Inventory Models for Probabilistic Demand: Cost & Service Trade-offs

MIT Center for Transportation & Logistics

Items to Cover

- Comparing Inventory Performance Metrics
- Inputted versus Implied Metrics
- Periodic Review Policies
- Example Problem: ShopCo
- Trading off Lead Time and Review Period

Notation

D = Average Demand (units/time)

c = Variable (Purchase) Cost ($/unit)

h = Carrying or Holding Charge ($/inventory $/time)

c_t = Fixed Ordering Cost ($/order)

$c_e = c*h$ = Excess Holding Cost ($/unit/time)

c_s = Shortage Cost ($/unit/time)

Q = Replenishment Order Quantity (units/order)

L = Replenishment Lead Time (time)

T = Order Cycle Time (time/order)

$N = 1/T$ = Orders per Time (order/time)

IP = Inventory Position (units)

IOH = Inventory on Hand (units)

IOO = Inventory On Order (units)

μ_{DL} , σ_{DL} = Expected and Standard Deviation of Demand over Lead Time (units)

μ_{DL+R} , σ_{DL+R} = Expected and Standard Deviation of Demand over Lead Time plus Review Period (units)

k = Safety Factor

s = Reorder point (units)

S = Order up to Point (units)

R = Review Period (time)

IFR = Item Fill Rate (%)

CSL = Cycle Service Level (%)

CSOE = Cost of Stock Out Event ($/event)

CSI = Cost per Item Short

E[US] = Expected Units Short (units)

$G(k)$ = Unit Normal Loss Function

Comparing Inventory Performance Metrics

Inventory Performance Metrics

- Establishes Safety Stock
  - Finds k value
  - Expected Cost of Safety Stock = $c_e k\sigma_{DL}$

$$s = \mu_{DL} + k\sigma_{DL}$$

- Service Based Metrics – set k to meet expected LOS
  - Cycle Service Level (CSL)
    - Probability of not stocking out during cycle
  - Item Fill Rate (IFR)
    - Expected percentage of demand met during each cycle

$$CSL = P\left[x \le k\right]$$

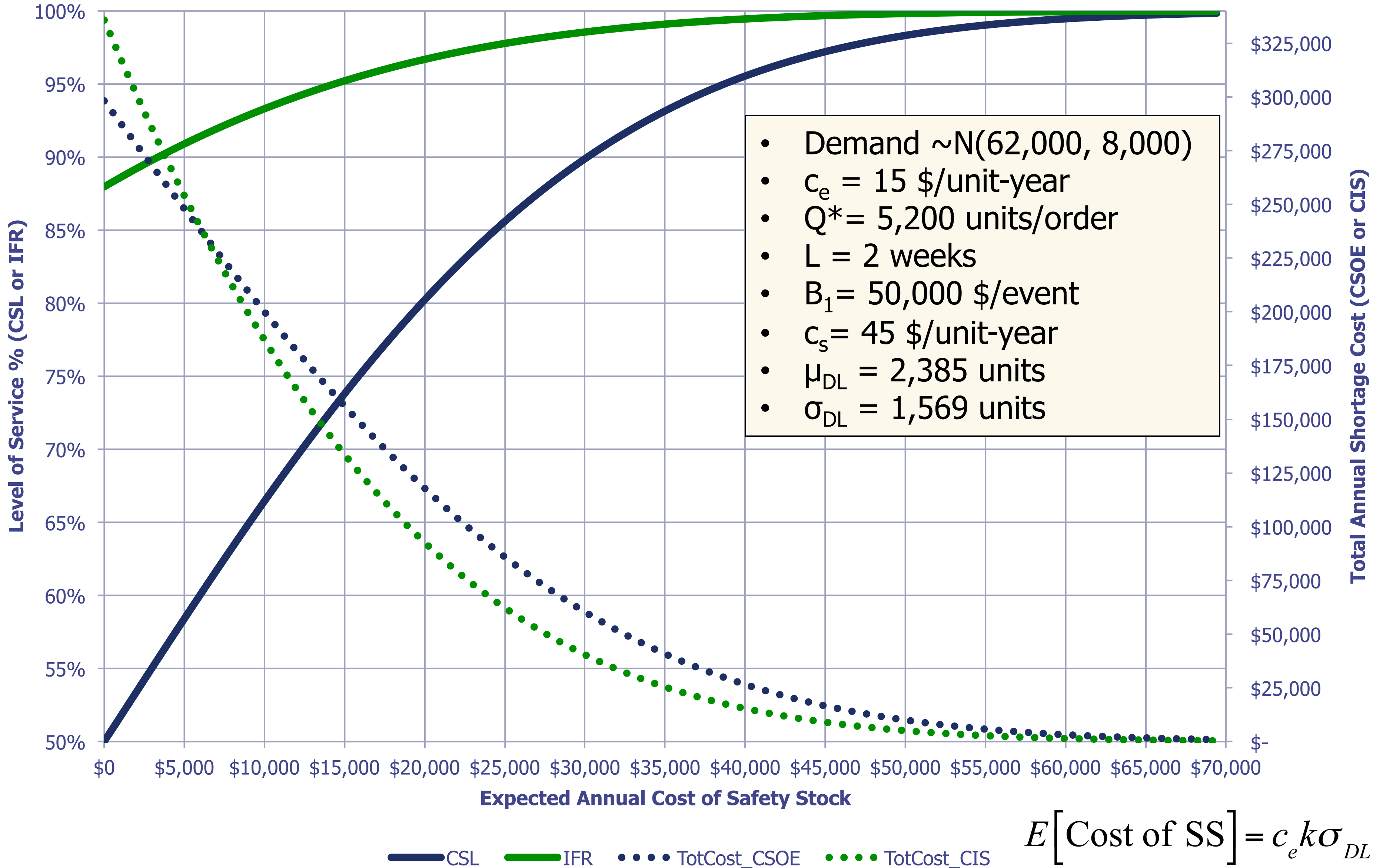
$$IFR = 1 - \frac{\sigma_{DL} G[k]}{Q}$$

- Cost Based Metrics – find k that minimizes total costs
  - Cost per Stockout Event (CSOE)
    - Penalty of B_1 if any stock out occurs
  - Cost per Item Short (CIS)
    - Penalty of c_s for each item short per cycle

$$E\left[CSOE\right] = (B_1)P\left[x \ge k\right]\left(\frac{D}{Q}\right)$$

$$E\left[CIS\right] = c_s \sigma_{DL} G(k)\left(\frac{D}{Q}\right)$$

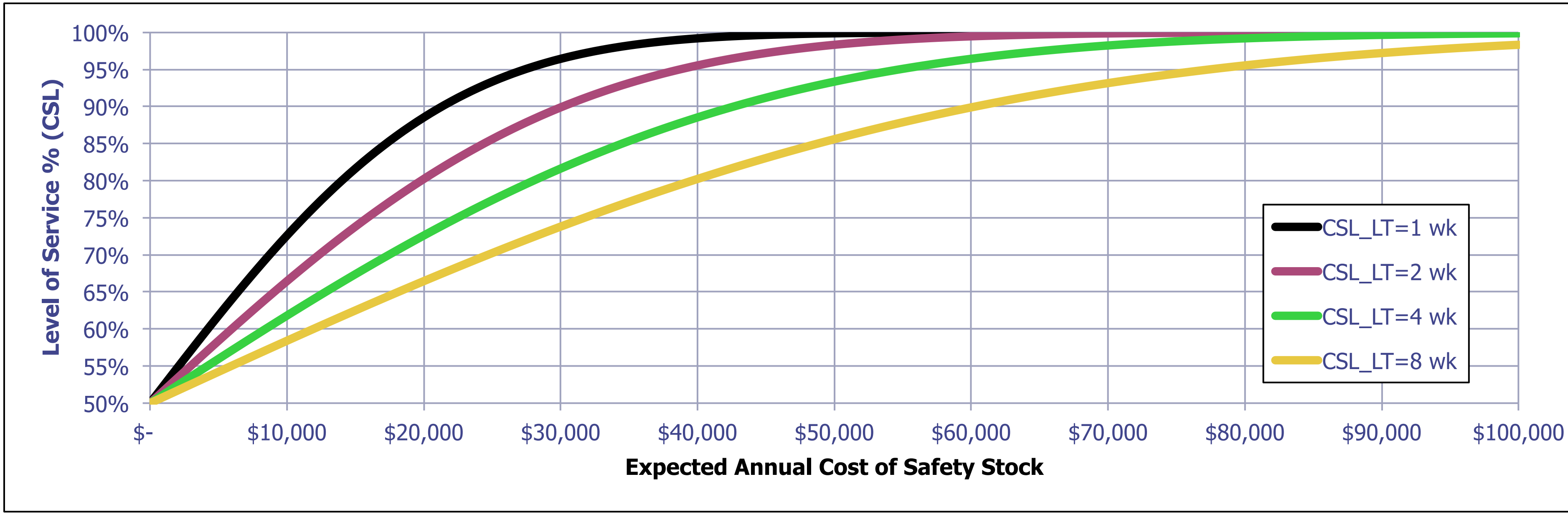
Performance Metrics v. Safety Stock Costs

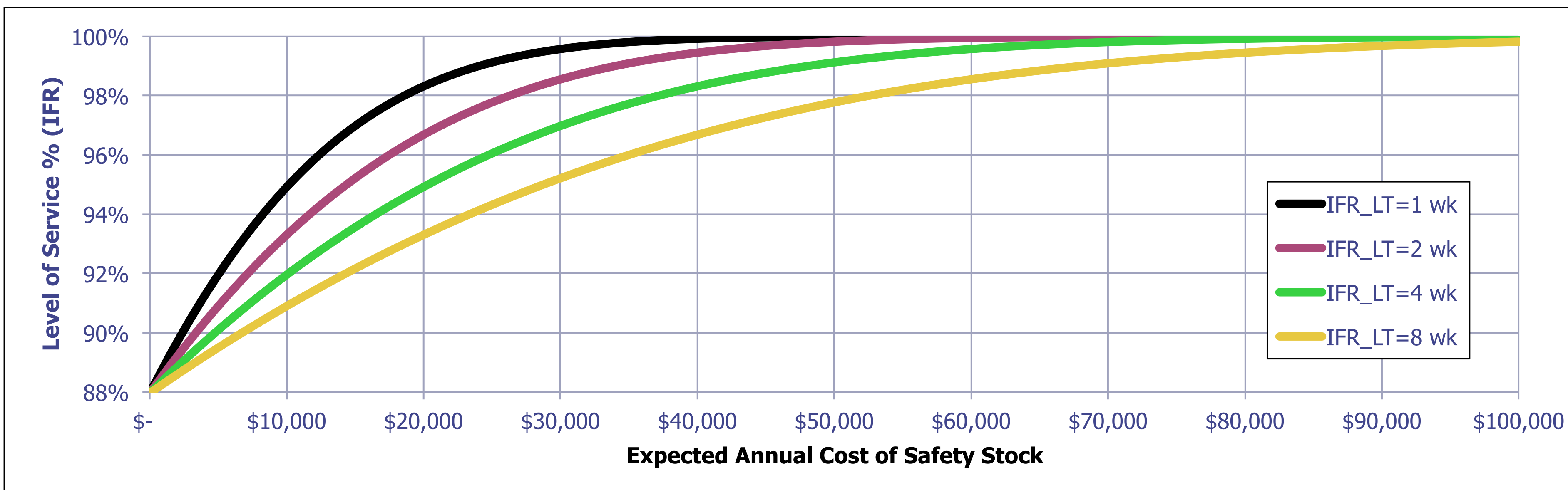


- Demand ~N(62,000, 8,000)
- c_e = 15 $/unit-year
- Q*= 5,200 units/order
- L = 2 weeks
- B_1= 50,000 $/event
- c_s= 45 $/unit-year
- μ_{DL} = 2,385 units
- σ_{DL} = 1,569 units

$$E\left[\text{Cost of SS}\right] = c_e k \sigma_{DL}$$

Lead Time v. Safety Stock Costs





Inputted vs. Implied Metrics

Safety Stock Logic

$$E\left[\text{Cost of SS}\right] = c_e k \sigma_{DL}$$

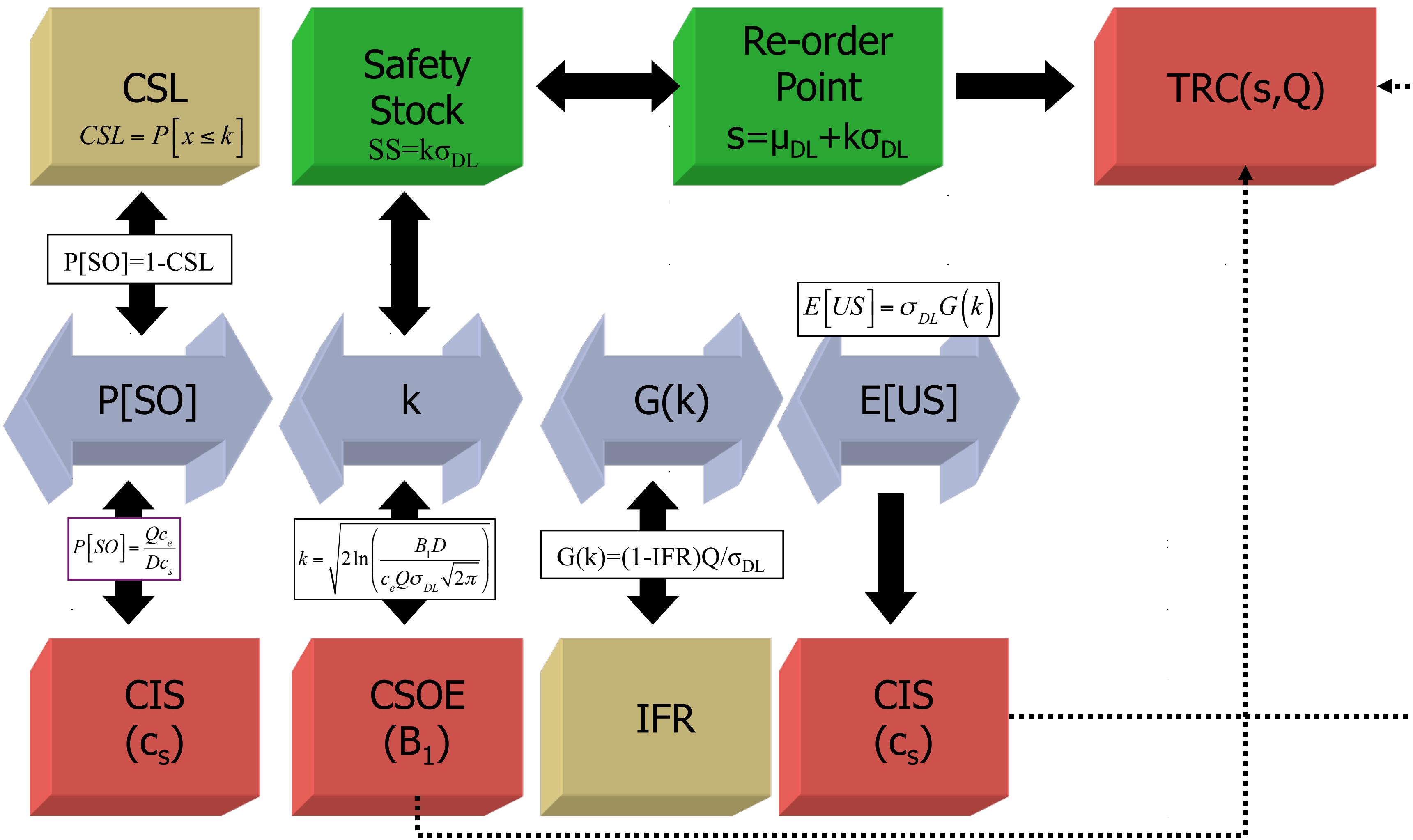


Figure adapted from Dr. Jim Master's ESD.260 Course Notes (2002)

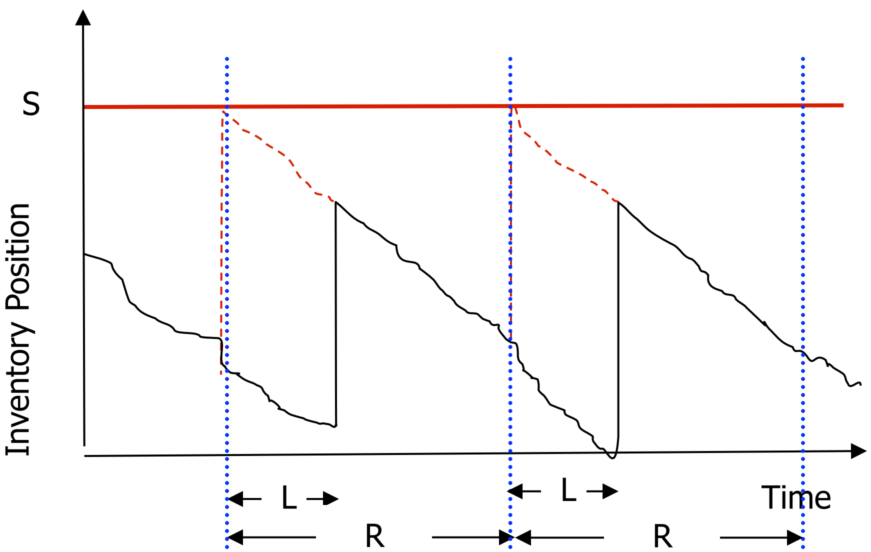
Periodic Review Policies

Assumptions: Periodic Review Policies

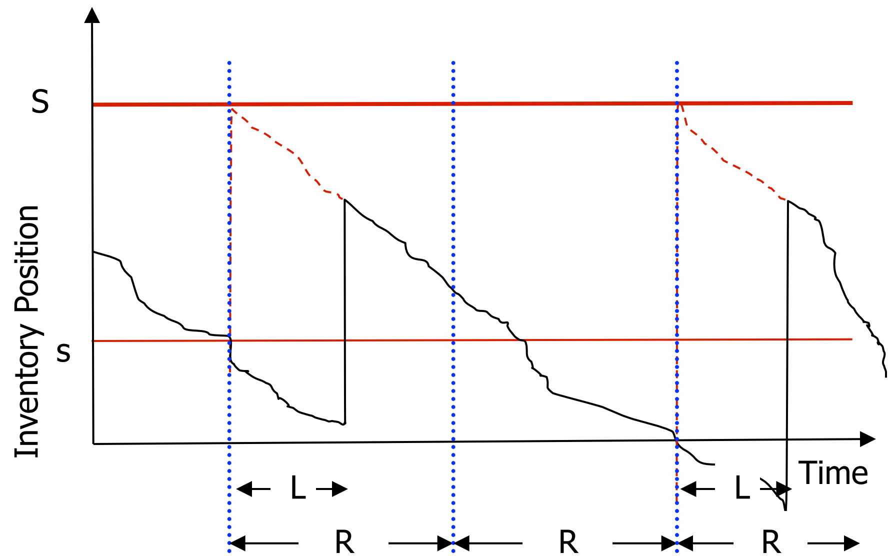
- Demand
  - Constant vs **Variable**
  - Known vs **Random**
  - **Continuous** vs Discrete
- Lead Time
  - Instantaneous
  - **Constant** vs Variable
  - **Deterministic** vs Stochastic
  - Internally Replenished
- Dependence of Items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - Continuous vs **Periodic**
- Number of Locations
  - **One** vs Multi vs Multi-Echelon
- Capacity / Resources
  - **Unlimited**
  - Limited / Constrained
- Discounts
  - **None**
  - All Units vs Incremental vs One Time
- Excess Demand
  - None
  - All orders are backordered
  - **Lost orders**
  - Substitution
- Perishability
  - **None**
  - Uniform with time
  - Non-linear with time
- Planning Horizon
  - Single Period
  - Finite Period
  - **Infinite**
- Number of Items
  - **One** vs Many
- Form of Product
  - **Single Stage**
  - Multi-Stage

Periodic Review Policies

- Order-Up-To-Level (R, S)
  - Policy: **Order S-IP every R time periods**
  - Replenishment cycle system



- Hybrid (R, s, S) System
  - Policy: **Every R time periods, Order S-IP if IP ≤ s, if IP>s then do not order**
  - General case for many policies

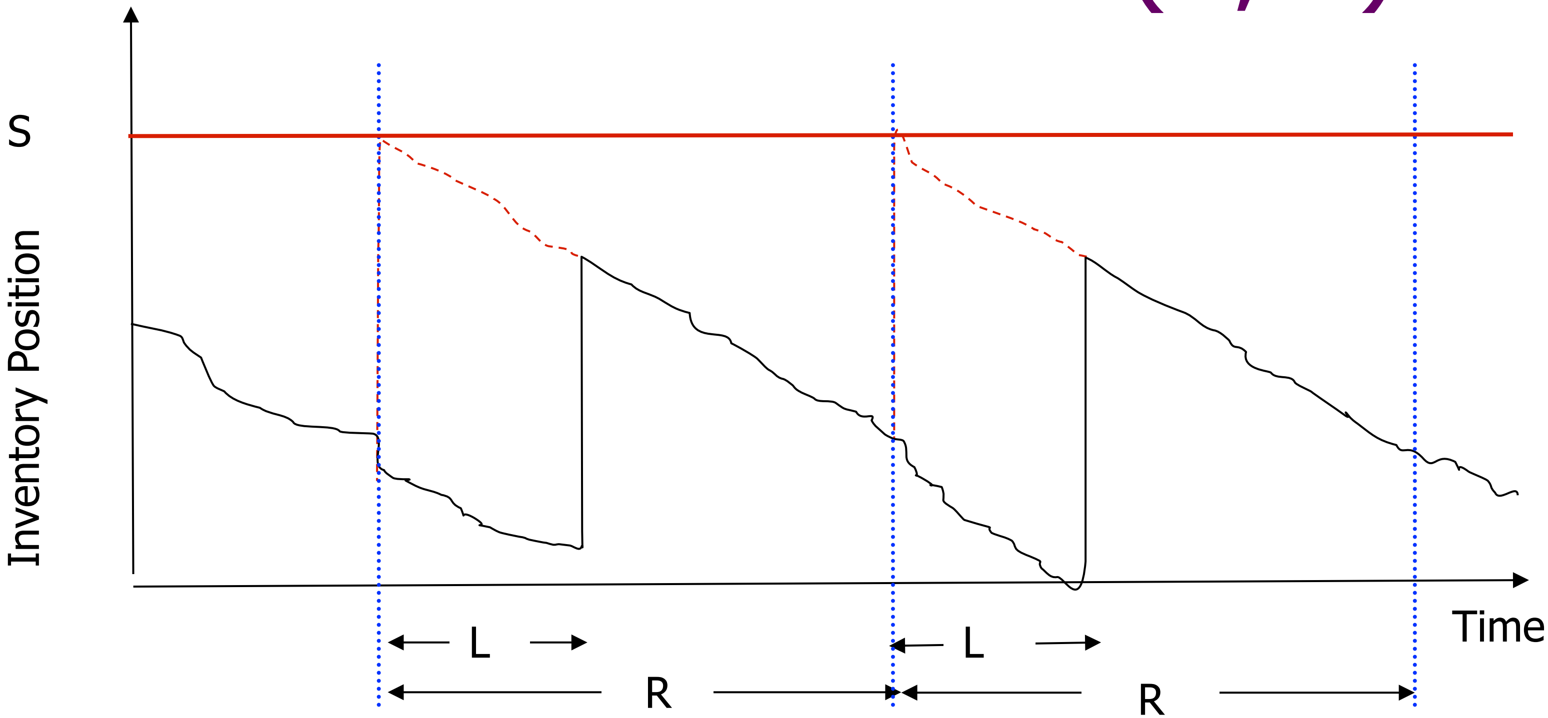


Notation

s = Reorder Point	S = Order-up-to Level	L = Replenishment Lead Time
Q = Order Quantity	R = Review Period	IOH= Inventory on Hand

IP = Inventory Position = (IOH) + (Inventory On Order) – (Backorders)

Periodic Review Policies (R, S)



Differences from Continuous Review Policy (s, Q)

- How much to order?
- How long should safety stock cover?

Periodic vs Continuous Review

- Convenient transformation of (s, Q) to (R, S)
  - (s, Q)= Continuous, order Q when IP≤s
  - (R, S)= Periodic, order up to S every R time periods
- Allows for the use of all previous (s, Q) decision rules
  - s for continuous system becomes S for periodic system
  - Q for continuous system becomes D*R for periodic system
  - L for a continuous system becomes R+L for periodic system
- Approach
  - Make transformations
  - Solve for (s, Q) using transformations
  - Determine final policy, so that

$$S = x_{DL+R} + k\sigma_{DL+R}$$

(s, Q)		(R, S)
s	⇔	S
Q	⇔	D*R
L	⇔	R+L

Example: ShopCo

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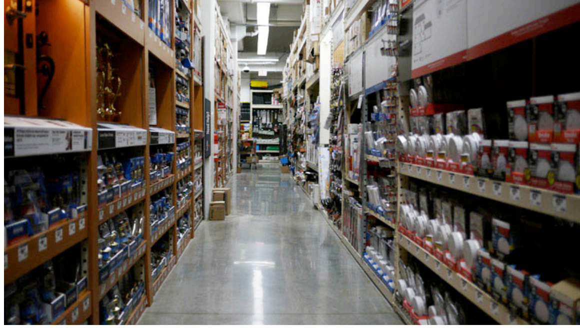


- Background:
  - ShopCo is a North America based large store format retailer of home improvement products with >2,000 stores. Each ShopCo store generally operates independently: ordering and receiving product directly from its suppliers.
  - One supplier (Hurricane Drills) sells a portfolio of electric drills that, on average, cost ShopCo $75 each. Each store uses periodic review policies to order directly from Hurricane and uses an annual holding charge of 15%. Assume 52 week year.
- Problem:
  - Find the (R, S) ordering policy for Hurricane drills for store #1301 given:
    - Forecasted annual demand of Hurricane drills is ~N(3,400, 400)
    - Lead Time is 1 week
    - Review Period is 4 weeks
    - Desired CSL = 95%
    - Hurricane has a minimum order quantity (MOQ) of 240 drills
    - Orders need to be in multiples of 12 drills to fit on pallets
  - What is the expected annual cost of cycle and safety stock?

Case adapted from Anand, S. and Song, X. (2011) "Supply Chain Responsiveness for a Large Retailer," MIT Supply Chain Management Program Thesis.
Image Source:http://commons.wikimedia.org/wiki/File:Hardware_Store.jpg

Example: ShopCo

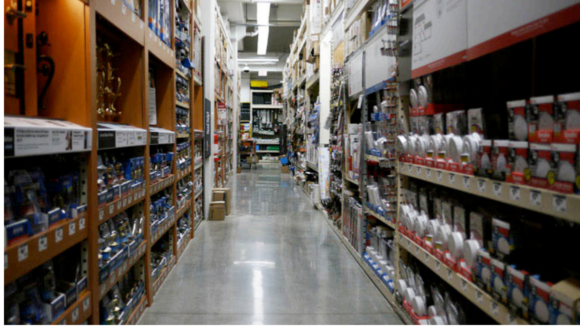
- Finding Order Policy:
  - Find Q = D*R = (3,400 units/year)(4/52 years) = 261.5 ≈ 264 units *(why?)*
  - Find R+L = 4 weeks + 1 week = 5 weeks or 0.0962 years
    so that, n= 52/5 = 10.4 "coverage" periods per year
  - Find μ_{DL+R} = (3,400)/(10.4) = 326.9 ≈ 327 units
  - Find σ_{DL+R} = (400)/(√10.4) = 124.03 ≈124 units
  - Find k where CSL = 0.95 or P[x≤k] = 0.95, k=1.644 = 1.64
  - Find S = $\mu_{DL+R} + k\sigma_{DL+R}$ = 327 + (1.64)(124) = 530.4 ≈ 530 units

Policy: Order up to 530 units every 4 weeks.

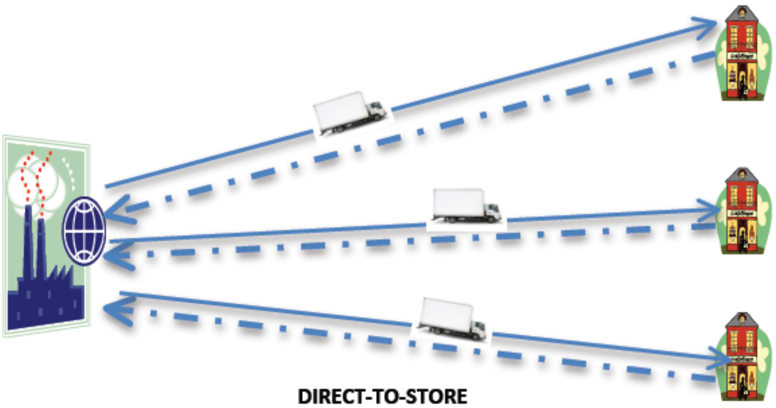
- Finding Cost of Cycle & Safety Stock:
  - Cost of Cycle Stock = $c_e(DR/2)$ = (75)(0.15)(264/2) = $1,485 per year
  - Cost of Safety Stock = $c_e k\sigma_{DL+R}$ = (75)(0.15)(1.64)(124) = $2,288 per year

Trading Off Lead Time and Review Period

Example: ShopCo continued



- New Mixing Center Strategy:
  - ShopCo has decided to deploy a fulfillment strategy where each store orders from its Regional Distribution Center (RDC), instead of directly to the supplier.
  - Each ShopCo RDC then consolidates orders from its dedicated stores and places a combined order to the vendor. The vendor will then ship to each of the RDCs where ShopCo "mixes" the products from multiple suppliers to distribute a single combined load to each store.



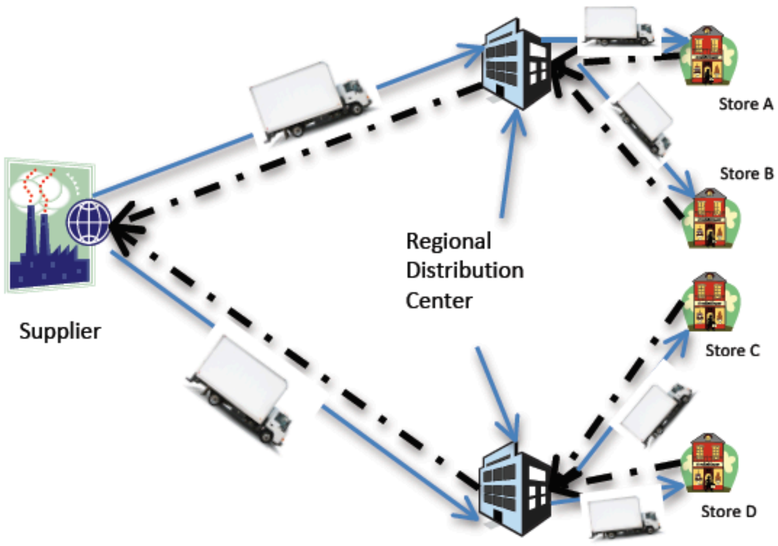


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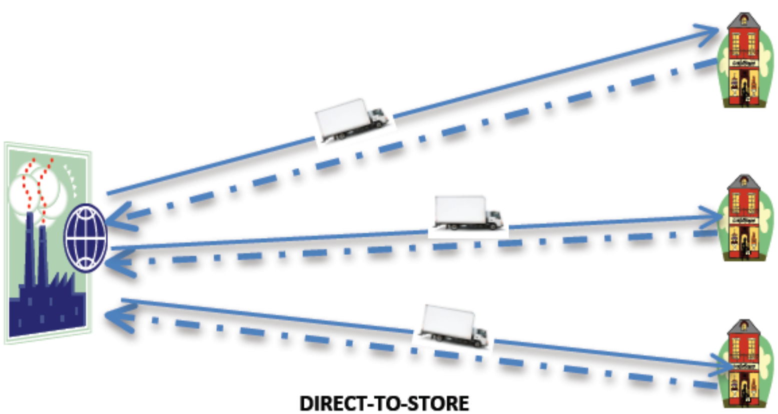
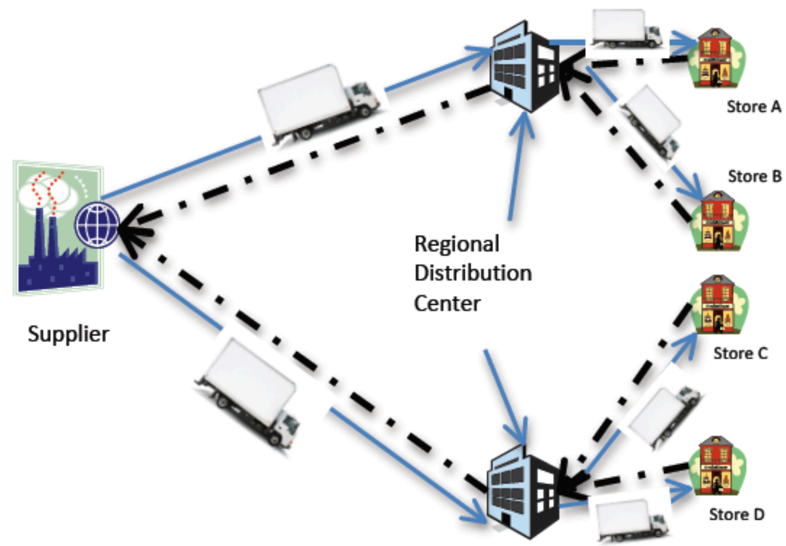
Image Sources: http://commons.wikimedia.org/wiki/File:Hardware_Store.jpg and Anand & Song (2011)

Example: ShopCo continued

- Revised Problem with Mixing Centers:
  - Find the (R, S) ordering policy for Hurricanes for store #1301 given:
    - Forecasted annual demand of Hurricane drills is ~N(3,400, 400)
    - Desired CSL = 95%
    - Lead Time is now 10 days (call this 1.5 weeks for simplicity)
    - Review Period is reduced to 2 weeks
    - ShopCo's RDCs do not have a minimum order quantity (MOQ) to stores *(why?)*
    - Orders still need to be in multiples of 12 drills to fit on pallets *(why?)*
  - What is the expected annual cost of cycle and safety stock?





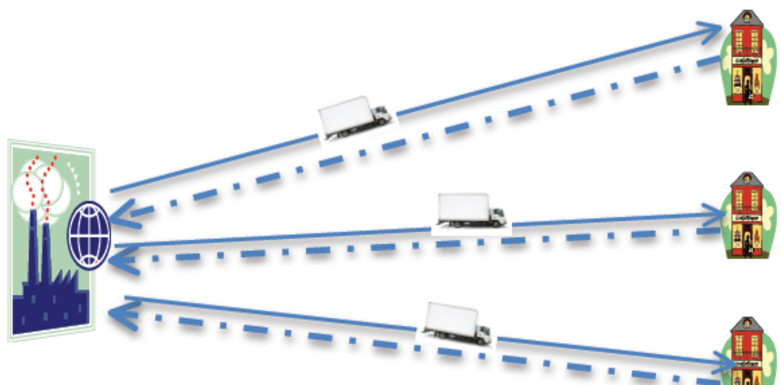
Example: ShopCo with Mixing Strategy

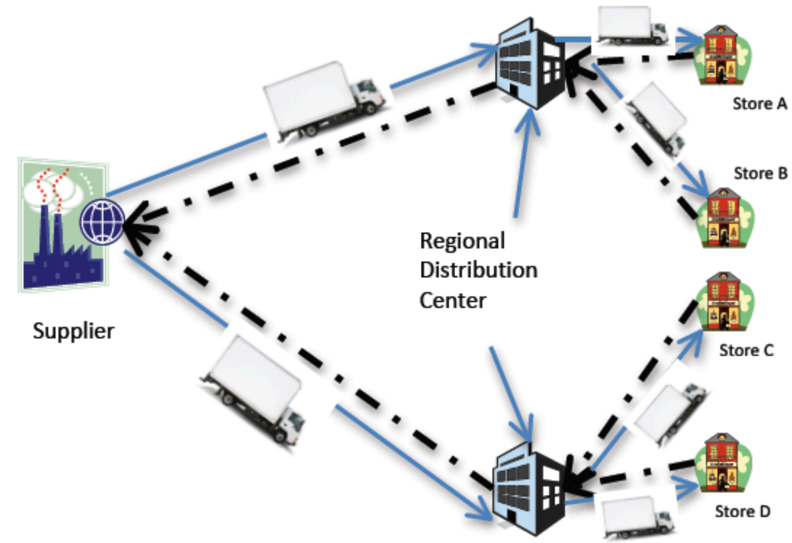
- Finding Order Policy:
  - Find Q = D*R = (3,400 units/year)(2/52 years) = 130.8 ≈ 132 units *(why?)*
  - Find R+L = 2 weeks + 1.5 week = 3.5 weeks or 0.0673 years
    so that, n= 52/3.5 = 14.86 "coverage" periods per year
  - Find μ_{DL+R} = (3,400)/(14.86) = 228.8 ≈ 229 units
  - Find σ_{DL+R} = (400)/(√14.86) = 103.76 ≈ 104 units
  - Find k where CSL = 0.95 or P[x≤k] = 0.95, k=1.644 = 1.64
  - Find S = $\mu_{DL+R} + k\sigma_{DL+R}$ = 229+ (1.64)(104) = 399.56 ≈ 400 units

Policy: Order up to 400 units every 2 weeks.

- Finding Cost of Cycle & Safety Stock:
  - Cost of Cycle Stock = $c_e(DR/2)$ = (75)(0.15)(132/2) ≈ $743 per year
  - Cost of Safety Stock = $c_e k\sigma_{DL+R}$ = (75)(0.15)(1.64)(104) ≈ $1,919 per year

Example: ShopCo continued





Strategy	Lead Time (weeks)	Review Period (weeks)	Cycle Stock ($/year)	Safety Stock ($/year)	Avg. Inventory Costs ($/year)
Direct-to-Store	1	4	1,485	2,288	3,773
Mixing Centers	1.5	2	743	1,919	2,662

Which is better? Which did the store managers prefer?

Relationship Between L & R

- Average Inventory Costs $= c_e[DR/2 + k\sigma_{DL+R} + LD]$

- Individual Impacts
  - Increasing Lead Time L
    - ➔ Increases Safety Stock non-linearly
    - ➔ Increases Pipeline Stock linearly
  - Increasing Review Period, R:
    - ➔ Increases Safety Stock non-linearly
    - ➔ Increases Cycle Stock linearly

- Combined Impacts
  - Can be used to trade Replenishment speed (L) for frequency (R)
  - Determine which is the right mix

Key Points from Lesson

Key Points

- Inventory Performance Metrics
  - Service Based: IFR vs. CSL
  - Cost Based: CSOE vs. CIS
- Inputted vs. Implied Metrics
  - Designing to one metric sets the others
  - Can backwards calculate *implied* values
- Periodic Review (R, S)
  - Very commonly used
  - Use the (s, Q) rules with simple transformations

    $Q \rightarrow D*R, \ \ s \rightarrow S, \ \ \ L \rightarrow L+R$
  - Changing L & R have different impacts on inventory

Questions, Comments, Suggestions?
Use the Discussion!

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