

### PROBLEM SET 3

Due March 2, 2006

#### Section 3.1.

From Folland's book # 3.1 page 88. Do problems 2, 3, 4.

#### Section 3.2.

From Folland's book # 3.2 page 92. Do problems 8, 9, 10, 13, 16 (correction: need both measures to be  $\sigma$ -finite), 17 (correction: need  $\nu$  to be  $\sigma$ -finite also).

**Extra Problem 1 :** Let  $\mu$  be a positive measure over  $(X, \mathcal{M})$  and let  $f$  real be an *extended*  $\mu$ -integrable function on  $X$ . Define

$$\nu(E) := \int_E f d\mu \quad E \in \mathcal{M}$$

Show that  $\nu$  is a *signed measure* on  $(X, \mathcal{M})$

#### Section 3.3.

**Extra Problem 2 :** Let  $\mu$  be a positive measure over  $(X, \mathcal{M})$  and let  $f : X \rightarrow \mathbb{C}$  be a  $\mu$ -integrable function on  $X$ ;— i.e.  $f \in L^1(\mu)$ . Define

$$\nu(E) := \int_E f d\mu \quad E \in \mathcal{M}$$

(a) Show that  $\nu$  is a *complex measure* on  $(X, \mathcal{M})$

(b) In particular, show that if  $f \in L^1(\mu)$  takes only *real* values —i.e.  $f : X \rightarrow \mathbb{R}$ — then  $\nu$  as defined here is a **finite signed measure** (here use the Extra Problem 1 above).

From Folland's book # 3.3 page 94. Do problem 20.