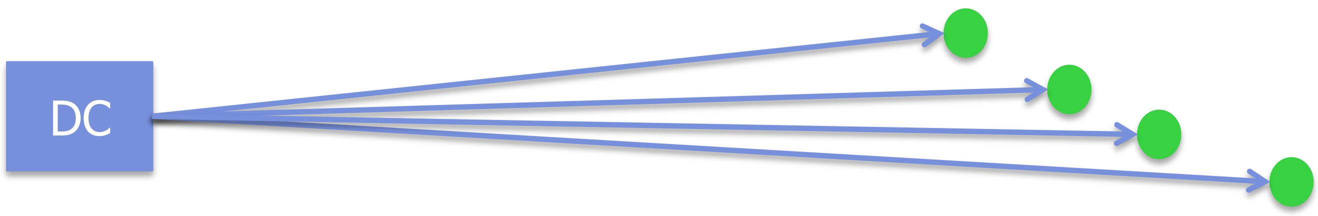
One to Many Distribution

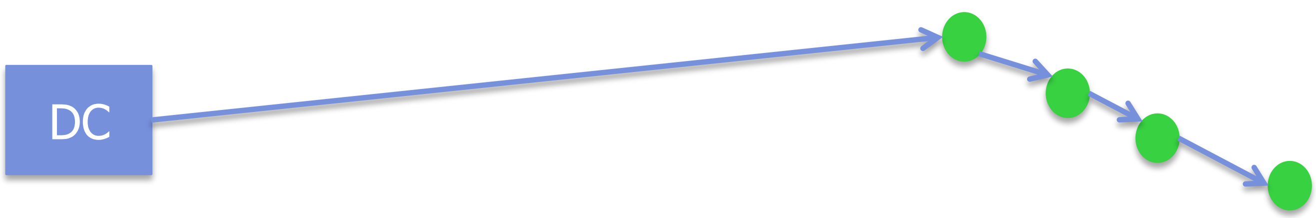
MIT Center for
Transportation & Logistics

How can I distribute products?

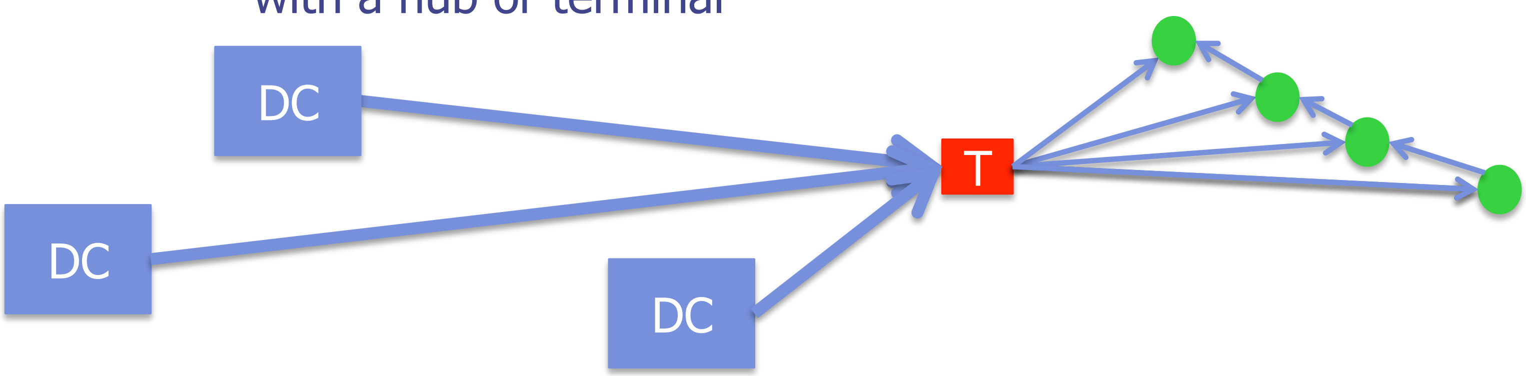
One-to-One – direct or point to point movements from origin to destination



One-to-Many – multi-stop moves from a single origin to many destinations



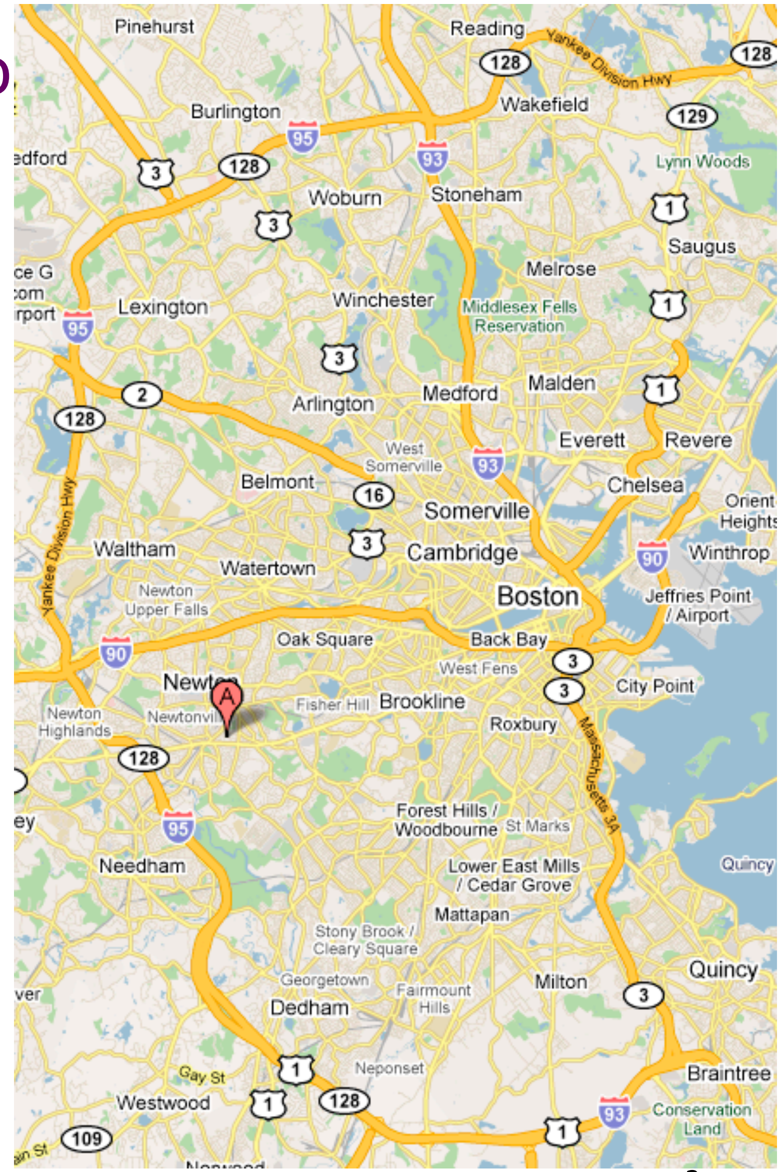
Many-to-Many – moving from multiple origins to multiple destinations usually with a hub or terminal



Example: OfficeMin

- Your firm delivers office supplies to firms within the I-95 highway loop around Boston from your distribution center located in Newton.
- You want to estimate:
  1. Expected cost per day,
  2. Expected truck fleet size, and
  3. Sensitivity of the solution.

- What information do I need?
- What methodology should I use?

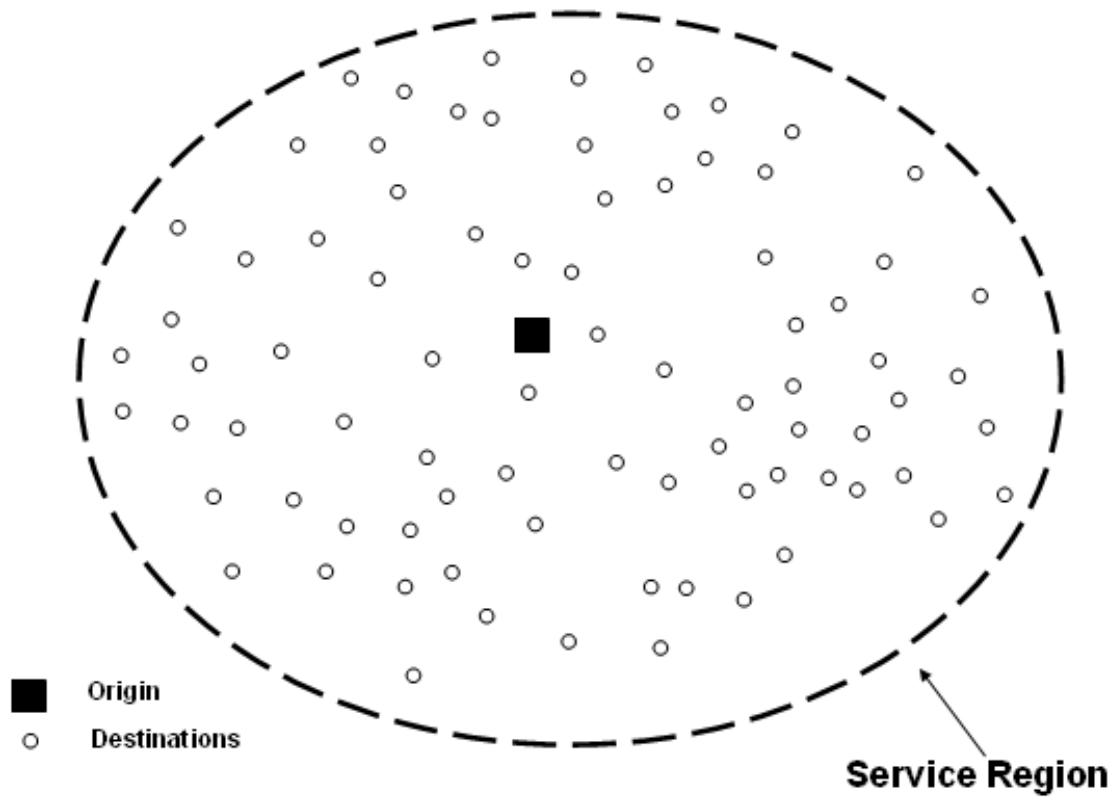


Defining Delivery Districts

One to Many System

Single Distribution Center:
- Products originate from one origin
- Products are demanded at many destinations
- All destinations are within a specified Service Region
- Ignore inventory (same day delivery)

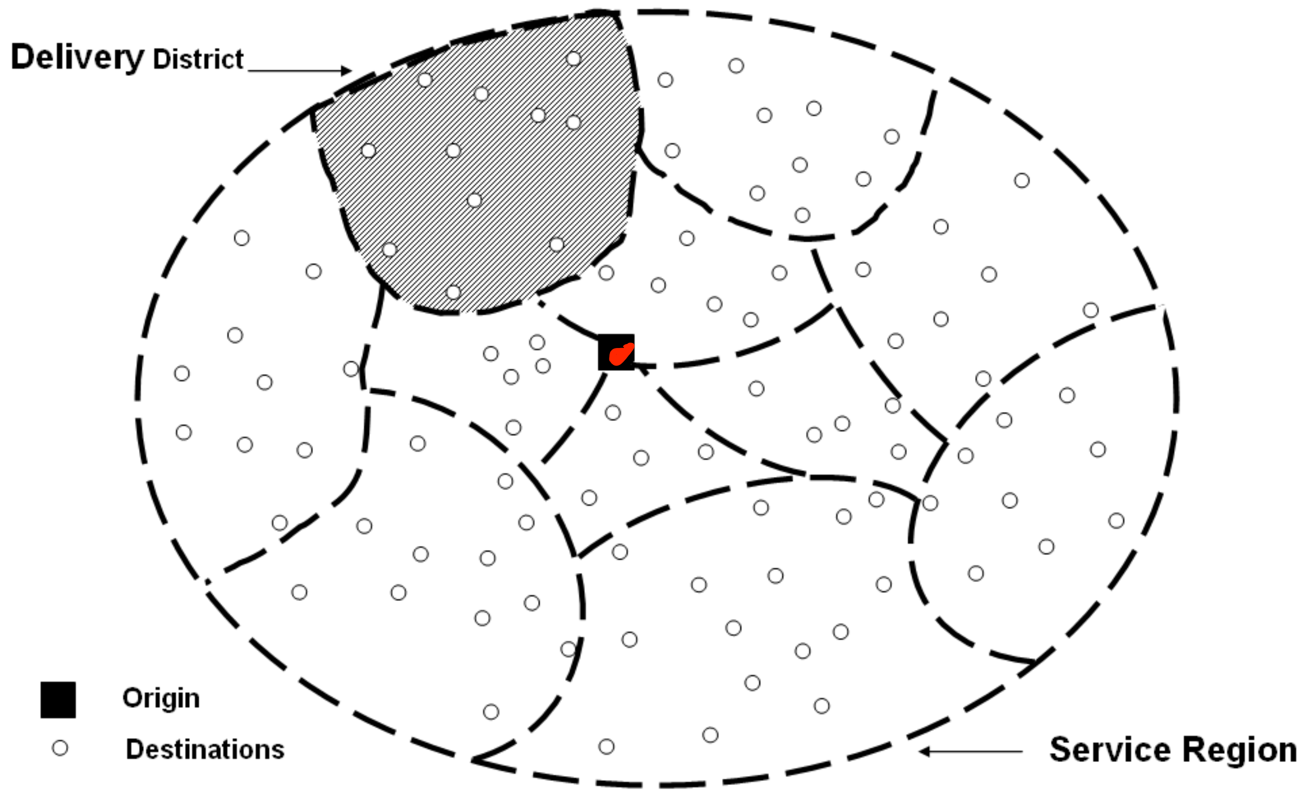
Assumptions:
- Vehicles are homogenous
- Same capacity, Q_{MAX}
- Fleet size is constant



Case adapted from Hernandez Lopez, J.J. (2003) "Evaluation of Bulk and Packaged Distribution Strategies in a Specialty Chemical Company," MIT Supply Chain Management Program Thesis.

One to Many System

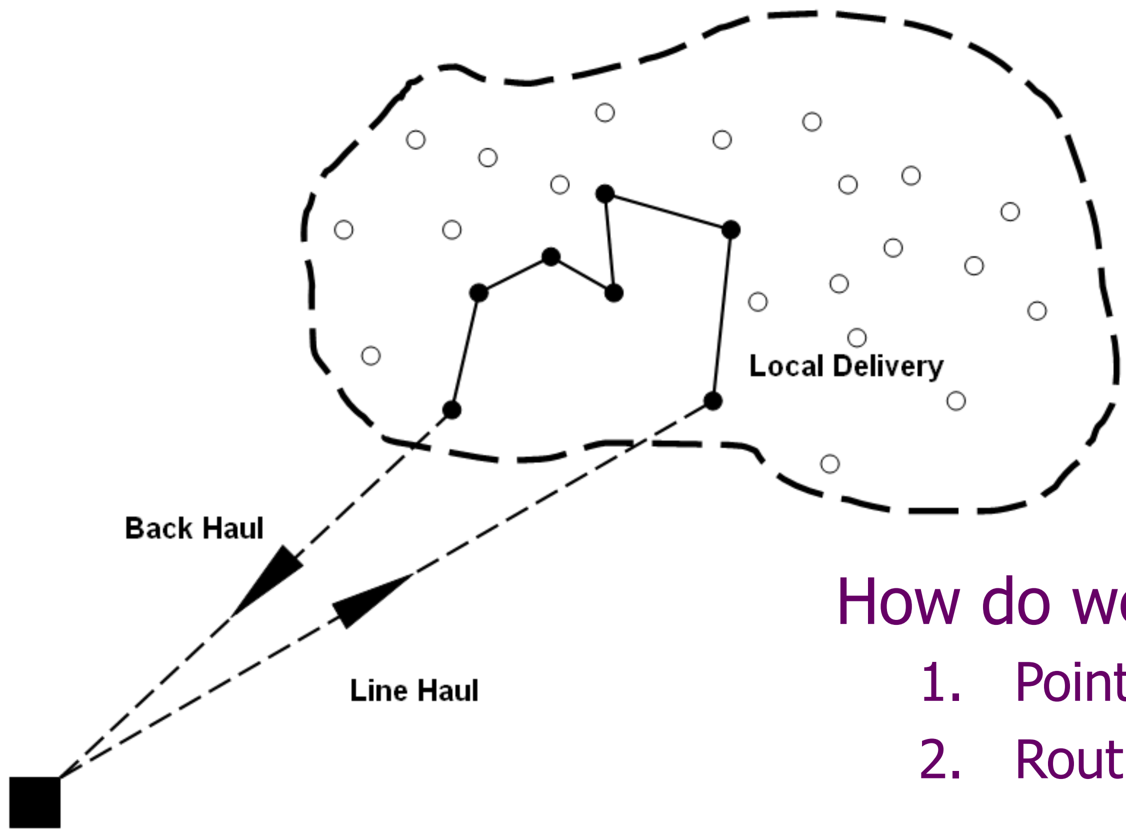
Finding the estimated total distance:
- Divide the Service Region into Delivery Districts
- Estimate the distance required to service each district



One to Many System

Route to serve a specific district:

- Line haul from origin to the 1st customer in the district
- Local delivery from 1st to last customer in the district
- Back haul (empty) from the last customer to the origin



$$d_{TOUR} \approx 2d_{LineHaul} + d_{Local}$$

$d_{LineHaul}$ = Distance from origin to center of gravity (centroid) of delivery district

d_{Local} = Local delivery between c customers in one district

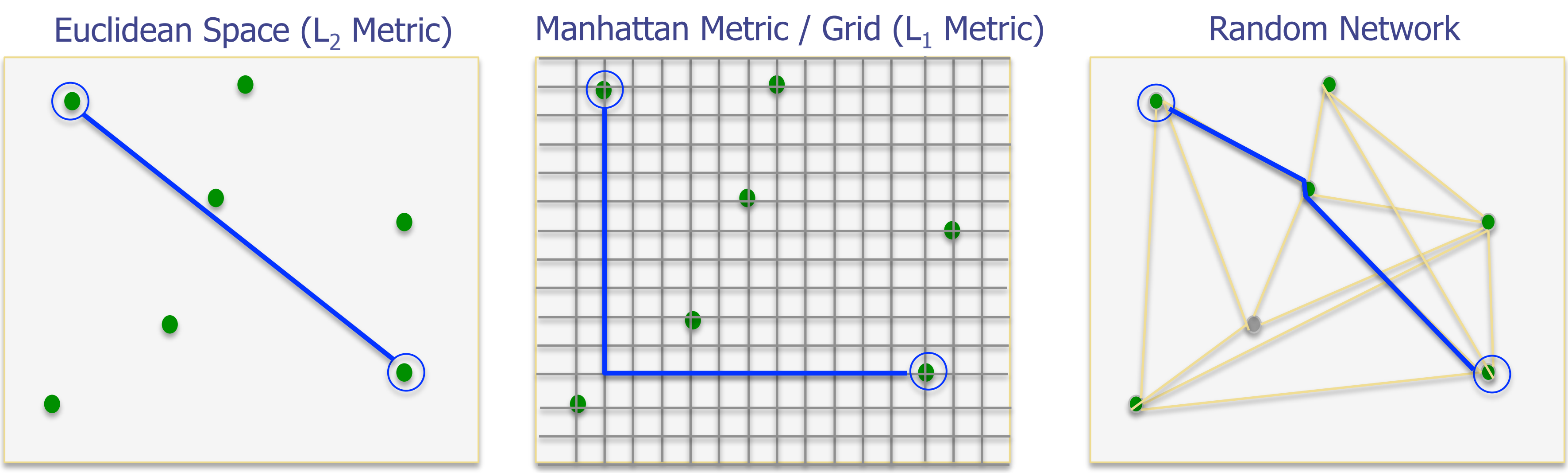
How do we estimate distances?

1. Point to Point
2. Routing or within a Tour

Estimating Point to Point Distances

Distance Estimation: Point to Point

- Why bother?
- How to do it?
  - Depends on the topography of the underlying region
    - Euclidean Space: $d_{A\text{-}B} = \sqrt{[(x_A - x_B)^2 + (y_A - y_B)^2]}$
    - Grid: $d_{A\text{-}B} = |x_A - x_B| + |y_A - y_B|$
    - Random Network: different approach



Distance Estimation: Point to Point

- For Random (real) Networks use: $D_{A-B} = k_{CF}\, d_{A-B}$
- Find d_{A-B} - the "as crow flies" distance.
  - Euclidean: for really short distances
    - $d_{A-B} = SQRT((x_A-x_B)2+(y_A-y_B)2)$
  - Great Circle: for locations within the same hemisphere
    - $d_{A-B} = 3959(\arccos[\sin[LAT_A]\sin[LAT_B] + \cos[LAT_A]\cos[LAT_B]\cos[LONG_A-LONG_B]])$

    Where:
    - LAT_i = Latitude of point i in radians
    - $LONG_i$ = Longitude of point i in radians
    - Radians = (Angle in Degrees)($\pi/180^o$)
- Apply an appropriate circuity factor (k_{CF})
  - How do you get this value?
  - What do you think the ranges are?
  - What are some cautions for this approach?

Selected Values of k_{CF}

Country	k_{CF}	StdDev
Argentina	1.22	0.15
Australia	1.28	0.17
Belarus	1.12	0.05
Brazil	1.23	0.11
Canada	1.30	0.10
China	1.33	0.34
Egypt	2.10	1.96
Europe	1.46	0.58
England	1.40	0.66
France	1.65	0.46
Germany	1.32	0.95
Italy	1.18	0.10
Spain	1.58	0.80
Hungary	1.35	0.25
India	1.31	0.21
Indonesia	1.43	0.34

Country	k_{CF}	StdDev
Japan	1.41	0.15
Mexico	1.46	0.43
New Zealand	2.05	1.63
Poland	1.21	0.09
Russia	1.37	0.26
Saudi Arabia	1.34	0.19
South Africa	1.23	0.12
Thailand	1.42	0.44
Turkey	1.36	0.34
Ukraine	1.29	0.12
United States	1.20	0.17
Alaska	1.79	0.87
US East	1.20	0.16
US West	1.21	0.17

Source: Ballou, R. (2002) "*Selected country circuity factors for road travel distance estimation," Transportation Research Part A, p 843-848.*

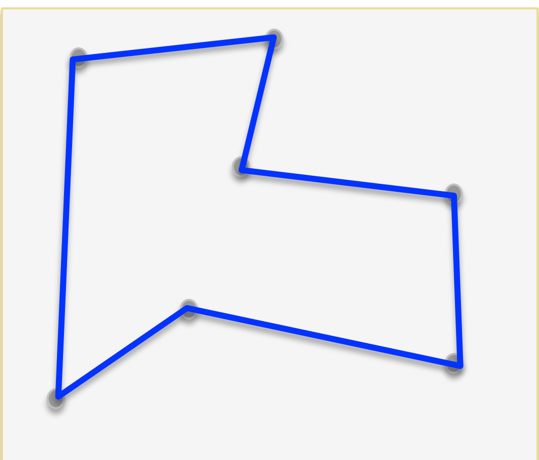
Estimating Route Distances

Distance Estimation: Routing

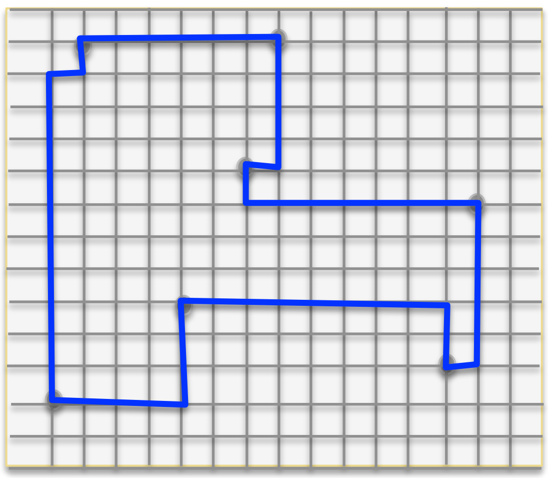
$d_{TSP} = k_{TSP} \sqrt{(nA)}$

- Traveling Salesman Problem
  - Starting from an origin, find the minimum distance required to visit each destination once and only once and return to origin.
  - The expected TSP distance, d_{TSP}, is proportional to $\sqrt{(nA)}$ where n= number of stops and A=area of district
  - The factor (k_{TSP}) is a function of the topology

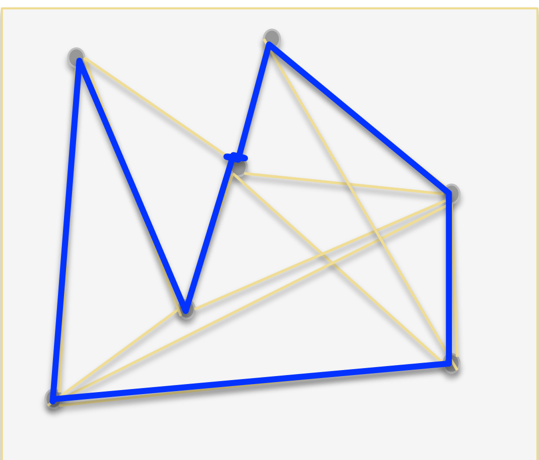
Euclidean Space (L_2 Metric)



Manhattan Metric / Grid (L_1 Metric)



Random Network



One to Many System

A=Area of district
n=Number of stops in district
δ=Density (# stops/Area)
k=VRP network factor (unitless)
d_{TSP}=Traveling Salesman Distance
d_{stop}=Average distance per stop

- What can we say about the expected TSP distance to cover n stops in district with an area of A?
- A good approximation, assuming a "fairly compact and fairly convex" area, is:

$$E\left[d_{TSP}\right] \approx k_{TSP}\sqrt{nA} = k_{TSP}\sqrt{n\left(\frac{n}{\delta}\right)} = \frac{k_{TSP}n}{\sqrt{\delta}}$$

- What values of k_{TSP} should we use?
  - Lots of research on this for L_1 and L_2 networks - depends on district shape, approach to routing, etc.
  - Euclidean (L_2) Networks
    - k_{TSP} = 0.57 to 0.99 depending on clustering & size of N (MAPE~4%, MPE~-1%)
    - k_{TSP}=0.765 commonly used
  - Grid (L_1) Networks
    - k_{TSP} = 0.97 to 1.15 depending on clustering and partitioning of district

References: Daganzo, C.. (2010) Logistics Systems Analysis, 4th Edition Springer-Verlag.
Larson, R. and Odoni, A. (1981) Urban Operations Research, http://web.mit.edu/urban_or_book/www/book/

Estimating Tour Distances

Estimating Tour Distance

- Finding the total distance traveled on all tours, where:
  - l = number of tours
  - c = number of customer stops per tour and
  - n=total number of stops = c*l

$$E\left[d_{TOUR}\right] = 2d_{LineHaul} + \frac{ck_{TSP}}{\sqrt{\delta}}$$

$$E\left[d_{AllTours}\right] = lE\left[d_{TOUR}\right] = 2ld_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}$$

- Minimize number of tours by maximizing vehicle capacity

$$l = \left[\frac{D}{Q_{MAX}}\right]^{+}$$

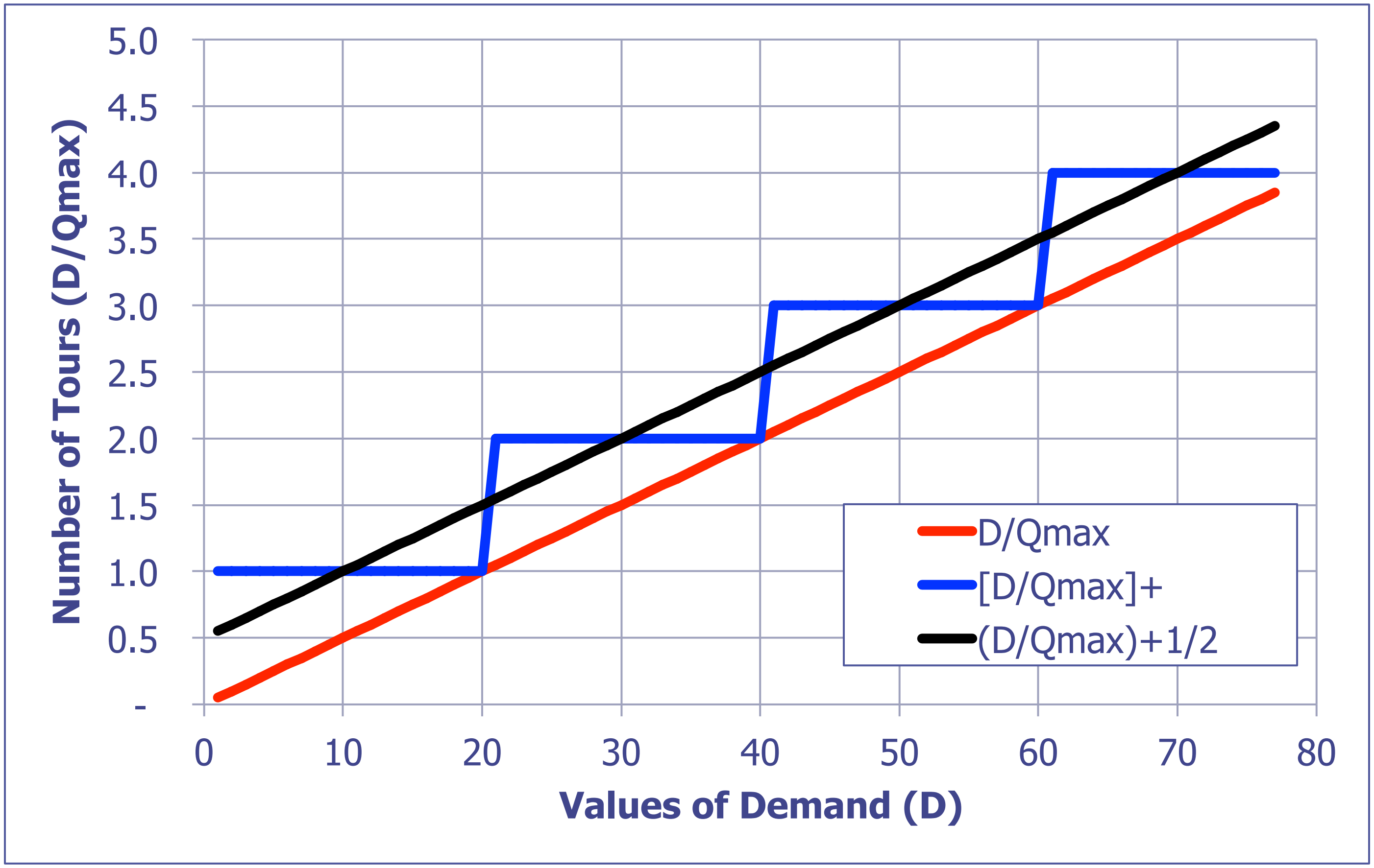
$$E\left[d_{AllTours}\right] = 2\left[\frac{D}{Q_{MAX}}\right]^{+} d_{LineHaul} + \frac{nk_{TSP}}{\sqrt{\delta}}$$

$[x]^{+}$ = lowest integer value > x.
This is a step function

Estimate this with continuous function:
$E([x]^{+}) \sim E(x) + ½$

Continuous Approximation

In this example, Q_{MAX}=20. The number of tours, l, would be $[D/Q_{MAX}]^{+}$ which is a step function. Step functions are not continuous – lets create a continuous approximation of this function that we can use.



One to Many System

- So that expected distance for all tours becomes:

$$E\left[d_{AllTours}\right] = 2\left[\frac{E\left[D\right]}{Q_{MAX}}\right]^{+} d_{LineHaul} + \frac{E\left[n\right]k_{TSP}}{\sqrt{\delta}} = 2\left[\frac{E\left[D\right]}{Q_{MAX}} + \frac{1}{2}\right]d_{LineHaul} + \frac{E\left[n\right]k_{TSP}}{\sqrt{\delta}}$$

- Note that if each delivery district has a different density, then:

$$E\left[d_{AllTours}\right] = 2\sum_i \left[\frac{E\left[D_i\right]}{Q_{MAX}} + \frac{1}{2}\right]d_{LineHaul_i} + k_{TSP}\sum_i \frac{E\left[n_i\right]}{\sqrt{\delta_i}}$$

Putting it all together

For identical districts, the approximate transportation cost to deliver to each customer becomes:

Expected number of loads/unloads (customer + origin)

Expected number of linehaul moves or tours

Expected distance for the linehaul portion. How do I get this?

Expected local delivery distance regardless of number of tours

$$TransportCost = c_s\left[E[n] + \frac{E[D]}{Q_{MAX}} + \frac{1}{2}\right] + c_d\left(2\left[\frac{E[D]}{Q_{MAX}} + \frac{1}{2}\right]d_{LineHaul} + \frac{E[n]k_{TSP}}{\sqrt{\delta}}\right) + c_{vs}E[D]$$

Cost per item per stop

$E[n]$=Expected number of stops in district
$E[D]$=Expected demand in district
Q_{MAX}=Capacity of each truck
c_s=Cost per stop ($/stop)
c_d=Cost per distance ($/mile)
c_{vs}=Cost per unit per stop ($/item-stop)

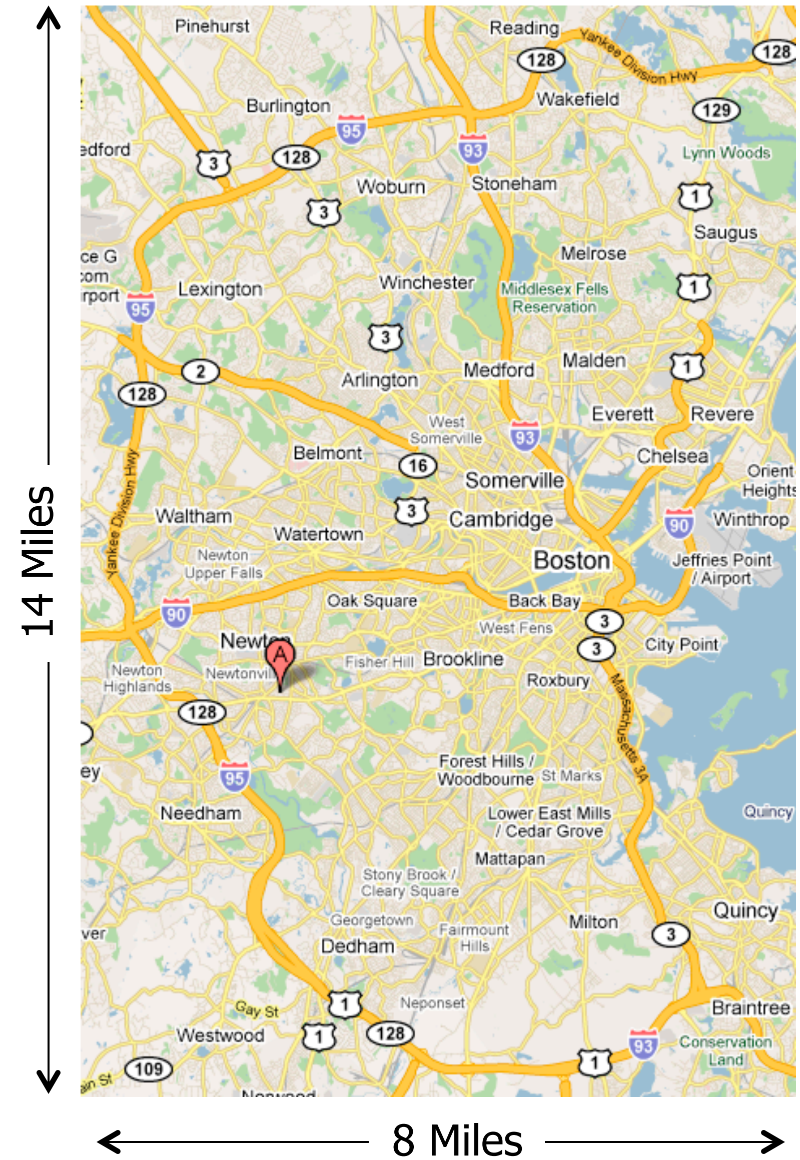
δ=Density (# stops/Area)
k_{TSP} =TSP network factor (unitless)
d_{TSP}=Traveling Salesman Distance
d_{stop}=Average distance per stop

Solution OfficeMin

OfficeMin Problem

- You deliver office supplies to firms within the I95 loop around Boston from your DC in Newton. This region is about 8 miles by 14 miles.
- You expect ~ 100 customer orders per day – for about 1 to 2 pallets of product each. Local vans can handle 5 pallets at most.
- You estimate it costs about $10 per stop (to load or unload), about $5 per pallet to deliver to end customer, and about $1 a mile for driving.

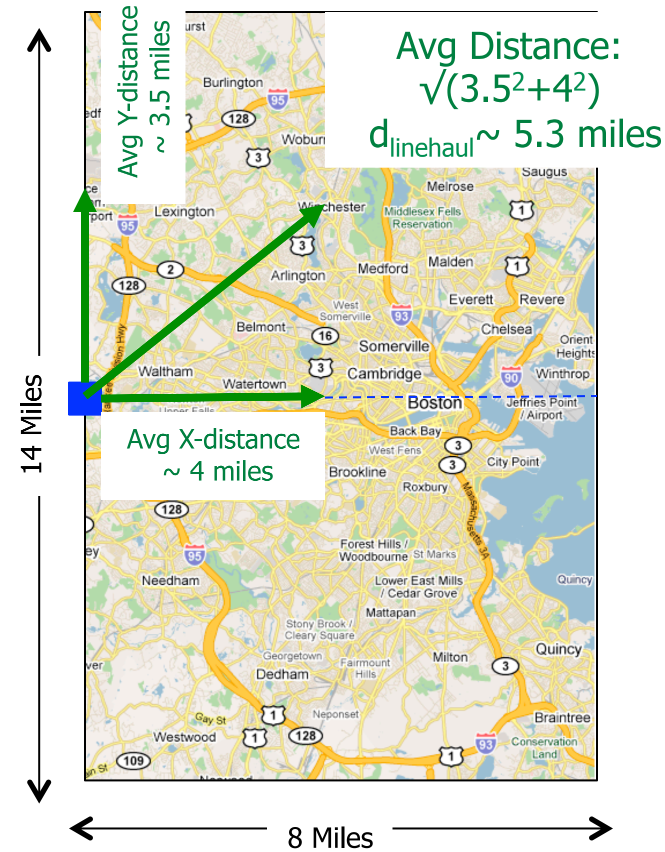
- What is the expected daily transportation cost?



OfficeMin

$$TransportCost = c_s\left[E[n]+\frac{E[D]}{Q_{MAX}}+\frac{1}{2}\right]+c_d\left(2\left[\frac{E[D]}{Q_{MAX}}+\frac{1}{2}\right]d_{LineHaul}+\frac{E[n]k_{TSP}}{\sqrt{\delta}}\right)+c_{vs}E[D]$$

- What do we know?
  - c_s = 10 \$/stop
  - c_d = 1 \$/mile
  - c_{vs} = 5 \$/pallet
  - E[n] = 100
  - E[D] = 150
  - Q_{MAX} = 5 pallets
- What do we need to find?
  - k = 1.15 (estimate)
  - δ = 100/(8)(14) = 0.89 ~ 1
  - $d_{linehaul}$ = ??



OfficeMin

c_s = 10 \$/stop	E[n] = 100	k = 1.15
c_d = 1 \$/mile	E[D] = 150	δ = 1
c_{vs} = 5 \$/pallet	Q_{MAX} = 5 pallets	$d_{linehaul}$ = 5

$$TransportCost = c_s\left[E[n]+\frac{E[D]}{Q_{MAX}}+\frac{1}{2}\right]+c_d\left(2\left[\frac{E[D]}{Q_{MAX}}+\frac{1}{2}\right]d_{LineHaul}+\frac{E[n]k_{TSP}}{\sqrt{\delta}}\right)+c_{vs}E[D]$$

Estimated number of tours per day:

$$l=\frac{E[D]}{Q_{MAX}}+\frac{1}{2}=\frac{150}{5}+.5=30.5$$

Estimated stop (load/unload) cost per day:

$$c_s\left[E[n]+E[l]\right]=10(100+30.5)=\$1305$$

Estimated distance (driving) cost per day:

$$c_d\left(2E[l]d_{LineHaul}+\frac{E[n]k_{TSP}}{\sqrt{\delta}}\right)=1\left(2(30.5)(5)+\frac{100(1.15)}{\sqrt{1}}\right)=305+115=\$420$$

Estimated stop-pallet costs per day:

$$c_{vs}E[D]=5(150)=\$750$$

Estimated total daily cost ~ \$2400 to \$2500

Estimating Fleet Size
[OPTIONAL – MORE ADVANCED]

Estimating the Fleet Size

- Find minimum number of vehicles required based on the amount of required work time each day where
  - M = minimum number of vehicles needed in fleet
  - t_w = available worktime for each vehicle per period
  - W = required amount of work time each day
  - s = average vehicle speed
  - l = number of shipments per period
  - t_l = loading time per shipment
  - t_s = unloading time per stop

$$Mt_w \geq W$$

$$W = \frac{d_{AllTours}}{s} + lt_l + nt_s$$

$$W = \frac{\left(2E\left[l\right]d_{LineHaul} + \frac{E\left[n\right]k_{TSP}}{\sqrt{\delta}}\right)}{s} + E\left[l\right]t_l + E\left[n\right]t_s$$

$$W = \left(\frac{2d_{LineHaul}}{s} + t_l\right)E\left[l\right] + E\left[n\right]\left(\frac{k_{TSP}}{s\sqrt{\delta}} + t_s\right)$$

Fleet Size

c_s = 10 \$/stop	E[n] = 100	k = 1.15	t_w = 10 hrs
c_d = 1 \$/mile	E[D] = 150	δ = 1	s = 45 mph
c_{vs} = 5 \$/pallet	Q_{MAX} = 5 pallets	$d_{linehaul}$ = 5	t_l = 0.5 hr
			t_s = 0.5 hr

Note that W is a linear combination of two random variables, **n** and **D.** But, they are not independent, in fact, $D = nD_c$ where D_c is the number of pallets per customer

$$W = \left(\frac{2d_{LineHaul}}{s} + t_l\right)\left[\frac{D}{Q_{MAX}} + \frac{1}{2}\right] + n\left(\frac{k_{TSP}}{s\sqrt{\delta}} + t_s\right)$$

$$E\left[D_c\right] = 1.5$$

$$Var[D_c] = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n} = 0.25$$

$$a = \left(\frac{2d_{LineHaul}}{s} + t_l\right)\left[\frac{1}{Q_{MAX}}\right] = 0.144\,hrs$$

$$b = \left(\frac{k_{TSP}}{s\sqrt{\delta}} + t_s\right) = 0.525\,hrs$$

$$c = \frac{1}{2}\left(\frac{2d_{Linehaul}}{s} + t_l\right) = 0.361\,hrs$$

$$W = aD + bn + c$$
$$W = aD_c n + bn + c$$
$$W = (aD_c + b)n + c$$

Setting $X=aD_c+b$, W is now a function of random variables, X and n.

$$W = Xn + c$$
$$E[W] = E[X]E[n] + c$$
$$Var[W] = E[n]Var[X] + E[X]^2 Var[n]$$

Fleet Size

c_s = 10 \$/stop	E[n] = 100	k = 1.15	t_W = 10 hrs
c_d = 1 \$/mile	E[D] = 150	δ = 1	s = 45 mph
c_{vs} = 5 \$/pallet	Q_{MAX} = 5 pallets	$d_{linehaul}$ = 5	t_l = 0.5 hr
			t_s = 0.5 hr

$$W = \left(\frac{2d_{LineHaul}}{s} + t_l\right)\left[\frac{D}{Q_{MAX}} + \frac{1}{2}\right] + n\left(\frac{k_{TSP}}{s\sqrt{\delta}} + t_s\right)$$

$$a = 0.144\,hrs \quad b = 0.525\,hrs \quad c = 0.361\,hrs$$

$$E[D_c] = 1.5\ pallets \quad Var[D_c] = 0.25$$

$$W = (aD_c + b)n + c = (0.144D_c + 0.525)n + 0.361 = Xn + c$$

$$E\left[X\right] = aE\left[D_c\right] + b = 0.144(1.5) + 0.525 = 0.741$$

$$Var[X] = a^2Var[D_c] = 0.021(0.25) = 0.00525$$

$$E[n] = 100\ customers$$

$$Var[n] = 400\ (assume\ \sigma_n = 20)$$

$$E[W] = E\left[X\right]E\left[n\right] + c$$

$$E[W] = (0.741)(100) + 0.361 = 74.46$$

$$Var[W] = E\left[n\right]Var\left[X\right] + E\left[X\right]^2 Var\left[n\right]$$

$$Var[W] = (100)(0.00525) + (0.741)^2(400) = 220$$

Distribution of required daily work hours:
μ_W ~ 75 hrs
σ_W ~ 15 hrs

Fleet Size

- Daily distribution of required work time ~N(75, 15)
- Set the fleet size (M) to match our level of risk – how?

Note: This is not the TSP k!

- Select a cycle service level (CSL) equal to $P[W<Mt_w]$
  - Set M= $(\mu_W + k_{CSL}\,\sigma_W) / t_w$
  - M(80%) = (75 hrs + 0.84(15 hrs))/ (10 hrs/veh) = 8.00 = 8
  - M(90%) = (75 hrs + 1.28(15 hrs))/ (10 hrs/veh) = 9.42 = 10
  - M(95%) = (75 hrs + 1.64(15 hrs))/ (10 hrs/veh) = 9.96 = 10
  - M(99%) = (75 hrs + 2.33(15 hrs))/ (10 hrs/veh) = 10.99 = 11

- Using very few, very rough estimates of input values, we can get a feel for the trade-offs between costs and service.
- Approximations can be used for sensitivity analysis.

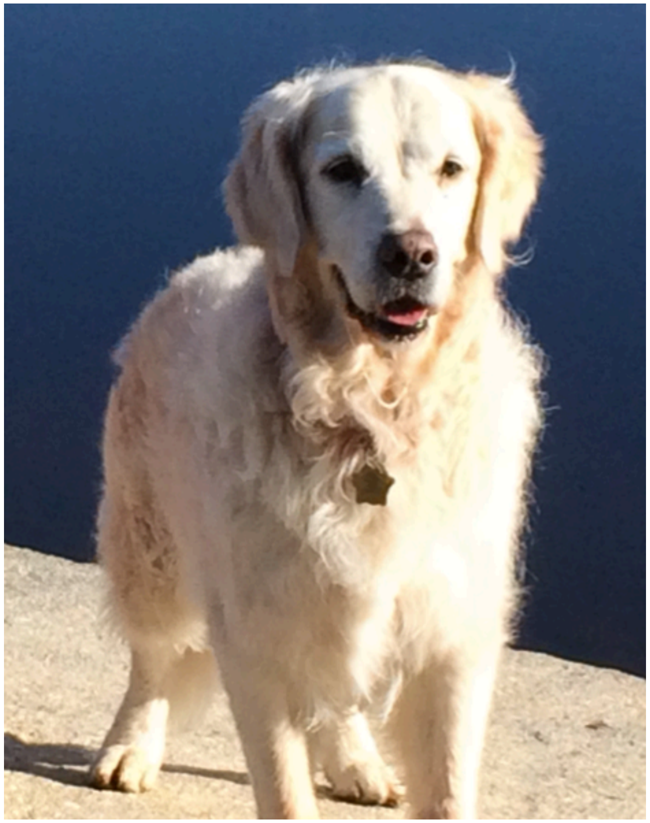
Key Take Aways

Key Take Aways

- Many forms of product flow: One-to-One, One-to-Many, Many-to-One, and Many-to-Many
- Approximations are good "first steps"
  - Require minimal data
  - Allow for fast sensitivity analysis
  - Enables quick scoping of the solution space
- Optimal methods usually require tremendous amounts of detailed information
- Planning problems usually have lots of uncertainty – the actual conditions are unknown
- Before deciding to spend the time and energy to find an optimal solution, it is helpful to see if it is worth it.

CTL.SC1x -Supply Chain & Logistics Fundamentals

Questions, Comments, Suggestions? Use the Discussion!



"Dexter – continuously approximating"
Yankee Golden Retriever Rescued Dog
(www.ygrr.org)

MIT Center for Transportation & Logistics

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