PROBLEM SET 2

Due October 4th, 2001

Problem 1 (#2 from Folland p.24). *Complete the reminding cases of the Proof of Proposition 1.2 (Folland p. 22) by proving that* $\mathcal{B}_{\mathbb{R}} \subseteq \mathcal{M}(\mathcal{E}_j)$ *for all* $j \geq 3$.

Problem 2 (#4 from Folland p.24). *An algebra* $\mathcal{A}$ *is a* σ*-algebra if and only if* $\mathcal{A}$ *is closed under countable increasing unions (by the latter it is meant that if* $\{E_j\}_{j=1}^{\infty} \subset \mathcal{A}$ *and* $E_1 \subset E_2 \subset E_3 \subset \cdots$, *then* $\bigcup_{j=1}^{\infty} E_j \in \mathcal{A}$ *).*

Problem 3 (#5 from Folland p.24). *If* $\mathcal{M}$ *is the* σ*-algebra generated by* $\mathcal{E}$, *then* $\mathcal{M}$ *is the union of the* σ*-algebras generated by* $\mathcal{F}$ *as* $\mathcal{F}$ *ranges over all countable subsets of* $\mathcal{E}$. *(Hint: show that the latter object is a* σ*-algebra).*

Problem 4 (#7 from Folland p.27). *If* $\mu_1, \mu_2, \ldots, \mu_n$ *are measures on* $(X, \mathcal{M})$ *and* $a_1, a_2, \ldots, a_n \in [0, \infty)$, *then* $\sum_{j=1}^{n} a_j \mu_j$ *is a measure on* $(X, \mathcal{M})$.

Problem 5 (#9 from Folland p.27). *If* $(X, \mathcal{M}, \mu)$ *is a measure space and* $E, F \in \mathcal{M}$, *then* $\mu(E) + \mu(F) = \mu(E \cup F) + \mu(E \cap F)$

Problem 6 (#10 from Folland p.27). *Given a measure space* $(X, \mathcal{M}, \mu)$ *abd* $E \in \mathcal{M}$, *define*

$$\mu_E(A) = \mu(A \cap E) \qquad \text{for } A \in \mathcal{M}.$$

Prove that μ_E *is a measure.*

Problem 7. *Provide a complete proof of Proposition 1.4 p. 23 of Folland. That is prove the following:*

Suppose that $\mathcal{M}_\alpha$ *is generated by* $\mathcal{E}_\alpha$, $\alpha \in A$. *Then* $\bigoplus_{\alpha \in A} \mathcal{M}_\alpha$ *is generated by*

$$\mathcal{F}_1 := \{\pi_\alpha^{-1}(E_\alpha) \,:\, E_\alpha \in \mathcal{E}_\alpha, \alpha \in A\}.$$

If A *is countable and* $X_\alpha \in \mathcal{E}_\alpha$ *for all* α, $\bigoplus_{\alpha \in A} \mathcal{M}_\alpha$ *is generated by*

$$\mathcal{F}_2 := \{\Pi_{\alpha \in A}(E_\alpha) \,:\, E_\alpha \in \mathcal{E}_\alpha\}.$$

Note : Verbatim reproduction of the proof in book with no further explanations or justifications of claims is not acceptable. If you use a previous result please state clearly what are you using instead of just refering to numbering in the book.