M534H HOMEWORKS– Spring 2008

Prof. Andrea R. Nahmod

• Set 7. Due date: Tuesday May 6

Section 5.2 3, 4, 7, 9, 10, 11, 17.

Section 5.3 1, 2a)b), 3, 4a)b), 5a), 6, 10.

Section 5.4 1, 5, 6, 7 , 8a)

• Set 6. Due date: Thursday April 24

Section 4.3 2, 4, 6, 11

Section 5.1 3, 5, 6, 7, 9.

Additional Problems for 4.3 and 5.1

• Set 5. Due date: Thursday April 10

Section 2.4: 1, 2, 7 (here you can use that the integral is $\sqrt{\pi}$ which we proved in class) 8, 14, 15 16, 18

Section 2.5: 1

Section 4.1: 2, 3

Section 4.2: 1, 3

Additional Problems Separate variables to solve the following problems.

A1. $u_{tt} - u_{xx} = 0$ in $0 < x < 3$ with boundary conditions $u(0,t) = u(3,t) = 0$

A2. $u_t = u_{xx}$ in $0 < x < L$ with boundary conditions $u_x(0,t) = u(L,t) = 0$

A3. $u_t = u_{xx} + u$ in $0 < x < \pi$ with boundary conditions $u(0,t) = u(\pi,t) = 0$

• Set 4. Due date: Thursday March 13th

Section 2.3: 2, 4, 5, 6, 7.

• Set 3. Due date: Thursday March 6th. Postponed to March 11th

Section 2.1: 1, 2, 5, 8, 9, 11

Problem 11. Find the general solution of

$$3u_{tt} + 10u_{xt} + 3u_{xx} = \sin(x+t)$$

Section 2.2: 1, 2, 3, 5*

Problem 5* Consider the wave equation in 1D with *damping*

$$u_{tt} = c^2\, u_{xx} - ku - ru_t \qquad k, r > 0$$

show that the *energy functional*

$$E(t) = \frac{1}{2}\int_{-\infty}^{\infty} |u_t|^2 + c^2|u_x|^2 + k|u|^2\, dx$$

satisfies $dE/dt \le 0$; that is *energy decreases.* Assume u and its derivatives vanish as $x \to \pm\infty$.

Additional Problems.

Problem A.1. Find the solution to wave equation with initial conditions $u(x,0) = \sin x$, $u_t(x,0) = 0$. Calculate then $u_t(0,t)$.

Problem A.2. Solve the PDE $\quad u_{xx} + u_{xt} - 10u_{tt} = 0$

- **Set 2. Due date: Thursday February 28th**

Section 1.4: 1

Section 1.5: 1, 4, 5, 6 (modified; see below)

Problem 5 states: Consider the equation

$$u_x + yu_y = 0$$

with boundary condition $u(x,0) = \phi(x)$.
(a) For $\phi(x) = x$, for all x, show that no solution exists.
(b) For $\phi(x) = 1$ for all x, show that there are many solutions.

Problem 6 states: Solve the equation $u_x + 2x\,y^2\,u_y = 0$ and find a solution that satisfies the auxiliary condition $u(0,y) = y$.

Section 1.6: 1, 2, 4

Additional Problems:

(1) Find the general solution of $u_x - \sin(x)\, u_y \;=\; 0$. Then find the special solution that satisfies $f(0, e^y) = e^y$.

(2) Find the regions in 2d-space where $x^2\, u_{xx} + 4u_{xy} + y^2\, u_{yy} \;=\; 0$ is respectively elliptic, parabolic, hyperbolic. Plot these regions.

- **Set 1. Due date: Thursday February 14th**

Section 1.1: 2, 3, 4, 11

Section 1.2: 2, 3, 4, 5.

Section 1.3: 7, 9, 10 (in $\mathbb{R}^3$), 11.

Additional Problem:

Let $u \;=\; u(\mathbf{x}, t)$ be a solution to the wave equation $u_{tt} - \Delta u = 0$ in $\mathbb{R}^2$. Assuming that $\nabla u \to 0$ fast enough as $|\mathbf{x}| \to \infty$ prove that

$$E(t) \;=\; \int_{\mathbb{R}^2} |u_t|^2 \;+\; |\nabla\, u|^2 \; dxdy$$

is constant in t for all time t.