Wheeled Kinematics

Autonomous Mobile Robots

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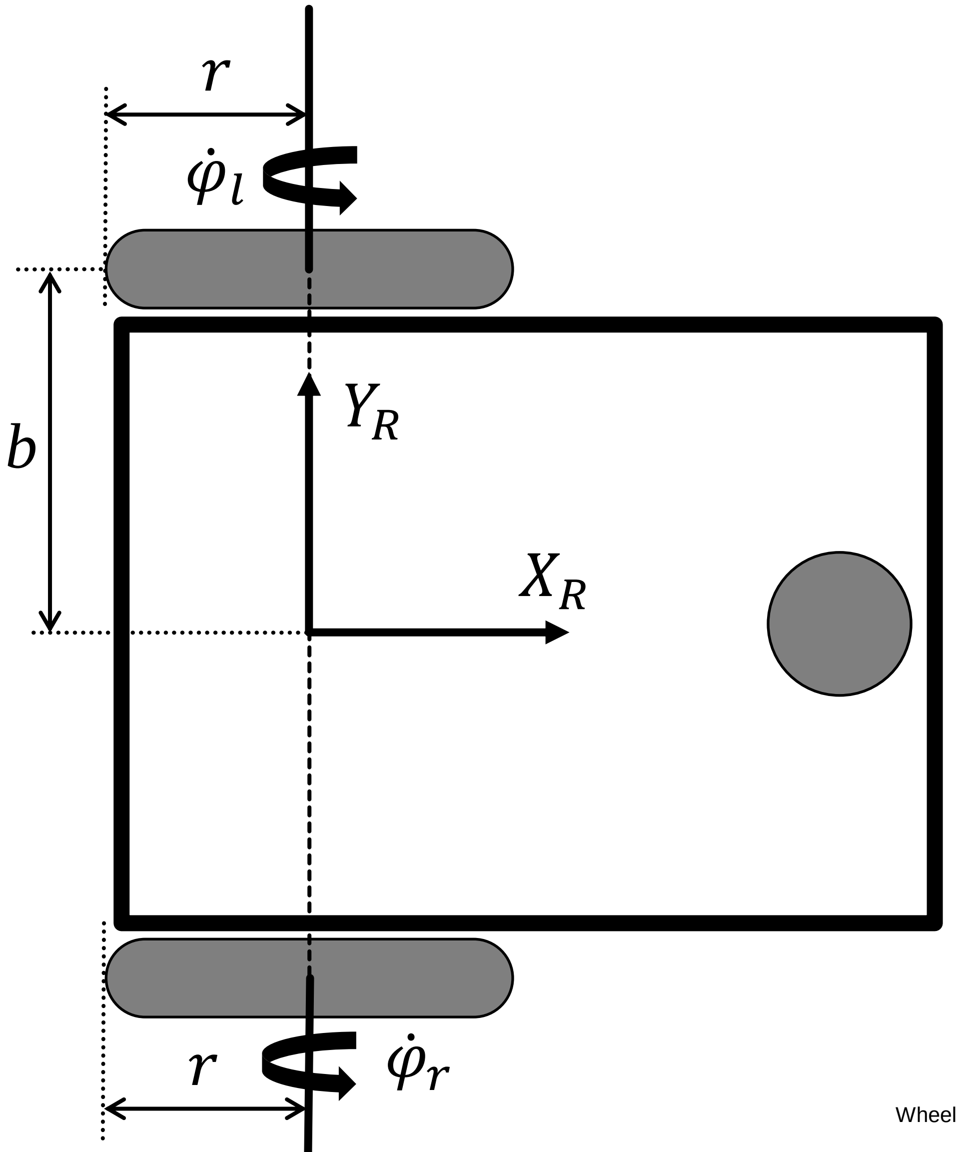
Worked Exercise | A Differential Drive Robot

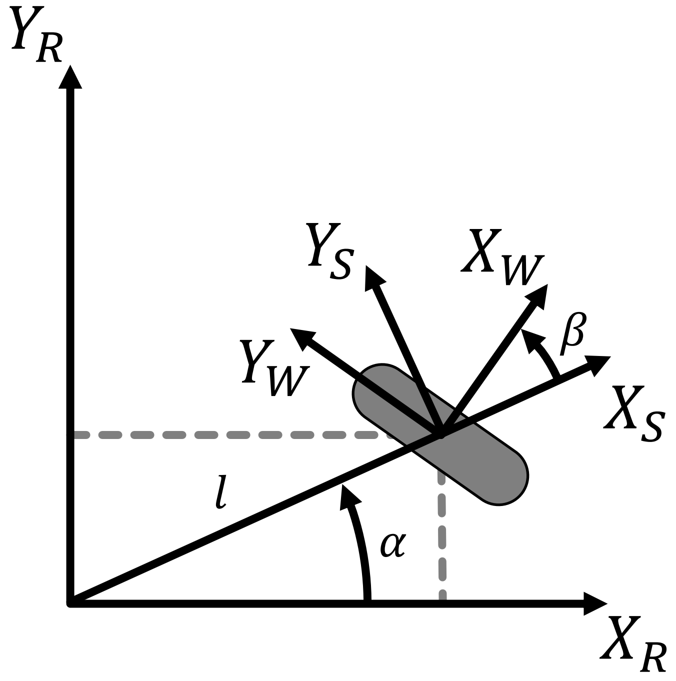
- Two fixed standard wheels
- The robot frame (R) in between the wheels
- Stack the wheel equations for this configuration

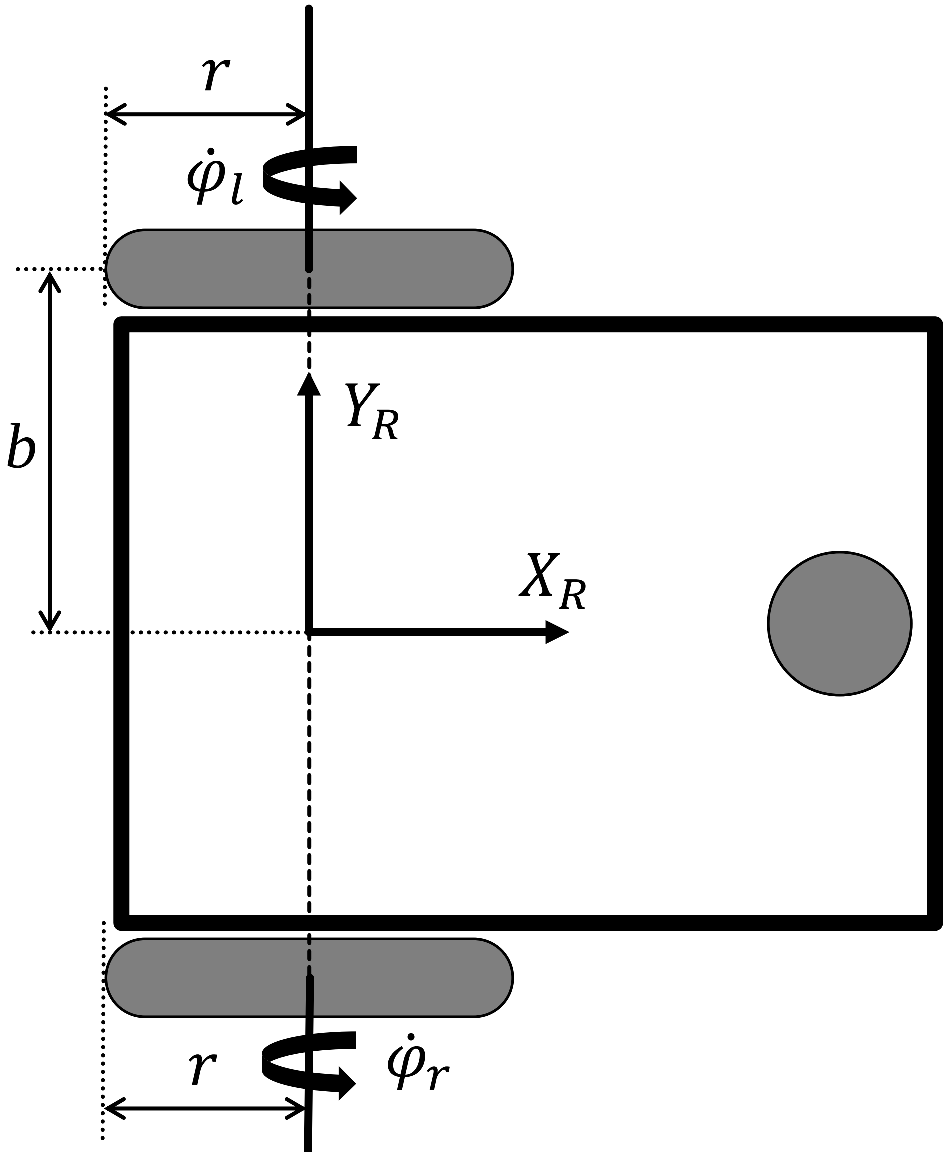


A Differential Drive Robot | Writing the constraint equations

- Rolling constraint
  $$[-\sin\alpha+\beta \quad \cos\alpha+\beta \quad l\cos\beta]R(\theta)\dot{\xi}_I=\dot{\varphi}r$$

- No-sliding constraint
  $$[\cos\alpha+\beta \quad \sin\alpha+\beta \quad l\sin\beta]R(\theta)\dot{\xi}_I=0$$





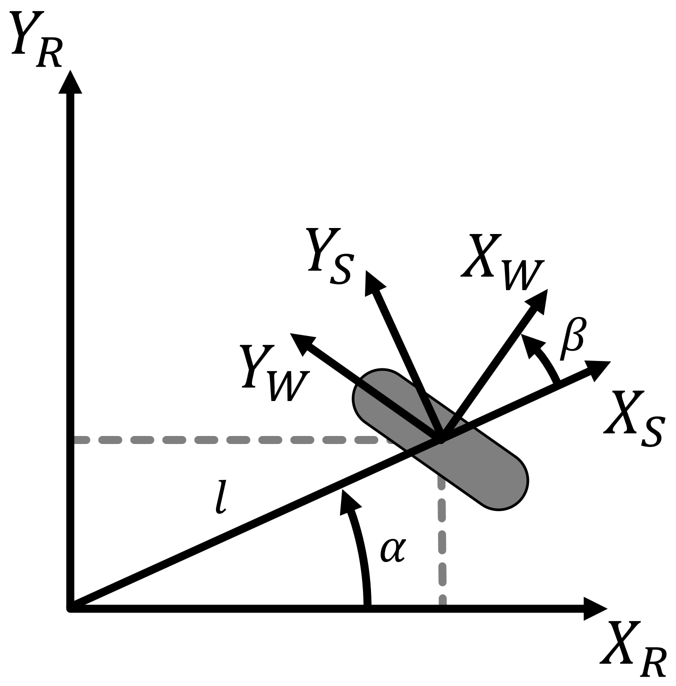
A Differential Drive Robot | Writing the constraint equations

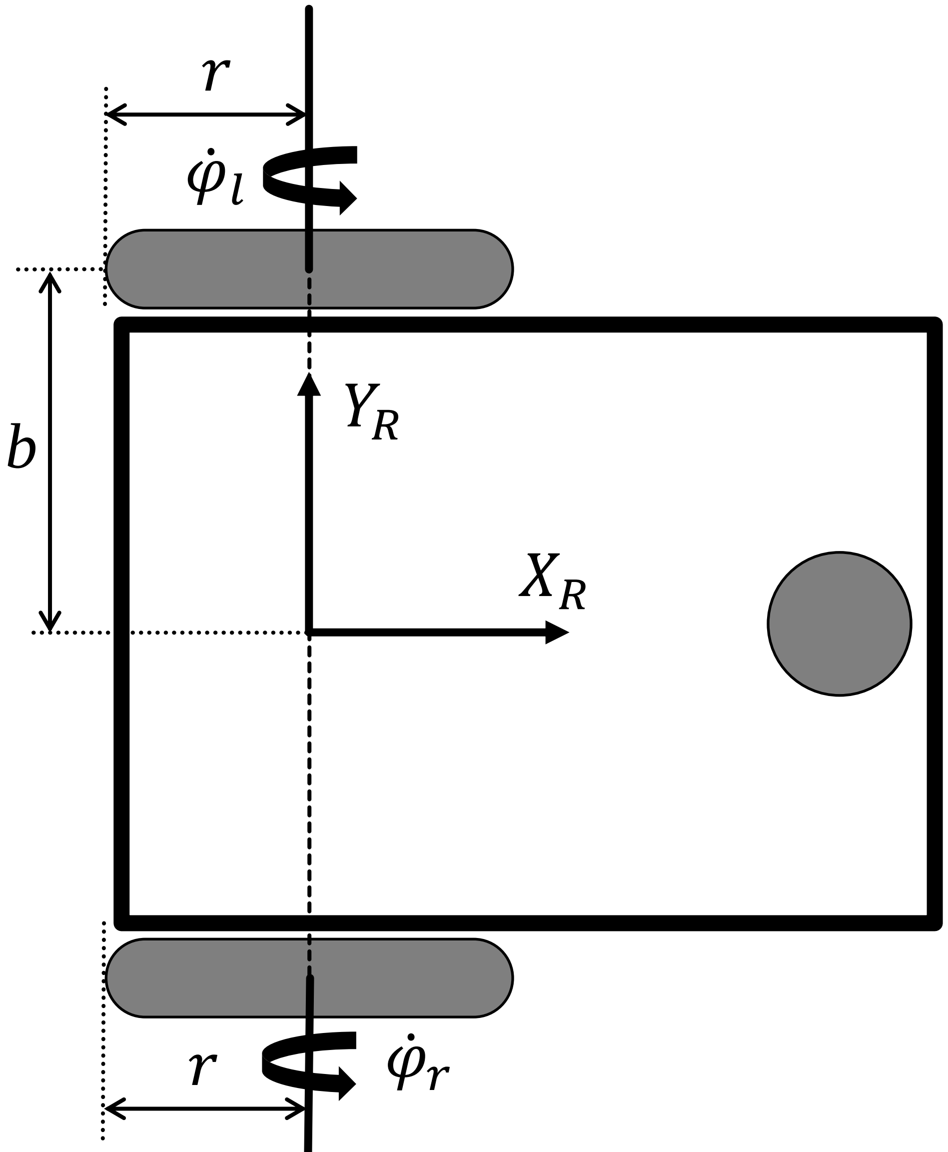
- Rolling constraint

$$[-\sin \alpha + \beta \quad \cos \alpha + \beta \quad l \cos \beta]\dot{\xi}_R = \dot{\varphi} r$$

- No-sliding constraint

$$[\cos \alpha + \beta \quad \sin \alpha + \beta \quad l \sin \beta]\dot{\xi}_R = 0$$





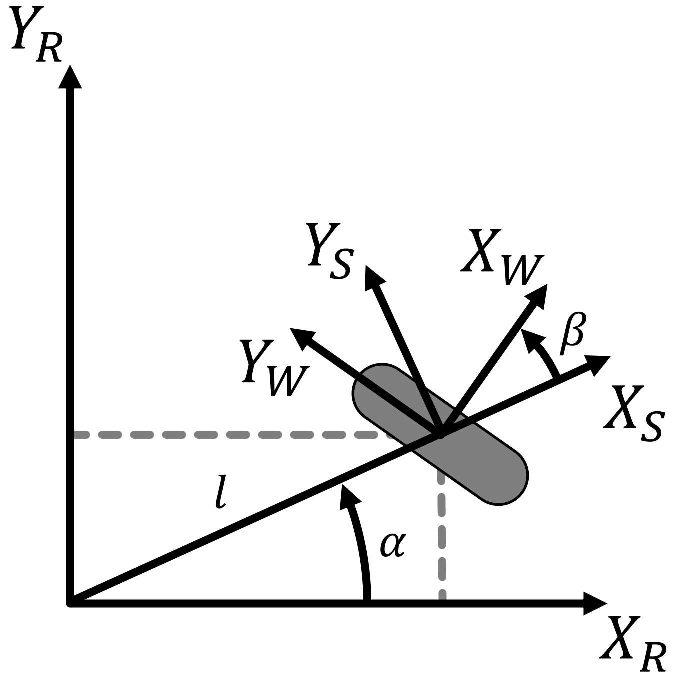
A Differential Drive Robot | Writing the constraint equations

- Rolling constraint

$$[-\sin \alpha + \beta \quad \cos \alpha + \beta \quad l \cos \beta]\dot{\xi}_R = \dot{\varphi} r$$

- No-sliding constraint

$$[\cos \alpha + \beta \quad \sin \alpha + \beta \quad l \sin \beta]\dot{\xi}_R = 0$$



- For the right wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = b$$

- For the left wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = -b$$

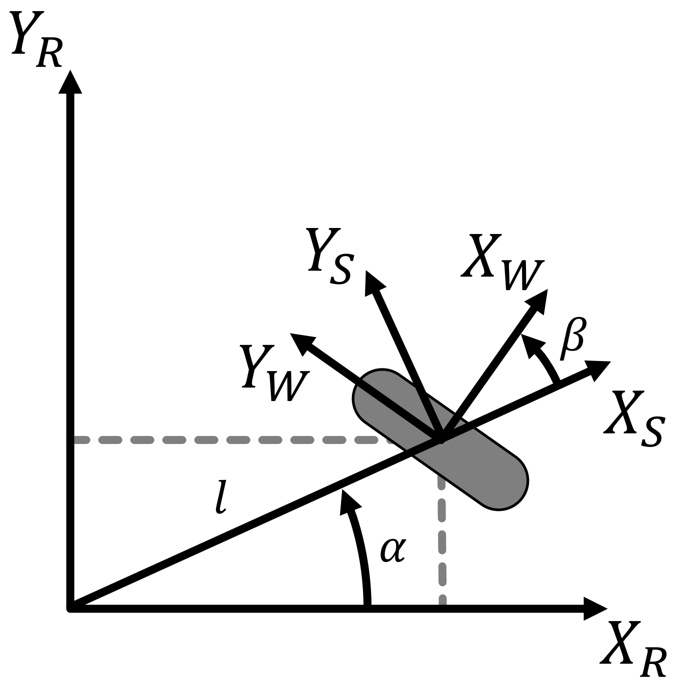
A Differential Drive Robot | Writing the constraint equations

- Rolling constraint

$$[-\sin\alpha+\beta \quad \cos\alpha+\beta \quad l\cos\beta]\dot{\xi}_R = \dot{\varphi}r$$

- No-sliding constraint

$$[\cos\alpha+\beta \quad \sin\alpha+\beta \quad l\sin\beta]\dot{\xi}_R = 0$$



- For the right wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = b$$

- For the left wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = -b$$

- Rolling constraints

$$\begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \end{bmatrix}\dot{\xi}_R = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

- No-sliding constraints

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}\dot{\xi}_R = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

A Differential Drive Robot | Writing the constraint equations

- Rolling constraints

$$J_1 = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \end{bmatrix}$$

- No-sliding constraints

$$C_1 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

- For the right wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = b$$

- For the left wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = -b$$

- Rolling constraints

$$\begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \end{bmatrix} \dot{\xi}_R = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

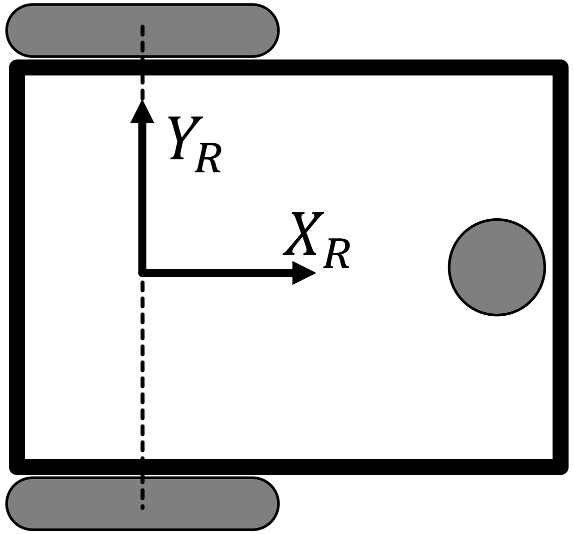
- No-sliding constraints

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \dot{\xi}_R = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

A Differential Drive Robot | Degree of Maneuverability

- No-sliding constraints

$$C_1 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$



- Degree of Steerability
  - No steerable wheels
  - $\delta_s = 0$
- Degree of Mobility
  - $\delta_m = 3 - \mathrm{rank}[C_1]$
  - C_1 clearly has 1 independent column (rank 1)
  - $\delta_m = 2$
- Degree of Maneuverability
  - $\delta_M = \delta_m + \delta_s$
  - $\delta_M = 2$

A Differential Drive Robot | Differential Kinematics

- For the right wheel

$$\alpha = -\pi/2,\ \beta = 0,\ l = b$$

- For the left wheel

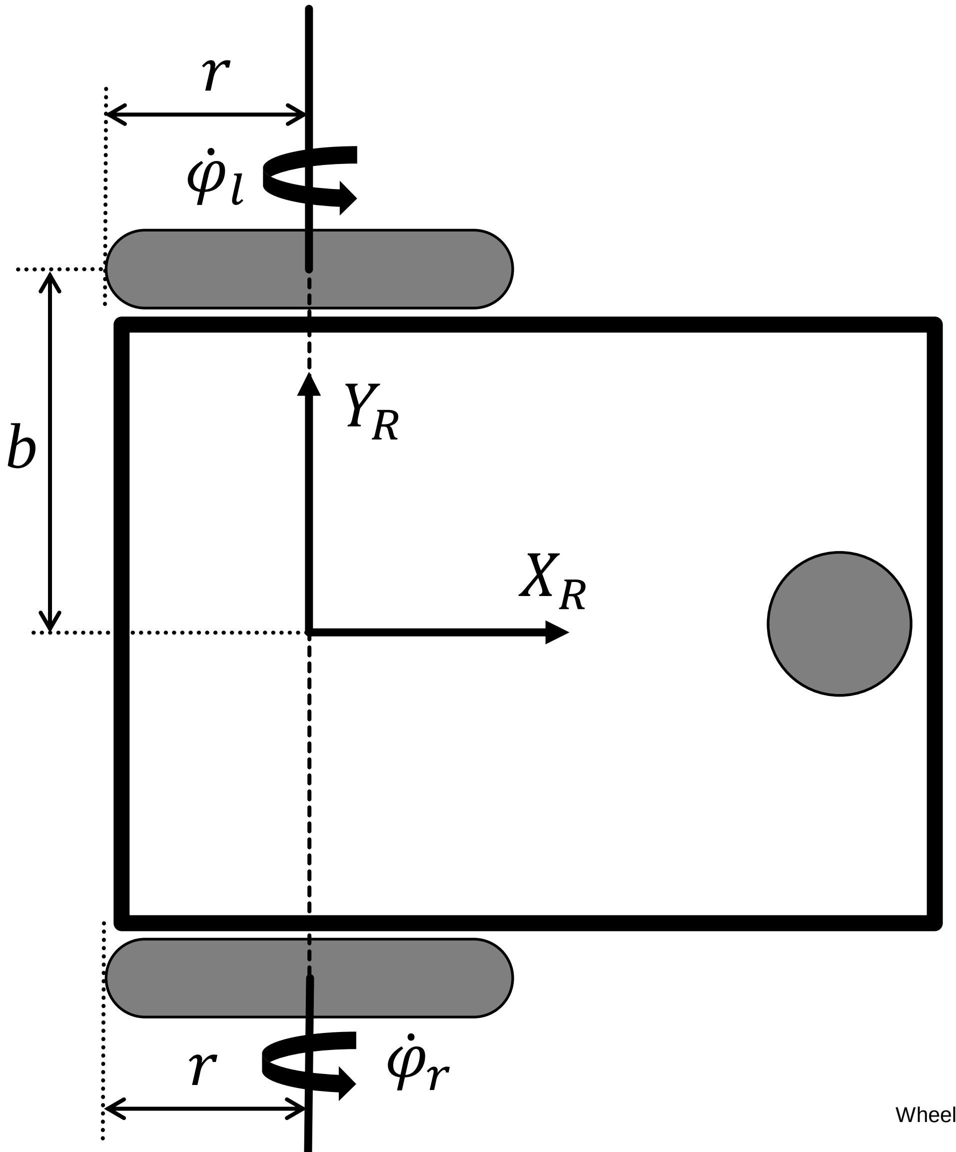
$$\alpha = -\pi/2,\ \beta = 0,\ l = -b$$

- Rolling constraints

$$\begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \end{bmatrix} \dot{\xi}_R = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

- No-sliding constraints

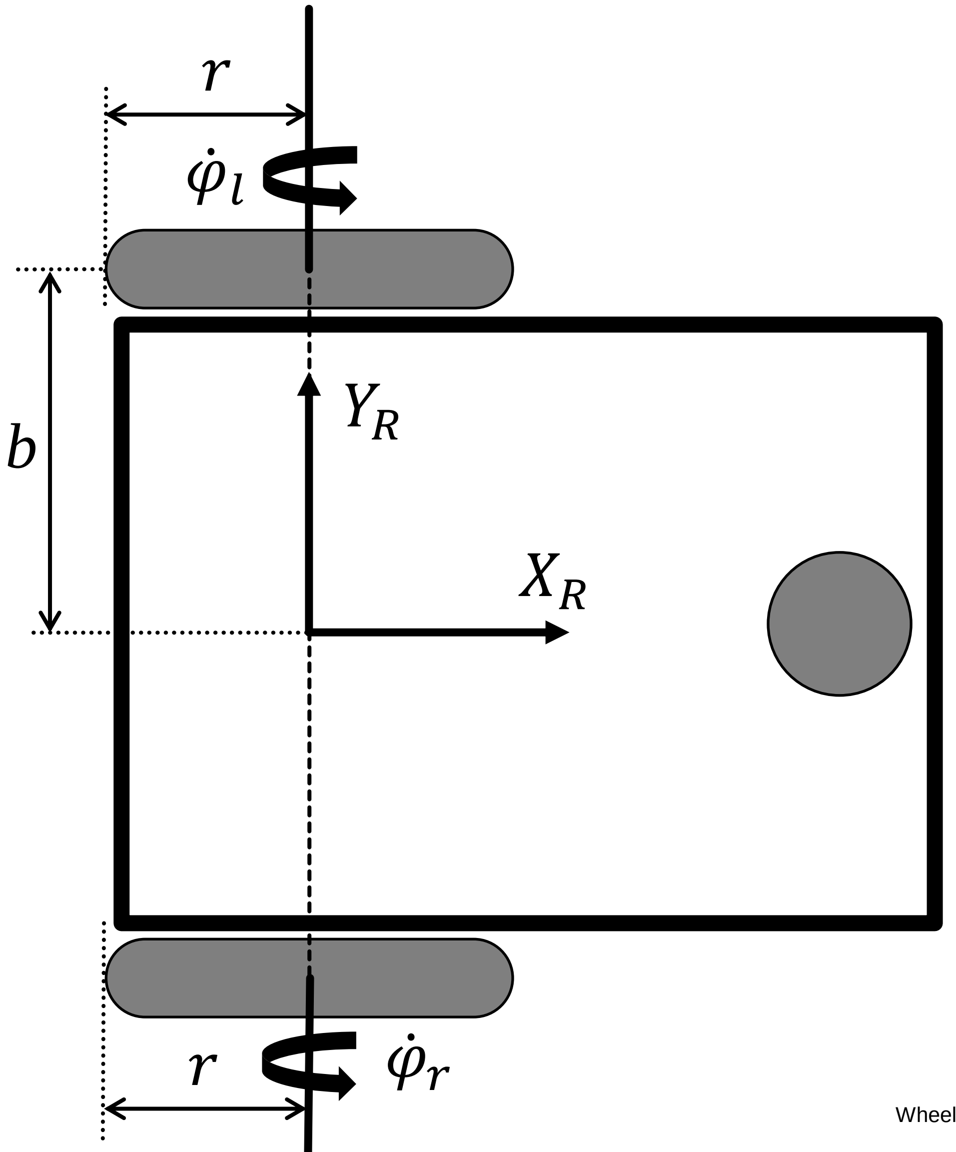
$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \dot{\xi}_R = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



A Differential Drive Robot | Differential Kinematics

- Stacked equations of motion

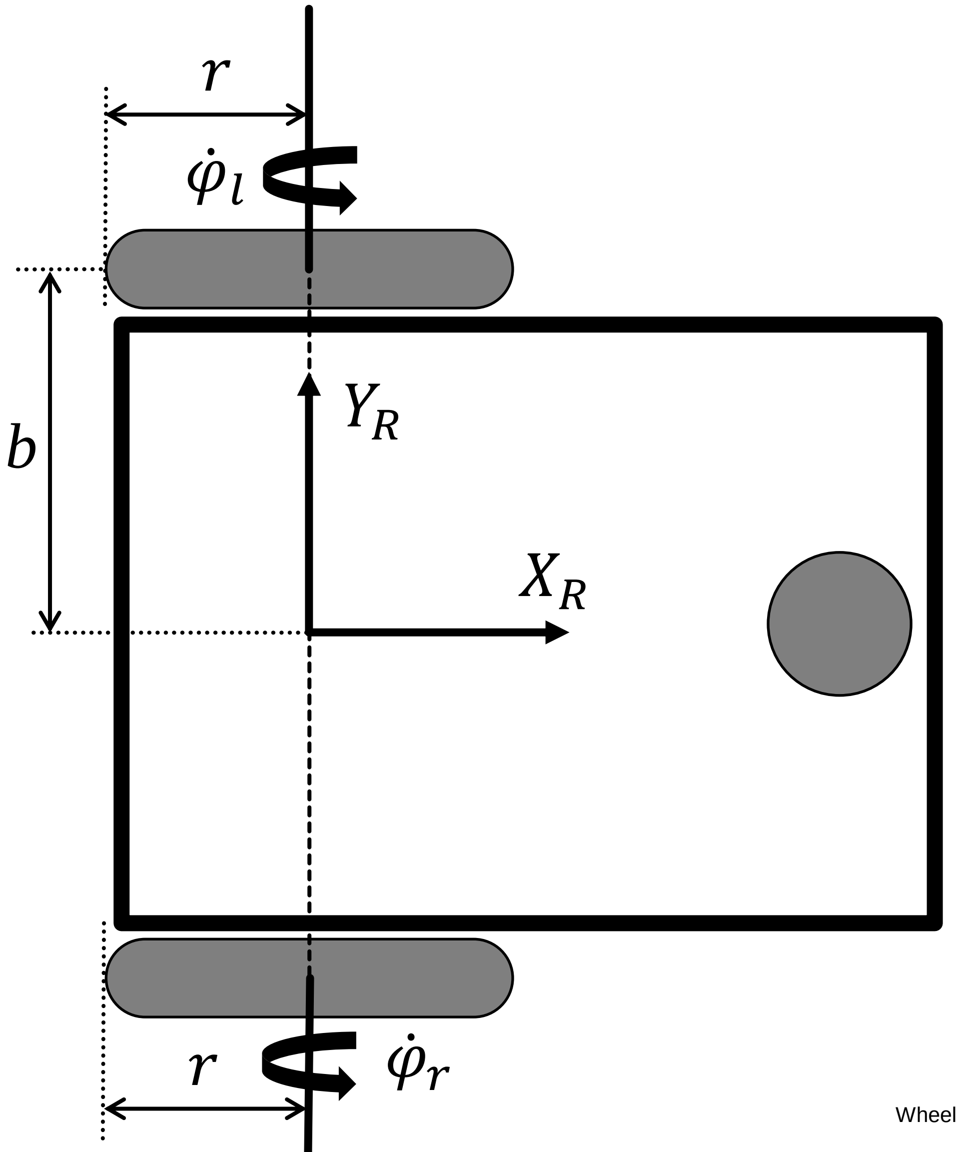
$$\begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \dot{\xi}_R = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$



A Differential Drive Robot | Differential Kinematics

- Stacked equations of motion

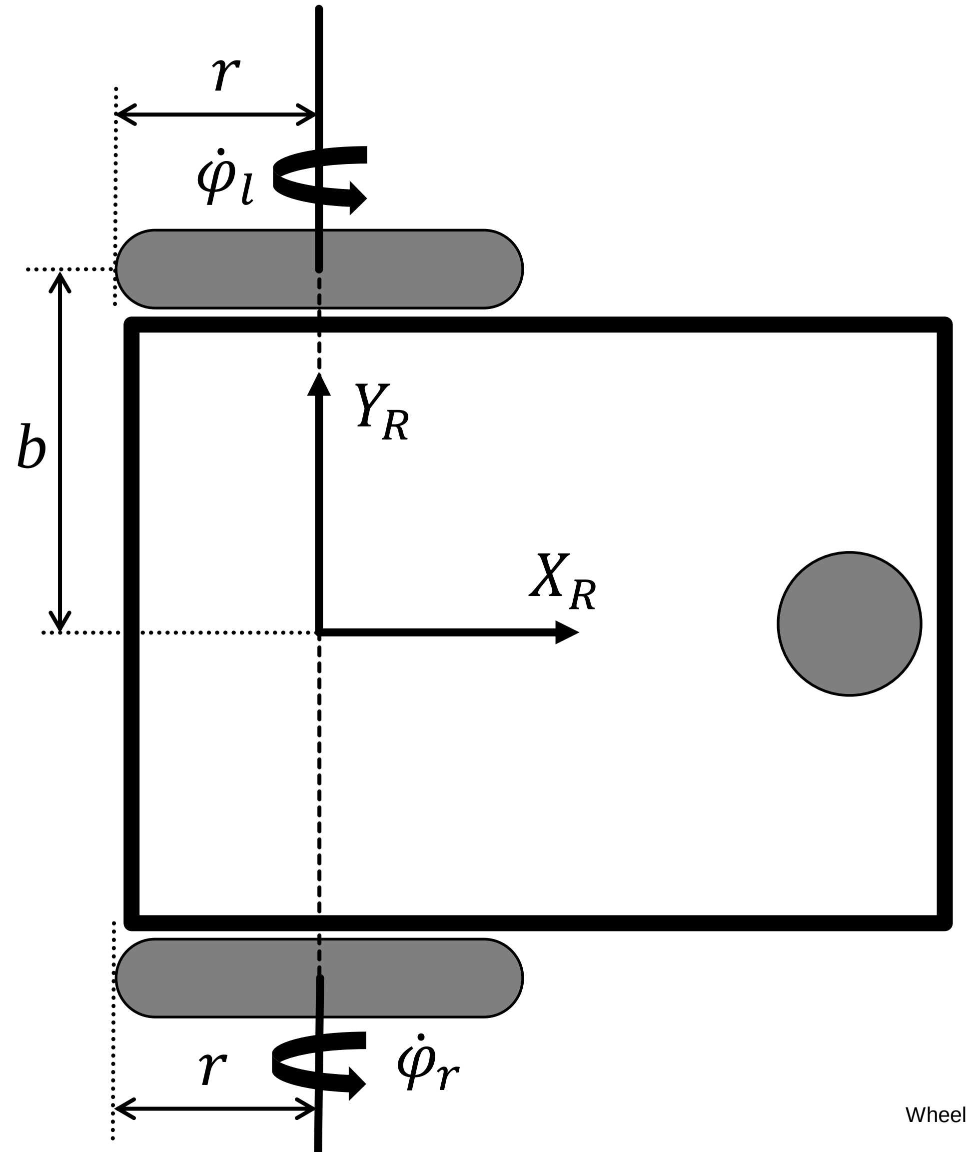
$$\underbrace{\begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}}_{=:A} \dot{\xi}_R = \underbrace{\begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{=:B} \underbrace{\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}}_{=:\dot{\varphi}}$$



A Differential Drive Robot | Differential Kinematics

- Stacked equations of motion

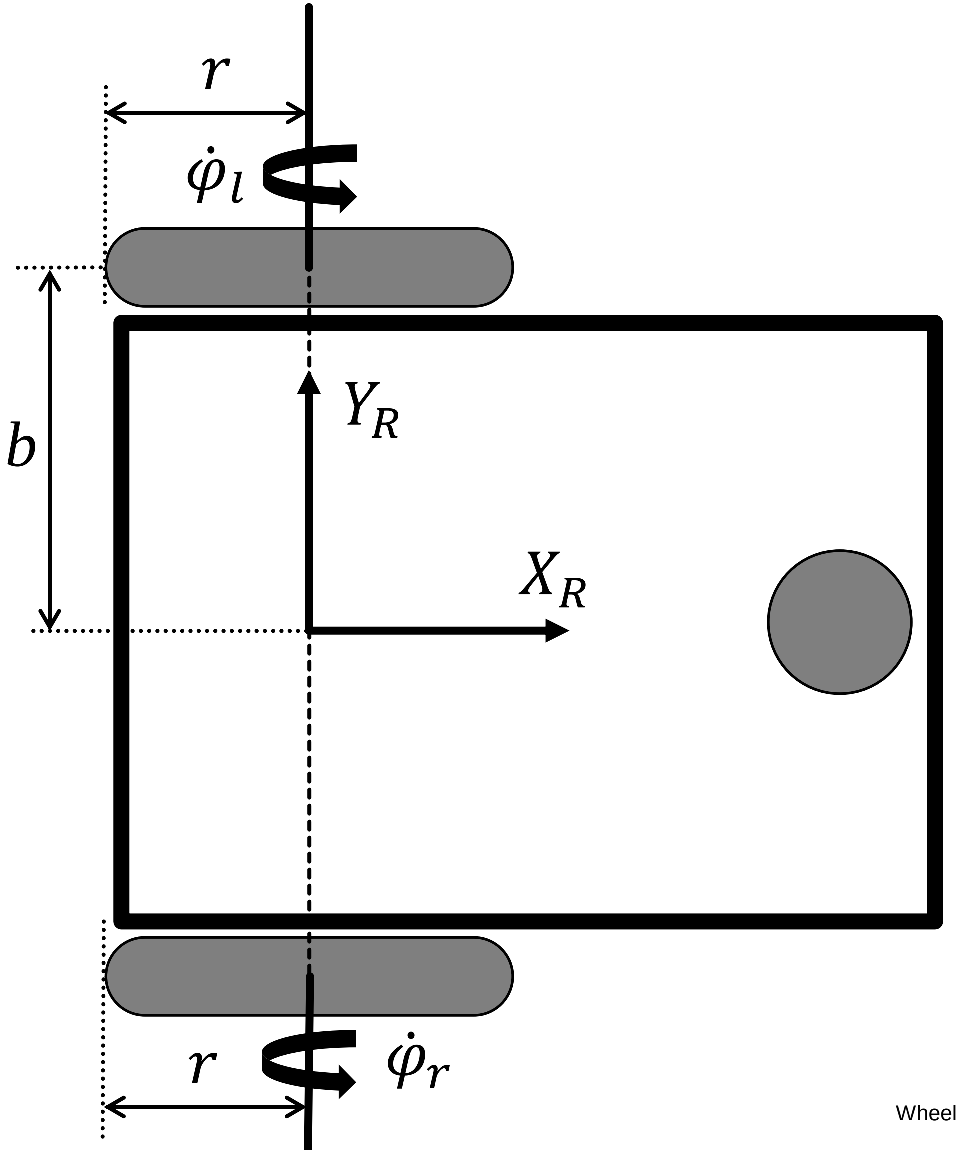
$$A\dot{\xi}_R = B\dot{\varphi}$$



A Differential Drive Robot | Differential Kinematics

- Stacked equations of motion

$$\underbrace{A}_{4\times 3}\ \dot{\xi}_R = \underbrace{B}_{4\times 2}\ \dot{\varphi}$$



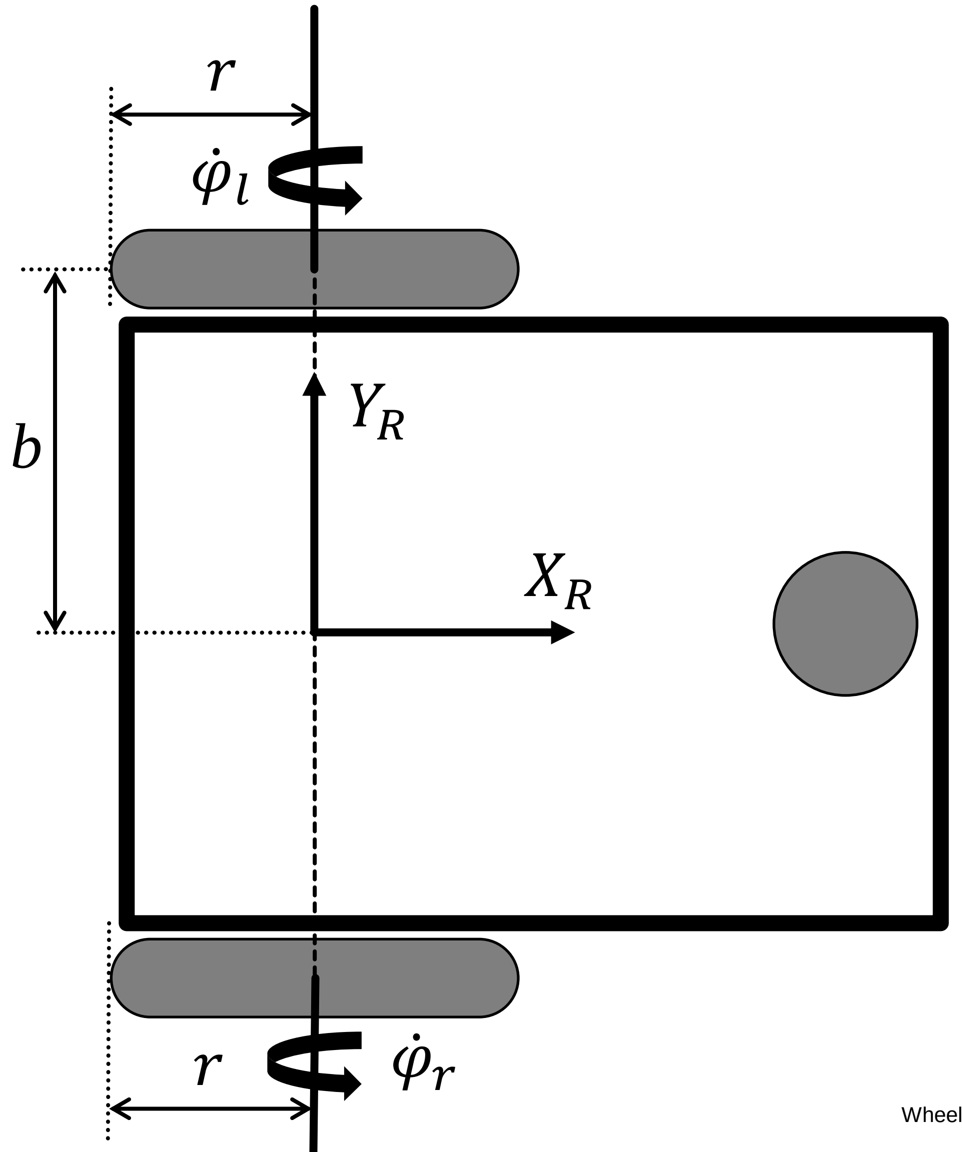
A Differential Drive Robot | Differential Kinematics

- Stacked equations of motion

$$A\dot{\xi}_R = B\dot{\varphi}$$

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B\dot{\varphi}$$



A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ b & -b & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ b & -b & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ b & -b & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ b & -b & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}, \qquad A^T B = \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}, \qquad A^T B = \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

$$(\mathrm{A}^\mathrm{T}\mathrm{A})^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2b^2 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}, \quad A^T B = \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

$$(\mathrm{A}^\mathrm{T}\mathrm{A})^{-1} A^T B = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2b^2 \end{bmatrix} \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\dot{\xi}_R = (A^T A)^{-1} A^T B \dot{\varphi}$$

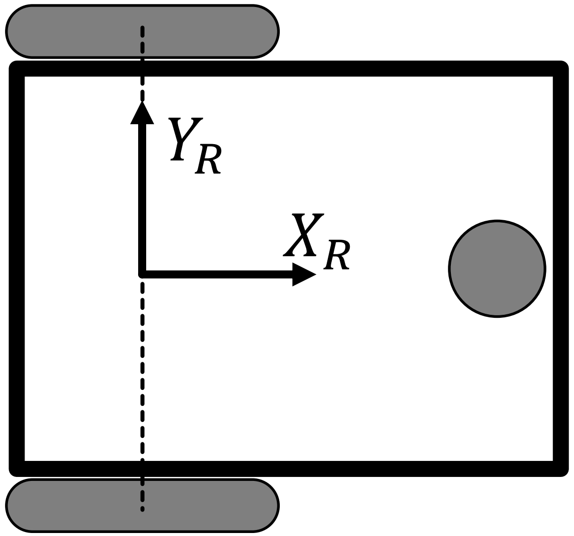
$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad A^T A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}, \quad A^T B = \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix}$$

$$(\mathrm{A^T A})^{-1} A^T B = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2b^2 \end{bmatrix} \begin{bmatrix} r & r \\ 0 & 0 \\ br & -br \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

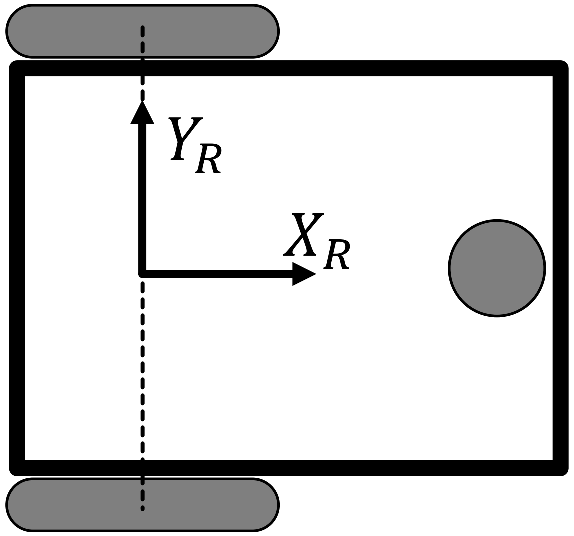


A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

- Forward velocity: $\dot{x} = r\frac{(\dot{\varphi}_r + \dot{\varphi}_l)}{2}$



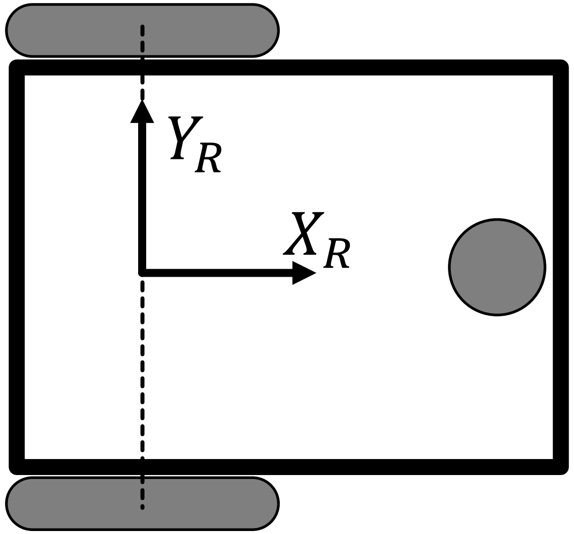
A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

- Forward velocity: $\dot{x} = r\frac{(\dot{\varphi}_r + \dot{\varphi}_l)}{2}$

- No-sliding: $\dot{y} = 0$

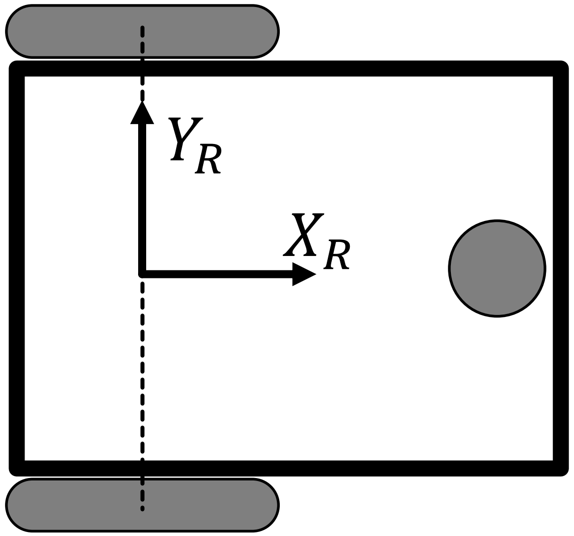


A Differential Drive Robot | Differential Kinematics

- Forward kinematics solution

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix}\dot{\varphi}_r \\ \dot{\varphi}_l\end{bmatrix}$$

- Forward velocity: $\dot{x} = r\frac{(\dot{\varphi}_r+\dot{\varphi}_l)}{2}$
- No-sliding: $\dot{y} = 0$
- Angular velocity: $\dot{\theta} = r\frac{(\dot{\varphi}_r-\dot{\varphi}_l)}{2b}$



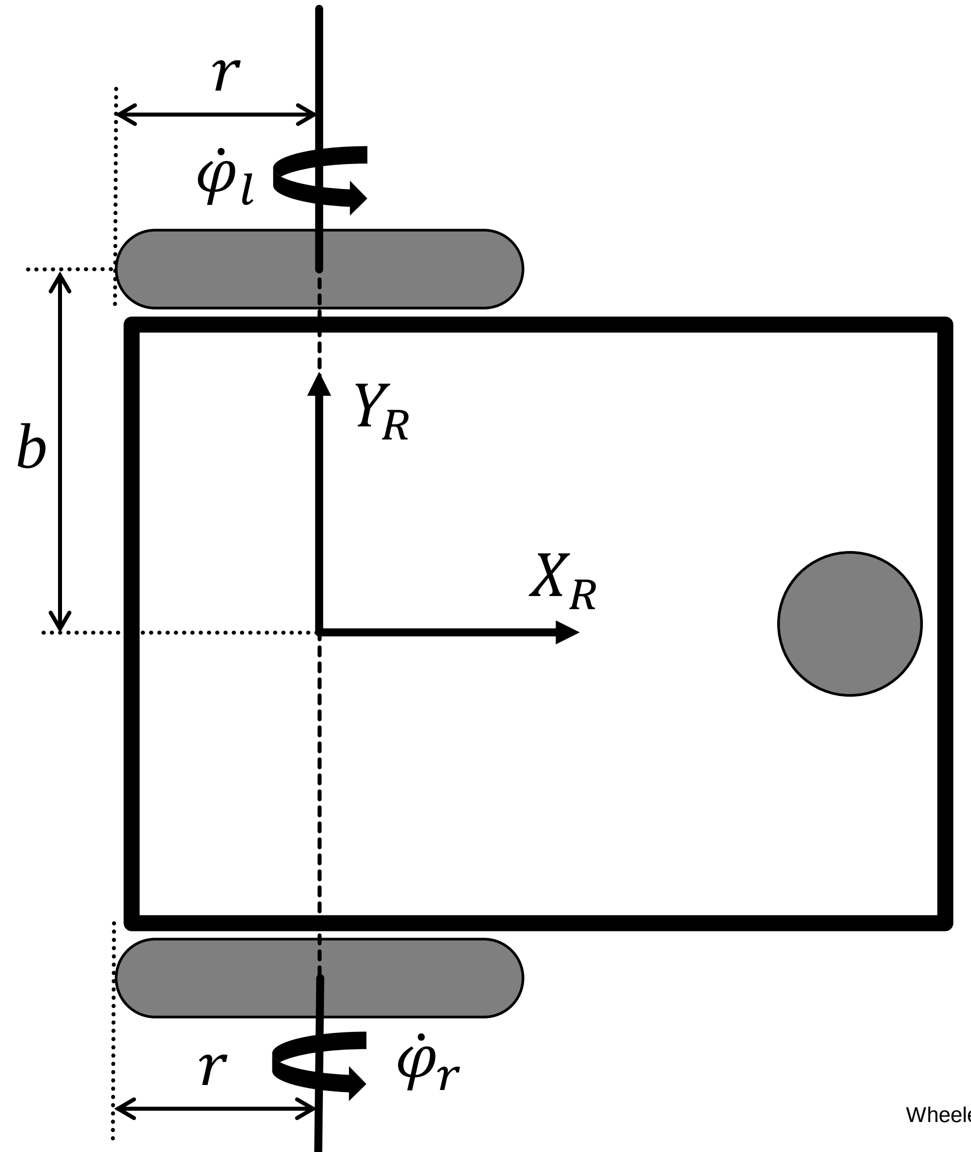
A Differential Drive Robot | Differential Kinematics

- Stacked equations of motion

$$A\dot{\xi}_R = B\dot{\varphi}$$

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$



A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B^T B = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B^T B = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}$$

$$B^T A = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}$$

$$B^T A = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & r & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}, \qquad B^T A = \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}, \qquad B^T A = \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

$$(B^T B)^{-1} = \begin{bmatrix} 1/r^2 & 0 \\ 0 & 1/r^2 \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}, \qquad B^T A = \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

$$(B^T B)^{-1} B^T A = \begin{bmatrix} 1/r^2 & 0 \\ 0 & 1/r^2 \end{bmatrix} \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\dot{\varphi} = (B^T B)^{-1} B^T A \dot{\xi}_R$$

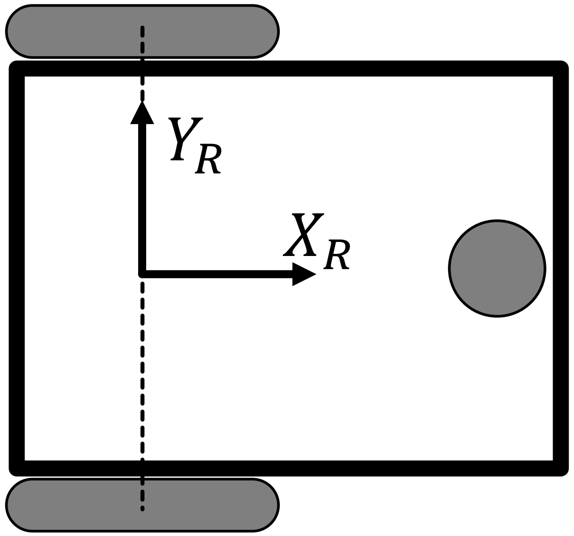
$$A = \begin{bmatrix} 1 & 0 & b \\ 1 & 0 & -b \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} r & 0 \\ 0 & r \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B^T B = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}, \qquad B^T A = \begin{bmatrix} r & 0 & br \\ r & 0 & -br \end{bmatrix}$$

$$(B^T B)^{-1} B^T A = \begin{bmatrix} 1/r^2 & 0 \\ 0 & 1/r^2 \end{bmatrix} \begin{bmatrix} r & 0 & -br \\ r & 0 & br \end{bmatrix} = \begin{bmatrix} 1/r & 0 & b/r \\ 1/r & 0 & -b/r \end{bmatrix}$$

A Differential Drive Robot | Differential Kinematics

- Inverse kinematics solution

$$\begin{bmatrix}\dot{\varphi}_r \\ \dot{\varphi}_l\end{bmatrix} = \begin{bmatrix}1/r & 0 & b/r \\ 1/r & 0 & -b/r\end{bmatrix}\begin{bmatrix}\dot{x} \\ \dot{y} \\ \dot{\theta}\end{bmatrix}$$



A Differential Drive Robot | Summary

- Degree of Maneuverability

$$\delta_m = 2, \qquad \delta_s = 0, \qquad \delta_M = 2$$

- Forward differential kinematics

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix}$$

- Inverse differential kinematics

$$\begin{bmatrix} \dot{\varphi}_r \\ \dot{\varphi}_l \end{bmatrix} = \begin{bmatrix} 1/r & 0 & b/r \\ 1/r & 0 & -b/r \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$