MATH 331.1, Fall 2007 : Midterm Exam

Your Name: _____

The exam consists of 5 questions, each worth 20 points.

1.(20 points) Solve the initial value problem $\frac{dx}{dt} = x^2\,[t + \sin(t)]$ with $x(0) = 6$.

2. Mary initially deposits \$1000 in a savings account that pays interest at the rate of 5% per year (compounded continuously). She also arranges for \$25 per week to be deposited automatically into her account.

(a) Assume that weekly deposits can be approximated by continuous deposits. Write down an initial value problem for her account balance $S(t)$ over time (t measured in years).

(b) How long does she needs to save to buy a \$5000 car?

3. Consider the following equation for a certain population of squirrels given by $P(t)$ (t is measured in years).

$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{2}\right)(P - 1)$$

(a) Find all the equilibrium points of the equations. Draw the phase line and indicate the type of each equilibrium points (i.e., sink, source, or node).

(b) Make a graph of the solutions with initial conditions $P(0) = 1/4$, $P(0) = 3/2$, and $P(0) = 3$.

(c) At a certain time the hunting of squirrels become permitted and the law allows that a certain percentage α of the squirrel population be eliminated every year. A new equation for the squirrel population is then

$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{2}\right)(P-1) - \alpha P$$

The IALS (International Association for the Liberation of Squirrels) asserts than no more than 10% of squirrels should be eliminated every year (i.e $\alpha = 0.1$), otherwise the population would go extinct. On the contrary the UHA (United Hunters of America) asserts that it is safe to hunt half of the squirrel population every year (i.e. $\alpha = 0.5$).

Analyze the bifurcations of the systems as α varies and determine who is right from the IALS or the UHA.

4. Solve the linear system

$$\begin{aligned} \frac{dx}{dt} &= 4x + 2y \\ \frac{dy}{dt} &= x + 3y \end{aligned}$$

with initial conditions

$$x(0) = 4\,, \quad y(0) = -2\,.$$

5. Find the solution for the system

$$\begin{aligned} \frac{dx}{dt} &= xy + y \\ \frac{dy}{dt} &= 2 \end{aligned}$$

with $x(0) = 3$ and $y(0) = 0$.