Introduction to the Error Function

Alexander S. Madsen

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1 Introduction

The following is a brief introduction to the error function $erf(x)$. The article is based the corresponding Wikipedia article about the subject. Furthermore it is written by me and the content contained within it should subsequently be read with a healthy degree of skepticismsm.

2 The Error Function

The error function is a non-elementary function defined by the integral

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} \exp\left(-t^2\right) \mathrm{d}t \,. \tag{1}$$

As seen on 1 it as a characteristic sigmoid shape (s-curve). The error function was developed by the English mathematician and astronomer J. W. L. Glaisher in 1871 in connection with "the theory of Probability, and notably the theory of Errors. It is thus closely

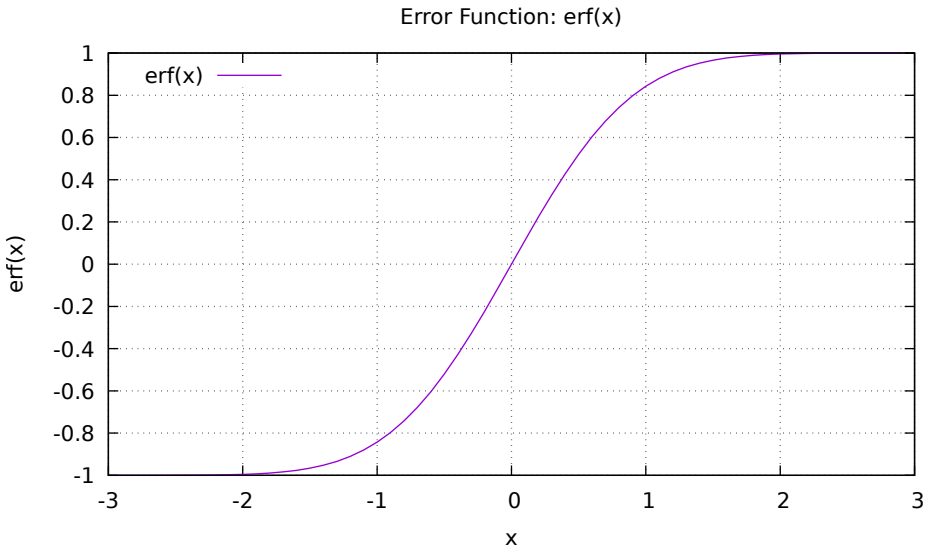

Figure 1: Plot of the error function in the interval $[-3, 3]$ made with gnuplot.

related to the normal, or Gaussian, distribution and is predominately used in probability theory. More precisely, for positive values of $x > 0$ the error function describes the probability of the error being within the interval $[-x, x]$ for a normally distrusted variable X with expectation value 0. That is

$$\operatorname{erf}\left(\frac{a}{\sigma\sqrt{2}}\right) \tag{2}$$