Inventory Models for Special Cases: A & C Items and Challenges

MIT Center for Transportation & Logistics

Inventory Management by Segment

	A Items	B Items	C Items
Type of records	Extensive, Transactional	Moderate	None – use a rule
Level of Management Reporting	Frequent (Monthly or more)	Infrequently - Aggregated	Only as Aggregate
Interaction w/ Demand	Direct Input High Data Integrity Manipulate (pricing etc.)	Modified Forecast (promotions etc.)	Simple Forecast at best
Interaction w/ Supply	Actively Manage	Manage by Exception	None
Initial Deployment	Minimize exposure (high v)	Steady State	Steady State
Frequency of Policy Review	Very Frequent (monthly or more)	Moderate (Annually/Event Based)	Very Infrequent
Importance of Parameter Precision	Very High – accuracy worthwhile	Moderate – rounding & approximation is ok	Very Low
Shortage Strategy	Actively manage (confront)	Set service levels & manage by exception	Set & forget service levels
Demand Distribution	Consider alternatives to Normal as situation fits	Normal	N/A

ACTIVE **AUTOMATIC** **PASSIVE**

Inventory Policies By Segment

- No hard and fast rules, but some rules of thumb

When & how to spend more time to manage A' inventory

Type of Item,	Continuous Review	Periodic Review
A Items	(s, S)	(R, s, S)
B Items	(s, Q)	(R, S)
C Items		Manual ~ (R,S)

When & how to spend less time to manage or reduce 'C' inventory

Agenda

- Class A Policies
  - Fast & Slow Moving Items
  - Poisson Distributions
- Class C Policies
- Real-World Implications

Inventory Policies for A Items

Managing Class A Inventory

- When does it make sense to spend more time?
  - Tradeoff between complexity and ‘other’ costs
  - Is the savings worth the extra effort?
- Adding precision
  - Finding ‘optimal’ parameters
  - Using more complex policies

Dictates whether item is Class A or not

$$TC = cD + c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + k\sigma_{DL}\right) + B_1\left(\frac{D}{Q}\right)P[SO]$$

$$TC = cD + c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + k\sigma_{DL}\right) + c_s\left(\frac{D}{Q}\right)\sigma_{DL}G(k)$$

Managing Class A Inventory

- Two Types of Class A items:
  - Fast moving but cheap (large D small c → Q>1)
  - Slow moving but expensive (large c small D → Q=1)

- Impacts the probability distribution used
  - Fast Movers - Normal or Lognormal Distribution
    - Good enough for B items
    - OK for A items if μ_{DL} or $\mu_{DL+R} \geq 10$
  - Slow Movers – Poisson Distribution
    - More complicated to handle
    - Ok for A items if μ_{DL} or $\mu_{DL+R} < 10$

Fast Moving A Items

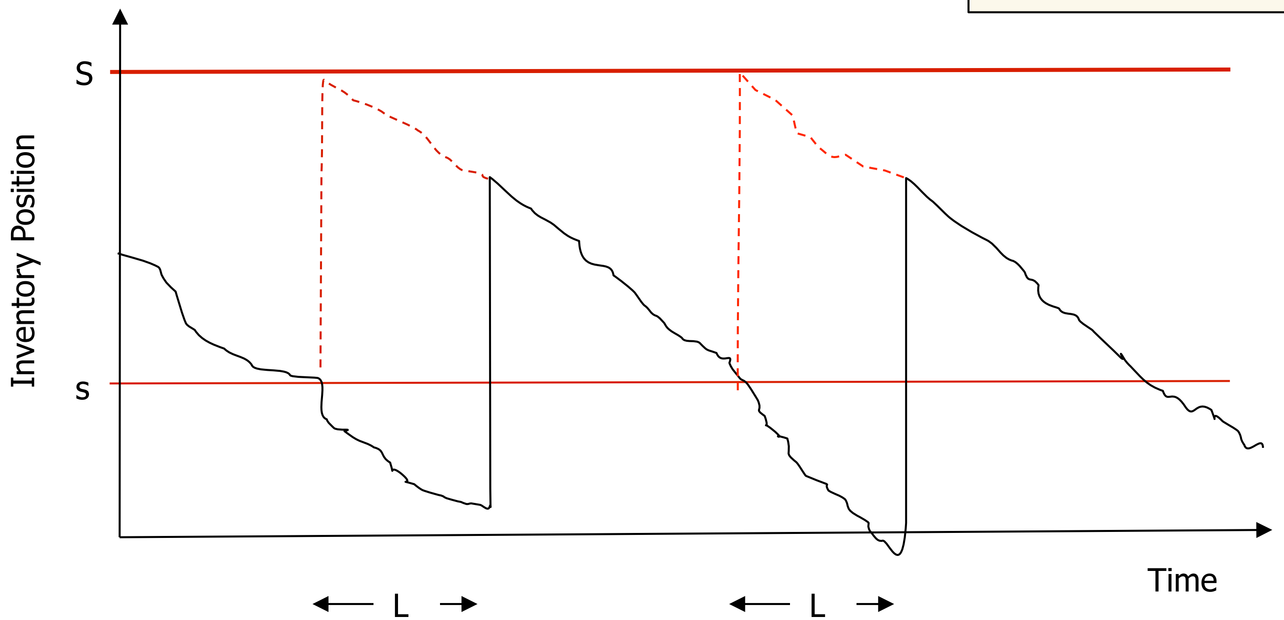
Fast Moving A Items

Order-Point, Order-Up-To-Level (s, S)

- Policy: **Order (S-IP) if IP ≤ s**
- Min-Max system
- Continuous Review

Note on Undershoots:

- Number of units of IP below reorder point, s, at time the order is placed, s-IP
- Only matters if demand is non-unit sized transactions
- If demand is always in units then (s, Q) = (s, S) where Q = S - s



Notation

s = Reorder Point | S = Order-up-to Level | L = Replenishment Lead Time

Q = Order Quantity | R = Review Period | IOH= Inventory on Hand

IP = Inventory Position = (IOH) + (Inventory On Order) – (Backorders)

Fast Moving A Items

- Suppose we have a Cost per Stock Out Event or B_1

$$TRC = c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + k\sigma_{DL}\right) + B_1\left(\frac{D}{Q}\right)P\left[x > k\right]$$

- How did we set (s, Q) policy for B items?
- Sequentially!
  - Set Q = EOQ
  - Found k that minimizes TRC

$$Q^* = \sqrt{\frac{2c_tD}{c_e}}$$

$$k^* = \sqrt{2\ln\left(\frac{DB_1}{\sqrt{2\pi}Qc_e\sigma_{DL}}\right)}$$

- Is it worth looking for better parameters?

Fast Moving A Items

$$TRC = c_t\left(\frac{D}{Q}\right) + c_e\left(\frac{Q}{2} + k\sigma_{DL}\right) + B_1\left(\frac{D}{Q}\right)P\left[x > k\right]$$

- Finding Better Parameters
  - Solve for k* and Q* simultaneously
  - Take partial differentials wrt Q and k
  - End up with two equations

$$Q^* = EOQ\sqrt{1 + \frac{B_1 P\left[x > k\right]}{c_t}}$$

$$k^* = \sqrt{2\ln\left(\frac{DB_1}{\sqrt{2\pi}Qc_e\sigma_{DL}}\right)}$$

- How do we solve it?
  - Iteratively solve the two equations
  - Stop when Q* and k* converge within acceptable range

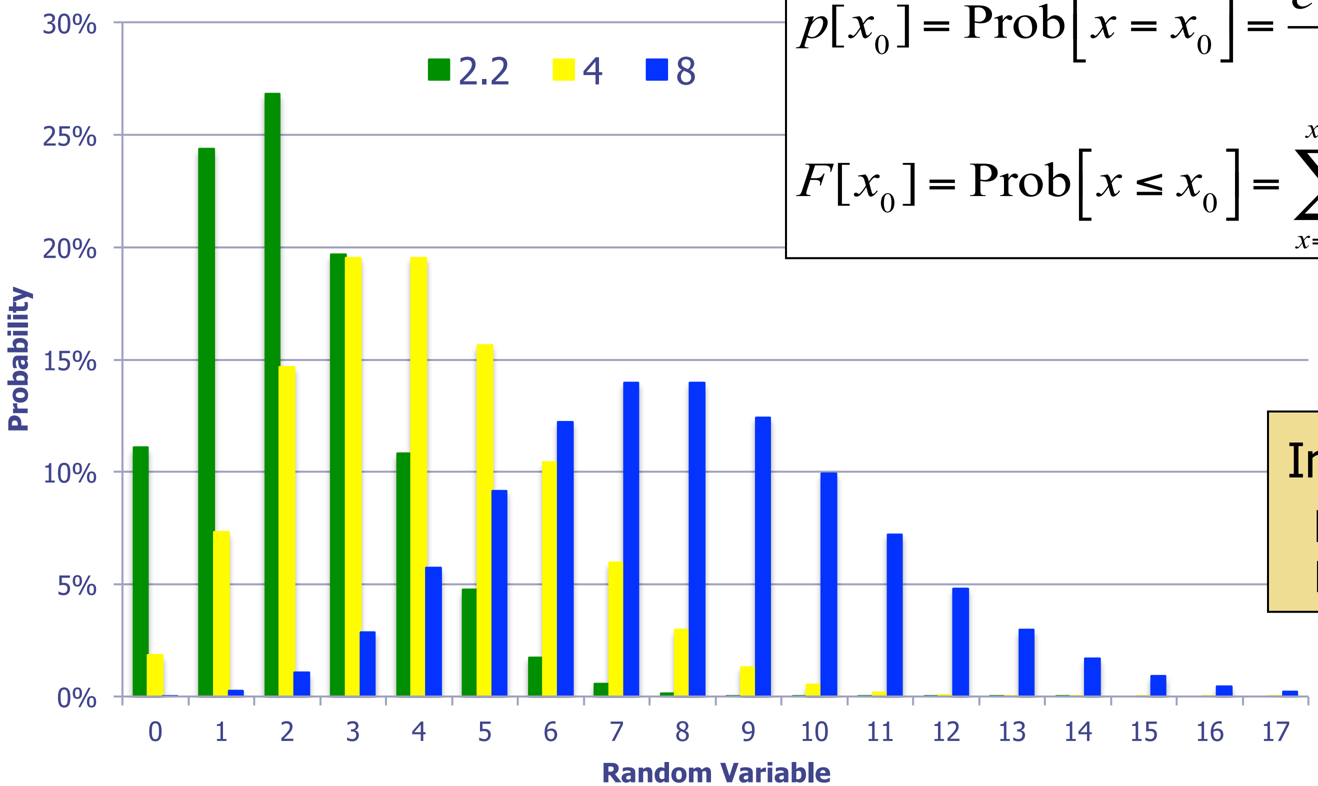
Slow Moving A Items

Slow Moving A Items

- Normal distribution may not make sense – why?
- Poisson distribution
  - Probability of x events occurring w/in a time period
  - Mean = Variance = λ

$$p[x_0] = \text{Prob}\left[x = x_0\right] = \frac{e^{-\lambda}\lambda^{x_0}}{x_0!} \quad for\ x_0 = 0,1,2,\ldots$$

$$F[x_0] = \text{Prob}\left[x \le x_0\right] = \sum_{x=0}^{x_0} \frac{e^{-\lambda}\lambda^{x}}{x!}$$



In Spreadsheets:
$p(x_0)$ = POISSON(x_0,λ,0)
$F(x_0)$ = POISSON(x_0,λ,1)

Example

- Problem:
  - Suppose that you want to set up a (s, Q) policy for an A item. Demand over lead time is Poisson distributed with a mean of 2.6 and you have already determined Q*=6 units. What re-order point would you use if you wanted to achieve a CSL of 95%?

- Solution
  - We want to find:

$$F[x_0] = \sum_{x=0}^{x_0} \frac{e^{-\lambda}\lambda^x}{x!} \geq 0.95$$

  - Simply build a table with pdf and cdf
  - Select s where F[x]≥CSL

Demand	p[x]	F[x]
0	7%	7%
1	19%	27%
2	25%	52%
3	22%	74%
4	14%	88%
5	7%	95%
6	3%	98%
7	1%	99%
8	0%	100%

- But what is the expected IFR?
  - IFR = 1 – E[US]/Q

Order 6 units when IP≤5

Loss Function for Discrete Function

- For any discrete function we can find the loss function, $L[X_i]$, for each value of X given the cumulative probability $F[X_i]$.
- Start with first value
  - $L[X_1] = \text{mean} - X_1$
  - $L[X_2] = L[X_1] - (X_2 - X_1)(1\text{-}F[X_1])$
  - $L[X_3] = L[X_2] - (X_3 - X_2)(1\text{-}F[X_2])$
  - . . . .
  - $L[X_i] = L[X_{i-1}] - (X_i - X_{i-1})(1\text{-}F[X_{i-1}])$
- For our problem:
  - $L[X_1] = L[0] = 2.60 - 0 = 2.60$
  - $L[X_2] = L[1] = 2.60 - (1 - 0)(1 - .074) = 1.67$
  - $L[X_3] = L[2] = 1.67 - (2 - 1)(1 - .267) = .94$
  - etc.

Loss Function for ~P(λ=2.6)

i	Demand (Xi)	p[x]	F[x]	L[x]
1	0	7.4%	7.4%	2.60
2	1	19.3%	26.7%	1.67
3	2	25.1%	51.8%	0.94
4	3	21.8%	73.6%	0.46
5	4	14.1%	87.7%	0.20
6	5	7.4%	95.1%	0.07
7	6	3.2%	98.3%	0.02
8	7	1.2%	99.5%	0.01
9	8	0.4%	99.9%	0.00

At s=5, E[US] = 0.07
IFR=1-(0.07/6) = 98.8%

What s for IFR=80%
since E[US]=Q(1-IFR) = 1.2
then select s=2

Method adapted from Cachon & Terwiesch (2005), Matching Supply & Demand

Managing Class C Inventories

Managing Class C Inventories

- What are C items?
  - Typically low cD values
  - Large number, low total value items
  - Need to consider implicit & explicit costs

- Objective: minimize management attention
  - Regardless of policy, savings not significant
  - Design simple rules to follow
  - Explore opportunities for disposing of inventory

Material adopted from Silver, Pyke, & Peterson (1999), Inventory Management and Production Planning

Simple Reorder Rules

- Set Common Reorder Quantities
  - Assume common c_t and h values
  - Find $D_i c_i$ values for ordering frequencies
  - Example:
    - Select between monthly, quarterly, semi-annual, or annual so that $w_1=1$, $w_2=3$, $w_3=6$, $w_4=12$

$$c_t D_i/Q_{i1} + (c_i h Q_{i1})/2 = c_t D_i/Q_{i2} + (c_i h Q_{i2})/2$$

$$12 c_t D_i/D_i w_1 + c_i h D_i w_1/24 = 12 c_t D_i/D_i w_2 + c_i h D_i w_2/24$$

$$(c_i h D_i/24)(w_1 - w_2) = (12 c_t)(1/w_2 - 1/w_1)$$

$$D_i c_i = [(24)(12 c_t)/(h(w_1 - w_2))]\ (1/w_2 - 1/w_1)$$

$$D_i c_i = 288 c_t / (h w_1 w_2)$$

Rule if $D_i c_i \geq 96(c_t/h)$ then order Monthly

Else: if $D_i c_i \geq 16(c_t/h)$ then order Quarterly

Else: if $D_i c_i \geq 4(c_t/h)$ then order Semi-Annually

Else: Order Annually

Disposing of Excess Inventory

- Why does excess inventory occur?
  - SKU portfolios tend to grow
  - Poor forecasts - Shorter lifecycles

- Which items to dispose?
  - Look at DOS (days of supply) for each item = IOH/D
  - Consider getting rid of items that have DOS > x years

- What actions to take?
  - Convert to other uses
  - Ship to more desired location
  - Mark down price
  - Auction

Real-World Challenges

Sadly the world is not so simple

- Reality is often ugly!
  - Models are not used exactly as in textbooks
  - Data is not always available or correct
  - Technology matters
  - Business processes matter even more
- Inventory policies try to answer three questions:
  - How often should I check my inventory?
  - How do I know if I should order more?
  - How much to order?
- All inventory models use two key numbers
  - Inventory Position
  - Order Point

Material adopted from Blanco, E. E. (2005), MIT Course Notes, ESD.260 - Logistics Systems

Inventory Position – how much do I have?

Data Collection?
- Number of item-location combinations
  - (10^3 locations)($10^{4\text{-}5}$ SKUs) ≈ **10^8!**
- Database processing power
  - @ ~10^3 transactions/second **> 24 hrs**
- Business process cycles
  - hourly / 3-4x daily / daily / weekly

When is an item "On-Order"?
- Order has been generated by the system?
- Order has been transmitted to the supplier?
- Order has been accepted by the supplier?
- Order has been shipped by the supplier?

Inventory Position (IP) = Inventory On Hand (IOH) + Inventory On Order (IOO) - Backorders & Commitments

Data Integrity?
- Wrong/missing product codes
- "Fat Finger" data entry
- Scanner/reader problems
- Shrinkage & Returns

What is the lead time?

Orders have expected arrival dates, but
- Is it updated? By whom?
- How about partial orders?
- How about multiple vendors?

Challenges?
- Cancellation policy?
- "Phantom Orders"?
- Ordering grace periods

Order Point – when should I place an order?

- Who collects lead times?
- How often are they updated?

- How was forecast error estimated?
- Initial vs. final forecasts?
- Level of detail (SKU-location/aggregate)
- Includes lead time variability?

Order Point (s) = Expected Demand over Lead and Review Time (μ_{DL+R}) + Safety Factor (k) * Std. Dev. of Demand, or Forecast Error, over Lead and Review Time (σ_{DL+R})

- Are forecasts accurate?
- Demand vs. Sales?
- Promotions?
- Trends or Seasonality?

- What demand PDF was assumed?
- Who defines/owns service levels?
- How often are they updated?
- How is this entered into the ERP?

Other Challenges

- Inventory decisions @ item-location level
  - Local "Optimal" $\neq$ System Optimal
  - Are items really independent?

- Technology
  - Homegrown (Legacy) vs. ERP vs Niche systems
  - Parameter configuration & installation (Daunting!)
  - Integration with forecasting systems (data level)
  - Implicit assumptions and parameter updates

- Demand Forecasting
  - Assumed stationary demand
  - Recalculate parameters on a regular basis (s[t], S[t], Q[t])
  - Need to deal with SKU life-cycle

Key Points from Lesson

Key Points from Lesson

- Manage Inventory by Segment
  - Class A items → Active
  - Class B items → Automatic
  - Class C (and lower) items → Passive

- Real-World Challenges
  - Data availability and integrity
  - Technology capabilities and limitations
  - Avoid relying on "magic tools" that no one understands
  - Be cognizant of business process and rules

Questions, Comments, Suggestions?
Use the Discussion!

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