Supply Chain Fundamentals: Inventory Management

Notes from “Supply Chain & Logistics Fundamentals”

CTL.SC1X (offered by MITx)

weeks 5-8

# Concepts & Definitions

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| **Safety Stock**  **Cycle**  **Stock**  **On Order**    Cycle Time (T)  Lead Time (L)  Order Qty (Q)  Re-order Point (s)  Review Time (R) | **Inventory On Hand** (IOH) = Physical inventory available to meet demand. This can be broken down into **Cycle Stock** (inventory used to meet expected demand) and **Safety Stock** (inventory used as a buffer for uncertainty).  **Inventory On Order** (IOO) = Inventory ordered but not yet delivered. Also called “Pipeline Inventory.”  **Inventory Position** (IP) = IOH + IOO |

Three levels of inventory decisions:

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| Strategic  (Supply Chain Decisions) | What are the potential alternatives to inventory?  How should product be designed? |
| Tactical  (Deployment Decisions) | What items should be carried as inventory?  In what form should they be maintained?  How much of each should be held and where? |
| Operational  (Replenishment Decisions) | How often should inventory status be determined?  When should a replenishment decision be made?  How large should the replenishment be? |

These lessons examine operational replenishment decisions with emphasis on optimal ordering policies. The ordering policies examined include:

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| EOQ Models | Simple models that assumes constant demand and attempt to find optimal order quantities. Balances inventory costs with ordering costs. |
| Single Period Model | The “newspaper problem.” Maximize profit for one-time buy & sell scenarios. |
| Base Stock Policy | One-for-one replenishment. This is “replacement ordering,” a continuous review policy that constantly places orders for whatever has just been sold. |
| Continuous Review (s, Q) | Order an EOQ quantity (*Q*) whenever inventory reaches a specified re-order point (*s*). (Orders can be placed at any time) |
| Periodic Review (R, S) | Order-up-to a specified level (*S*) every *R* periods. (Orders can only be placed at fixed intervals). |

# EOQ Models

The “Economic Order Quantity” calculates an optimal ordering policy under a set of simple assumptions. The solution balances the costs of ordering with the cost of holding inventory and is very robust with respect to how these costs are estimated. Intuitively:

* High ordering costs lead to larger orders (order less often, but carry more inventory)
* High inventory costs lead to smaller orders (order more often, but carry less inventory)

The EOQ model tries to find the balance between these and provides an order quantity that is near-optimal even when applied under less restrictive assumptions.

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| Q  T  μD | Assumptions:   * Constant demand * Instantaneous delivery   Optimal policy is to order *Q* units every *T* periods of time (where ). In this case, total costs are:  = Purchase Cost (cost per unit \* units ordered)  = Ordering Cost (cost per order \* # of orders)  = Inventory Cost (holding cost \* avg. inventory)  Avg. Inventory = |

*Optimal Solution:*

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| --- | --- |
|  | NOTE: This is found by taking the derivative of TRC(Q) and setting it equal to zero. Also, be careful of units. If *ct* and *ce* are annual costs, *D* must be annual demand. |
|  |  |

This solution is very robust. Even if there are errors estimating the input variables or errors in implementing the policy (ordering in quantities other than *Q\**). It can be shown that over-estimating demand (*D*) or ordering costs (*ct*) by as much as 50% still produces a solution that is within 6% of optimal. Over-ordering by as much as 50% produces a solution that is 25% worse than optimal. In all of these cases, over-estimating is better than under-estimating. Inventory costs (*ce*) are the exception: under-estimating these is better than over-estimating.

*EOQ with Non-Zero Lead Time*

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| Q  T  μD  L | If orders are delivered with lead time *L* (rather than being delivered instantaneously) the optimal order quantity and total cost equations do not change. We also will still place orders every *T* units of time, but instead of waiting until inventory reaches zero to place an order, we place it when .  Inventory levels are:  NOTE: *L/T* is the percentage of time that we have an order in transit and *Q* is the amount in transit. Pipeline inventory does not normally incur holding costs, but even if it did the value does not change with Q so it is not included in *TRC*. This is why the optimal solution from the normal EOQ model is still optimal. |

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# *EOQ with Planned Backorders*

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| Q  T  μD  b  T1  T2 | We can also extend the EOQ model to allow back-orders. This means that any shortage is back-ordered with cost/unit/time = cs.  The optimal ordering policy in this case changes to:   * Order Q\* Units Every T\* time periods * Order Q\* Units When back-orders = b |

“CR” is the “critical ratio,” and it expresses the cost of shortages relative to other inventory costs in the model. If this is low (shortages do not cost much), the optimal policy will be to delay ordering, accumulating shortages for as long as possible before placing one big order (bigger than *Q\**) to catch up. If this is high (shortages cost much more than excess inventory) the optimal order policy will be close to *Q\** and will not allow shortages. The optimal solution is trying to find a balance between excess inventory and shortage costs.

# Single Period Model

The single period model covers the case where we must make a one-time buy to meet unknown demand. There is no inventory, and any excess product must be discarded if it is not sold. This is the classic “newspaper problem” where today’s newspaper must be produced in the morning, and at the end of the day must be thrown away (since nobody wants a newspaper from yesterday).

If we buy quantity *Q* at cost *c* and sell it at price *p*, our profit is:

*x* = demand (random, or unknown)

The optimal solution is one that maximizes the expected profit:

We can generalize the problem by introducing salvage costs and additional penalties for not meeting demand. In this case, we have:

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| Variable | Name | Units |
| 𝑔 | Salvage value. Excess product can be sold at this lower price if it is not sold at price *p*. | $ / units |
| *B* | Penalty for not satisfying demand | $ / units |

An analytical solution can be found in both cases by examining the marginal cost of ordering an extra unit when order quantity is *Q*:

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|  | Expected excess cost of Qth unit ordered | |
|  | Expected shortage cost of the Qth unit ordered | |
| where: | |  | |

Lost profit (*p – c*) and additional penalty (*B*) for not satisfying demand

Purchase cost (*c*) of excess product minus salvage value (*g*)

The optimal solution (regardless of the distribution of *x*) is the point at which the marginal values are equal:

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|  | NOTE: This is optimal because we should always order more as long as the cost of excess product is less than the cost of a shortage. We would continue to increase *Q* until the cost of excess product becomes larger than the cost of a shortage. This transition occurs exactly when the two marginal costs are equal. |

*Optimal Solution if Demand is Normally Distributed*

For any probability distribution, the optimal solution is the point *Q\** at which:

For the normal distribution where it is sometimes helpful to transform a value *Q* to the unit normal distribution:

The optimal solution *k\** can be found using the inverse cumulative density function. In Excel, this is simply:

For a given value of *k* we also have:

*G(k)* is the “Unit Normal Loss Function.” It provides the expected units short using a unit normal distribution. This can be calculated using a lookup table or in Excel as:

G(k) = NORMDIST(k,0,1,0) - k\*(1-NORMSDIST(k))

It can be scaled by the standard deviation (σ) in order to get expected units short for any normal distribution.

**Base Stock Policy**

The “base stock” inventory policy sets a base stock level (*S*) and orders up to that level. Any time a unit is sold another one is re-ordered. This is a continuous review policy since it assumes that we can place an order at any time and it will always be delivered with lead time (*L*). The effect of this policy is that the inventory is constantly being re-ordered. Inventory Position (or base stock) remains constant although Inventory on Hand and Inventory on Order will fluctuate.



To use the base stock policy, we first must determine the desired Level of Service (LOS). The base stock level will then be set to meet this LOS. LOS is usually set arbitrarily (i.e. management says we must be in-stock 95% of the time). However, the optimal service level is one that balances costs of inventory with costs of shortages, and just like the single period model this is equal to the critical ratio:

= quantity demanded over lead time *L*

If demand is normally distributed (), we can calculate the *k*-value corresponding to the desired level of service just as we did in the Single Period Model. We can then set *S\** to:

**Continuous & Periodic Review Policies**

Most inventory replenishment policies can be divided into two groups:

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| Continuous Review Policies | Orders can be placed at any time |
| Periodic Review Policies | Orders can only be placed at specific times |

A few of the more common replenishment policies include:

*Continuous Review Policies*

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| Base Stock Policy (*S*)   * Always order up to *S* * “Replacement ordering” | Effectively: place an order for 1 unit every time that we sell a unit. This will keep the inventory position equal to the base stock level *S*. |
| Order-Point, Order-Quantity (*s,Q*)   * Order *Q* if *IP <= s* * “Two-bin system” | This will order a fixed quantity (*Q*) whenever inventory reaches the re-order point *s*. *Q* is typically set to *Q\** from EOQ. |
| Order-Point, Order-Up-To-Level (*s,S*)   * Order *Q = S - IP* if *IP <= s* * “Min-max system” | Orders up to target level *S* whenever inventory reaches re-order point *s*. As a continuous review policy this is equivalent to the (*s, Q*) model with . |

*Periodic Review Policies*

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| Order-Up-To-Level (*R, S*)   * Order *Q = S - IP* every *R* time periods. * “Replenishment Cycle System” | This will place an order each time period and order-up-to the target inventory level *S*. It is similar to a base stock policy but orders periodically rather than continuously. |
| Hybrid (*R, s, S*)   * Every *R* time periods, order *Q = S – IP* if *IP <= s* * General case for many policies | This will place an order each time period only if inventory falls below the re-order point *s*. When it does, it will place an order that will bring inventory to the target inventory level *S*. This is a generic model that includes all other models as special cases. |

We’ve already examined the Base Stock Policy (*S*). We will also look in detail at the Order-Point, Order Quantity (*s, Q*) model and the Order-Up-To-Level (*R, S*) model. In doing so, it will be useful to define a few other terms related to level of service and expected shortages:

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| Cycle Service Level (*CSL*) | The percentage of order cycles that will be completed without a stockout/shortage event. |
| Item Fill Rate (*IFR*) | The percentage of customer demand that is filled. |
| Cost per Stock Out Event (*CSOE*) | The cost incurred per stock out event ($/event). This cost is the same regardless of how many units short we were. |
| Cost per Item Short (*CIS*) | The cost incurred for each unit of unmet demand ($/unit). |

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# Order-Point, Order-Quantity (*s, Q*)

The (*s, Q*) policy is a model that places an order of size *Q* whenever inventory reaches the re-order point *s*. *Q* is usually determined using an EOQ model to determine optimal order quantities. *s* can then be set in various ways:

*Method 1: Cycle Service Level (CSL)*

If we specify a target Cycle Service Level (i.e. management says that orders should cover expected demand 90% of the time) the optimal order point is determined such that:

As before, this can be solved in Excel using: .

*Method 2: Item Fill Rate (IFR)*

If we specify the desired Item Fill Rate (i.e. management says that orders should cover 95% of expected demand) the optimal order point can be set to achieve this policy. If *Q* is expected demand for the order cycle, the Item Fill Rate is given as:

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| --- | --- |
| (general definition) | (normally distributed demand) |

There is no analytical solution for *k*, but we can use “guess and check” or recursive search algorithms to find the value of *k* that produces the target IFR. Note that once we find the value of *k*, the Cycle Service Level can be calculated as well. It is interesting to note that the IFR produced by an ordering policy is always greater than CSL. This is because the CSL gets penalized any time an out-of-stock event occurs. It does not get credit for demand that was met during the order cycle before the out-of-stock occurred. This difference between CSL and IFR can be very large.

*Method 3: Total Cost Optimization*

We can also look for solutions that minimize total costs:

The first few components of this equation are known:

PurchaseCost = (cost per unit \* units ordered)

OrderCost = (cost per order \* expected number of orders)

HoldingCost = (cost per unit/year of inventory \* avg. inventory level)

However, the StockOutCost is a bit more subjective. If we have a fixed cost per stock out event (CSOE = *B1*) this would be:

StockOutCost =

E[StockOutEvents] =

This can be solved analytically to get value of *k* that minimizes costs:

|  |  |
| --- | --- |
|  | Note that this is only valid when: |

Alternately, we might prefer to specify a cost per item short (CIS = ). In this case we would have:

StockOutCost =

E[UnitsShort] =

This can also be solved analytically by finding the optimal value of *k* such that:

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|  | Note that this is only valid when:  Note also that *D* should be annual demand (unit/year) if *ce* and *cs* are in annual terms ($/unit/year) |

# Order-Up-To-Level (*R, S*)

The order-up-to-level policy places an order every *R* period, ordering up to the target inventory level *S*. This is actually very similar to the (*s, Q*) ordering policy, but with a few changes:

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| Value | (s,Q) | (R,S) |
| Expected Sales per Order Cycle | Q |  |
| Time that safety stock must cover sales | L | R + L |

Since we will be covering sales over period *R + L* instead of just *L* we will also assume that sales are drawn from the normal distribution instead of .

This means that if we solve for *k* in an (*s, Q*) model using and and substitute:

the *k* that we obtain will be the optimal *k* that we want to use in the (*R, S*) model as well. The only difference is that we will have to convert it back using to units using:

We can make similar substitutions into the other relevant terms of the (*s, Q*) model. Some outputs of the model remain unchanged, but some are slightly different. The table below summarizes important values:

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| --- | --- | --- |
| Value | (*s, Q*) | (*R, S*) |
| PurchaseCost |  |  |
| OrderCost |  |  |
| HoldingCost |  |  |
| E[StockOutEvents] |  |  |
| E[UnitsShort] |  |  |
| CSL | CDF(k) | CDF(k) |
| IFR |  |  |

# Additional Concepts

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| Risk Pooling | Safety stock levels are driven by the standard deviation of demand (or the RMSE/accuracy of forecasts). If we aggregate items in 1 location instead of *N*, the standard deviation should decrease by a factor of . Safety stock levels should also decrease proportionately. This is one reason to consider shared inventory locations and distribution centers. |
| Item Segmentation | Items can be segmented into ABC categories based on annual purchase cost (*cD*). The methods discussed so far are suitable to class B items. Class A items will require special attention and advanced modeling. Class C items can be managed passively. Class A items typically fall into two categories:   * Fast moving but cheap (large order quantities) * Slow moving but expensive (small orders: maybe only 1 unit) |
| Addtl. Distributions | Slow-moving items (especially in class A) might require special modeling. In general, a Poisson distribution is a better model for items where average sales over time *R+L* are less than 10. |

# Reference: Variables & Notation

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| --- | --- | --- | --- |
| Symbol | Name | Description | Units |
| D | Total Demand (over period of time being examined) | Customer demand for an item. This demand must be met or there will be a shortage. | units / time |
| c | Variable (purchase) cost | Cost of purchasing 1 unit | $ / unit |
| ct | Fixed Order (transaction) Cost | Cost of placing an order | $ / order |
| ce = c ∙ h | Excess Holding Cost | Cost of holding excess inventory | $ / unit / time |
| h | Carrying or Holding Charge | Excess holding cost (expressed as a percentage of unit cost) | $ / unit / time  (% of unit cost) |
| cs | Cost of a shortage | Cost of being short and not meeting demand | $ / units / time |
| L | Lead Time | Time between when an order is placed and when it is delivered | time |
| R | Review Time | Time between orders (in periodic review system) | time |
| Q | Replenishment Order Quantity | Quantity ordered | units / order |
| T | Order Cycle Time | Time between deliveries where inventory is used to meet demand. | time / order |
| N = 1/T | Orders per Time | Average number of orders placed in a given time period | order / time |
| TC | Total Cost | Total cost of an inventory management policy | $ / time |
| TRC | Total Relevant Cost | The portion of Total Cost that can be influenced by one more variables under our control. This is different for each model. | $ / time |