Pricing Complex Derivatives

Team DataMafia

Indian Statistical Institute

Workflow

Hypothesis about Distribution

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Integral Calculation

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Optimisation of Total SSE

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Parameter Estimation

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Final Payoff Calculation

Pricing Complex Derivatives

- Price of an call option

$$C(X,K,T) = \mathbb{E}[\max(X_T - K, 0)]$$

$$C(X,K,T) = \int_0^{\infty} \max(X_T - K, 0) f_X(X_T)\, dX_T$$

$$C(X,K,T) = \int_K^{\infty} (X_T - K) f_X(X_T) dX_T$$

Where $f_X(X_T)$ is the probability density function of time T, asset price X_T

Objective Function

- In this context we need to estimate the probability distribution of the asset prices.
- That means with an assumption of the distribution of the asset prices we need to estimate the parameter for the model.

- The objective function :

$$\min \sum(C(X, K, T, parameters) \text{ - Price Given})^{2} \text{ w.r.to parameters}$$

with proper constraints for distributional assumptions

Distribution of Asset Price

- The first assumption is the normality of the price distribution
- Our assumption of normality passes the test as minimised Total sum of squares achieved for oil options case was 0.0002
- The total sum of squares achieved for forex options is 8.2916
- The pdf of normal distribution will look like:

$$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Optimal Parameters

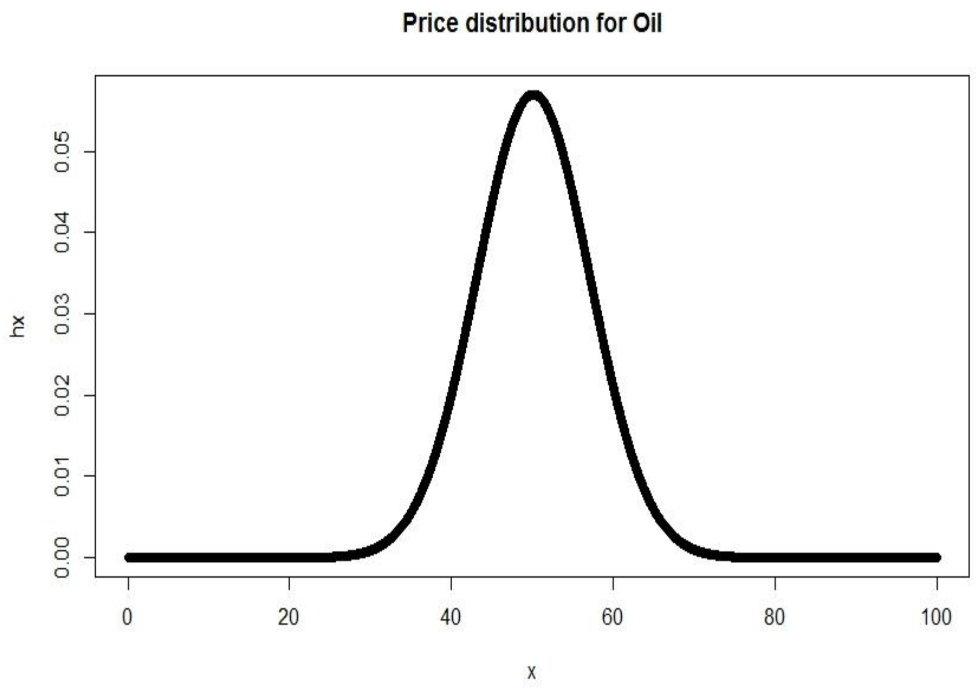
- Total SEE for oil call option prices came to be 0.00002.

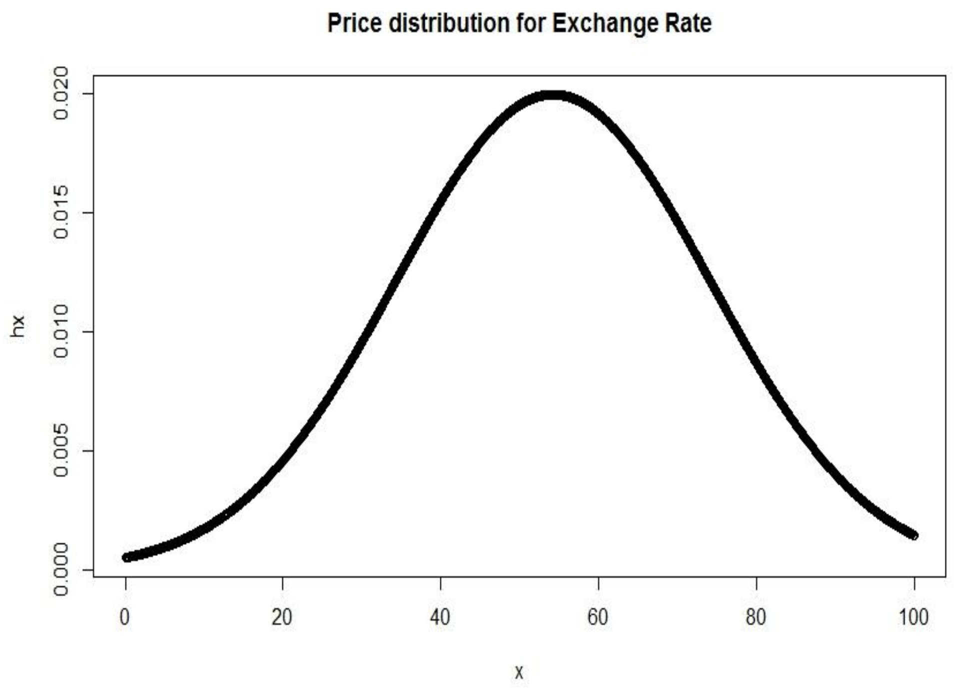
The optimal parameter came to be: mean = 50.01165, Standard deviation = 7.00022

- The total SSE for FX call option 8.2916.

The optimal parameter for FX prices came to be: mean = 54.21735, standard deviation = 19.9633

The distributions





Digital and Exotic Payoffs

- The payoffs have been calculated using optimal distributional parameters.

1. Digital payoff $D(K,T;X) = \mathbb{E}[\mathbb{I}(X_T > K)]$
2. Exotic payoff $P(K,T;X) = \mathbb{E}[(\mathrm{Max}(X_T - K, 0))^2]$

Estimating Joint Distribution

- Assumption:

1. The oil option prices and FX option process are independent.
2. The joint distribution follows a bivariate normal distribution.

Reason: If both of the asset follows bivariate normal distribution jointly, then their conditional (& marginal as well) distribution will also follow normal distribution, which we had already indicated in previous cases. The assumption of independence has been taken because the it is difficult to estimate the correlation between asset prices from the option data

Individual Payoffs

- Let's assume Q_1 and Q_2 are the payoff prices of two assets.

Hence, according to the question,

$$Q_1 = \int_{0}^{\infty} \int_{K_2}^{\infty} x_1 (x_2 - K_2) \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x_2 - \mu_2)^2}{2{\sigma_2}^2}} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x_1 - \mu_1)^2}{2{\sigma_1}^2}} dx_2 dx_1$$

$$= \int_{K_2}^{\infty} (x_2 - K_2) \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x_2 - \mu_2)^2}{2{\sigma_2}^2}} dx_2 \int_{0}^{\infty} x_1 \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x_1 - \mu_1)^2}{2{\sigma_1}^2}} dx_1$$

[because both of the asset prices are independent]

Individual Payoff and Joint Payoff

- Similarly Q_2 has been estimated using the strike price of Asset 1.
- The final pay off in terms of Joint Distribution would be:

$$\int_{K_2}^{\infty} (x_2 - K_2) \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x_2 - \mu_2)^2}{2\sigma_2{}^2}} dx_2 * \int_{K_1}^{\infty} (x_1 - K_1) \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x_1 - \mu_1)^2}{2\sigma_1{}^2}} dx_1$$

[because both of the asset prices are independent]

Optimization with Four parameters

- For this optimization the objective is

$$\min \sum (C(X, K, T, parameters) - \text{Price Given})^{2} \text{ w.r.to parameters}$$

with proper constraints for distributional assumptions

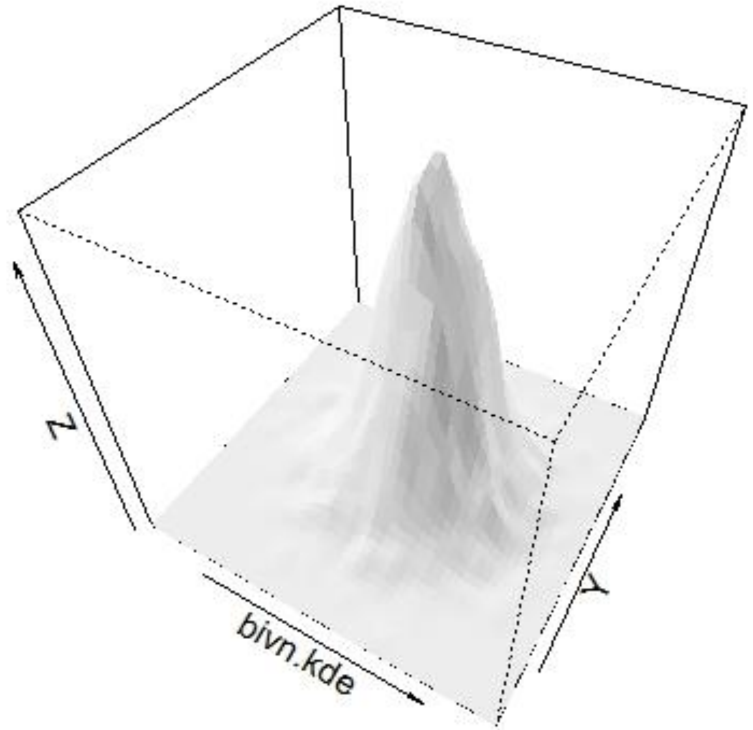
Here, we are optimizing all the four parameters to minimise the total SSE which has four components,

a) Estimation error of Oil option prices conditional on FX option strike prices, b) Estimation error of FX option prices conditional on Oil option strike prices, c) Estimation error of Oil option prices marginally (from Problem 1 & 2), d) Estimation error of FX option prices marginally (from problem 1 & 2)

Final Optimal Parameters

- The minimised Total SSE came out to be: 8.289797

where as the optimal parameters are (54.22097,19.95393) & (45.27257, 9.016578)



Discussion on Optimization Technique

- We have used BOBYQA optimization technique for bound constrained optimization without derivatives. [1]
- BOBYQA performs derivative-free bound-constrained optimization using an iteratively constructed quadratic approximation for the objective function.
- The difficulty of calculating derivatives of objective function containing integrals lead to this option.
- The quadratic approximation of the objective function which makes it suitable because of the quadratic nature of the total SSE function

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[1] M. J. D. Powell, "The BOBYQA algorithm for bound constrained optimization without derivatives”

Future Possibilities

- If the correlation between Oil and Exchange Rate can be determined or estimated, the assumption of independence could be relaxed. A general bivariate normal distribution could be fitted.