of their assets in either stocks or bonds, and leave the proportions invested in each asset constant throughout retirement. They examine five portfolios composed of stocks and bonds with fixed allocations of: 100% stocks, 75% stocks, 50% stocks, 25% stocks, and 100% bonds. They then examined the performance of these portfolios over time periods of

15, 20, 25, and 30 years with withdrawal rates ranging from 3% to 12%. While their approach examined the performance using historical returns, we will produce similar results for the simulated Markowitz models in Tables 1 & 2. Tables 3 & 4 provide a summary of the previous tables by collecting the maximum PoS for each time period / withdrawal rate combination and comparing the results with those obtained by the optimal control method.

We find that for a cautious investor who requires a 99% chance of surviving retirement without running out money, the results are similar for both methods. In both cases the investor is well-advised to limit himself to a withdrawal rate between 3% and 4% of his initial retirement balance. We also note that the optimal control method provides a 99% PoS for a retiree withdrawing 3% over 30 years whereas this had not been possible using the preset, constant allocation portfolios.

The impact on investors who are willing to withdrawal more from their portfolios is much more pronounced. At withdrawal rates higher than 4% we find that the PoS increases by as much as 9% using the optimal control methods. On average, an investor can increase his PoS for each time horizon by retirement by 4.5% using these methods.

In the three-asset market the results are similar. A cautious investor would again be well-advised to limit his retirement withdrawals to between 3% and 4% of his initial retirement balance. However, we see that withdrawals can actually go as high as 6% for an investor planning a 15-year retirement. We also see that the average PoS performance using the optimal control method in this market is now a 5.4% increase over the results of the preset, constant allocation portfolios. The performance increases can be attributed to two factors: 1) the ability of the optimal control portfolio to re-allocate the investments during the course of the simulation, and 2) the ability of the optimal control method to select portfolios of any asset allocation instead of limiting ourselves to 5 pre-determined portfolios.

Funding a Retirement Portfolio

The next problem we address is that of an investor trying to adequately fund a retirement portfolio. In this case, the investor must determine how much to contribute to his retireme each year and how to allocate the portfolio amongst a set of assets. As an example, we take an investor with an annual salary of $50,000. We assume that this investor will contribute 10% of his earnings ($5,000) to his retirement portfolio each year until he retires and that he will require 80% of his pre-retirement income ($40,000) when he is in retirement. We also assume that this investor is expecting a 20 year retirement, and that he requires a 95% chance of surviving retirement without running out of money. The investor's cash flow is given by the function:

$$C(t) = \begin{cases} +5{,}000 & t < t_R \\ -40{,}000 & t \geq t_R \end{cases}$$

where t_R is again the time of retirement, and we will use Market B in our simulation since it offers a more diverse set of assets for our hypothetical investor.

We can solve this problem completely by building a grid-based optimizer over the relevant time and wealth domain. Determining the grid size can be a bit problematic. We need to ensure that this grid is large enough in the time and wealth domains to completely capture the range of possibilities in the simulation. The tables calculated in the previous section can assist us with defining the size of the wealth axis. We note that a 5% withdrawal rate is sustainable over a 20 year retirement, which implies that our investor will require about $800,000 at the time of retirement to meet his goals. If his account reaches this balance at any time in the simulation, he should be able to preserve his balance by investing in low-risk assets, thus making the PoS for any wealth point above this level equal to 100%. To be safe, we will extend the grid from 0 to $1,000,000 to encapsulate this space.

We also need to determine how many years to include in the simulation. Since our grid-based solver works backwards through time we can do this incrementally. We just need to extend back to a point where the expected PoS of a $0 balance is 95%. This represents the point in time where the investor must begin investing in order to meet his goals. Prior to this point, he does not need to invest anything, and the expected PoS at all points on the grid will be greater than 95%. For this simulation we will find that the investor must begin saving at least 49 years before his retirement, so that the time of our simulation will have to be at least 69 years. We will extend this to 70 (with $t_R = 50$) just to keep the numbers simple.

Building a grid-based optimizer for the simulation defined above produces estimates of the optimal control function (the optimal portfolio allocation), and the expected utility (the expected PoS) for each time and wealth combination. An interesting (and useful) way to examine this data is to look at each time in the simulation grid and find the minimum amount of wealth an investor must have at that point in time in order to guarantee a fixed PoS for retirement. We have assumed our investor requires a 95% PoS, and we can thus look at the minimum suggested account balances at each point in the investor's life that would be required to meet this goal. The results are summarized in Table 5 where we provide the minimum suggested balance at each point in time and also examine the optimal portfolio allocation for that time and balance. The evaluation of the portfolio over time is also illustrated in Figure 1.

We find that the investor should begin saving 45 to 50 years before retirement (actually 49 years), and that he is required to have $723,823 at retirement (implying a sustainable withdrawal rate of 5.5%. We also see that the investor is well-advised to begin by investing 100% in stocks, and aggressively seek high returns. When he is 36 years away from retirement he will begin investing in bonds, and he will not invest anything in T-Bills until he is 10 years from retirement. At this point T-Bills (the lowest risk asset) soon begin to dominate the portfolio as the investor is more concerned with conserving wealth than with reaching for higher returns. This analysis is much in line with conventional wisdom on the subject as investors are invariably counseled to invest aggressively when young and then steadily move into more conservative, wealth-preserving assets as they age.

The right-most column in Table 5 provides the expected terminal value of the portfolio. We see that this is around $300,000 for an investor who starts investing at the suggested time point: 49 years before retirement. This emphasizes the point that the method we are using is planning for a worst-case scenario by ensuring that our portfolio succeeds in all but 5% of circumstances.

However, in the majority of circumstances we will have a surplus in this account which can be withdrawn or left to posterity.

A Comparison with Fixed Return Planning

The results of the previous section are a set of investment guidelines for our hypothetical investor, advising him on how much he should have saved for retirement at each point in his life and also advising him on the optimal allocation of his portfolio in the hypothetical market we examined. It is interesting to compare these results with other planning tools, specifically that of assuming a fixed return on one's assets. This type of planning is easy to do, but it can be disastrous to an investor with the above goals. If we assume that our investments earn a fixed rate of return, we are eliminating the elements of risk from our investment, and thus the possibility of ruin is entirely ignored. We also have to determine a suitable rate of return to use, and this can be difficult as well.

A truly risk-free investment in our simulation would have a return of less than 0.72% (the return on T-bills), and even using this rate to plan for retirement implies that an investor has zero chance of saving enough for retirement. He would be required to save for 102 years in order to fund the 20 year retirement described above. If we instead assume a return of 8.64% (the return on the S&P 500) in this planning process, we will be told that we only need to save for 26 years to fund our retirement, and we run the risk of almost certainly underfunding our retirement using these numbers.

Table 6 shows a comparison of minimum suggested balances using various fixed rates of return and Figure 2 illustrates these same values. We find that a reasonable expectation for returns is somewhere between 1% and 3%, and that while the 3% return provides a good approximation of the required portfolio balances early in life, these expectations need to be reduced downwards towards 1% as we near retirement. Using the wrong return at any point in this calculation will cause an investor to significantly underfund or overfund a retirement portfolio. As an example, an investor assuming an 8% return on his investments would be told that he only needs to accumulate $8,559 by the time he is 25 years from retirement – when in fact the required balance is $239,135 for a 95% success rate. Below-average investment returns would be disastrous for the this investor.

The optimal control method offers a significant improvement by incorporating risk into the model and also allowing an investor to respond to that risk by allocating his portfolio in different ways. In practice, while it still may be possible to use the fixed return model, it should be modified to use different expected returns (ranging from 1% to 3%) at different points in time. It may also be possible to approximate the optimal portfolio from the target returns of the optimal control method (ranging from 0.72% to 8.64%) and constructing Markowitz optimal portfolios for each target return. This would be a much less computationally expensive process than using the optimal control grid calculations, but it would simply be trying to reproduce the benchmarks created by the optimal control method.

Summary

This paper has described a grid-based method for approximating the optimal control solution for an investor saving for retirement with the goal of withdrawing a fixed, annual amount during retirement. We have assumed that the investor is trying to maximize the probability of survival (PoS), or the probability of making it through retirement without running out of money. We have used this method to analyze sustainable withdrawal rates during retirement and found results that largely agree with those of the Trinity studies, i.e. that a withdrawal rate of 4% - 5% of an investor's initial retirement portfolio is sustainable over a typical retirement period. We have also been able to improve the results of the Trinity study by allowing an investor to re-allocate his portfolio over time.

We also extended our analysis out to the years before retirement as an investor is trying to adequately fund a retirement portfolio. We examined a hypothetical investor and used the optimal control method to determine the time at which this investor should begin saving for retirement, the minimum balance an investor is required to have at each point in time in order to meet his goals, and the optimal allocation of assets in this portfolio at each time. We have shown that the optimal allocation of assets follows conventional wisdom – i.e., that an investor should aggressively seek high returns early in life while later moving into more conservative assets as they near retirement – and we have also provided exact numbers that could be used to guide this investor throughout life.

It is also important to remember that the numbers used in this paper (the investment amounts, withdrawal amounts, portfolio balances, and asset returns) are all adjusted for inflation, and that that all of these numbers should be adjusted for inflation in practice. The hypothetical required minimum balance for our investor at retirement ($720,823) is in present dollars. For an investor retiring today, this number would be an accurate goal. However, for an investor retiring in 20 years (and assuming fixed inflation of 2%) this number would be closer to $1,071,106. For an investor retiring in 30 years this would be $1,305,672. The tables in this paper do not include these results (since they differ depending on one's age and assumption), and simplify calculations by dealing with everything in terms of present dollar amounts. The results will still hold, however, as long as one adjusts their contributions, withdrawals, and balances for inflation as it is observed each year.

In the practical application of this model there are several possibilities (and needs) for extension. First, and most obviously, the expected retirement duration, income, and the amount one will contribute for retirement should be tailored to suit an individual investor. The hypothetical investor in this paper had a constant level of income over his working life and always invested 10% of it into the retirement portfolio. In practice, an increasing level of income would be more appropriate, and perhaps one would also want to vary the contribution to the retirement portfolio as well. It also would be desirable for the income level to be a stochastic variable that could be modeled to include the risks of unemployment or periods of low earnings in the calculation.

It would also be advisable to model an investor's lifespan as a stochastic variable and to include the possibility that the investor lives longer than the assumed retirement period – or that he/she doesn't even make it to the planned retirement at all. Unfortunately, this means we will not know at the end of the simulation time frame if the investor is alive or dead, and thus we cannot work backwards in time to calculate the optimal control grid and cannot apply the optimal control grid method as described. To overcome this problem an investor would be well-advised to plan on a retirement period that makes sense to them, and then insure themselves against the possibility of out-living that time period by purchasing a longevity annuity at the time of retirement. This will provide a steady stream of income beyond the range of the simulation considered here, and the only modification required is to increase the required cash outflow at time t_R by the expected purchase price of the annuity.

Finally, it would also be interesting to investigate the relationships between this model – where an investor must self-fund their own retirement and guarantee a certain PoS – versus that of a traditional pension or defined-benefit plan. In the former, an investor must protect themselves against a worst-case scenario by guaranteeing a 95% PoS. In doing so, they will almost certainly reach the end of life with a surplus account balance with an expected value of about $300,000. This amount exceeds the entire sum that our hypothetical investor contributed to his retirement account. In a shared plan, this excess amount could be used to fund another retirement account or cover shortfalls and deficits in other investors' accounts. The diversification of risk would allow a shared plan to obtain the same PoS for its members with a lower level of required contributions. This would offer a strong argument in favor of shared retirement plans that would allow investors to exchange their expectations of account surpluses for lower required contributions while still obtaining their desired retirement income.

Appendix: Tables and Figures

Table 1
Portfolio Success Rate with Inflation Adjusted Annual Withdrawals
Market A: Stocks & Bonds

	Annualized Withdrawal Rate as a % of Initial Portfolio Value									
Payout Period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100 % Stocks										
15 Years	99	98	94	89	81	71	60	49	39	29
20 Years	98	94	87	78	67	55	43	33	25	17
25 Years	96	89	80	69	56	45	34	25	18	12
30 Years	94	85	75	62	50	39	29	21	14	9
75% Stocks										
15 Years	100	99	98	93	85	73	59	45	32	21
20 Years	99	97	91	81	67	52	37	25	15	9
25 Years	98	93	84	70	54	39	26	16	9	5
30 Years	97	89	77	61	45	31	19	12	7	3
50% Stocks										
15 Years	100	100	99	96	88	73	53	34	19	10
20 Years	100	99	94	82	63	42	25	12	6	2
25 Years	100	96	85	66	44	25	13	6	2	1
30 Years	98	91	75	53	32	17	8	3	1	0
25% Stocks										
15 Years	100	100	99	96	84	63	39	19	8	3
20 Years	100	99	92	74	49	25	10	3	1	0
25 Years	99	94	77	51	25	10	3	1	0	0
30 Years	98	87	62	34	14	5	1	0	0	0
100% Bonds										
15 Years	100	100	97	87	68	44	24	10	4	1
20 Years	99	95	79	53	29	12	4	1	0	0
25 Years	96	81	54	28	12	4	1	0	0	0
30 Years	90	65	36	15	5	1	0	0	0	0

Table 2
Portfolio Success Rate with Inflation Adjusted Annual Withdrawals
Market B: Stocks, Bonds, & T-Bills

	Annualized Withdrawal Rate as a % of Initial Portfolio Value									
Payout Period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100 % Investment										
15 Years	99	98	94	89	81	71	60	49	39	30
20 Years	98	94	87	78	66	55	44	33	24	17
25 Years	96	89	80	69	57	45	34	25	18	12
30 Years	94	86	75	63	50	39	29	21	14	10
75% Investment										
15 Years	100	100	98	94	86	73	58	42	27	17
20 Years	100	98	92	82	66	49	33	20	12	6
25 Years	99	95	84	69	52	35	21	12	6	3
30 Years	98	90	77	60	41	26	15	8	4	2
50% Investment										
15 Years	100	100	100	97	87	68	44	23	9	3
20 Years	100	99	95	79	54	29	13	4	1	0
25 Years	100	96	82	57	30	13	4	1	0	0
30 Years	99	90	69	40	18	6	2	0	0	0
25% Investment										
15 Years	100	100	100	99	84	45	12	2	0	0
20 Years	100	100	95	62	20	3	0	0	0	0
25 Years	100	96	65	20	2	0	0	0	0	0
30 Years	99	82	33	5	0	0	0	0	0	0
100% Investment										
15 Years	100	100	100	96	48	4	0	0	0	0
20 Years	100	100	73	9	0	0	0	0	0	0
25 Years	100	75	8	0	0	0	0	0	0	0
30 Years	95	21	0	0	0	0	0	0	0	0

Table 3
Portfolio Success Rate with Inflation Adjusted Annual Withdrawals
Comparison of Trinity Portfolios with Optimal Portfolios
Market A: Stocks & Bonds

	Annualized Withdrawal Rate as a % of Initial Portfolio Value									
Payout Period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
Composite										
15 Years	100	100	99	96	88	73	60	49	39	29
20 Years	100	99	94	82	67	55	43	33	25	17
25 Years	100	96	85	70	56	45	34	25	18	12
30 Years	98	91	77	62	50	39	29	21	14	9
Optimal										
15 Years	100	100	100	98	93	82	70	57	45	34
20 Years	100	100	97	89	76	63	50	38	28	20
25 Years	100	98	91	78	65	51	39	29	20	14
30 Years	99	95	85	71	57	44	33	24	17	11

Table 4
Portfolio Success Rate with Inflation Adjusted Annual Withdrawals
Comparison of Trinity Portfolios with Optimal Portfolios
Market B: Stocks, Bonds & T-Bills

	Annualized Withdrawal Rate as a % of Initial Portfolio Value									
Payout Period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
Composite										
15 Years	100	100	100	99	87	73	60	49	39	30
20 Years	100	100	95	82	66	55	44	33	24	17
25 Years	100	96	84	69	57	45	34	25	18	12
30 Years	99	90	77	63	50	39	29	21	14	10
Optimal										
15 Years	100	100	100	100	95	85	72	59	46	35
20 Years	100	100	99	91	79	65	51	40	29	21
25 Years	100	99	92	80	66	53	40	30	21	14
30 Years	100	96	86	72	58	45	34	24	17	11

Table 5
Optimal Portfolio Details (PoS = 95%)

Simulation Time (t)	Years to Retirement	Suggested Balance	Portfolio Allocation			Expected Return	Expected Value
			T-Bills	Bonds	Stocks		
0	50	0	0.00	0.00	1.00	8.64%	335,961
5	45	20,429	0.00	0.00	1.00	8.64%	307,150
10	40	58,908	0.00	0.00	1.00	8.64%	305,045
15	35	108,091	0.00	0.05	0.95	8.33%	299,992
20	30	168,028	0.00	0.20	0.80	7.50%	277,364
25	25	239,135	0.00	0.30	0.70	6.94%	256,689
30	20	321,737	0.00	0.37	0.63	6.52%	239,005
35	15	415,380	0.00	0.43	0.57	6.17%	211,673
40	10	517,997	0.01	0.46	0.53	5.92%	180,636
45	5	624,042	0.14	0.43	0.42	5.00%	146,473
50	0	720,823	0.40	0.29	0.31	3.79%	106,645
55	(5)	571,789	0.51	0.23	0.26	3.30%	70,503
60	(10)	400,446	0.66	0.14	0.20	2.61%	38,796
65	(15)	207,165	0.86	0.03	0.11	1.69%	13,982

Figure 1
Optimal Portfolio Dynamics (PoS = 95%)

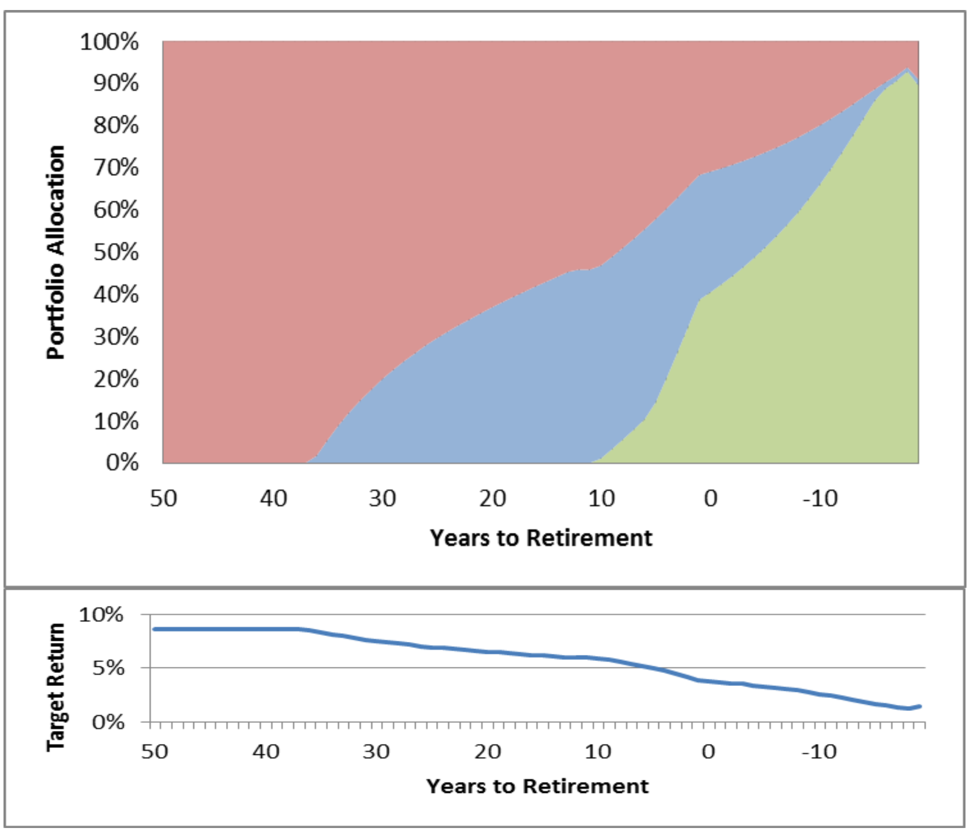
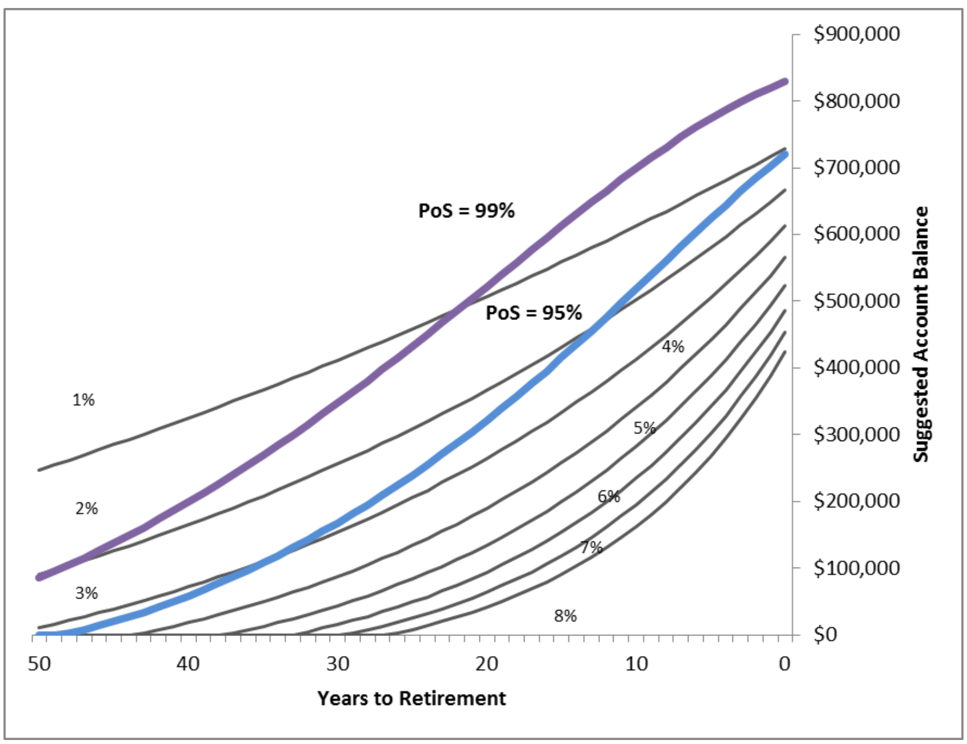


Table 6
Minimum Required Portfolio Balances:
Comparison of optimal portfolio (PoS = 95%) with fixed-rate portfolios

Years to Retirement	Optimal Balance	Assumed Fixed Return							
		1%	2%	3%	4%	5%	6%	7%	8%
50	0	247,304	90,743	11,170	0	0	0	0	0
45	20,429	285,424	126,207	39,495	0	0	0	0	0
40	58,908	325,489	165,363	72,331	18,794	0	0	0	0
35	108,091	367,597	208,595	110,397	49,947	13,019	0	0	0
30	168,028	411,853	256,325	154,526	87,850	44,244	15,850	0	0
25	239,135	458,367	309,024	205,684	133,965	84,096	49,396	25,275	8,559
20	321,737	507,253	367,208	264,989	190,070	134,958	94,289	64,203	41,909
15	415,380	558,633	431,447	333,741	258,331	199,872	154,365	118,802	90,910
10	517,997	612,634	502,373	413,443	341,381	282,721	234,761	195,380	162,910
5	624,042	669,390	580,681	505,839	442,424	388,460	342,348	302,784	268,702
0	720,823	729,040	667,138	612,952	565,358	523,413	486,325	453,424	424,144

Figure 2
Minimum Required Portfolio Balances:
Comparison of optimal portfolios (PoS = 95% and 99%) with fixed-rate portfolios



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(in order of relevance, and yes, I know it's formatting wrong)

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NOTE: I should mention this one in the paper when I set the 99/95% PoS. They suggested a 75% PoS is OK. I don't think it would be OK for me.

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(NOTE: I don't have access to this one. It's not free ☹)

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