PROBLEM SET 2

Due October 3, 2002

Problem 1 (#2 from Folland p.24). *Complete the reminding cases of the Proof of Proposition 1.2 (Folland p. 22) by proving that* $\mathcal{B}_{\mathbb{R}} \subseteq \mathcal{M}(\mathcal{E}_j)$ *for all* $j \geq 3$.

Problem 2 (#4 from Folland p.24). *An algebra* $\mathcal{A}$ *is a* σ*-algebra if and only if* $\mathcal{A}$ *is closed under countable increasing unions (by the latter it is meant that if* $\{E_j\}_{j=1}^{\infty} \subset \mathcal{A}$ *and* $E_1 \subset E_2 \subset E_3 \subset \cdots$*, then* $\bigcup_{j=1}^{\infty} E_j \in \mathcal{A}$ *).*

Problem 3 (#5 from Folland p.24). *If* $\mathcal{M}$ *is the* σ*-algebra generated by* $\mathcal{E}$*, then* $\mathcal{M}$ *is the union of the* σ*-algebras generated by* $\mathcal{F}$ *as* $\mathcal{F}$ *ranges over all countable subsets of* $\mathcal{E}$*. (Hint: show that the latter object is a* σ*-algebra).*

Problem 4. *Provide a* **complete** *proof of Proposition 1.4 p. 23 of Folland. That is prove the following:*

Suppose that $\mathcal{M}_\alpha$ *is generated by* $\mathcal{E}_\alpha$*,* $\alpha \in A$*. Then* $\bigoplus_{\alpha \in A} \mathcal{M}_\alpha$ *is generated by*

$$\mathcal{F}_1 := \{\pi_\alpha^{-1}(E_\alpha) \,:\, E_\alpha \in \mathcal{E}_\alpha, \alpha \in A\}.$$

If A *is countable and* $X_\alpha \in \mathcal{E}_\alpha$ *for all* α*,* $\bigoplus_{\alpha \in A} \mathcal{M}_\alpha$ *is generated by*

$$\mathcal{F}_2 := \{\Pi_{\alpha \in A}(E_\alpha) \,:\, E_\alpha \in \mathcal{E}_\alpha\}.$$

Note : Verbatim reproduction of the (partial) proof in book with no further explanations or justifications of claims is not acceptable. If you use a previous result please state clearly what are you using instead of just refering to numbering in the book. Also, provide a **full proof** of the 'second assertion'.

Problem 5. **# 7 from Folland's page 27.**

Problem 6. **# 9 from Folland's page 27.**

Problem 7. **# 10 from Folland's page 27.**

Problem 8. **# 13 from Folland's page 27.**

Problem 9. **# 14 from Folland's page 27.**