Inferring Expected Returns from Option Prices

Overview

Since college, I've wondered if there's a way to infer future expectations about stocks from options prices. Options prices should be tied to future expectations of the underlying stock's price (I think… although this isn't an input into the Black-Scholes equation), and the variety of strike prices should tell us something about the distribution at various future price points. This paper presents a simple model for doing just that. I don't know if this is really good or not, but it seems interesting.

Data & Approach

The following screenshot shows options prices for the VOO on 8/5. The options expire on 9/15 (30 trading days in the future), and the current price of VOO was 227.26.

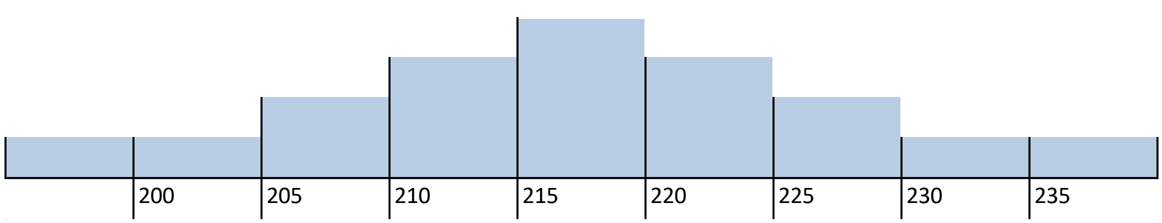
Calls for September 15, 2017

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
VOO170915C00230000	2017-08-04 9:48AM EDT	230.00	1.20	0.95	1.15	0.30	33.33%	3	24	7.30%
VOO170915C00235000	2017-07-28 11:46PM EDT	235.00	0.55	0.00	0.25	0.00	-	3	3	7.44%

Puts for September 15, 2017

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
VOO170915P00199000	2017-07-28 11:47PM EDT	199.00	0.10	0.10	0.35	0.00	-	1	1	23.68%
VOO170915P00200000	2017-07-28 11:47PM EDT	200.00	0.10	0.10	0.40	0.00	-	1	1	23.61%
VOO170915P00205000	2017-07-28 11:47PM EDT	205.00	0.15	0.25	0.50	0.00	-	1	1	20.92%
VOO170915P00210000	2017-07-28 11:47PM EDT	210.00	0.40	0.40	0.60	-0.05	-25.00%	1	2	17.88%
VOO170915P00215000	2017-07-28 11:47PM EDT	215.00	0.60	0.65	0.90	0.00	-	9	10	15.67%
VOO170915P00220000	2017-08-02 9:30AM EDT	220.00	1.05	0.90	1.20	0.00	-	4	8	12.46%
VOO170915P00225000	2017-07-31 3:55PM EDT	225.00	2.01	1.70	2.25	-0.04	-1.95%	5	15	10.65%
VOO170915P00230000	2017-08-04 3:55PM EDT	230.00	3.60	3.30	3.80	-0.30	-7.69%	2	13	6.98%

The goal of this analysis is to fit a simple distribution of the form shown below to this:



We will divide the distribution into intervals defined by the strike prices of the various options. We then want to calculate the probability that the future price will fall into each of these bins.

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Estimating the Extreme Values

To begin, we look at the extremes. The ask price on a 235 call option is \$0.25. This means that the expected payoff of this option should be about \$0.25. A trader buying the option would thus have a zero expected profit. (NOTE: Theory predicts that the expected profit should be near-zero. There are two sides to each trade and neither should have an advantage over the other. The buyer is thus essentially loaning money to the seller, and the return to the buyer should be the risk-free interest rate over the holding period of the option. We'll just assume that is near-zero since the options are short-term options.) The payoff on the option depends on its price. It is:

Case	Payoff
S <= 235	0
S > 235	S – 235

The expected payoff of the option is thus:

$$E[S-235 \mid S\ >235]*\Pr(S>235)+0*\Pr(S\leq 2350)=$$
$$E[S-235 \mid S\ >235]*\Pr(S>235)=\$0.25$$

So how do we build a distribution with an expected value of \$0.25? If we define an interval from 235 to 240 with a constant probability for all points in between, the expected payoff if the price exceeds 235 is \$2.50. That is:

$$E[S-235 \mid S\ >235]=E[S|S>235]-235=237.5-235=2.50$$

Note that we are assuming there is zero probability of the stock price exceeded 240. The expected value is then 237.5 and the payoff on the option is \$2.50. In order to make this worth \$0.25, the probability of the price exceeding 235 must be 10%.

$$E[S-235 \mid S\ >235]*\Pr(S>235)=0.25$$
$$2.50*\Pr(S>235)=0.25$$
$$\Pr(S>235)=.25/2.50=0.1=10\%$$

A more realistic estimate might be that the stock could increase as high as 285. The expected payoff is then the midpoint of the interval (260) minus the lower range (235), which is \$25. To provide an expected value of 0.25, the probability in this interval would need to be 1%. Alternately, we might build an interval that extends all the way to 335. The midpoint is then 285. The payoff is \$50, and the inferred probability is 0.5%. As we can see, the choice of how wide we make this interval can change the probability quite a bit. However, we'll see later that this might influence our results less than we'd expect.

We'll proceed using the 235 to 285 interval, implying that the probability for the price exceeding 235 is 1%. We can perform this same operation on the other extreme using the put option with a strike price of 200. Its current price is $0.40. If we build an interval from 150 to 200, the expected in-the-money payoff is $25, and the inferred probability of landing in this interval is $0.40 / $25 = 0.016 = 1.6%.

Estimating the Internal Intervals

Now that we have the tails estimated, we can proceed to estimate the internal intervals. This gets a little bit trickier. Let’s examine the interval from 230 to 235. If we bought a 230 call option for $1.15 and sell a 235 option for $0.25, the cost of this spread will be $0.90. The payoff will depend on the price of the stock:

Case	Payoff
S <= 230	0
230 <= S < 235	S - 235
S > 235	5

Notice that our profit is capped at $5.00 if the price exceeds 235. We also know the probability of the price exceeding 235. We calculated that earlier and estimated it to be 1%. So the value of this option derived from this case is $5.00 * 1% = $0.05. This leaves $0.85 to be explained by the case where the price falls between 230 and 235. If the price falls between 230 and 235, the expected price is 232.50 and the expected value is $2.50. In order to make this outcome worth $0.85, the probability of the stock’s price falling within this interval must be 34%. More formally:

$$\Pr(S > 235) * 5 + \Pr(230 \leq S \leq 235)\, E[S - 230 \mid 230 \leq S < 235] + \Pr\,(S > 230) * 0 =$$
$$0.01 * 5 \quad + \Pr(230 \leq S \leq 235)\, 2.50 \quad + \quad 0 = 0.90$$

$$\Pr(230 \leq S \leq 235) = \frac{.90 - .05}{2.50} = .34 = 34\%$$

If we had a call option at a strike price of 225, we could repeat this process. We now know the probability of S exceeding 230. It’s the probability of it exceeding 235 (1%) added to the probability of it falling in the range of 230 and 235 (34%), which is 35%. However, we don’t have a call option at a strike price of 225. Instead, we have to go to the other end of the distribution and look at the price of puts.

We already have the probability of the price falling below 200 (1.6%). We can construct a spread by buying the 205 put for $0.50 and selling the 200 put for $0.40. The cost of the spread is thus $0.10 and $0.08 of that is accounted for by the probability of the price falling below 200 ($5.00 * .016 = $0.08). If the price falls between 200 and 205, the expected payoff is $2.50. To make this worth $0.02, the inferred probability is .02 / 2.50 = 0.008 = 0.8%.

Repeating this process produces the following table:

Strike Price Range			Using Call Options			Using Put Options		
Min	Mid	Max	Spread Cost	Pr(S>x2)	Pr(x1<S<x2)	Spread Cost	Pr(S<x1)	Pr(x1<S<x2)
150	175.0	200				0.40	0.000	0.016
200	202.5	205				0.10	0.016	0.008
205	207.5	210				0.10	0.024	0.000
210	212.5	215				0.30	0.024	0.072
215	217.5	220				0.30	0.096	0.000
220	222.5	225				1.05	0.096	0.228
225	227.5	230				1.55	0.324	0.000
230	232.5	235	0.90	0.01	0.34			
235	260.0	285	0.25	0.00	0.01			

Finally, we are able to infer the probability distribution for each of the intervals we have defined. Now comes the fun part: we can use this distribution to calculate the expected price of the asset. We have:

Mid-Point	Probability
175.0	0.0160
202.5	0.0080
207.5	0.0000
212.5	0.0720
217.5	0.0000
222.5	0.2280
227.5	0.0000
232.5	0.3400
260.0	0.0100

These probabilities do not add to 100%. Instead, they add to 67.4%, but we can normalize the distribution so that they do add to 100%. This is the same as doing a weighted average using the probabilities as the weights. The expected value in this case is $225.67. If correct, the market is expecting the price to fall from its current price of $227.26 by $1.59. Since these options expire on September 15, we would also assume that this is the date of the predicted price.

Sensitivity to End Points

Earlier, we had to make an arbitrary decision about how far to extend the ends of our distribution beyond the highest call option and the lowest put option. We created intervals that were $5, $50, and $100 wide. The expected price of $225.67 was obtained using widths of $50. If we increase these to $100, the expected price becomes $225.69. This does not have much effect. If we decrease the width to $5, it becomes $221.85. Bringing the endpoints in too closely really skews the inferred distribution as well. The following table shows the different distributions based on end-point width:

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Mid-Point	$100-interval	$50-interval	$5 interval
175.0	0.0080	0.0160	0.1600
202.5	0.0240	0.0080	0.0000
207.5	0.0000	0.0000	0.0000
212.5	0.0560	0.0720	0.0000
217.5	0.0000	0.0000	0.0000
222.5	0.2440	0.2280	0.1000
227.5	0.0000	0.0000	0.1000
232.5	0.3500	0.3400	0.1600
260.0	0.0050	0.0100	0.1000

In general, we would not expect to see many intervals with a zero probability of occurring. This would lead us to prefer the wider intervals over the smaller ones. Some realistic boundary based upon the time-frame being examined would make sense. If you're looking at 1-month options it would make sense to take the largest 1-month movement observed in the stock to set the upper and lower endpoints of the distribution.

Fidelity Data versus Yahoo Data

Fidelity offered slightly different prices for these same options:

Call Price (Ask)	Strike	Put Price (Ask)
23.00	205	0.35
18.10	210	0.45
13.30	215	0.65
8.70	220	1.05
4.60	225	1.8
1.15	230	3.8
0.20	235	8.5
0.15	240	13.2
0.10	245	18.5
0.10	250	23.4

The predicted price based on the put option was $228.33. Predicting off the call options gave an expectation of $230.77. However, the distributions of both of these were far from ideal, and I don't know a good way of combining them:

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Mid	Probability using calls	Probability using puts
180.0		0.014
207.5	0	0.012
212.5	0	0.028
217.5	0	0.052
222.5	0	0.088
227.5	0.700	0.412
232.5	0.300	0.668
237.5	0	0
242.5	0	0
247.5	0	0
275.0	0.040	0
Total	1.040	1.247

I also calculated the expectation using the bid price of the put options rather than the asking price. The resulting expectation was $221.09. Averaging this with the estimate using call prices gives $224.71. A method that uses the mid-point of the bid and ask prices might be a good approach to this. I don't know how to reconcile the differences between call and put options, but using the mid-point might make these differences smaller.