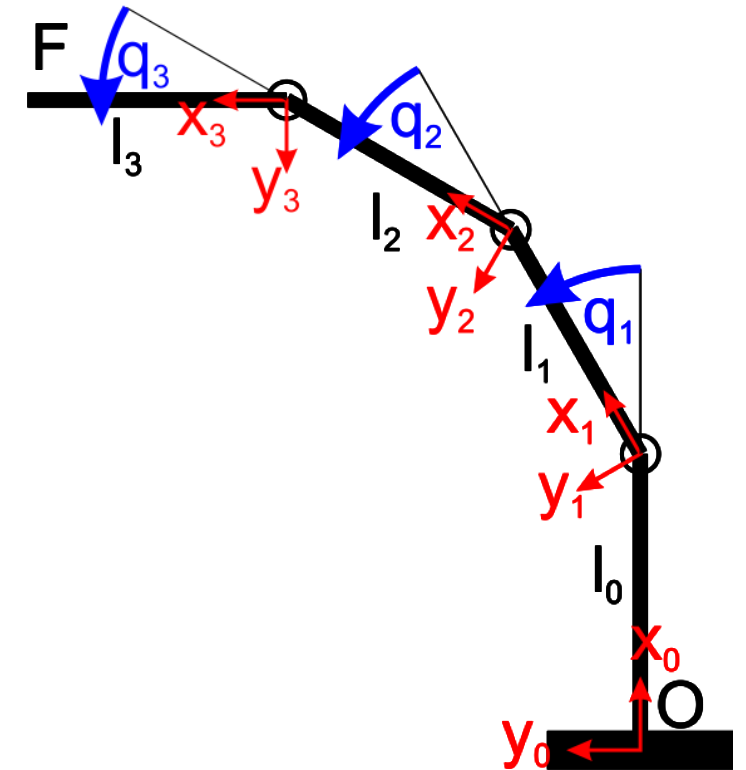# Kinematics | **Introduction Example**
Autonomous Mobile Robots

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Kinematics of a planar 3-link arm

- ## Arm with 3 DoF
  - Joint 1: $rot\left({}_0\mathbf{e}_z, q_1 \right)$
  - Joint 2: $rot\left({}_1\mathbf{e}_z, q_2 \right)$
  - Joint 3: $rot\left({}_2\mathbf{e}_z, q_3 \right)$

- ## Determine:
  1. End-effector position in CS 0 ${}_0\mathbf{r}_{OF}$
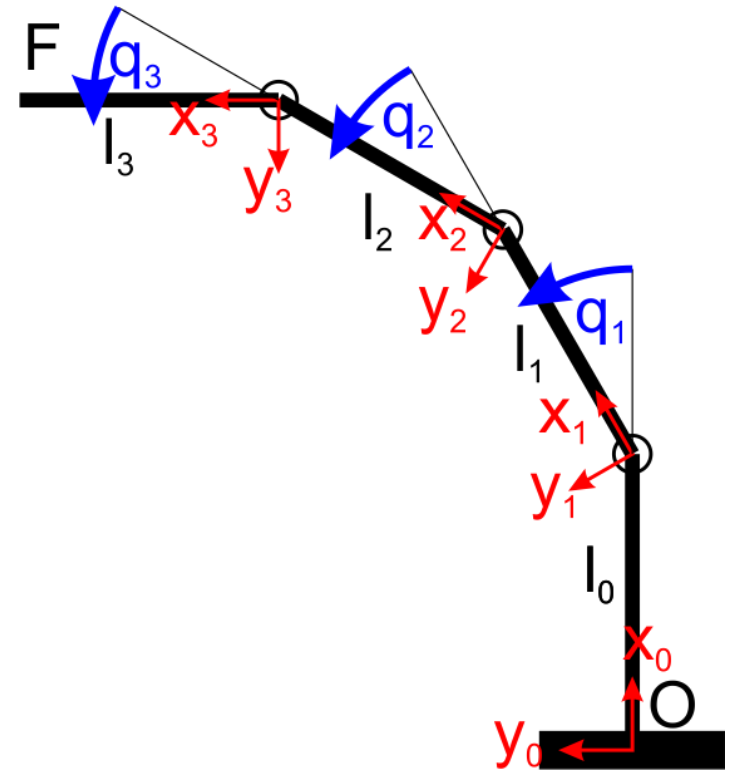 2. Angular velocity of link 3 in CS 0 ${}_0\boldsymbol{\omega}_{03}$

End-effector position

$${}_0\mathbf{r}_{OF}\left(\mathbf{q}\right) = {}_0\mathbf{r}_{O1} + {}_0\mathbf{r}_{12} + {}_0\mathbf{r}_{23} + {}_0\mathbf{r}_{3F}$$

$${}_0\mathbf{r}_{12} = \mathbf{R}_{01}{}_1\mathbf{r}_{12} = \begin{bmatrix} \cos\left(q_1\right) & -\sin\left(q_1\right) & 0 \\ \sin\left(q_1\right) & \cos\left(q_1\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{pmatrix} l_1 \\ 0 \\ 0 \end{pmatrix}$$

$$= {}_0\mathbf{r}_{O1} + \mathbf{R}_{01}{}_1\mathbf{r}_{12} + \mathbf{R}_{01}\mathbf{R}_{12}{}_2\mathbf{r}_{23} + \mathbf{R}_{01}\mathbf{R}_{12}\mathbf{R}_{23}{}_3\mathbf{r}_{3F}$$

$$= \ldots = \begin{pmatrix} l_0 + l_1\cos\left(q_1\right) + l_2\cos\left(q_1 + q_2\right) + l_3\cos\left(q_1 + q_2 + q_3\right) \\ l_1\sin\left(q_1\right) + l_2\sin\left(q_1 + q_2\right) + l_3\sin\left(q_1 + q_2 + q_3\right) \\ 0 \end{pmatrix}$$

Angular velocity

$$_0\boldsymbol{\omega}_{03} = {}_0\boldsymbol{\omega}_{01} + \boxed{{}_0\boldsymbol{\omega}_{12}} + {}_0\boldsymbol{\omega}_{23}$$

$$_0\boldsymbol{\omega}_{12} = \mathbf{R}_{01}\,{}_1\boldsymbol{\omega}_{12} = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{q}_1 \end{pmatrix}$$

$$= {}_0\boldsymbol{\omega}_{B1} + \mathbf{R}_{01}\,{}_1\boldsymbol{\omega}_{12} + \mathbf{R}_{01}\mathbf{R}_{12}\,{}_2\boldsymbol{\omega}_{23}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 + \dot{q}_3 \end{pmatrix}$$