Hint for 7) We seek solutions to $u_t - k u_{xx} = 0$ of the form $u(x,t) = v\left(\frac{x}{\sqrt{t}}\right)$

(You may let $y = \frac{x}{\sqrt{t}}$ from where

$$\frac{\partial y}{\partial t} = \frac{x}{2t\sqrt{t}} \, ; \quad \frac{\partial y}{\partial x} = \frac{1}{\sqrt{t}} \, ; \quad \frac{\partial^2 y}{\partial x^2} = 0 \,)$$

Or you may proceed directly. Either way you have:

$$u_t(x,t) = -\frac{x}{2t\sqrt{t}} \, v'\left(\frac{x}{\sqrt{t}}\right)$$

$$u_x(x,t) = \frac{1}{\sqrt{t}} \, v'\left(\frac{x}{\sqrt{t}}\right)$$

$$u_{xx}(x,t) = \frac{1}{t} \, v''\left(\frac{x}{\sqrt{t}}\right)$$

Hence $u_t - k u_{xx} = 0$ gives

$$\frac{-x}{2t\sqrt{t}} \, v'\left(\frac{x}{\sqrt{t}}\right) - k \frac{1}{t} \, v''\left(\frac{x}{\sqrt{t}}\right) = 0$$

multiply by $-\frac{t}{k}$

$$\frac{x}{2k\sqrt{t}} \, v'\left(\frac{x}{\sqrt{t}}\right) + v''\left(\frac{x}{\sqrt{t}}\right) = 0$$

Which in terms of y would read

$$(\dagger) \qquad \frac{y}{2k} v'(y) + v''(y) = 0 .$$

Suppose we want to solve $(\dagger)$. Then it is easier to let $w(y) = v'(y)$ whence $(\dagger)$ becomes

(or just use an integrating factor)

$$(\dagger\dagger) \qquad \frac{y}{2k} w(y) + w'(y) = 0$$

and a solution of $(\dagger\dagger)$ is

$$w(y) = c\, e^{-y^2/4k}$$

$\Rightarrow$ integrating get

$$v(y) = c_1 + c_2 \int_0^y e^{-\tilde{y}^2/4k} \, d\tilde{y}$$

or

$$v\left(\frac{x}{\sqrt{t}}\right) = c_1 + c_2 \int_0^{\frac{x}{\sqrt{4kt}}} e^{-z^2} \, dz$$

$$\overset{\shortparallel}{u(x,t)} = c_1 + c_2 \, \mathcal{E}rf\left(\frac{x}{\sqrt{4kt}}\right) \qquad \#$$