M534H HOMEWORKS– Spring 2009

• **Set 1. Due date: Thursday February 12th**

Section 1.1: 2, 3, 4, 11

Section 1.2: 1, 2, 3, 4, 5.

Section 1.3: 7, 9, 10 (in $\mathbb{R}^3$), 11.

Additional Problem:

Let $u = u(\mathbf{x}, t)$ be a solution to the wave equation $u_{tt} - \Delta u = 0$ in $\mathbb{R}^2$. Assuming that $\nabla u \to 0$ fast enough as $|\mathbf{x}| \to \infty$ prove that

$$E(t) = \int_{\mathbb{R}^2} |u_t|^2 + |\nabla u|^2 \, dxdy$$

is constant in t for all time t.

• **Set 2. Due date: Thursday February 26th**

Section 1.4: 1

Section 1.5: 1, 4 and then do 5, 6 as modified below.

Problem 5 states: Consider the equation

$$u_x + yu_y = 0$$

with boundary condition $u(x, 0) = \phi(x)$.
(a) For $\phi(x) = x$, for all x, show that no solution exists.
(b) For $\phi(x) = 1$ for all x, show that there are many solutions.

Problem 6 states: Solve the equation $u_x + 2x\,y^2\,u_y = 0$ and find a solution that satisfies the auxiliary condition $u(0, y) = y$.

Section 1.6: 1, 2, 4

Additional Problems:
(1) Find the general solution of $u_x - \sin(x)\,u_y = 0$. Then find the special solution that satisfies $f(0, e^y) = e^y$.

(2) Find the regions in 2d-space where $x^2\,u_{xx} + 4u_{xy} + y^2\,u_{yy} = 0$ is respectively elliptic, parabolic, hyperbolic. Plot these regions.