

Lead Time Variability & Mode Selection



Agenda

- Connections to Inventory Planning
- Transit Time Reliability
- Handling Lead Time Variability
- Mode Selection

Transportation Impact on Inventory

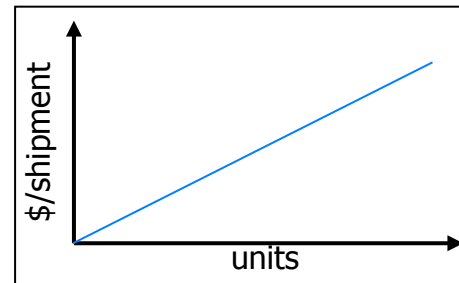
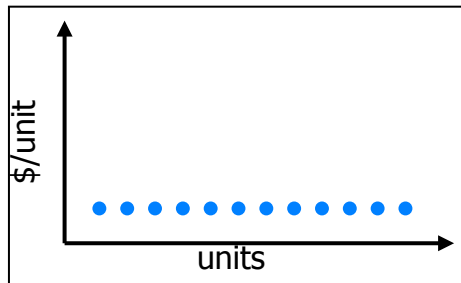
Impact on Inventory

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right) + B_{SO} \left(\frac{D}{Q} \right) \Pr[SO]$$

- How does transportation impact our total costs?
 - Cost of transportation
 - ◆ Value & Structure
 - Lead Time
 - ◆ Value & Variability & Schedule
 - Capacity
 - ◆ Limits on Q
 - Miscellaneous Factors
 - ◆ Special Cases

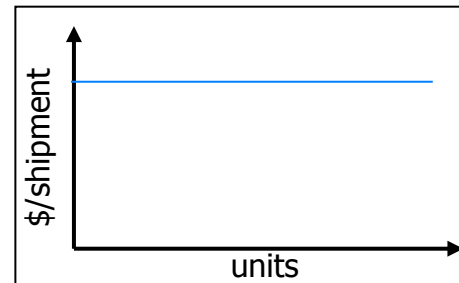
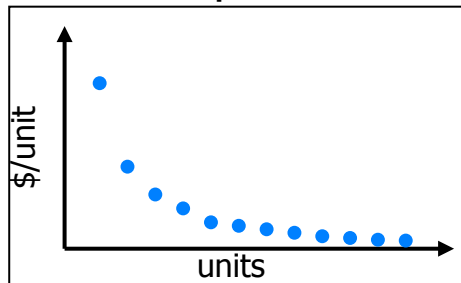
Transportation Cost Functions

Pure Variable Cost / Unit



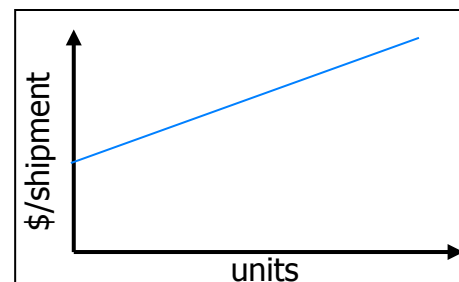
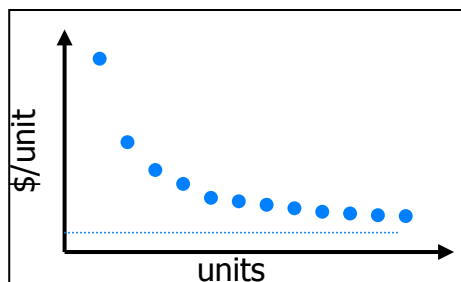
Modify unit cost (c) for Purchase Cost

Pure Fixed Cost / Shipment



Modify fixed order cost (c_t) for Ordering Cost

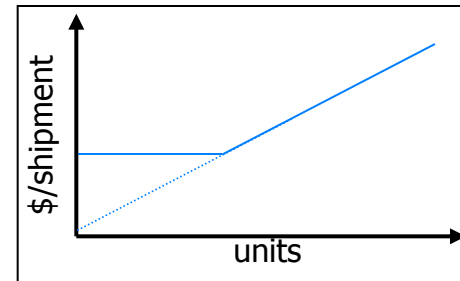
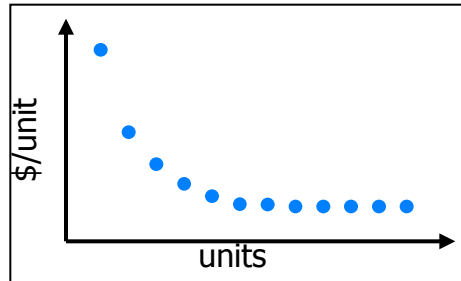
Mixed Variable & Fixed Cost



Modify both c_t and c

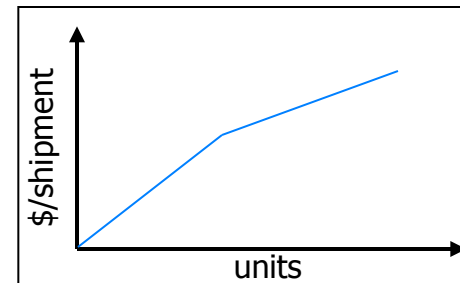
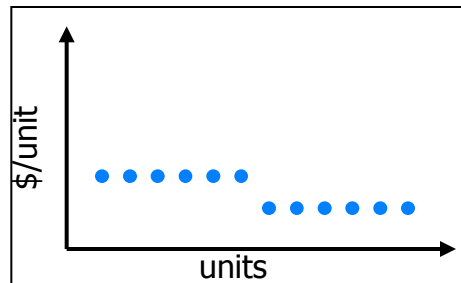
Complex Cost Functions

Variable Cost / Unit with a Minimum



algorithm

Incremental Discounts



algorithm

Note that approach will be similar to quantity discount analysis in deterministic EOQ

Shipping Shoes from Shenzhen II

- The Next Chapter

Shipping Shoes II

How should I ship my shoes from Shenzhen to Kansas City?

- General Information

- Shoes are manufactured, labeled, and packed at plant
- Demand $\sim N(4.5M, 0.54M)$ annual demand
- 3,000 shoe boxes fit into one TEU
- Average cost $\sim \$35$ per pair
- Cost of product in container \$105,000
- Average sales price $\sim \$75$ per pair
- Order for shipment cost \$5000 per order
- Holding costs are 15%
- Assume 50 weeks/year, 350 days/year
- Assume CSL 95%

Which option provides the lowest logistics cost?

- Transportation Options

Inland Origin: Shenzhen to Ports (\$/container)

- Yantian (\$35, 2 day)
- Hong Kong (\$30, 5 days)

Port to Port: China to US (\$/container)

- CSCL (AAC) Yantian to POLA (\$1100, 20 days)
- CSCL (AAS) Hong Kong to POLA (\$1025, 13 days)
- APL Hong Kong to New York (\$1200, 29 days)

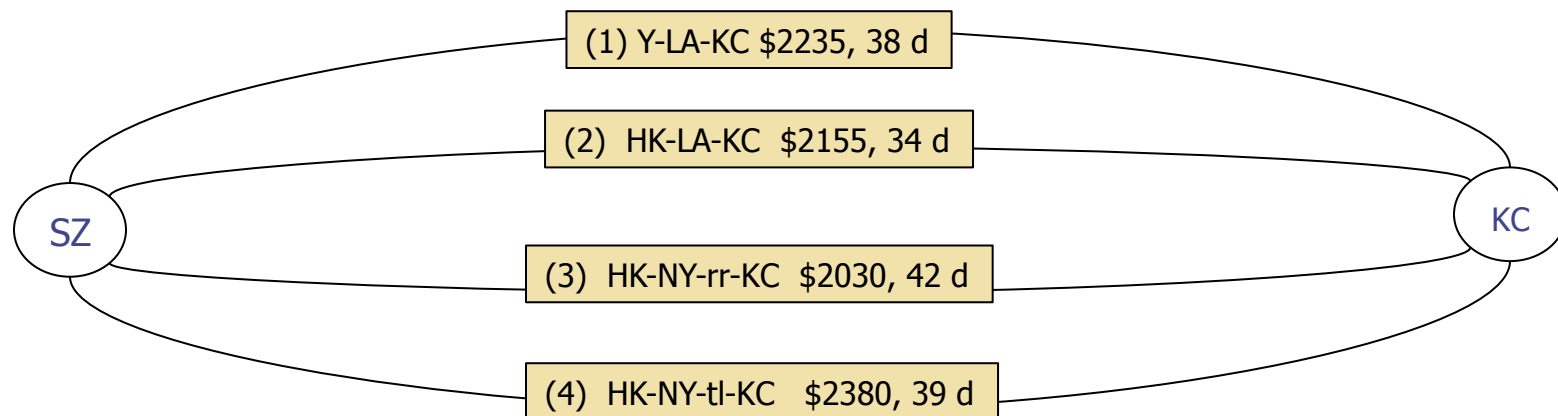
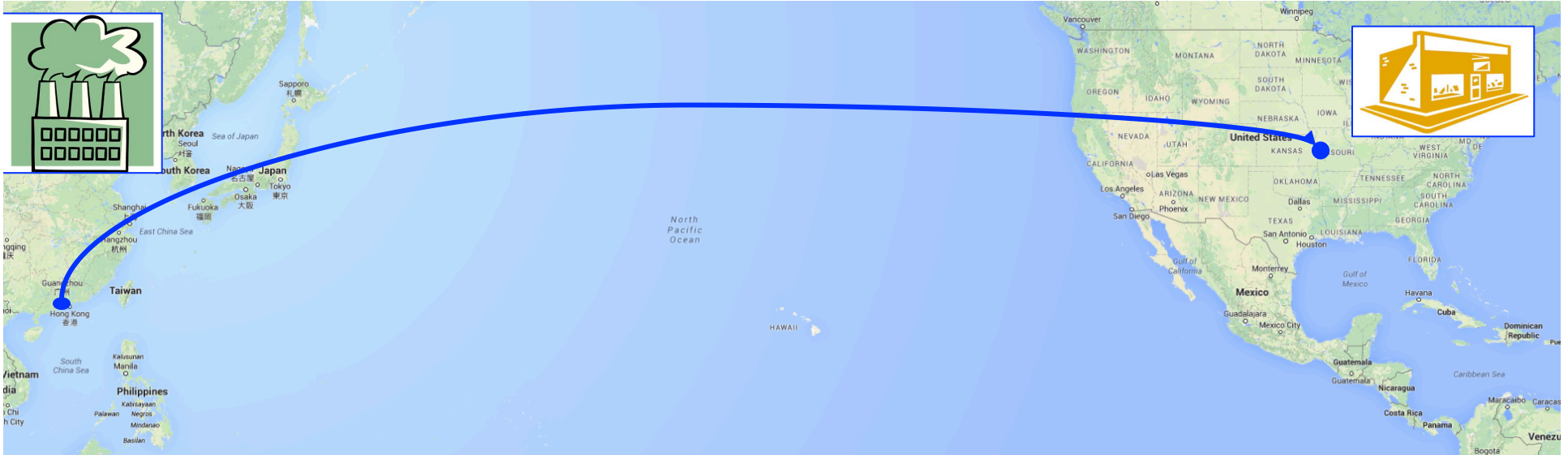
Destination Port: US Ports (\$/container)

- POLA (5 days)
- New York / New Jersey (3 days)

Inland Destination: To Kansas City (\$/container)

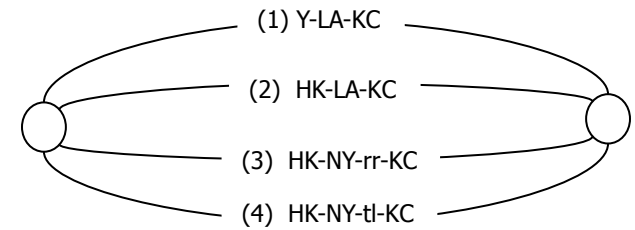
- POLA to KC by BNSF (\$1100, 11 days)
- PANYNJ to KC by NS (\$800, 5 days)
- PANYNJ to KC by HJBT Truckload (\$1150, 2 days)

Shipping Shoes



Shipping Shoes Part 2.

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$



$k=1.64$

$\sigma_{DL}=(180)\sqrt{(LT/350)}$ shoes

Path	L (days)	c_{trans} (\$/cnt)	c (\$/cnt)	c_t (\$/ord)	c_e (\$/cnt/yr)	D (cnt/yr)	Q (cnt/ord)	σ_{DL} (cnt)
1	38	\$2,235	\$107,235	\$5,000	\$16,085	1,500	30	59.3
2	34	\$2,155	\$107,155	\$5,000	\$16,073	1,500	30	56.1
3	42	\$2,030	\$107,030	\$5,000	\$16,055	1,500	30	62.4
4	39	\$2,380	\$107,380	\$5,000	\$16,107	1,500	30	60.1

Path	Purchase Cost (\$M)	Ordering Cost (\$K)	Cycle Stock Cost (\$K)	Safety Stock Cost (\$M)	Pipeline Inventory (\$M)	Total Cost (\$M)	Logistics Cost Per Shoe
1	\$160.8	\$250	\$241	\$1.57	\$2.62	\$165.5	\$1.77
2	\$160.7	\$250	\$241	\$1.48	\$2.34	\$165.0	\$1.67
3	\$160.5	\$250	\$241	\$1.65	\$2.89	\$165.5	\$1.78
4	\$161.1	\$250	\$242	\$1.59	\$2.69	\$165.9	\$1.86

Lowest **total** cost path is (2) at \$2155 /container = \$1.67 / pair of shoes

Transit Time Reliability

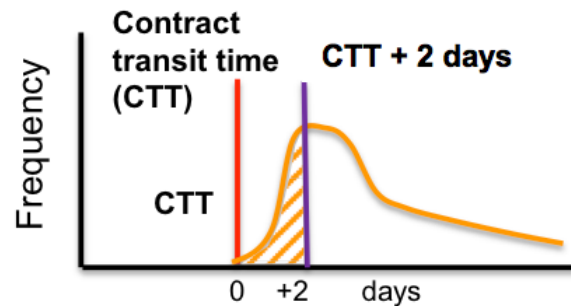
Lead / Transit Time Reliability

- Key Questions:
 - What is the definition of reliability within a firm?
 - What are the sources of unreliability/variability?
 - How can the current situation be improved?
- Two Dimensions of Reliability
 - Credibility
 - ◆ Did the carrier reserve slots as agreed to? (Rejections / Bumping)
 - ◆ Did the carrier stop at all ports agreed to? (Skipping)
 - ◆ Did the carrier load all containers committed? (Cut & Run)
 - Schedule Consistency
 - ◆ How close was the carrier's performance to their quoted schedule?
 - ◆ How consistent was the carrier's actual transit time?

Material adapted from Arntzen, B. (2011) "Global Ocean Transportation Project," Internal MIT Center for Transportation & Logistics (CTL) Report.

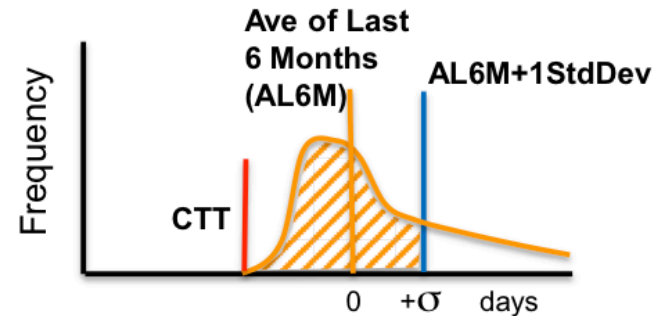
Definitions of Schedule Consistency

Compare actual transit time to the contract.



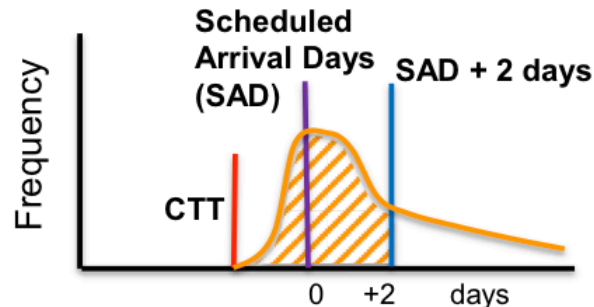
Reliability = % less than [CTT+2]

Compare actual transit time to the average of the last 6 months.



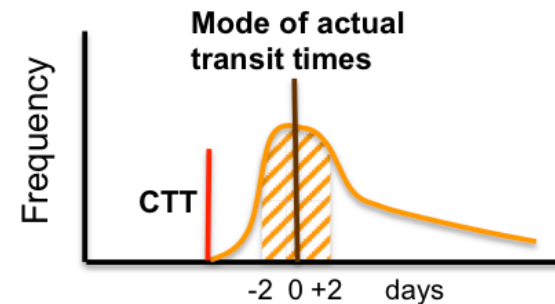
Reliability = % less than [AL6M+1StdDev]

Compare actual transit time to the published ship schedule.



Reliability = % less than [SAD+2]

Measure the “tightness” of the distribution of transit times.

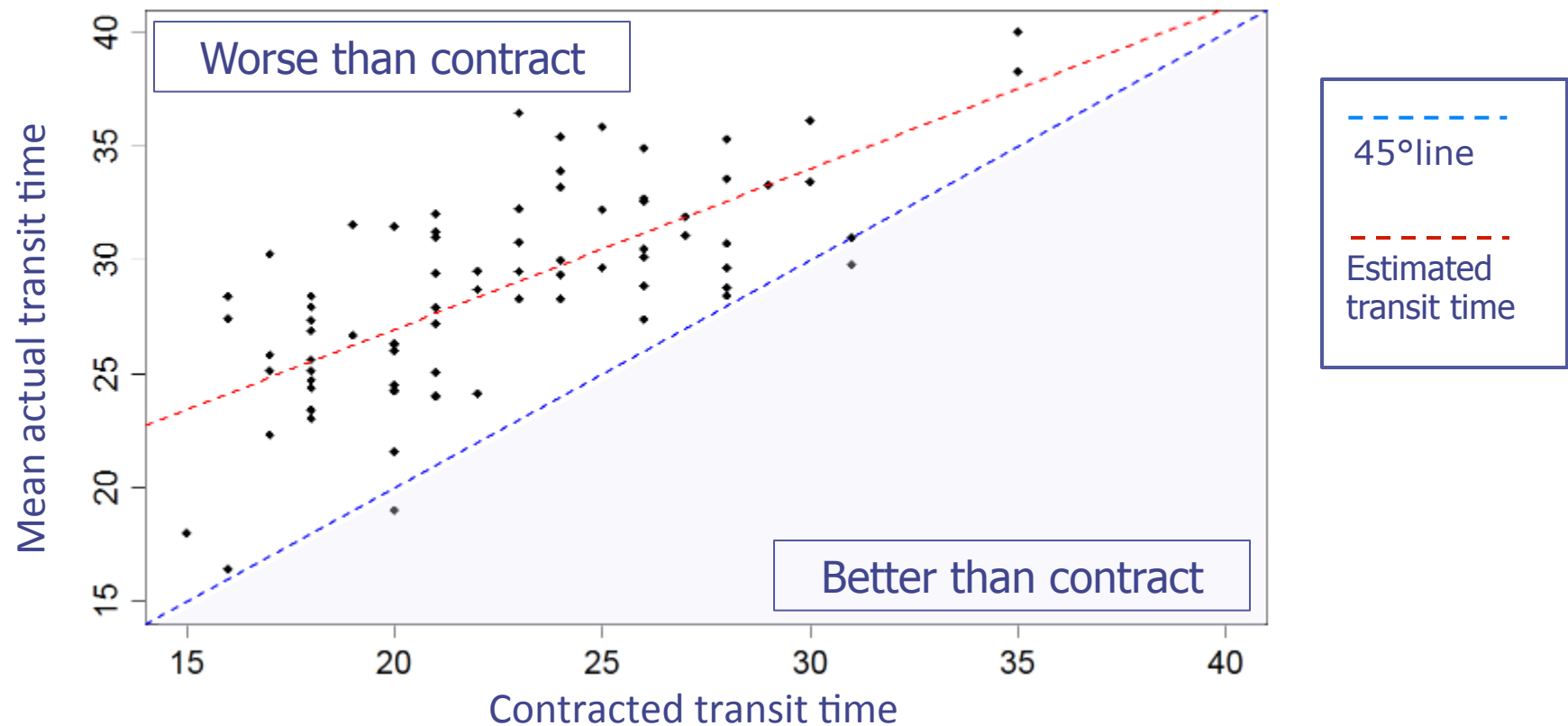


Reliability = % within 2 days of the mode

Material adapted from Arntzen, B. (2011) “Global Ocean Transportation Project,” Internal MIT Center for Transportation & Logistics (CTL) Report.

Three Observations from Practice

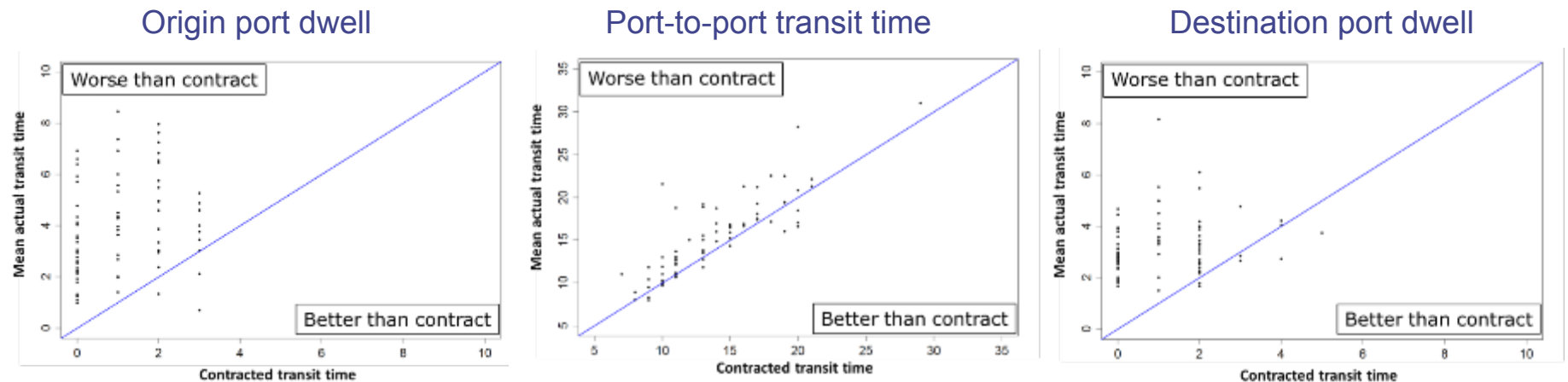
Observation 1: Contract reliability in procurement and operations do not always match



Material adapted from Caplice, C and Kalkanci, B. (2011) "Managing Global Supply Chains: Building end-to-end Reliability," Internal MIT Center for Transportation & Logistics (CTL) Report.

Three Observations from Practice

Observation 2: Contract reliability differs dramatically across different route segments



While accurate estimates of the port-to-port transit times exist, there is only limited information on port dwell times.

Three Observations from Practice

Observation 3: Most transit variability occurs in inland transportation and at the ports.

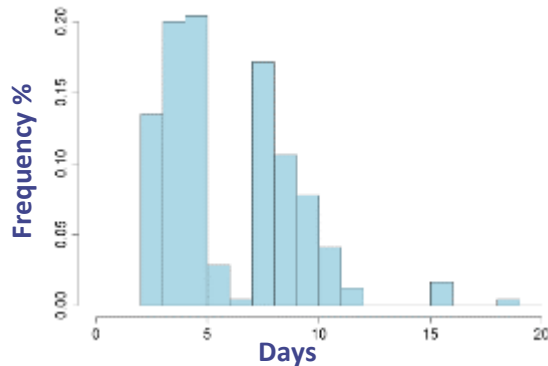
	Origin Landside Transit	Origin Port Dwell	Ocean Transit	Destination Port Dwell	Destination Landside Transit
Asia to North America	1.2	0.9	0.4	1.0	0.8
South America to North America	1.3	0.8	0.2	0.8	0.9
Europe to North America	0.7	0.7	0.3	0.7	0.7
North America to Europe	0.8	0.9	0.5	0.8	1.3

Coefficient of Variation of Time for Each Segment when $CV=\sigma/\mu$

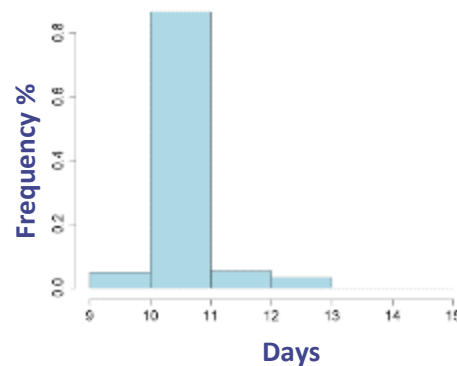
Lead Time Variability

Sample of Transit Time Distribution

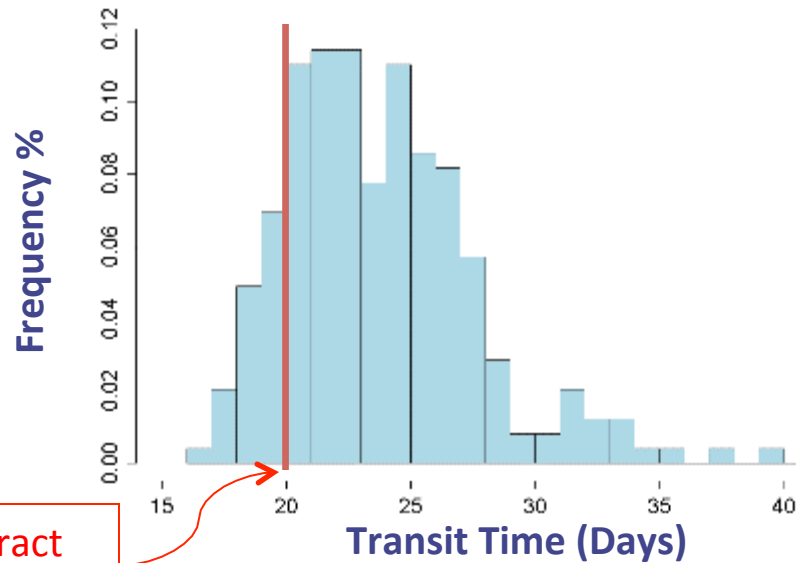
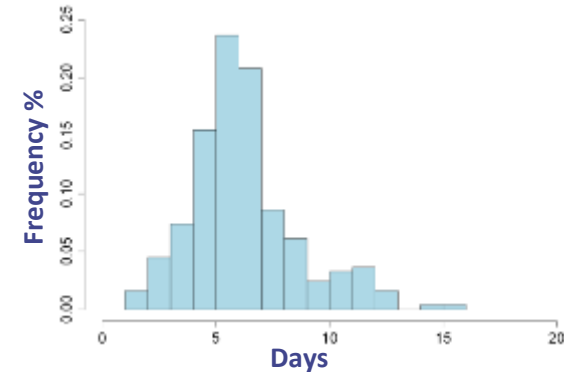
Origin-to-Port



Port-to-Port



Port-to-Destination

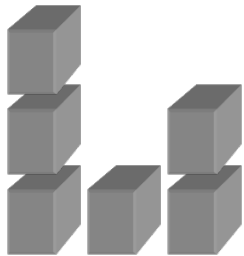


Sample of 245 container moves
Common trade lane

Lead Time Variability Impact

Weekly Demand $\sim U(1,3)$

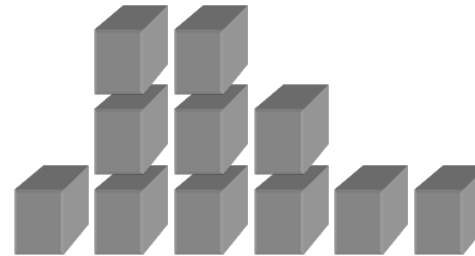
Lead Time 3 weeks



6 units

Weekly Demand $\sim U(1,3)$

Lead Time $\sim U(3,6)$ weeks

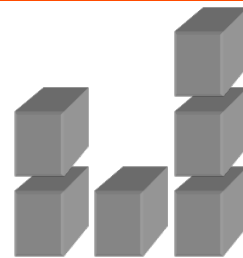


11 units

6 weeks

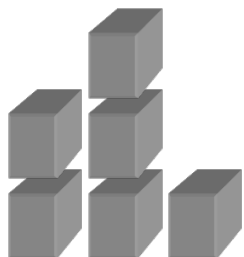


4 units

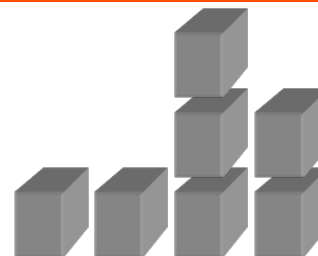


6 units

3 weeks



6 units



7 units

4 weeks

This is an example of Random Sums of Random Variables

Random Sums of Random Variables

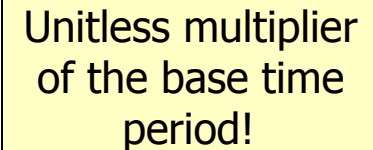
- Let
 - N = is a random variable assuming positive integer values 1, 2, 3....
 - X_i = independent random variables so that $E[X_i] = E[X]$
 - S = sum of X_i from $i=1$ to N
- Then
$$E[S] = E\left[\sum_{i=1}^N X_i\right] = E[N]E[X]$$
$$Var[S] = Var\left[\sum_{i=1}^N X_i\right] = E[N]Var[X] + (E[X])^2 Var[N]$$
- Simple Example
 - N has a mean of 28 and a standard deviation of 7
 - X has a mean of 180 and standard deviation of 68
- What is the mean, variance, and standard deviation of S ?
 - $E[S] = \mu_s = (28)(180) = 5040$
 - $Var[S] = \sigma_s^2 = (28)(68)^2 + (180)^2(7)^2 = 129472 + 1587600 = 1,717,072$
 - $StdDev[S] = \sigma_s = \sqrt{1,717,072} = 1310$

Full proof and discussion can be found at S. K. Ross, Introduction to Probability Models, 11th Edition, Academic Press, 2014, Chapter 3.

Lead Time Variability

- Sometimes referred to as Hadley-Whitin equation
 - Lead Time and Demand are independent RVs
 - μ_D = Expected demand (items) during one time period
 - σ_D = Standard deviation of demand (items) during one time period
 - μ_L = Expected number of time periods for lead time
 - σ_L = Standard deviation of time periods for lead time
 - μ_{DL} = Expected demand (items) over lead time
 - σ_{DL} = Standard deviation of demand (items) over lead time

Unitless multiplier
of the base time
period!



$$\mu_{DL} = \mu_L \mu_D \quad \sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

- Transportation Example
 - Suppose that lead time is 12 days on average with a standard deviation of 3 days. The daily demand for an item is 100 units with a standard deviation of 22.
 - What is my expected demand over lead time as well as standard deviation of demand over lead time.
 - ◆ $\mu_{DL} = (12)(100) = 1200$
 - ◆ $\sigma_{DL} = \sqrt{[(12)(22)^2 + (100)^2(3)^2]} = \sqrt{[5808 + 90000]} = 309.5 \sim 310$

I can now find set an inventory performance metric using this demand distribution!

Shipping Shoes from Shenzhen III

– The Final Chapter

Shipping Shoes III

How should I ship my shoes from Shenzhen to Kansas City?

- General Information

- Shoes are manufactured, labeled, and packed at plant
- Demand $\sim N(4.5M, 0.54M)$ annual demand
- 3,000 shoe boxes fit into one TEU
- Average cost $\sim \$35$ per pair
- Cost of product in container \$105,000
- Average sales price $\sim \$75$ per pair
- Order for shipment cost \$5000 per order
- Holding costs are 15%
- Assume 50 weeks/year, 350 days/year
- Assume CSL 95%

Which option provides the lowest logistics cost?

- Transportation Options

Inland Origin: Shenzhen to Ports (\$/cnt, μ_L , σ_L)

- Yantian (\$35, 2 days, 1 day)
- Hong Kong (\$30, 5 days, 5 days)

Port to Port: China to US (\$/cnt, μ_L , σ_L)

- CSCL (AAC) Yantian to POLA (\$1100, 20 days, 2 days)
- CSCL (AAS) Hong Kong to POLA (\$1025, 13 days, 13 days)
- APL Hong Kong to New York (\$1200, 29 days, 3 days)

Destination Port: US Ports (\$/cnt, μ_L , σ_L)

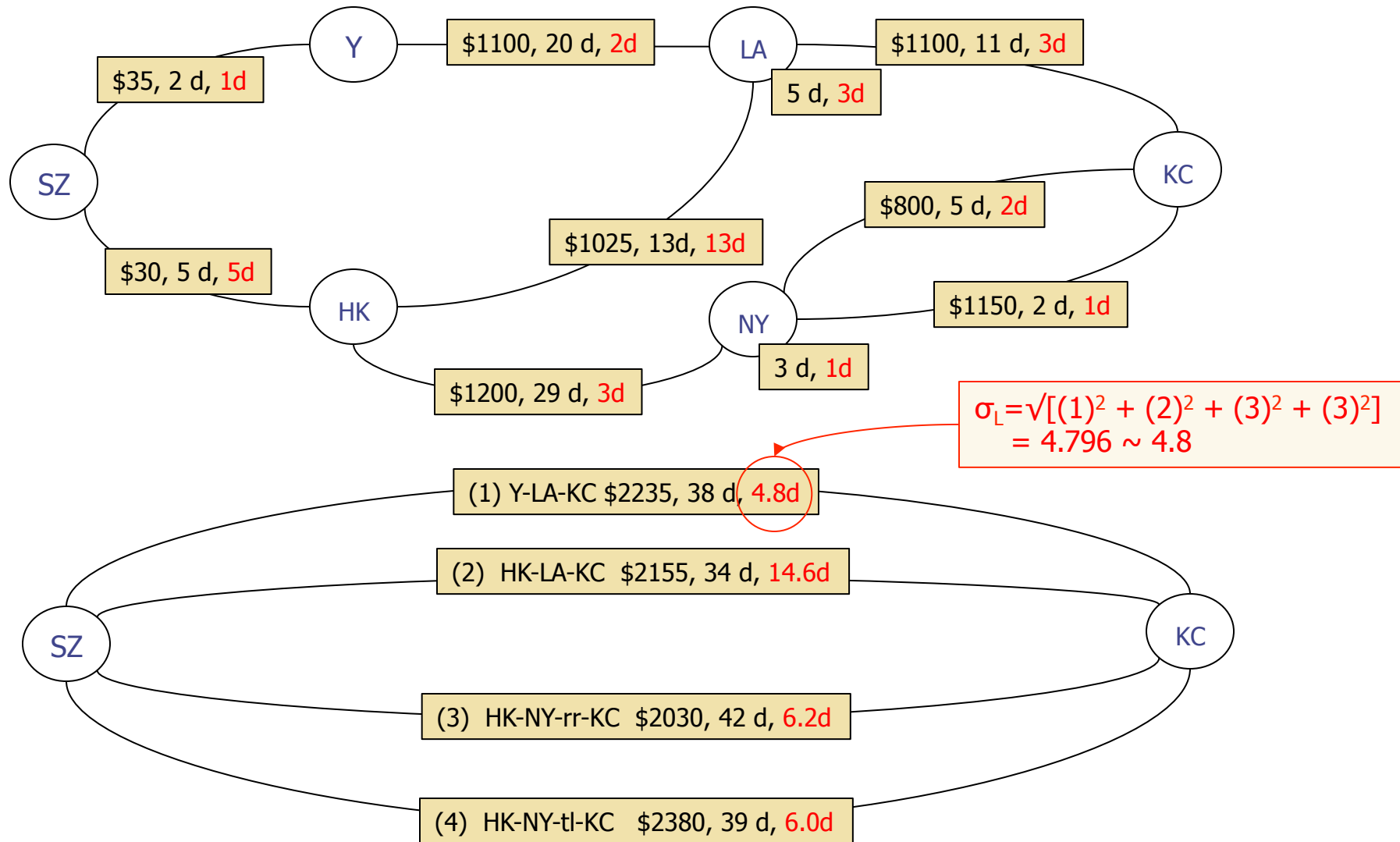
- POLA (\$0, 5 days, 3 days)
- New York / New Jersey (\$0, 3 days, 1 day)

Inland Destination: To Kansas City (\$/cnt, μ_L , σ_L)

- POLA to KC by BNSF (\$1100, 11 days, 3 days)
- PANYNJ to KC by NS (\$800, 5 days, 2 days)
- PANYNJ to KC by HJBT Truckload (\$1150, 2 days, 1 days)

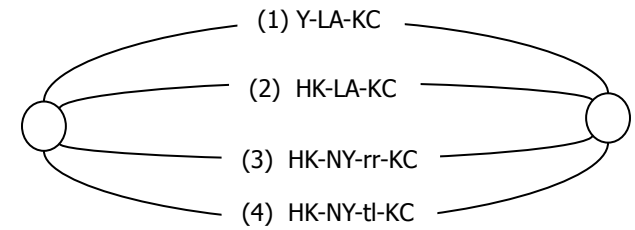
Shipping Shoes III

\$/cnt, μ_L , σ_L



Shipping Shoes III

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$



Only safety stock changes!

Path	μ_L	σ_L	μ_D	σ_D	μ_{DL}	σ_{DL}	New SS (\$M)	Old SS (\$M)
1	38	4.80	4.29	9.62	162.86	62.78	\$1.66	\$1.57
2	34	14.60	4.29	9.62	145.71	84.04	\$2.22	\$1.48
3	42	6.20	4.29	9.62	180.00	67.78	\$1.79	\$1.65
4	39	6.00	4.29	9.62	167.14	65.36	\$1.73	\$1.59

$$\mu_D = (1500 \text{ cnt/year}) / (350 \text{ days/year})$$

$$\sigma_D = (180 \text{ cnt/year}) / \sqrt{(350 \text{ days/year})}$$

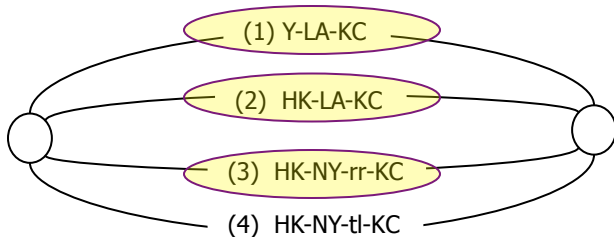
$$\mu_{DL} = \mu_L \mu_D \quad \sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

Shipping Shoes III

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

Path	Purchase Cost (\$M)	Ordering Cost (\$K)	Cycle Stock Cost (\$K)	Safety Stock Cost (\$M)	Pipeline Inventory (\$M)	Total Cost (\$M)	Logistics Cost Per Shoe
1	\$160.8	\$250	\$241	\$1.66	\$2.62	\$165.6	\$1.79
2	\$160.7	\$250	\$241	\$2.22	\$2.34	\$165.8	\$1.83
3	\$160.5	\$250	\$241	\$1.79	\$2.89	\$165.7	\$1.82
4	\$161.1	\$250	\$242	\$1.73	\$2.69	\$166.0	\$1.89

Lowest **total** cost path is (1) at \$2235 /container = \$1.79 / pair of shoes



#3 - Lowest transportation cost route @ ~\$0.68 \$/shoe

#2 - Lowest logistics cost route @ ~\$1.67 \$/shoe, not considering variability of transit time

#1 - Lowest logistics cost route @ ~\$1.79 \$/shoe, considering variability of transit time

Mode Selection

Mode Selection

- Criteria for selection between modes
 - Feasible choices:
 - ◆ By geography
 - Global: Air versus Ocean
 - Surface: Trucking (TL, LTL, parcel) vs. Rail vs. Intermodal vs. Barge
 - ◆ By required speed
 - >500 miles in 1 day – Air
 - <500 miles in 1 day – TL
 - ◆ By shipment size (weight/density/cube, etc.)
 - High weight, cube items cannot be moved by air
 - Large oversized shipments might be restricted to rail or barge
 - ◆ By other restrictions
 - Nuclear or hazardous materials (HazMat)
 - Product characteristics
 - Trade-offs within the set of feasible choices:
 - ◆ Cost
 - ◆ Time (mean transit time, variability of transit time, frequency)
 - ◆ Capacity
 - ◆ Loss and Damage

Mode Choice Example

- You are in charge of transportation planning for a manufacturer. One of the lanes you are managing brings raw material from a supplier into your plant. Your plant requires about $\sim N(3000, 750)$ pounds of the product per day. The product is valued at \$20 per lb with 20% annual holding cost. You assume a CSL of 95% and 250 working days per year. You take ownership of the product at the origin.
- You have two options for this inbound movement.
 - **Truckload** – Transit time is 3 days on average with a standard deviation of 0.5 days and it costs \$1800 per truckload (capacity of 40,000 lbs)
 - **Intermodal** – Transit time is 6 days on average with a standard deviation of 2 days and it costs \$1400 per container (capacity of 40,000 lbs)
- Questions:
 - Your company's policy is to always "weigh out" your shipments. That is, always ship in full truckload or container quantities. Following this policy, what mode should you select?

Solution: Mode Choice

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

$c = \$ 20$ per lb
 $h = 20\%$ per year
 $c_e = 20(0.20) = 4$ \$/yr
 $\mu_D = 3000$ lbs/day
 $\sigma_D = 750$ lbs/day
 $k = 1.64$

	TL	IM	
Lead Time (μ_L)	3	6	days
Std Dev Lead Time (σ_L)	0.5	2	days
Cost (c_t)	1800	1400	\$/load

$$\mu_{DL} = \mu_L \mu_D$$

$$\sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

	TL	IM
Average Demand over Lead Time (μ_{DL})	9000	18000
Std Dev Demand over Lead Time (σ_{DL})	1984	6275

	TL	IM
Capacity (Q)	40000	40000
Number of loads/year ($N = D/Q$)	18.75	18.75
Annual Ordering Cost	\$33,750	\$26,250
Annual Cycle Stock Cost	\$80,000	\$80,000
Annual Safety Stock Cost	\$13,017	\$41,164
Annual Pipeline Inventory Cost	\$36,000	\$72,000
Total Annual Logistics Cost	\$162,767	\$219,414

Select TL for this lane.

TL saves > \$56k /yr, but
 note – transport costs are
 higher but are trumped
 by safety stock and
 pipeline inventory.

Does the "weigh-out" policy make sense?

Solution: Mode Choice

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

$c = \$ 20$ per lb
 $h = 20\%$ per year
 $c_e = 20(0.20) = 4$ \$/yr
 $\mu_D = 3000$ lbs/day
 $\sigma_D = 750$ lbs/day
 $k = 1.64$

	TL	IM	
Lead Time (μ_L)	3	6	days
Std Dev Lead Time (σ_L)	0.5	2	days
Cost (c_t)	1800	1400	\$/load

$$Q^* = \sqrt{\frac{2c_t D}{c_e}}$$

	TL	IM
Average Demand over Lead Time (μ_{DL})	9000	18000
Std Dev Demand over Lead Time (σ_{DL})	1984	6275

	TL	IM
Capacity (Q)	25981	22913
Number of loads/year ($N = D/Q$)	28.87	32.73

Annual Ordering Cost	\$51,962	\$45,826
Annual Cycle Stock Cost	\$51,962	\$45,826
Annual Safety Stock Cost	\$13,017	\$41,164
Annual Pipeline Inventory Cost	\$36,000	\$72,000
Total Annual Logistics Cost	\$152,940	\$204,815

Still use TL

Shipping below max weight saves ~ \$10k per year! Why?

When would IM make sense for this lane?

- Lower value ($c \leq 0.73$ \$/lb)
- Better service ($\mu_L = 5$, $\sigma_L = 1$, & $c \leq 2.67$ \$/lb)
- Lower IM rate ($c_t \leq \$263$)

Key Points from Lesson

Key Points

$$TC(Q) = cD + c_t \left(\frac{D}{Q} \right) + c_e \left(\frac{Q}{2} + k\sigma_{DL} + LD \right)$$

- Mode/route/carrier selection is a trade-off between
 - Transportation costs
 - Inventory costs (cycle, safety, pipeline)
 - Level of service
- Need to consider more than just direct transport cost
- Lead time impacts safety stock levels and variability impacts it even more so!
- Be careful about shape of distribution for demand over lead time.

$$\mu_{DL} = \mu_L \mu_D$$

$$\sigma_{DL} = \sqrt{\mu_L \sigma_D^2 + (\mu_D)^2 \sigma_L^2}$$

CTL.SC1x -Supply Chain & Logistics Fundamentals

Questions, Comments, Suggestions?

Use the Discussion!



"Wilson – pondering the Hadley-Whitin Equation "
Yankee Golden Retriever Rescued Dog
(www.ygrr.org)



MIT Center for
Transportation & Logistics

caplice@mit.edu