Control of $X^{3/2,1/2,1}(-T,T)$ by $L^\infty H^{5/2}$.

First of all, I can easily put the apriori estimates in the space $Y^{3/2,1/2,1}(-T,T)$ (rather than $X^{3/2,1/2,1}(-T,T)$) with norm taken as

$$\|u\|_{Y_k^{3/2,1/2,1}} = \inf_{v:v|_{(-T,T)}=u} \sum_{j\leq k} 2^{j/2} 2^{3k/2} \left\| v_k^j \right\|_{L^2_{tx}} + 2^{3k/2} \left\| \sum_{j>k} v_k \right\|_{L^\infty_t L^2_x} + 2^{2k} \left\| \sum_{j>k} v_k \right\|_{L^2_{tx}},$$

$$\|u\|_{Y^{3/2,1/2,1}} = \left(\sum_k \|u\|_{Y_k^{3/2,1/2,1}} \right)^{1/2}.$$

That is instead of dealing with the space $X^{3/2,1/2,1}(-T,T)$, we replace it with the space $Y^{3/2,1/2,1}$ and now everything has to be done there.

This space is a bit bigger than $X^{3/2,1/2,1}(-T,T)$, i.e. the norm is smaller.
The claim then is that

$$\|u\|_{Y_k^{3/2,1/2,1}(-T,T)} \leq C\sqrt{T}\|u\|_{L^\infty H^{5/2}}.$$

Is that going to be enough?