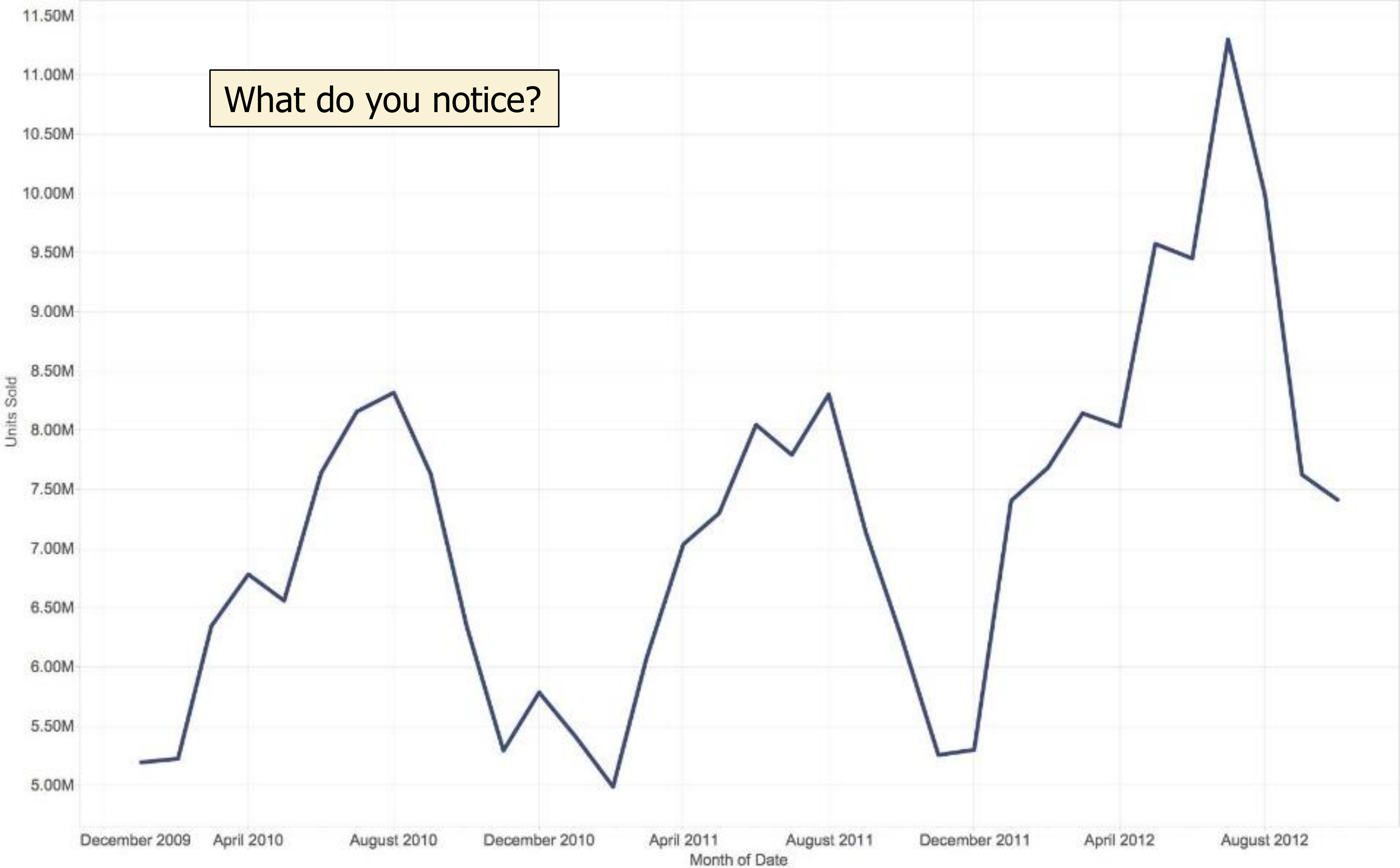
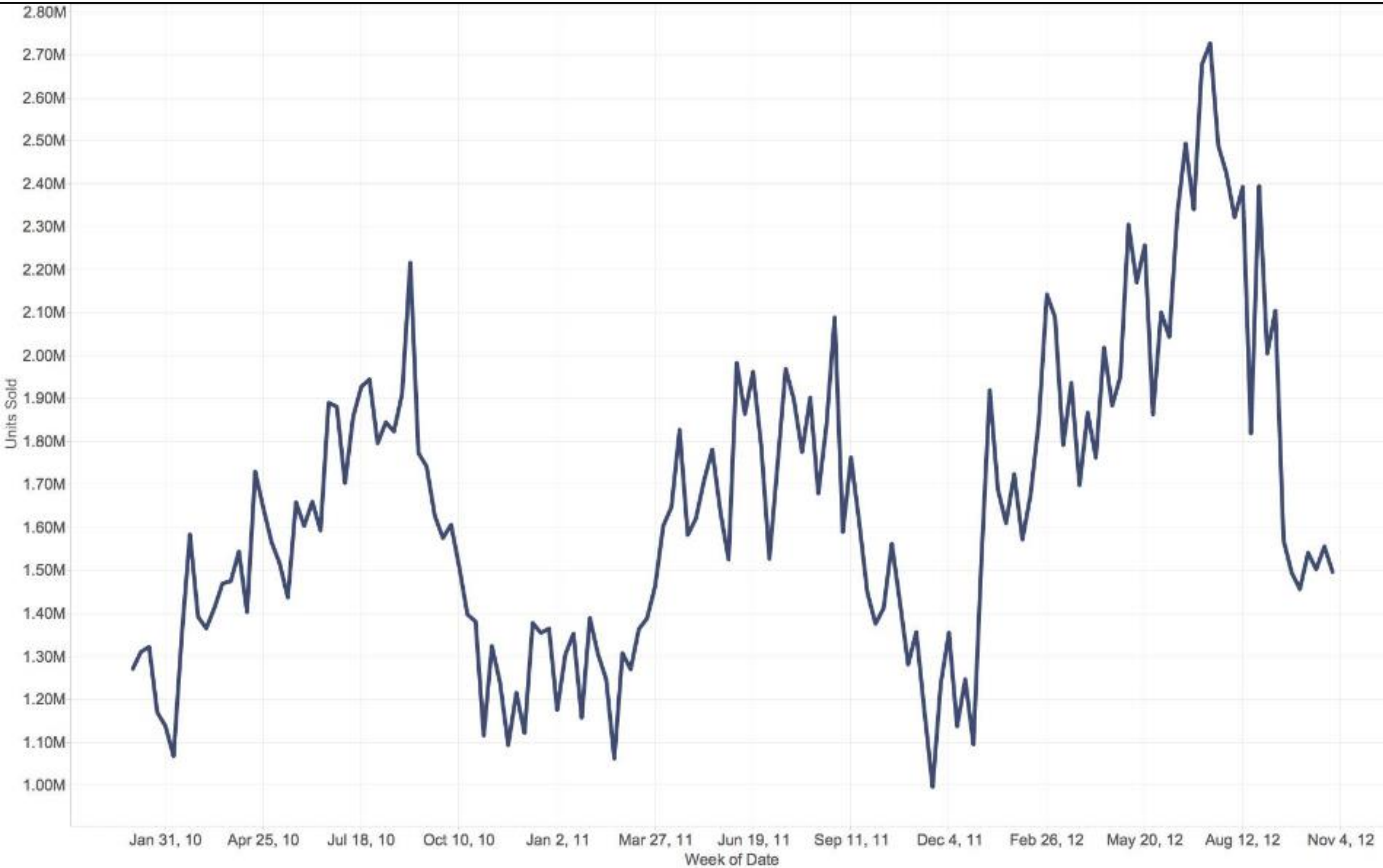
Time Series Analysis

MIT Center for Transportation & Logistics

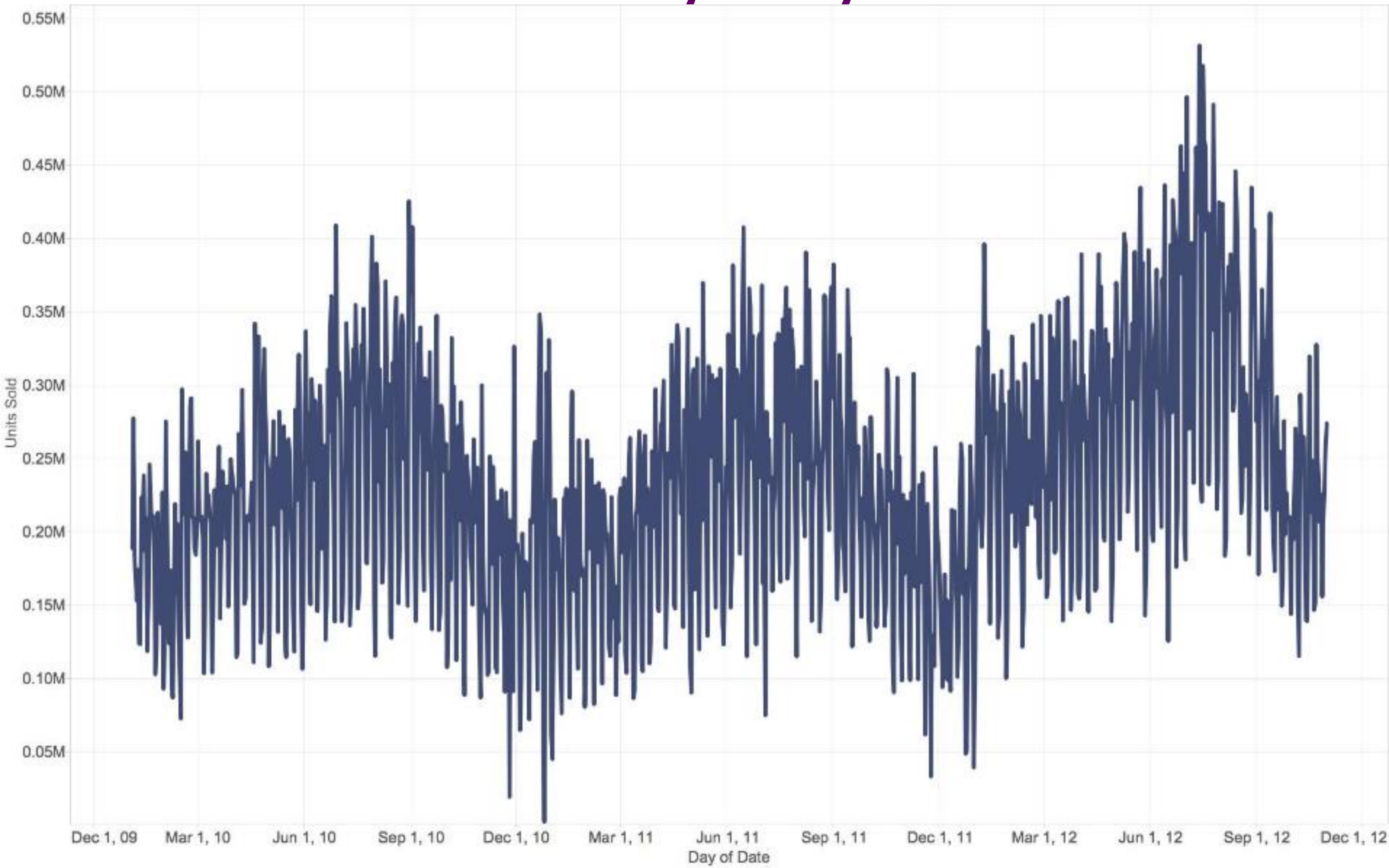
Demand – Sales By Month



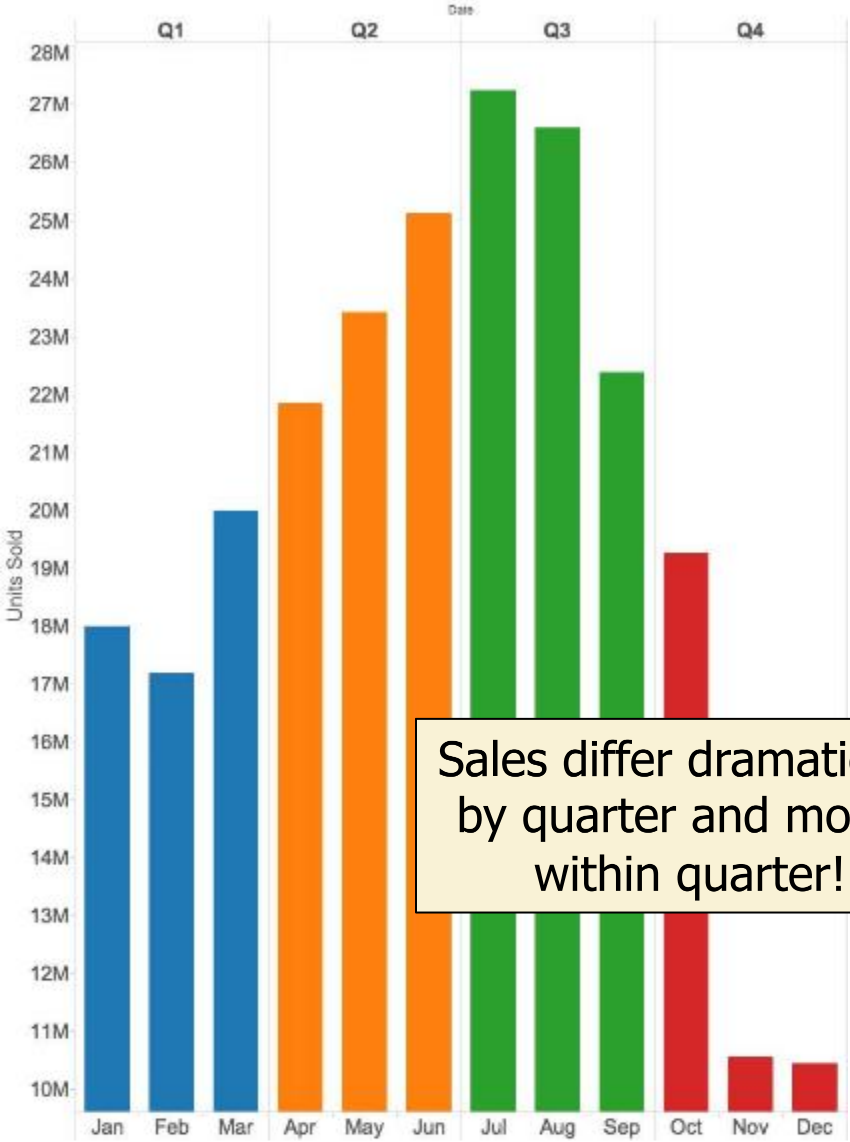
Demand – Sales by Week



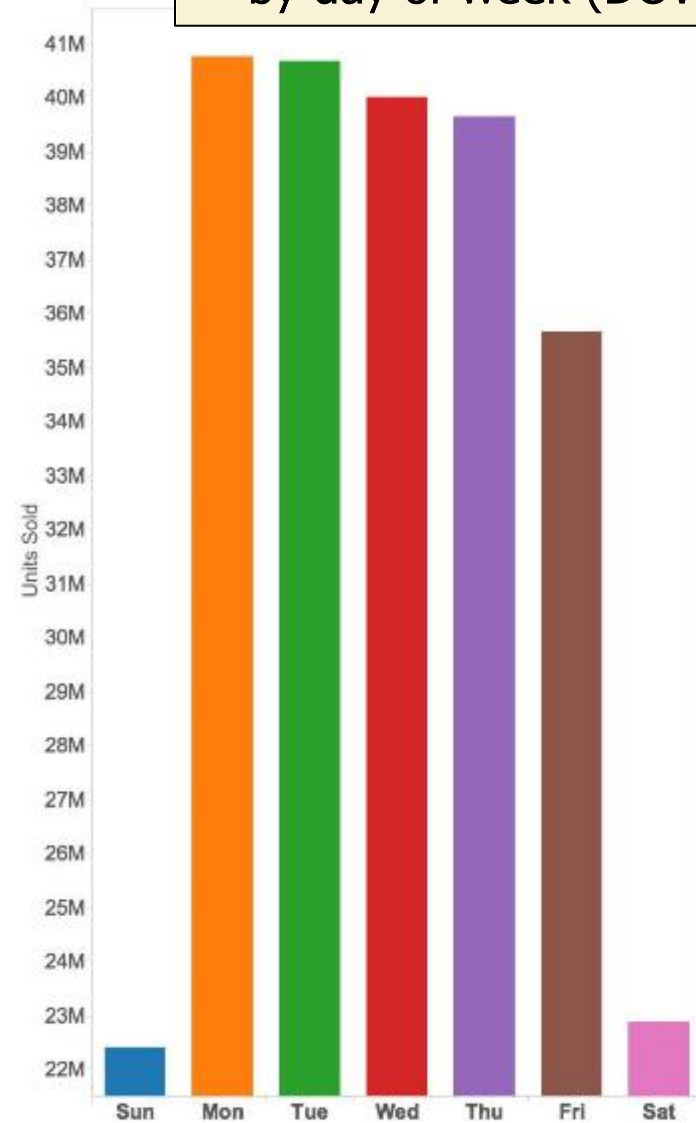
Demand – Sales by Day



Demand - Seasonality

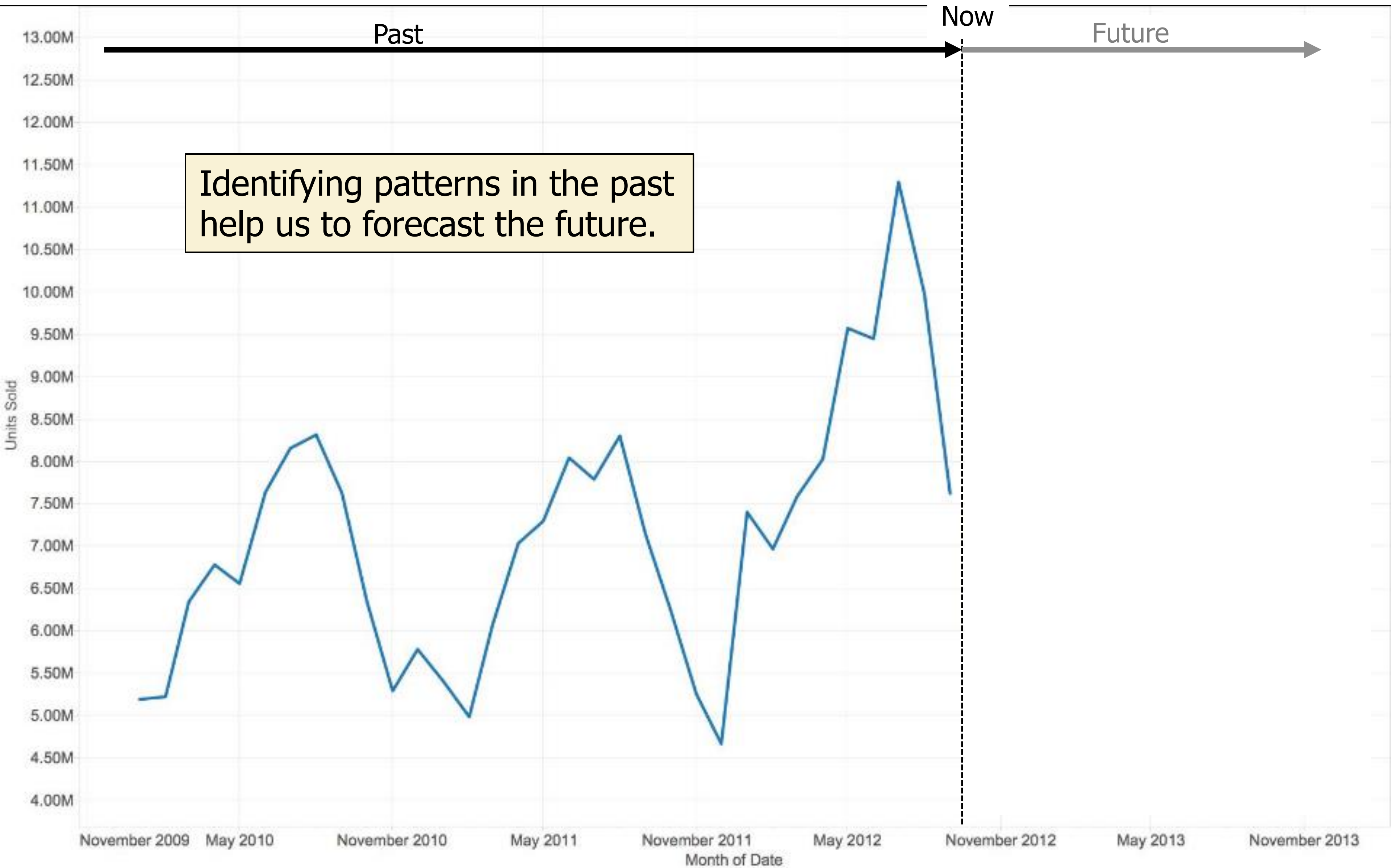


Sales differ dramatically by quarter and month within quarter!



Sales also differ dramatically by day of week (DOW)!

Demand Forecast



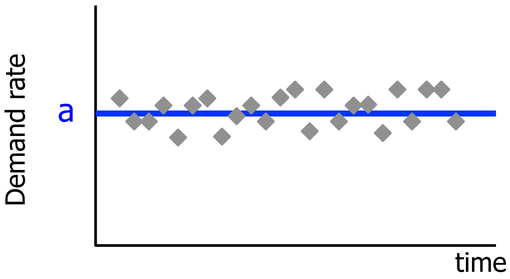
Agenda

- Time Series Components
- Cumulative Forecasts
- Naïve Forecasts
- Moving Average Forecasts

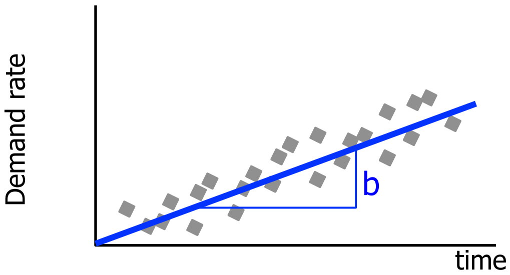
Time Series Components

Time Series Components

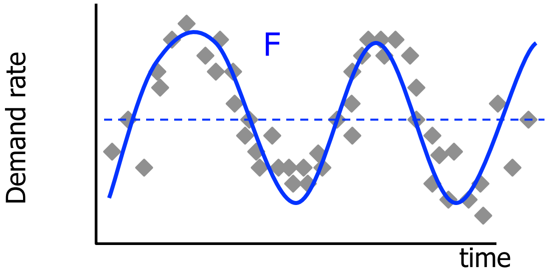
- Level (a)
  - Value where demand hovers around (mean)
  - Captures scale of the time series
  - With no other pattern present its a constant value



- Trend (b)
  - Rate of growth or decline
  - Persistent movement in one direction
  - Typically linear but can be exponential, quadratic, etc.



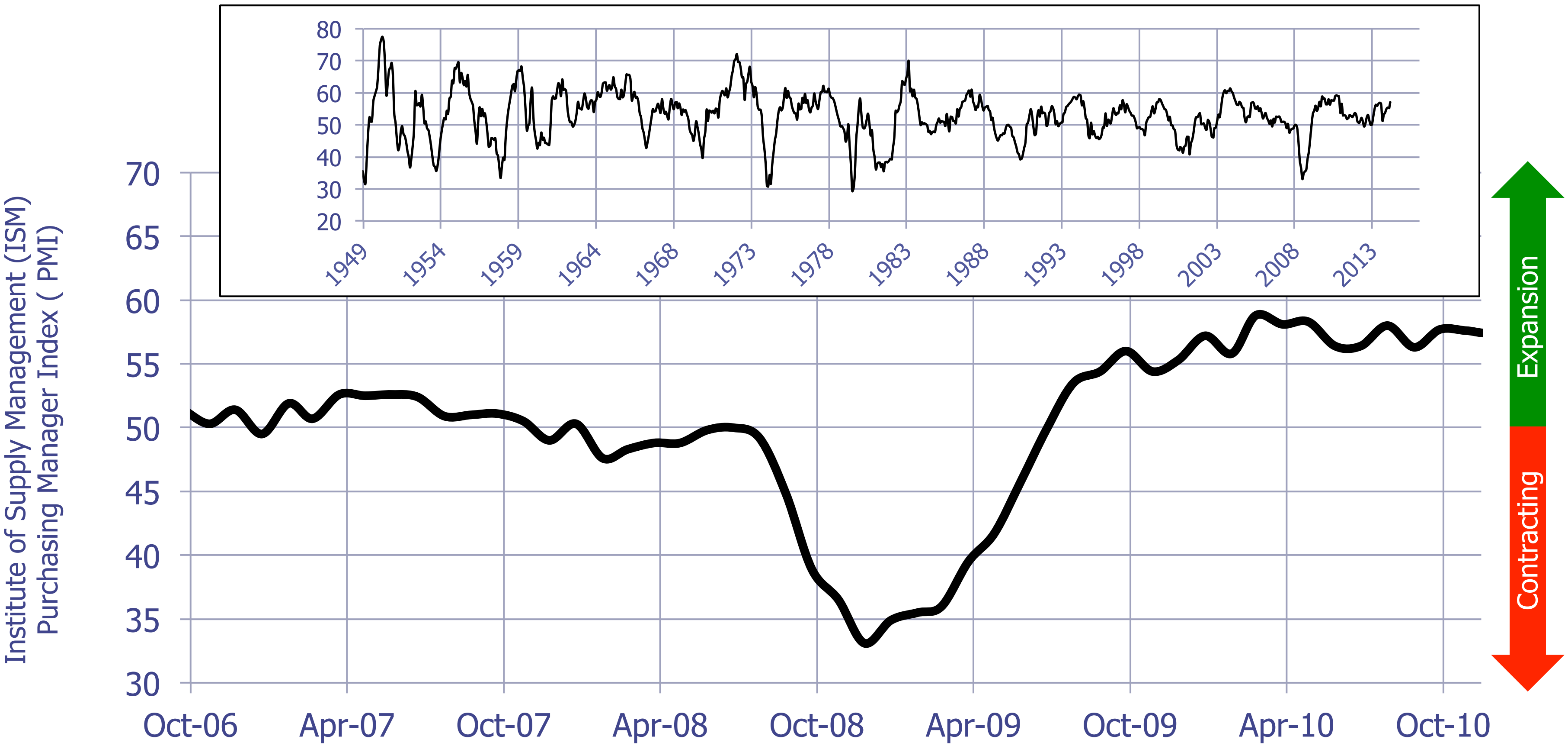
- Seasonal Variations (F)
  - Repeated cycle around a known and fixed period
  - Hourly, daily, weekly, monthly, quarterly, etc.
  - Can be caused by natural or man-made forces



- Random Fluctuations (e or ε)
  - Remainder of variability after other components
  - Irregular and unpredictable variations, noise

Time Series Components

- Cyclical Movements (C)
  - Periodic movement not of a fixed period
  - Duration can be of different lengths
  - Most often tied to longer term business cycles or economic conditions



Data source: Federal Reserve of St. Louis.
http://research.stlouisfed.org/fred2/series/NAPM

Time Series Models

- Components can be combined in different ways:
  - Multiplicative: $x_t = bF_tC_te_t$
  - Additive: $x_t = a + bt + F_t + C_t + e_t$
  - Mixed: $x_t = (a + bt)F_t + C_t + e_t$
    $x_t = a + btF_t + C_t + e_t$
    $x_t = aF_t + bt + C_t + e_t$

Note – we can transform the multiplicative $x_t = bF_tC_te_t$ to: $\ln(x_t)=\ln(b)+\ln(F_t)+\ln(C_t)+\ln(e_t)$

Model depends on how seasonality impacts trend and/or level?

- We will focus on four models
  - Level Model: $x_t = a + e_t$
  - Trend Model: $x_t = a + bt + e_t$
  - Mix Level-Seasonality Model: $x_t = aF_t + e_t$
  - Mix Level-Trend-Seasonality Model: $x_t = (a + bt)F_t + e_t$

Notation:

x_t = Actual demand in period t
a = Level component
F_t = Seasonal index appropriate for period t
e_t = Error – independent random variable (μ=0) and constant σ^2
t = time period (0, 1, 2,…n)
b = linear trend
C_t = Cyclical index for period t

Cumulative vs. Naïve Forecasts

Time Series Models

- Predominant use of Time Series is for forecasting product demand of . . .
  - Mature products at the SKU level over a . . .
    - Short time horizon (weeks, months, quarters, year) . . .
      - Where demand of items is independent.
- So, components used are level, trend, seasonality, and error.

- Simple Procedure
  1. Select an appropriate underlying model of the demand pattern over time
  2. Estimate and calibrate values for the model parameters
  3. Forecast future demand with the models and parameters selected
  4. Review model performance and adjust parameters and model accordingly

Time Series Analysis

- Critical assumption: How important is the history?
- Two extreme assumptions: Very Important or Not at All

Cumulative Forecast

- All history matters equally
- Pure stationary demand

Underlying Model:

$$x_t = a + e_t$$

where:

$$e_t \sim \text{iid}\ (\mu=0\ ,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = \frac{\sum_{i=1}^{t} x_i}{t}$$

Naïve Forecast

- Most recent dictates next
- Random Walk, Last is Next

Underlying Model:

$$x_t = x_{t-1} + e_t$$

where:

$$e_t \sim \text{iid}\ (\mu=0\ ,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = x_t$$

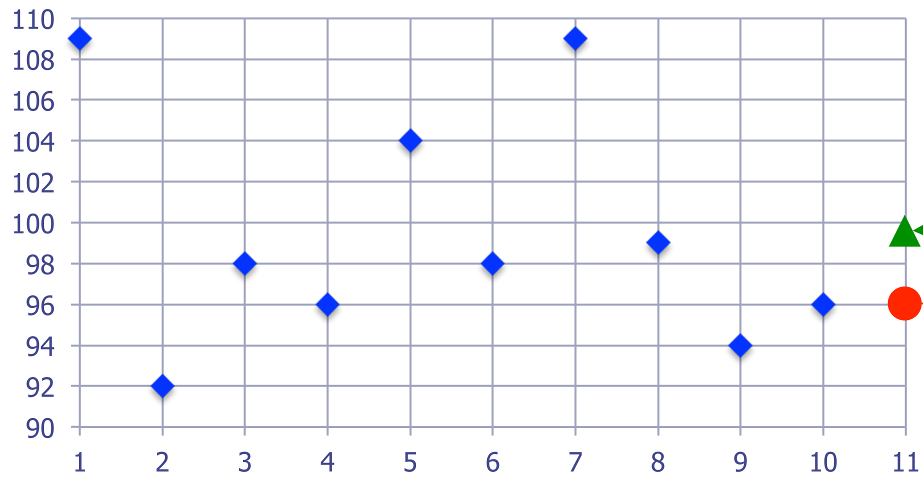
$\hat{x}_{t,t+\tau}$ = Forecast made at end of period t for demand in period t+τ, for τ=1,2,3 ...

x_t = Actual demand for period t

Cumulative vs. Naïve Forecasts

Suppose we are at time=10 and want to find forecast for time=11; $x^\wedge_{10,11}$

t	x_t
1	109
2	92
3	98
4	96
5	104
6	98
7	109
8	99
9	94
10	96



Cumulative Forecast:

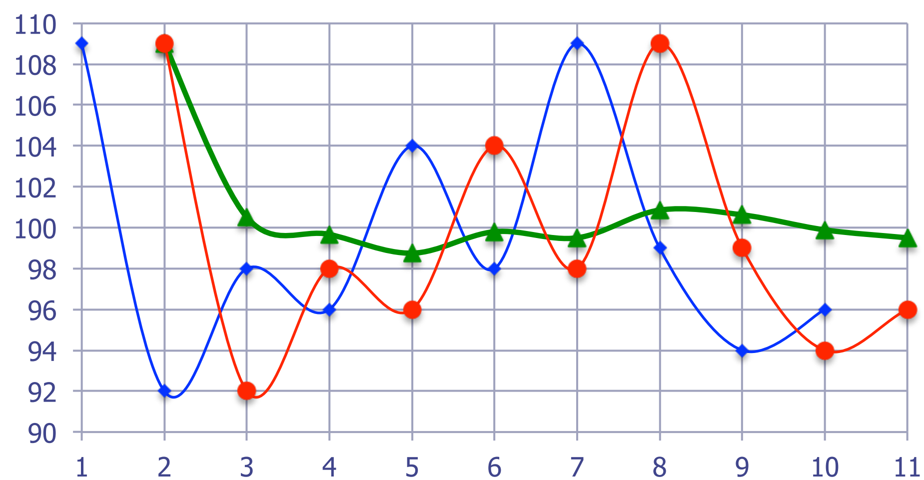
$$\hat{x}_{10,11} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{995}{10} = 99.5$$

Naïve Forecast:

$$\hat{x}_{10,11} = x_{10} = 96$$

Lets look at "next period" forecasts for cumulative and naïve models . . .

t	x_t	Cumul $x^\wedge_{t,t+1}$	Naïve $x^\wedge_{t,t+1}$
1	109	109	109
2	92	100.5	92
3	98	99.7	98
4	96	98.8	96
5	104	99.8	104
6	98	99.5	98
7	109	100.9	109
8	99	100.6	99
9	94	99.9	94
10	96	99.5	96



Note:

- Cumulative model is "calm" while the Naïve model is "nervous".
- Naïve model is more responsive than the cumulative model.

Moving Average Forecast

Time Series Models

- Moving Average
  - Only include the last M observations
  - Compromise between cumulative and naïve

Underlying Model:

$$x_t = a + e_t$$

where:

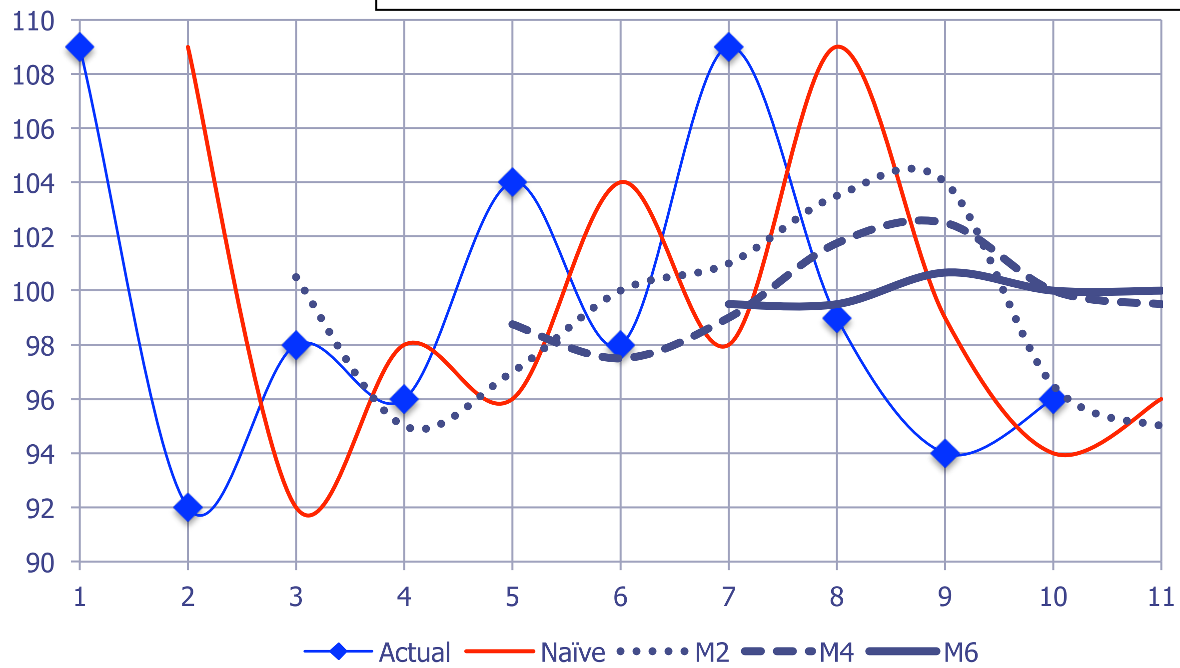
$$e_t \sim \text{iid}\ (\mu=0\ ,\ \sigma^2=V[e])$$

Forecasting Model:

$$\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^{t} x_i}{M}$$

$\hat{x}_{t,t+1}$

t	x_t	Naïve	M2	M4	M6	Cum
1	109	109				109.0
2	92	92	100.5			100.5
3	98	98	95.0			99.7
4	96	96	97.0	98.8		98.8
5	104	104	100.0	97.5		99.8
6	98	98	101.0	99.0	99.5	99.5
7	109	109	103.5	101.8	99.5	100.9
8	99	99	104.0	102.5	100.7	100.6
9	94	94	96.5	100.0	100.0	99.9
10	96	96	95.0	99.5	100.0	99.5



Moving Average Models

$$\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^{t} x_i}{M}$$

- Moving Average Model is a general model
  - Cumulative model (M=t)
  - Naïve model (M=1)

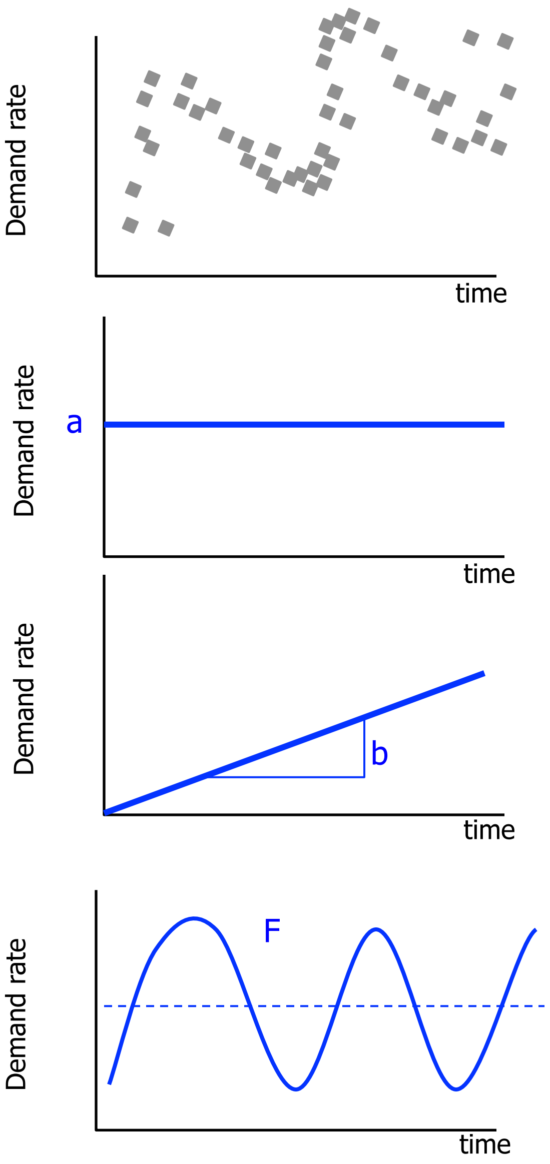
- How big should M be?
  - Too small? Overly responsive to noise, very nervous
  - Too big? Averages out noise, misses step changes in demand
  - Often use "practical" values of M (4, 6, 12, etc.)

- Note that Moving Average models always lag!
  - Assumes stationary demand
  - The larger the M, the longer the lag

Key Points from Lesson

Key Points from Lessons

- Time Series Analysis
  - Pattern matching of data that is distributed over time
- Five Components (focus on first four)
  - Level (a)
  - Trend (b)
  - Seasonality (F)
  - Error (e)
  - Cyclical (C)
- Decompose the Demand using Models
  - Level Model: $x_t = a + e_t$
  - Trend Model: $x_t = a + bt + e_t$
  - Seasonality Model: $x_t = (a + bt)F_t + e_t$



Key Points from Lessons

$$\hat{x}_{t,t+1} = \frac{\sum_{i=1}^{t} x_i}{t}$$

$$\hat{x}_{t,t+1} = x_t$$

$$\hat{x}_{t,t+1} = \frac{\sum_{i=t+1-M}^{t} x_i}{M}$$

- Three Models
  - Cumulative – "everything matters"
  - Naïve – "only yesterday matters"
  - Moving Average – "select how much matters"
- Differences
  - Level of volatility
    - Naïve (nervous) to Cumulative (calm) with MA in middle
  - Required amount of data to store
    - Naïve & Cumulative (1 per SKU)
    - Moving Average (M items for each SKU)
- Similarities
  - Assume level demand – no trends or steps or seasonality
  - All of these models lag to some degree
  - Equal weighting of observations regardless of time

CTL.SC1x -Supply Chain & Logistics Fundamentals

Questions, Comments, Suggestions? Use the Discussion!

"Dude"
Photo courtesy Yankee Golden Retriever Rescue (www.ygrr.org)

MIT Center for Transportation & Logistics

caplice@mit.edu