

Inference rules in natural deduction.

Name	rule	coq forward	coq backward
$\wedge I$	$\frac{H1 : A \quad H2 : B}{A \wedge B}$	<code>pose proof(conj H1 H2).</code>	<code>split.</code>
$\wedge E_1$	$\frac{H1 : A \wedge B}{A}$	<code>pose proof(proj1 H1).</code>	—
$\wedge E_2$	$\frac{H1 : A \wedge B}{B}$	<code>pose proof(proj2 H1).</code>	—
$\vee I_1$	$\frac{H1 : C}{C \vee X}$	<code>pose proof(or_introl (B:=X) H1).</code> <code>or pose proof(@or_introl _ X H1).</code>	<code>left.</code>
$\vee I_2$	$\frac{H1 : C}{X \vee C}$	<code>pose proof(or_intror (A:=X) H1).</code> <code>or pose proof(@or_intror X _ H1).</code>	<code>right.</code>
$\vee E$	$\frac{H1 : A \vee B \quad H2 : A \Rightarrow C \quad H3 : B \Rightarrow C}{C}$	<code>destruct H1.</code> (H2 and H3 are not needed)	—
$\Rightarrow I$	assumption/discharge	<code>assert(A->B). intros Hn.</code>	<code>intros Hn.</code>
$\Rightarrow E$	$\frac{H1 : A \Rightarrow B \quad H2 : A}{B}$	<code>pose proof(H1 H2).</code>	<code>apply H1.</code>
$\neg I$	$\frac{H1 : A \Rightarrow \perp}{\neg A}$	<code>pose proof (H1:~A).</code>	—
$\neg E$	$\frac{H1 : \neg A \quad A \Rightarrow \perp}{A \Rightarrow \perp}$	<code>unfold not in *.</code>	<code>unfold not.</code>
$\Leftrightarrow I$	$\frac{H1 : A \Rightarrow B \quad H2 : B \Rightarrow A}{A \Leftrightarrow B}$	<code>pose proof(conj H1 H2: A<=>B).</code>	<code>split.</code>
$\Leftrightarrow E$	$\frac{H1 : A \Leftrightarrow B \quad H2 : (\dots A \dots)}{(\dots B \dots)}$	<code>rewrite H1 in H2.</code> <code>or rewrite <- H1 in H2.</code>	<code>rewrite H1.</code> <code>or rewrite <- H1.</code>
$\perp E$	$\frac{H1 : \perp}{X}$	<code>destruct H1.</code>	<code>exfalso.</code>
$\top I$	$\frac{}{\top}$	<code>pose proof(I).</code>	—
$\forall I$	assumption/discharge	—	<code>intros c</code>
$\forall E$	$\frac{H1 : \forall x, Px \quad c \text{ is a constant}}{Pc}$	<code>pose proof(H1 c).</code>	—
$\exists I$	$\frac{c \text{ is a constant} \quad H1 : Pc}{\exists x, Px}$	<code>pose proof(ex.intro P c H1).</code>	<code>exists c.</code>
$\exists E$	$\frac{H1 : \exists x, Px \quad c \text{ is a constant (fresh)}}{Pc}$	<code>destruct H1.</code>	—
$=I$	$\frac{c \text{ is a constant}}{c = c}$	<code>pose proof(eq_refl c).</code>	<code>reflexivity</code>
$=E$	$\frac{c \text{ is a constant} \quad d \text{ is a constant} \quad H1 : c = d \quad H2 : Pc}{Pd}$	<code>rewrite H1 in H2.</code> <code>or rewrite <- H1 in H2.</code>	<code>rewrite H1.</code> <code>or rewrite <- H1.</code>