**M725–Functional Analysis**

**Homework– Spring 2019**

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Problems listed are from Schechter's Principles of Functional Analysis (2nd edition).

- **Set 1. Due date: 02/28/2019**

First, read carefully Sections 1.2; 1.3 and 1.4 in Chapter 1.

Chapter 1 Section 1.5: 5, 9, 10, 2, 3, 4, 14.

Additional Problem 1: Let $X$ be the vector space of all sequences $\{a_n\}_{n\geq 1}$of real numbers with only finitely many nonzero terms. Consider the $\ell^1$-norm $\|\{a_n\}\|_1 = \sum_{n=1}^{\infty} |a_n|$ and the $\ell^2$-norm, $\|\{a_n\}\|_2 = \left(\sum_{n=1}^{\infty} |a_n|^2\right)^{1/2}$. Prove that $\|\cdot\|_1$ and $\|\cdot\|_2$ are **not** equivalent norms.

Chapter 2 Section 2.5: 1, 2, 5, 9, 15, 18.

Additional Problem 2: Let $a$ and $b$ be two positive real numbers. Let $x_0$ be a vector in a normed space $X$. Assume that $|f(x_0)| \leq a$ for all $f \in X^*$ $(= X')$ with $\|f\|_{X^*} \leq b$. Probe that $\|x_0\|_X \leq \dfrac{a}{b}$

Optional: Do 23, 24 in Chapter 2 Section 2.5.

- **Set 2. Due date: 04/04/2019**

Note more problems will be added to this set soon so please start.

Chapter 3 Section 3.8: 1, 2, 4, 7, 8, 13, 15, 19, 25, 28.

- **Set 3. Due date: 04/30/2019**

Chapter 4 Section 4.5: 1, 2, 3, 4, 5, 6, 8, 11, 13, 18, 22.

Hint for 1: prove sequential compactness.

Hint for 11: use Gram-Schmidt

Hint for 18. Consider a set $L = \{x_k\}_{k\geq 1}$ of normalized (to have norm one) linearly independent vectors in $X$ (infinite dimensional normed v.s). Consider the subspace Y of X consisting of all finite linear combination so of vectors in L. Next consider a sequence of functions $\{h_\ell\}_{j\geq 1}$ defined on Y defined as $h_\ell(y) = a\ell$ if $y = ax_\ell$ and zer otherwise and let $H(y) = \sum_{\ell=1}^{\infty} h_\ell(y)$. Show that $H$ is well-defined in $Y$ but unbounded on the unit ball in $Y$.