Math 331 Section 1. Solutions to Quiz 1.

Problem 1. a) Find the general solution to the first order differential equation

$$\frac{dy}{dx} = e^{(3x+1)}\, y^2$$

Solution. By separation of variables and integration we have that

$$\int \frac{y'}{y^2}\, dx = \int e^{(3x+1)}\, dx \qquad \Longrightarrow$$

$$\int \frac{1}{y^2}\, dy = \int e^{(3x+1)}\, dx \qquad \text{since} \quad dy = y', dx$$

Hence,

$$-\frac{1}{y} = \frac{1}{3}\, e^{(3x+1)} + C \quad \text{for any constant } \; C \qquad \Longrightarrow$$

$$y(x) = \frac{-1}{\frac{1}{3}\, e^{(3x+1)} + C}$$

b) Is it linear? **No** due to the presence of the term y^2.

c) Find the particular solution $y = y(x)$ to the equation in part a) satisfying $y(0) = 3$ (express $y(x)$). underbarSolution. If $y(0) = 3$ then that means that

$$3 = y(0) = \frac{-1}{\frac{1}{3}\, e^{(3.0+1)} + C}$$

$$\Longrightarrow 3 = \frac{-1}{\frac{1}{3}\, e + C} \qquad \Longrightarrow C = -\frac{1}{3} - \frac{e}{3} = -(\frac{1}{3} + \frac{e}{3}).$$

Hence in this case,

$$y(x) = \frac{-1}{\frac{1}{3}\, e^{(3x+1)} - (\frac{1}{3} + \frac{e}{3})}$$

Problem 2 Solve the following initial value problem for the given linear differential equation

$$\frac{dy}{dt} = -2t\, y + 4e^{-t^2} \qquad y(0) = 3$$

Solution. We first rewrite as

$$\frac{dy}{dt} + 2t\, y = 4e^{-t^2}$$

and note that this is a linear equation with integrating factor $\mu = e^{\int 2t\, dt} = e^{t^2}$.

Multiplying both sides of the equation by μ we get that

$$\left(e^{t^2} y\right)' = e^{t^2} 4e^{-t^2} \quad \text{or} \quad \left(e^{t^2} y\right)' = 4.$$

Integrating both sides with respect to t gives

$$e^{t^2} y = 4t + C \quad \Longrightarrow \quad y(t) = 4te^{-t^2} + Ce^{-t^2} \quad \text{for any constant C.}$$