M624 HOMEWORKS– Spring 2009

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SET 5: DUE APRIL 23RD, 2009

From Folland's book # 5.5 page 177. Do problems 54, 55, 57b)d) - use 22) in conjunction with 57a)- , 58, 59, 60, 61.

SET 4: DUE APRIL 16TH, 2009

From Folland's book # 5.1 page 155. Do problems 3, 6, 7, 9, 12a)-d) For this read the discussion in middle of page 153 first).

From Folland's book # 5.2 page 159. Do problems 17, 22 a).

From Folland's book # 5.3 page 164. Do problem 29.

SET 3: DUE MARCH 26TH, 2009

From Folland's book # 3.5 page 107. Do problems 27, 28, 31, 32 33, 37, 42.

SET 2: DUE FEBRUARY 26TH 2009

From Folland's book # 3.3 page 94. Do problem 20.

Extra Problem 2 *: Let μ be a positive measure over $(X, \mathcal{M})$ and let $f : X \to \mathbb{C}$ be a μ-integrable function on X;– i.e. $f \in L^1(\mu)$. Define*

$$\nu(E) := \int_E f \, d\mu \qquad E \in \mathcal{M}$$

(a) *Show that ν is a complex measure on $(X, \mathcal{M})$*

(b) *In particular, show that if $f \in L^1(\mu)$ takes only real values –i.e. $f : X \to \mathbb{R}$ – then ν as defined here is a* **finite** *signed measure (here use the Extra Problem 1 above).*

Extra Problem 3*: Let μ and ν be two measures on $(X, \mathcal{M})$ defined by*

$$\mu(A) := \int_A e^{-x^2} \, dx \qquad A \in \mathcal{M}$$

$$\nu(A) = \int_A e^{-x^2+x} \, dx \qquad A \in \mathcal{M}.$$

Show that $\mu << \nu$ and compute the Radon-Nikodym derivative $\frac{d\mu}{d\nu}$.

From Folland's book # 3.4 page 100. Do problems 22, 23, 24, 25a).

SET 1: DUE FEBRUARY 12TH, 2009

From Folland's book # 3.1 page 88. Do problems 2, 3, 4.

From Folland's book # 3.2 page 92. Do problems 8, 9, 10, 13, 16 (correction: need both measures to be σ-finite), 17 (correction: need ν to be σ-finite also.

Extra Problem 1 : Let μ be a positive measure over $(X, \mathcal{M})$ and let f real be an *extended μ-integrable* function on X. Define

$$\nu(E) := \int_E f \, d\mu \qquad E \in \mathcal{M}$$

Show that ν is a *signed measure* on $(X, \mathcal{M})$