Wheeled Locomotion | Wheeled Kinematics

Autonomous Mobile Robots

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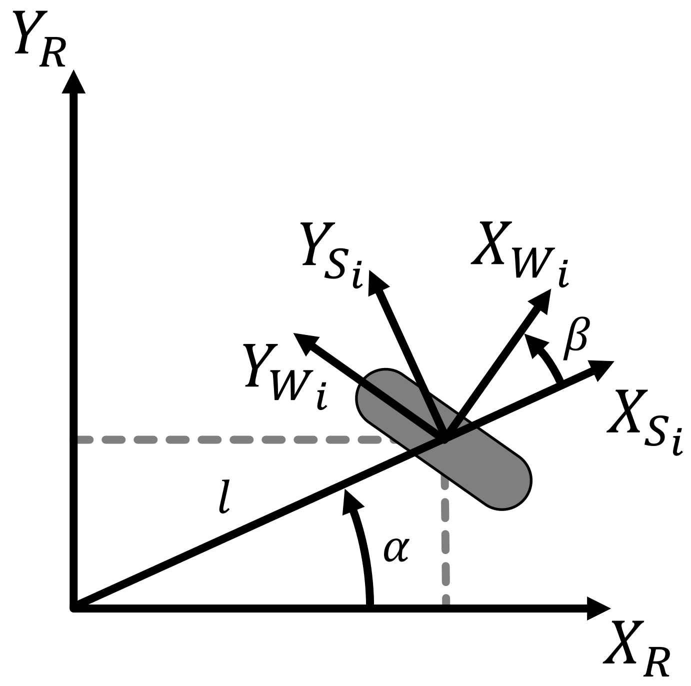
Review: Standard wheel



- The constraints imposed by a single wheel, W_i, can be expressed as follows
  - Rolling constraint

$$J_{1,i}(\beta_{s,i})R(\theta)\dot{\xi}_I - \dot{\varphi}_i r_i = 0$$

  - No-sliding constraint

$$C_{1,i}(\beta_{s,i})R(\theta)\dot{\xi}_I = 0$$

Differential Kinematics

- Given a wheeled robot, each wheel imposes n constraints
- Only fixed and steerable standard wheels impose no-sliding constraints
- Suppose a robot has n wheels of radius r_i, the individual wheel constraints can be concatenated in matrix form:
  - Rolling constraints

$$J_1(\beta_S)R(\theta)\dot{\xi}_I - J_2\dot{\varphi} = 0, \quad J_1(\beta_S) = \begin{bmatrix} J_{1f} \\ J_{1S}(\beta_S) \end{bmatrix}, \quad J_2 = diag(r_1, \cdots, r_N), \quad \dot{\varphi} = \begin{bmatrix} \dot{\varphi}_1 \\ \vdots \\ \dot{\varphi}_N \end{bmatrix}$$

  - No-sliding constraints

$$C_1(\beta_S)R(\theta)\dot{\xi}_I = 0, \quad C_1(\beta_S) = \begin{bmatrix} C_{1f} \\ C_{1S}(\beta_S) \end{bmatrix}$$

Differential Kinematics

- Stacking the rolling and no-sliding constraints gives an expression for the differential kinematics

$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2 \\ 0 \end{bmatrix} \dot{\varphi}$$

- Solving this equation for $\dot{\xi}_I$ yields the **forward differential kinematics** equation needed for computing wheel odometry
- Solving this equation for $\dot{\varphi}$ yields the **inverse differential kinematics** equation needed for control

Degree of Maneuverability

- The maneuverability of a robot is measured by different metrics
- Mobility – restricted by no-sliding constraints
  - δ_m : degree of mobility
- Steerability – additional freedom due to steering mechanisms
  - δ_s : degree of steerability

Degree of Mobility

- To avoid any lateral slip, the motion vector needs to satisfy:

$$C_1(\beta_S)R(\theta)\dot{\xi}_I = 0, \quad C_1(\beta_S) = \begin{bmatrix} C_{1f} \\ C_{1S}(\beta_S) \end{bmatrix}$$

- Hence, $R(\theta)\xi_I$ must belong to the null space, N, of $C_1(\beta_S)$
  - The null space of $C_1(\beta_S)$ is the set of all vectors, n, such that $C_1(\beta_S)n = 0$
  - Intuitively, this means that all the axes associated with the no-slip constraints from all standard wheels converge at a single point—the instantaneous center of rotation

Degree of Mobility

- The larger the rank of $C_1(\beta_S)$, the more constrained a robot's mobility. The degree of mobility, δ_m, reflects this:

$$\delta_m = \dim N[C_1(\beta_s)] = 3 - \mathrm{rank}[C_1(\beta_s)]$$

- For a robot with no standard wheels:

$$\mathrm{rank}[C_1(\beta_s)] = 0, \quad \delta_m = 3$$

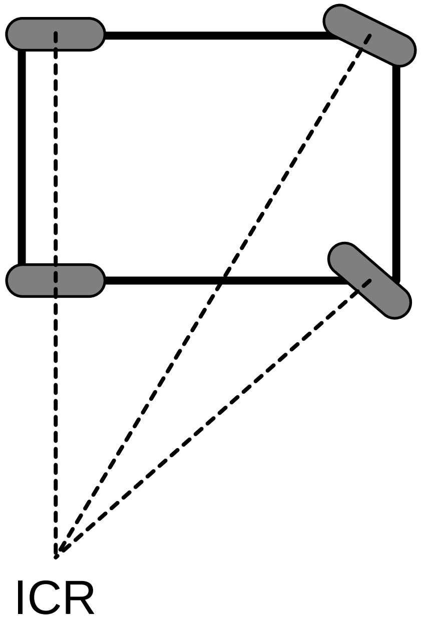
- For a robot with all directions constrained

$$\mathrm{rank}[C_1(\beta_s)] = 3, \quad \delta_m = 0$$

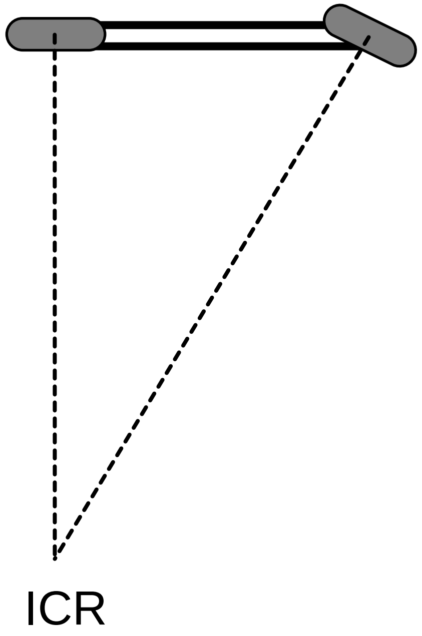
Degree of Mobility

- Instantaneous Center of Rotation (ICR)
  - For any robot with $\delta_m = 2$, the ICR is constrained to lie on a line
  - For any robot with $\delta_m = 3$, the ICR can be any point on the 2D plane

Ackerman Steering, $\delta_m = 2$



Bicycle, $\delta_m = 2$

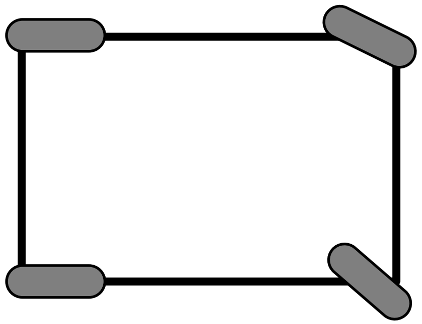


Degree of Steerability

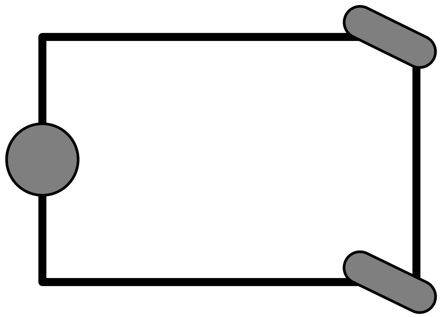
- Degree of steerability,
  - Represents an indirect degree of motion
  - The particular orientation at any instant imposes a kinematic constraint
  - The ability to change that orientation leads to an additional degree of maneuverability

$$\delta_s = \mathrm{rank}[C_{1s}(\beta_s)], \qquad 0 \le \delta_s \le 2$$

Ackermann Steering, $\delta_S = 1$



Two Steer, $\delta_S = 2$

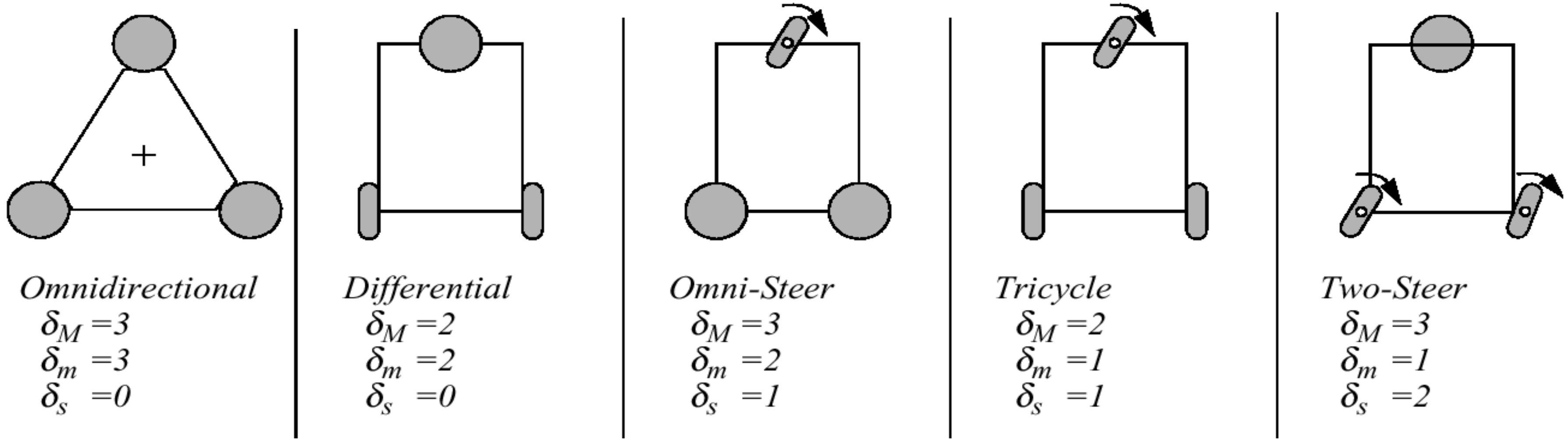


Degree of Maneuverability

- The Degree of Maneuverability, δ_M, combines mobility and steerability

$$\delta_M = \delta_m + \delta_s$$

- Examples

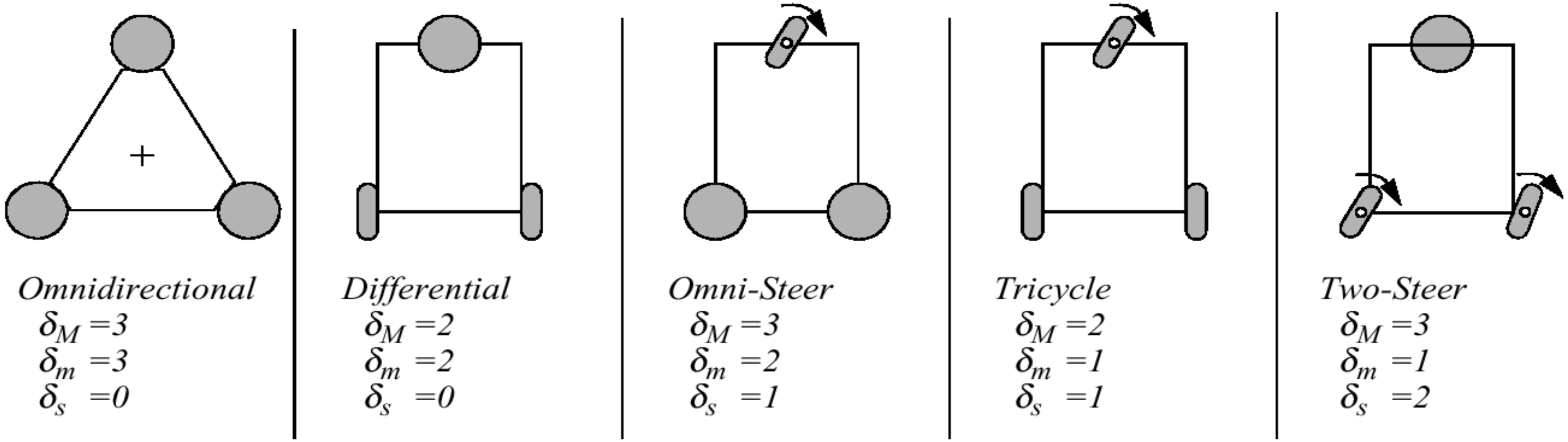


Degree of Maneuverability

- The Degree of Maneuverability, δ_M, combines mobility and steerability

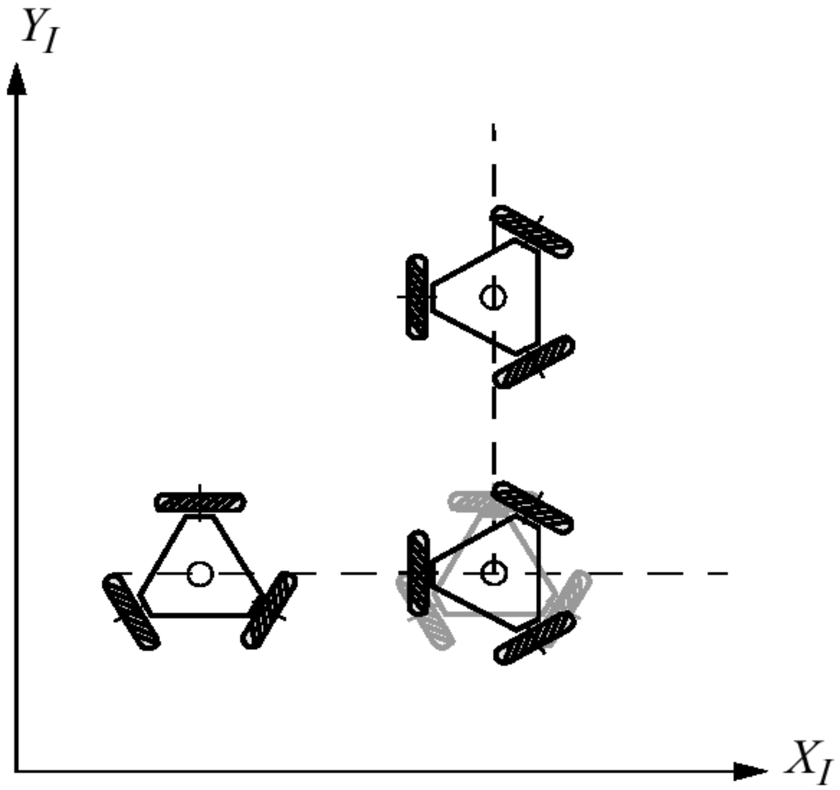
$$\delta_M = \delta_m + \delta_s$$

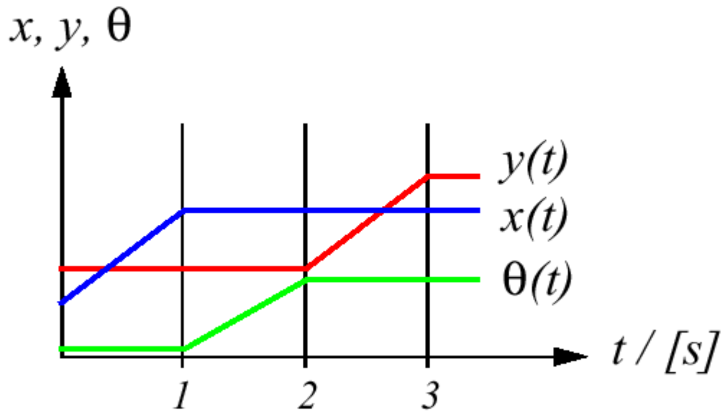
- Note: Two robots with the same δ_M may not be equivalent **instantaneously**



Degree of Maneuverability
Omni-Drive Example

- Three fixed Swedish wheels
  $\delta_M = 3$
  $\delta_m = 3$
  $\delta_s = 0$





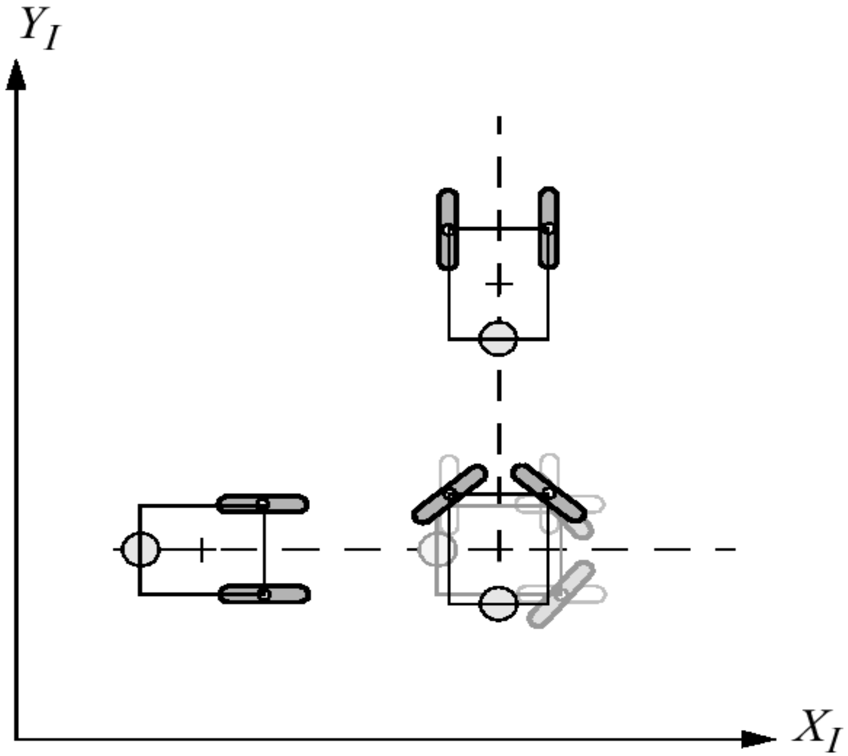
Degree of Maneuverability
Two-Steer Example

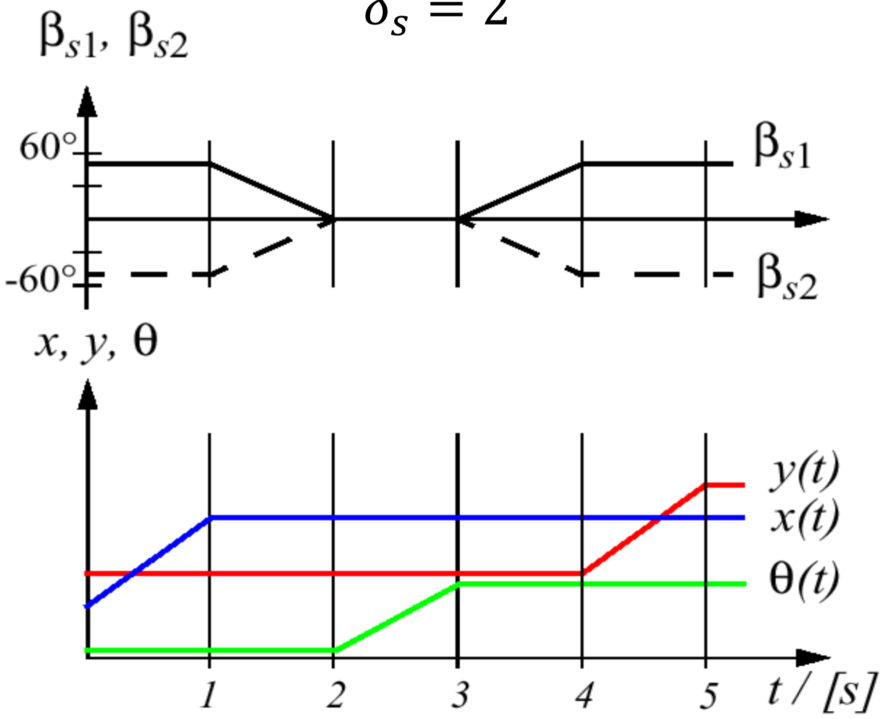
- Two steerable standard wheels

  $\delta_M = 3$

  $\delta_m = 1$

  $\delta_s = 2$





Summary

- Stacking individual wheel constraints gives an equation that may be manipulated to derive the forward and inverse differential kinematics equations for a wheeled mobile robot

$$\begin{bmatrix} J_1(\beta_s) \\ C_1(\beta_s) \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2 \\ 0 \end{bmatrix} \dot{\varphi}$$

- Analysis of the component parts of this equation lets us classify wheeled mobile robots in terms of maneuverability—a combination of mobility and steerability