Math 331 Section 3
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Quiz 1— Solutions

Problem 1. Consider the logistic population model

$$\frac{dP}{dt} = 0.4\,P\,(1 - \frac{P}{230})$$

where $P(t)$ is the population at time t.

(a) For what values of P is the population in *equilibrium* ?

Population is in equilibrium whenever $\frac{dP}{dt} = 0$.
This holds whenever either $P = 0$ or $P = 230$.

(b) For what values of P is the population in *increasing* ?

Here we need to find for what values of P $\frac{dP}{dt} \geq 0$.
This is when $0 \leq P \leq 230$

(c) For what values of P is the population in *decreasing* ?

Here we need to find for what values of P $\frac{dP}{dt} \leq 0$

Since negative population doesn't make sense, we always have $P \geq 0$. Hence population is decreasing when $P \geq 230$

Problem 2. Solve the following initial value problem

$$\frac{dy}{dt} = \frac{1}{2y+3}, \qquad y(0) = 1$$

First we note that there are no equilibrium solutions.
Next we solve using separation of variables. We write

$$(2y+3)\frac{dy}{dt} = 1 \Longrightarrow y^2 + 3y = t + C$$

To determine C we use the initial condition; set $t = 0$ and $y = 1$ to get $4 = C$ hence we have

$$y^2 + 3y = t + 4 \Longrightarrow y^2 + 3y - (t+4) = 0$$

Solving the quadratic equation we get that $y(t) = \frac{3}{2} \pm \sqrt{25+4t}$ But since we **must have** that $y(0) = 1$ the solution we seek is

$$y(t) = \frac{3}{2} + \sqrt{25+4t}$$

Problem 3 Given the initial value problem

$$\frac{dy}{dt} = y^3, \qquad y(0) = 1$$

(a) Find a formula for the solution

First we note that $y = 0$ is an equilibrium solution. For $y \neq 0$ we solve by separation of variables to get

$$\frac{1}{y^3}\frac{dy}{dt} = 1 \Longrightarrow -\frac{1}{2}\frac{1}{y^2} = t + C$$

Using the initial condition we can now find C; set $t = 0$ and $y = 1$ to get $-\frac{1}{2} = C$. Hence

$$-\frac{1}{y^2} = 2t - 1 \Longrightarrow y^2 = 1 - 2t \Longrightarrow y = \pm\sqrt{1-2t}$$

But since the solution we seek **must satisfy** $y(0) = 1$; we must have $y = \sqrt{1-2t}$.

(b) State the domain of definition of the solution

The solution $y(t)$ found in (a) has domain of definition the set of all $t \leq \frac{1}{2}$

(c) Describe what happens to the solution as it approaches the limits of its domain of definition.

Since the domain of definition are all $-\infty < t \leq \frac{1}{2}$; we need to compute :

$$\lim_{t \to \frac{1}{2}^-} \sqrt{1-2t} \; = \; 0$$

and

$$\lim_{t \to -\infty} \sqrt{1-2t} \; = \; +\infty$$