Inventory Models for Special Cases: Multiple Items & Locations

MIT Center for Transportation & Logistics

Agenda

- Inventory Policies for Multiple Items
  - Grouping Like Items
  - Exchange Curves
- Inventory Policies for Multiple Locations
  - Location Pooling

Model Assumptions: >1 Items

- Demand
  - Constant vs **Variable**
  - Known vs **Random**
  - **Continuous** vs Discrete
- Lead Time
  - Instantaneous
  - **Constant** vs Variable
  - **Deterministic** vs Stochastic
  - Internally Replenished
- Dependence of Items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous** vs Periodic
- Number of Locations
  - **One** vs Multi vs Multi-Echelon
- Capacity / Resources
  - **Unlimited**
  - Limited / Constrained
- Discounts
  - **None**
  - All Units vs Incremental vs One Time
- Excess Demand
  - None
  - All orders are backordered
  - **Lost orders**
  - Substitution
- Perishability
  - **None**
  - Uniform with time
  - Non-linear with time
- Planning Horizon
  - Single Period
  - Finite Period
  - **Infinite**
- Number of Items
  - One vs **Many**
- Form of Product
  - **Single Stage**
  - Multi-Stage

Managing Multiple Items

- What are the problems with managing items independently?
  - Lack of coordination – constantly ordering items
  - Ignores common constraints (e.g., financial budget or space)
  - Missed opportunities for consolidation / synergies
  - Waste of management time

- Two Issues to Solve
  - Can we aggregate SKUs to use similar operating policies?
    - Group using common cost characteristics or break points
    - Group using Power of Two Policies
  - How do we manage inventory under common constraints?
    - Exchange Curves for Cycle Stock
    - Exchange Curves for Safety Stock

Grouping Like Items – Break Points

Grouping Like Items – Break Points

- Basic Idea: Replenish higher value items faster

- Used for situations with multiple items that have:
  - Relatively stable demand
  - Common ordering costs, c_t, and holding charges, h
  - Different annual demands, D_i, and purchase costs, c_i

- Approach –
  - Pick a base time period, w_0, (typically a week)
  - Create a set of candidate ordering periods (w_1, $w_{2,}$ etc.)
  - Find D_ic_i values where $TRC(w_j)=TRC(w_{j+1})$
  - Group SKUs with that fall in common value (D_ic_i) buckets

Grouping Like Items - Example

- Selected w_0 = 1 week
- Number of weeks of supply (WOS) to order for item i ordering at time period j = $Q_{ij} = D_i(w_j/52)$
- Selecting between options w_1 & w_2 (where $w_1<w_2<w_3$ etc.) becomes:

$$c_t D_i/Q_{i1} + (c_i h Q_{i1})/2 = c_t D_i/Q_{i2} + (c_i h Q_{i2})/2$$

$$52c_t D_i/D_i w_1 + c_i h D_i w_1/104 = 52c_t D_i/D_i w_2 + c_i h D_i w_2/104$$

$$(c_i h D_i/104)(w_1-w_2) = (52c_t)(1/w_2 - 1/w_1)$$

$$D_i c_i = [(104)(52c_t)/(h(w_1-w_2))]\,(1/w_2 - 1/w_1)$$

$$D_i c_i = 5408c_t / (hw_1w_2)$$

Rule if $D_i c_i \geq 5408c_t /(hw_1w_2)$ then select w_1

Else: if $D_i c_i \geq 5408c_t /(hw_2w_3)$ then select w_2

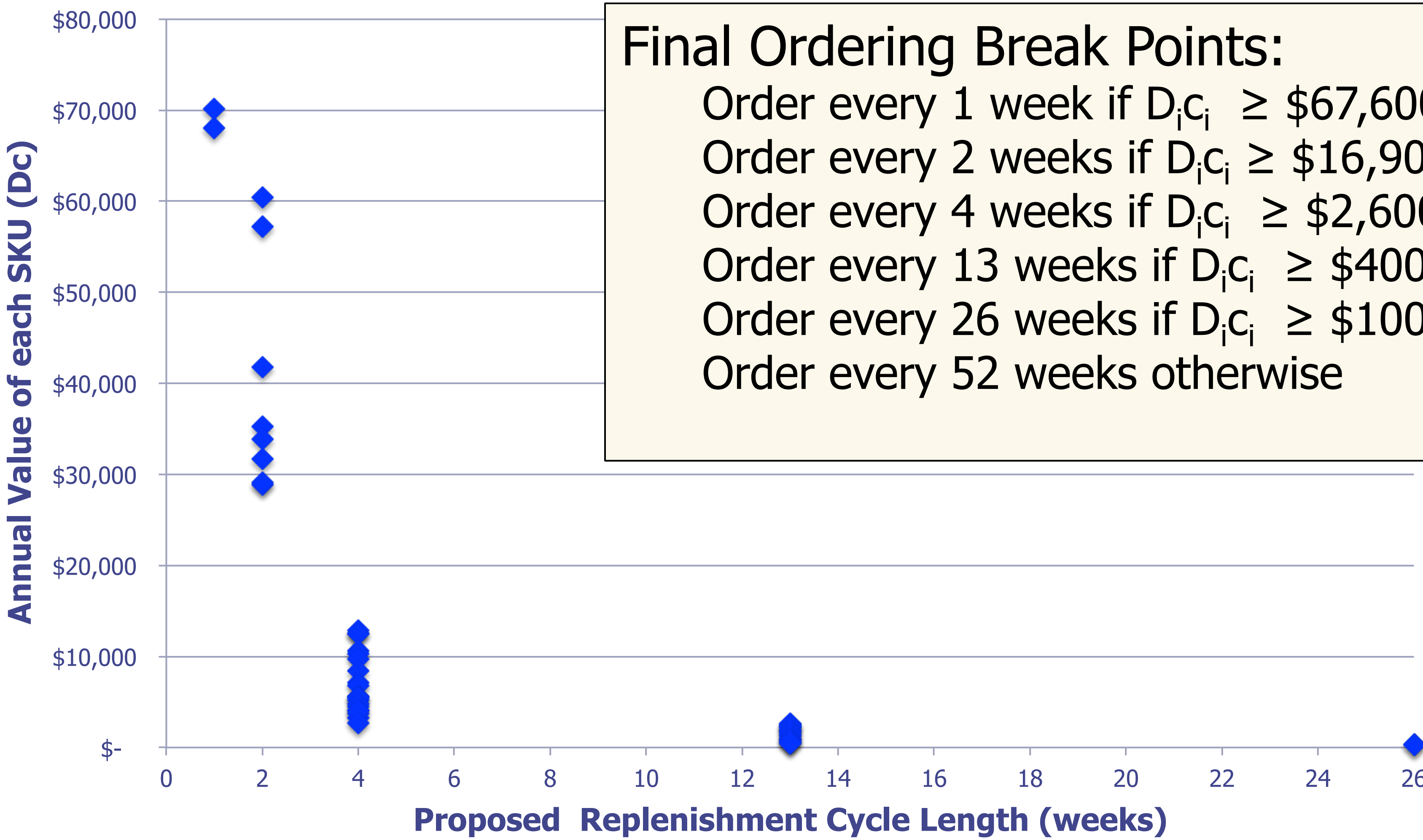
Else: if $D_i c_i \geq 5408c_t /(hw_3w_4)$ then select w_3

Else: . . . .

Grouping Like Items - Example

- Problem:
  - Suppose you need to set up replenishment schedules for several hundred parts that have relatively stable (yet not necessarily the same) demand. They all have similar order costs (c_t = \$5) and holding charge (h = 0.20).
  - You have the following potential ordering periods (in weeks): w_1=1, w_2=2, w_3=4, w_4=13, w_5=26, and w_6=52.
  - What break-even ordering points should you establish?
- Solution:
  - Break-point for selecting between 1 week or 2 weeks is:
    - $D_ic_i = 5408_t / (hw_1w_2) = 5408(5) / (.2)(1)(2) = \$67,600$
    - If $D_ic_i \geq \$67,600$ then order 1 week's worth each week
  - Break-point for selecting between 2 weeks or 4 weeks is:
    - $D_ic_i = 5408c_t / (hw_2w_3) = 5408(5) / (.2)(2)(4) = \$16,900$
    - If $\$67,600 > D_ic_i \geq \$16,900$ then order 2 week's worth every 2 weeks

Grouping Like Items - Example



Final Ordering Break Points:

- Order every 1 week if $D_i c_i \geq \$67,600$
- Order every 2 weeks if $D_i c_i \geq \$16,900$
- Order every 4 weeks if $D_i c_i \geq \$2,600$
- Order every 13 weeks if $D_i c_i \geq \$400$
- Order every 26 weeks if $D_i c_i \geq \$100$
- Order every 52 weeks otherwise

Grouping Using Power of Two Policies

Power of Two Policies

$$\frac{T^*}{\sqrt{2}} \le 2^k \le \sqrt{2}\,T^*$$

$$\frac{\ln\left(\frac{T^*}{\sqrt{2}}\right)}{\ln(2)} \le k \le \frac{\ln\left(T^*\sqrt{2}\right)}{\ln(2)}$$

- Recall from previous lesson:
  - Order in time intervals of powers of two
  - Select a realistic base period, T_{Base} (day, week, month)
  - Guarantees that TRC will be within 6% of optimal!

$$T^* = \frac{Q^*}{D} = \frac{\sqrt{\frac{2c_t D}{c_e}}}{D} = \sqrt{\frac{2c_t}{Dc_e}}$$

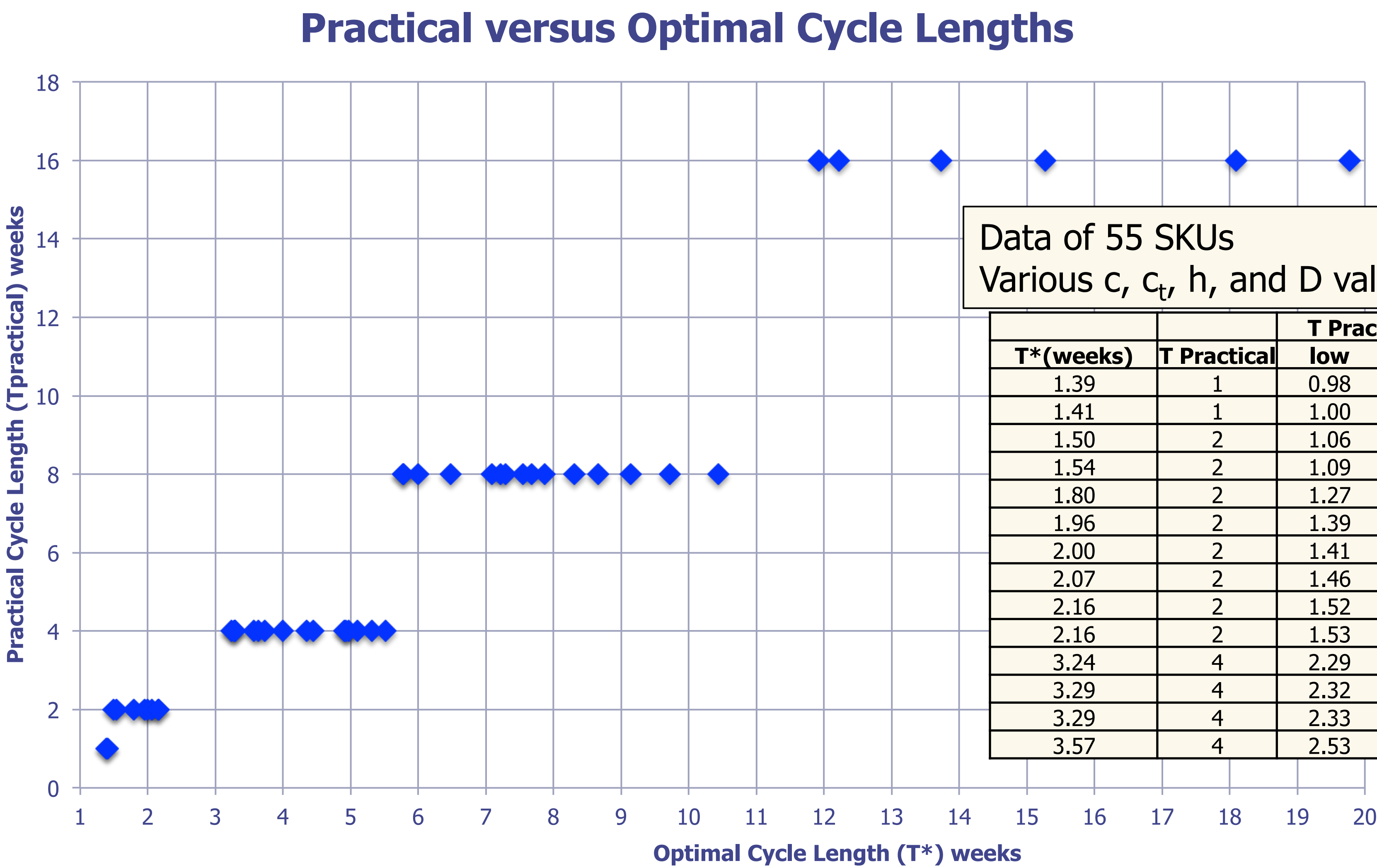
- Simple Process
  - Create table of SKUs
  - Calculate T^* for each SKU
  - Calculate $T_{practical}$ for each SKU

$$T_{practical} = 2^{\left\lceil \frac{\ln\left(\frac{T^*}{\sqrt{2}}\right)}{\ln(2)} \right\rceil}$$

In Spreadsheets:

$T_{practical}$=2^ROUNDUP[LN(T^*/SQRT(2)) / LN(2)]

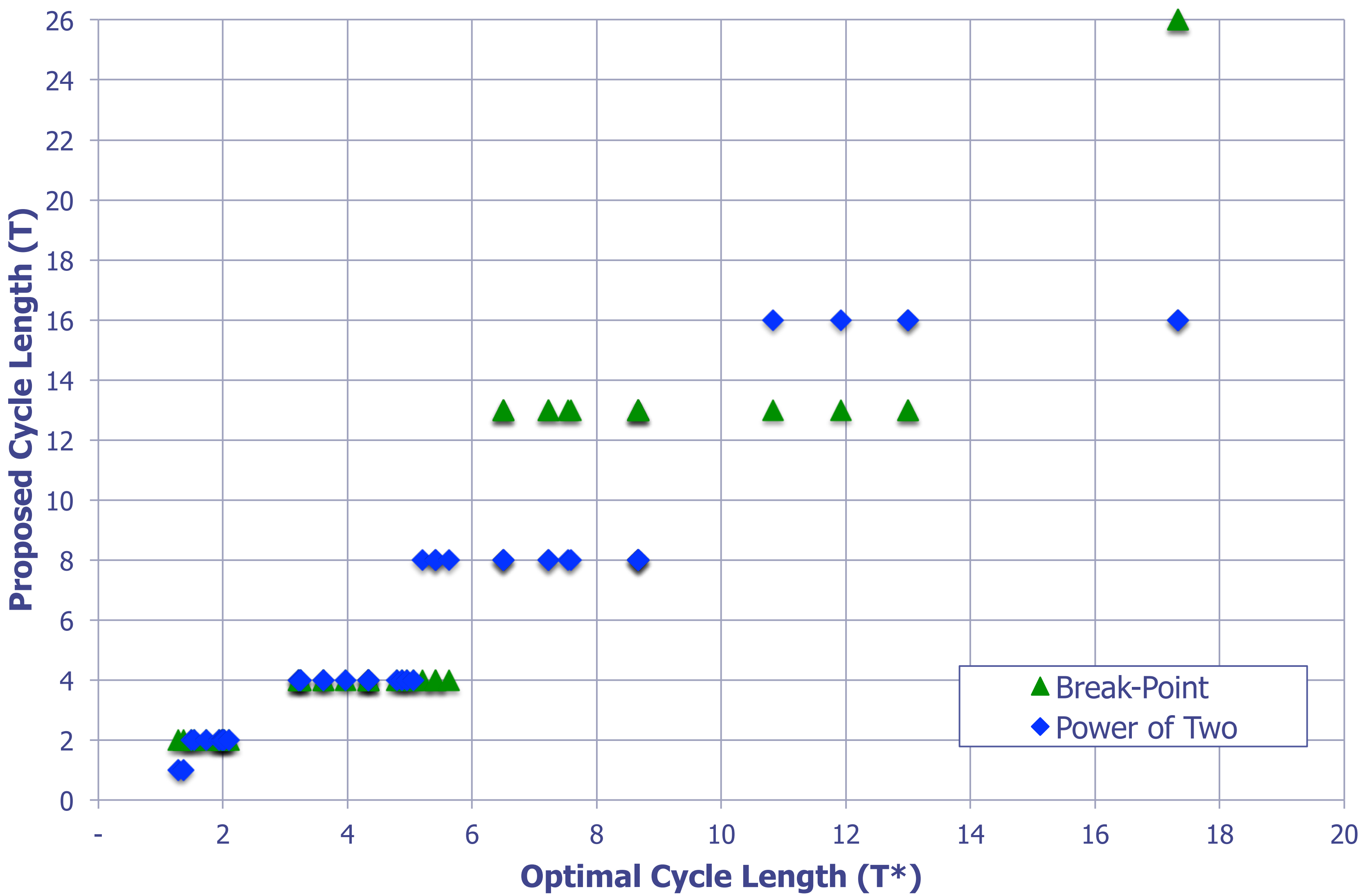
Grouping – Power of Two



Data of 55 SKUs
Various c, c_t, h, and D values

		T Practical	
T*(weeks)	T Practical	low	high
1.39	1	0.98	1.96
1.41	1	1.00	1.99
1.50	2	1.06	2.12
1.54	2	1.09	2.17
1.80	2	1.27	2.55
1.96	2	1.39	2.77
2.00	2	1.41	2.83
2.07	2	1.46	2.92
2.16	2	1.52	3.05
2.16	2	1.53	3.06
3.24	4	2.29	4.58
3.29	4	2.32	4.65
3.29	4	2.33	4.66
3.57	4	2.53	5.05

Comparing Both Methods



Exchange Curves for Cycle Stock

Exchange Curves – Cycle Stock

- What if I have a budget for inventory?
  - Find best allocation of inventory budget across multiple SKUs
  - Cost parameters are management policy levers!
- Relevant Cost Parameters
  - Holding Charge (h)
    - There is no single correct value
    - Reflection of management's investment and risk profile
  - Ordering Cost (c_t)
    - Not known with any precision
    - Cost allocations for time and systems differ between firms
- Exchange Curve
  - Trade-off between total annual cycle stock (TACS) and number of replenishments (N)
  - Determine the c_t/h value that meets budget constraints

$$TACS = \sum_{i=1}^{n} \frac{Q_i c_i}{2} \qquad N = \sum_{i=1}^{n} \frac{D_i}{Q_i}$$

Exchange Curves – Cycle Stock

$$TACS = \sum_{i=1}^{n} \frac{Q_i c_i}{2} = \sum_{i=1}^{n} \frac{\left(\sqrt{\frac{2c_t D_i}{hc_i}}\right) c_i}{2} = \sum_{i=1}^{n} \sqrt{\frac{c_t D_i c_i}{2h}} = \boxed{\sqrt{\frac{c_t}{h}} \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{D_i c_i}}$$

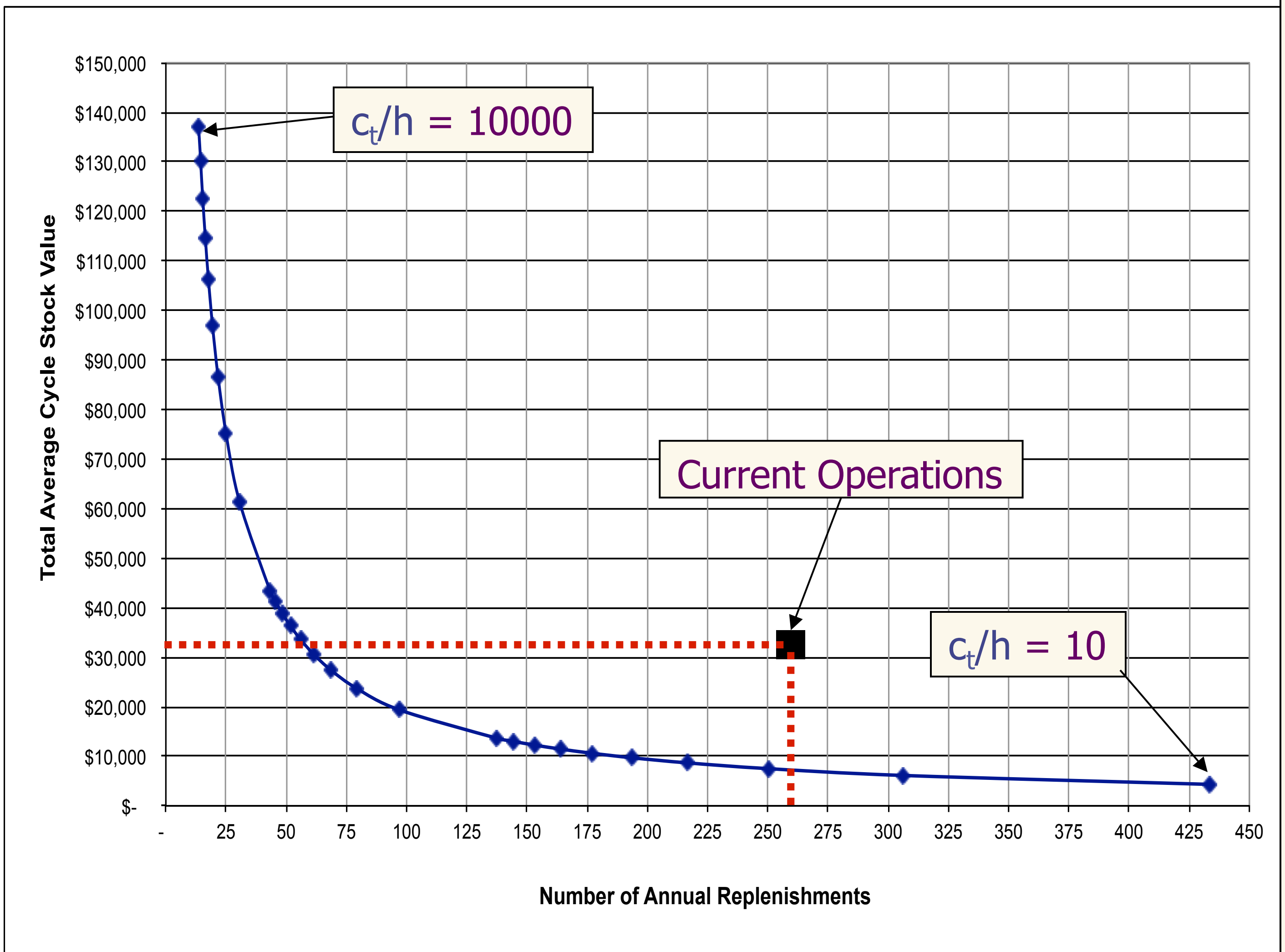
$$N = \sum_{i=1}^{n} \frac{D_i}{Q_i} = \sum_{i=1}^{n} \frac{D_i}{\sqrt{\frac{2c_t D_i}{hc_i}}} = \sum_{i=1}^{n} \sqrt{\frac{hD_i c_i}{2c_t}} = \boxed{\sqrt{\frac{h}{c_t}} \frac{1}{\sqrt{2}} \sum_{i=1}^{n} \sqrt{D_i c_i}}$$

- Approach:
  - Create table of SKUs with "Annual Value" ($D_i c_i$) and $\sqrt{(D_i c_i)}$
  - Find the sum of $\sqrt{(D_i c_i)}$ term for SKUs being analyzed
  - Calculate TACS and N for range of (c_t/h) values
  - Chart (N vs TACS)

Approach adapted from Silver, Pyke, Peterson (1998), Inventory Management and Production Planning and Scheduling

Exchange Curves – Cycle Stock

Data of 65 SKUs from Hospital Ward



c_t/h	N	TACS
10000	14	$137,043
9000	14	$130,011
8000	15	$122,575
7000	16	$114,659
6000	18	$106,153
5000	19	$96,904
4000	22	$86,674
3000	25	$75,062
2000	31	$61,288
1000	43	$43,337
900	46	$41,113
800	48	$38,762
700	52	$36,258
600	56	$33,569
500	61	$30,644
400	69	$27,409
300	79	$23,737
200	97	$19,381
100	137	$13,704
90	144	$13,001
80	153	$12,258
70	164	$11,466
60	177	$10,615
50	194	$9,690
40	217	$8,667
30	250	$7,506
20	306	$6,129
10	433	$4,334

Exchange Curves – Safety Stock

Exchange Curves – Safety Stock

- What if we have a safety stock budget?
  - Need to trade-off cost of <u>safety stock</u> and <u>level of service</u>
  - Key parameter is safety factor (k) – usually set by management
  - Estimate the aggregate service level for different budgets

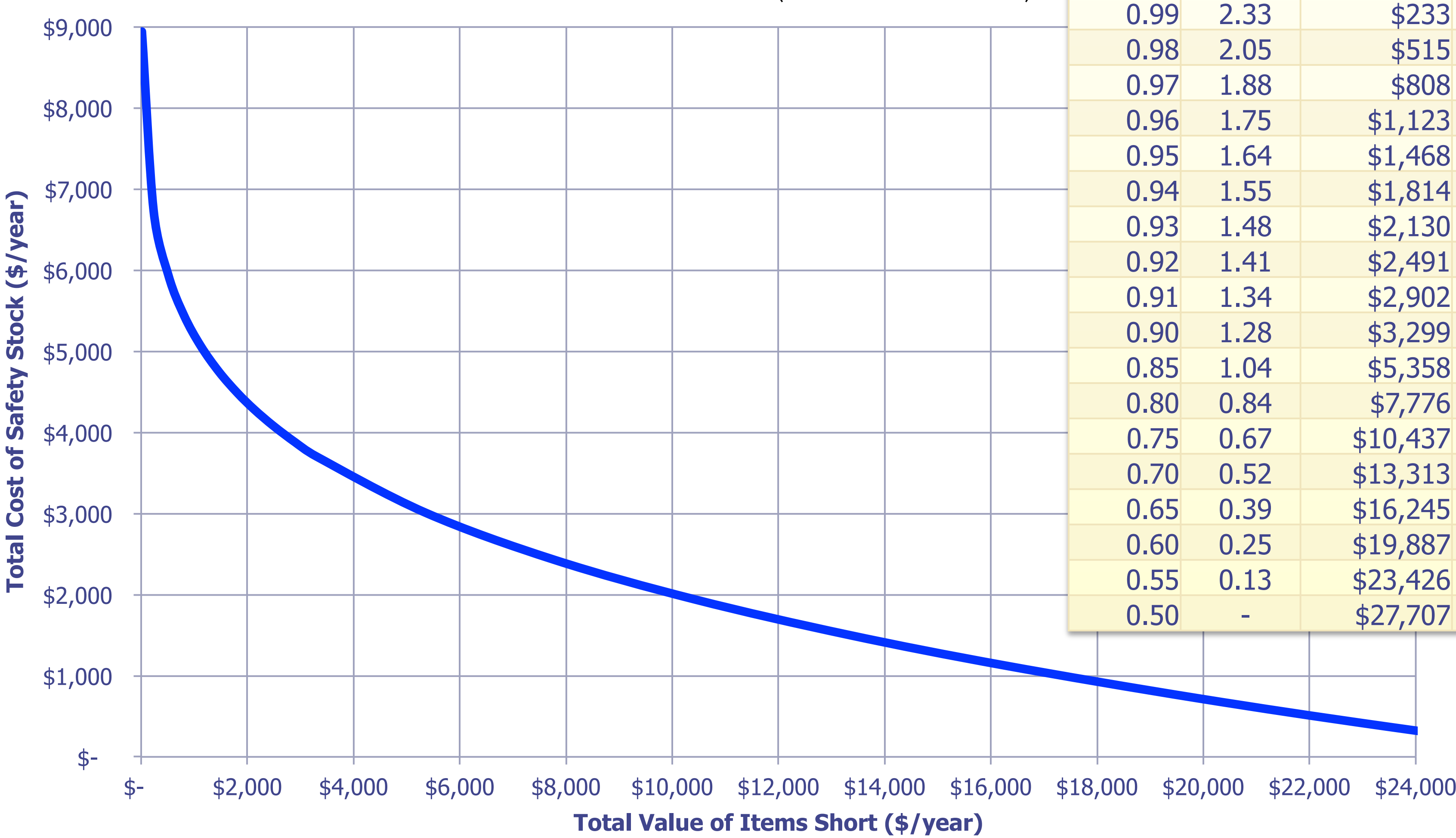
- Process:
  1. Select an inventory metric to target
  2. Starting with a high metric value calculate:
     - The required k_i to meet that target for each SKU
     - The resulting safety stock cost for each SKU and the total safety stock (TSS)
     - The other resulting inventory metrics of interest for each SKU and total
  3. Lower the metric value, go to step 2
  4. Chart resulting TSS versus Inventory Metrics

$$TSS = \sum_{i=1}^{n} k_i \sigma_{DLi} c_i \qquad TVIS = \sum_{i=1}^{n} \left(\frac{D_i}{Q_i} c_i \sigma_{DLi} G(k_i) \right)$$

Exchange Curves – Safety Stock

Data of 65 SKUs from Hospital Ward

$$TSS = \sum_{i=1}^{n} k_i \sigma_{DLi} c_i \qquad TVIS = \sum_{i=1}^{n} \left(\frac{D_i}{Q_i} c_i \sigma_{DLi} G(k_i) \right)$$



CSL	k	TVIS	TSS
0.999	3.09	$19	$8,941
0.99	2.33	$233	$6,742
0.98	2.05	$515	$5,932
0.97	1.88	$808	$5,440
0.96	1.75	$1,123	$5,064
0.95	1.64	$1,468	$4,745
0.94	1.55	$1,814	$4,485
0.93	1.48	$2,130	$4,282
0.92	1.41	$2,491	$4,080
0.91	1.34	$2,902	$3,877
0.90	1.28	$3,299	$3,704
0.85	1.04	$5,358	$3,009
0.80	0.84	$7,776	$2,430
0.75	0.67	$10,437	$1,939
0.70	0.52	$13,313	$1,505
0.65	0.39	$16,245	$1,129
0.60	0.25	$19,887	$723
0.55	0.13	$23,426	$376
0.50	-	$27,707	$-

Multiple Locations

Example: MedEx

- Situation:
  - MedEx is a medical device manufacturer that delivers products directly to hospitals wards. One item, the X104, is used by three different wards within Northwest Hospital with daily demand ~N(22, 4.6). The purchase cost (c) is \$156, the lead time (L) to replenish is 2 days, order cost (c_t) is \$40, annual holding charge (h) is 20%, and CSL is set at 99.9%. Assume a 365 day year.
  - Currently each ward manages their own inventory independently using an (s, Q) inventory replenishment policy.

- Problem:
  - How much cycle and safety stock should each ward hold?

Case adapted from DeScioli, D. (2005) "Differentiating the Hospital Supply Chain For Enhanced Performance," MIT Supply Chain Management Program Thesis.
Image Source: http://commons.wikimedia.org/wiki/File:KH_St_Elisabeth_RV_2013_Aufwachraum_02.jpg

Example: MedEx – individual wards

- Solution – for each ward:
  - Find Average Cycle Stock
    - $Q^* = \sqrt{[(2)(40)(365)(22)/(156)(.2)]} = 143.5 \approx 144$ units
    - Average cycle stock per ward = $Q^*/2 = (144/2) = 72$ units
  - Find Average Safety Stock
    - $\mu_{DL} = (22)(2) = 44$ units
    - $\sigma_{DL} = (4.6)(\sqrt{2}) = 6.51 \approx 6.5$ units
    - k = 3.09 for CSL = 99.9% (from table or spreadsheet)
    - Average safety stock per ward = $k\sigma_{DL} = (3.09)(6.5) = 20.1$ units
- Solution - across all three wards
  - Average total cycle stock = 3(72) = 216 units or $6,739
  - Average safety stock = (3)(20.1) = 60.3 units or $1,881
- What if they pool their inventories to a common location?

Example: MedEx – pooled inventory

- Solution:
  - Find Pooled Demand
    - Each ward has daily demand ~N(22, 4.6)
    - E[Daily Pooled Demand] = (3)(22) = 66 units
    - V[Daily Pooled Demand]

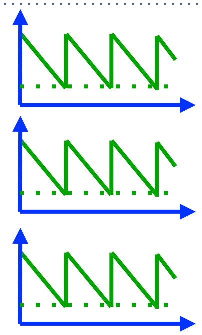
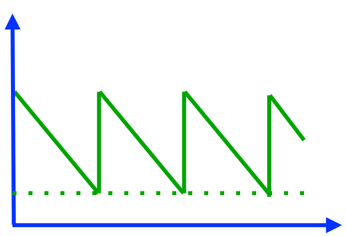
      = V[Ward$_1$ Demand]+V[Ward$_2$ Demand]+V[Ward$_3$ Demand]

      Or – we could just say $\sigma_{pooled} = \sigma_{wardi}\sqrt{n} = (4.6)(\sqrt{3}) = 7.967 \approx 8.0$
  - Find Average Cycle Stock
    - $Q^* = \sqrt{[(2)(40)\,(365)(66)/(156)(.2)]} = 248.5 \approx 250$ units
    - Average cycle stock = $Q^*/2 = (249/2) = 125$ units
  - Find Average Safety Stock
    - $\mu_{DL} = (66)(2) = 132$ units
    - $\sigma_{DL} = (8)(\sqrt{2}) = 11.31 \approx 11.3$ units
    - k = 3.09 for CSL = 99.9% (from table or spreadsheet)
    - Average safety stock = $k\sigma_{DL} = (3.09)(11.3) = 34.9 \approx 35$ units
- Solution - across all three wards
  - Average total cycle stock = 125 units or $3,900
  - Average safety stock = 35 units or $1,092

Example: MedEx – pooled inventory

Strategy	Average Cycle Stock	Average Safety Stock	Average Inventory
Independent	216	60.3	≈276
Pooled	125	35	≈160

Why did the inventory reduce by √3? Coincidence?

		Cycle Stock	Safety Stock
	Independent	$q_i^* = \sqrt{\frac{2c_t d_i}{c_e}} = \sqrt{\frac{2c_t D}{c_e n}}$ $\overline{IOH} = \sum_{i=1}^{n}\left(\frac{q_i^*}{2}\right) = \sqrt{n}\left(\frac{Q^*}{2}\right)$	$\overline{SS}_{independent} = k\sigma_{d_i} = k\sigma_D\sqrt{n}$
	Pooled	$Q^* = \sqrt{\frac{2c_t D}{c_e}}$ $\quad \overline{IOH} = \left(\frac{Q^*}{2}\right)$	$\overline{SS}_{pooled} = k\sigma_D$

Key Points from Lesson

Key Points

- Inventory Policies for Multiple Items
  - Grouping Like Items – Use common operating policies
    - Break-even Q points vs. Power of Two vs. ?????
  - Exchange Curves – Budget constraints
    - Use c_t/h and k as management levers
- Inventory Policies for Multiple Locations
  - Location Pooling – Square Root “Law”
    - Pooling inventory from n to 1 location reduces SS & CS by $\sqrt{n}$
    - Similarly, pooling from n to m locations reduces SS & CS by $\sqrt{(n/m)}$
  - !!!Caution!!! This is based on many assumptions . . .
    - Evenly distributed demand
    - Ordering follows EOQ with common c_t
    - Demand distribution in different locations is independent

Questions, Comments, Suggestions?
Use the Discussion!

MIT Center for
Transportation & Logistics

caplice@mit.edu