Kinematics | **Application Example**

Autonomous Mobile Robots

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Planar 3-link arm

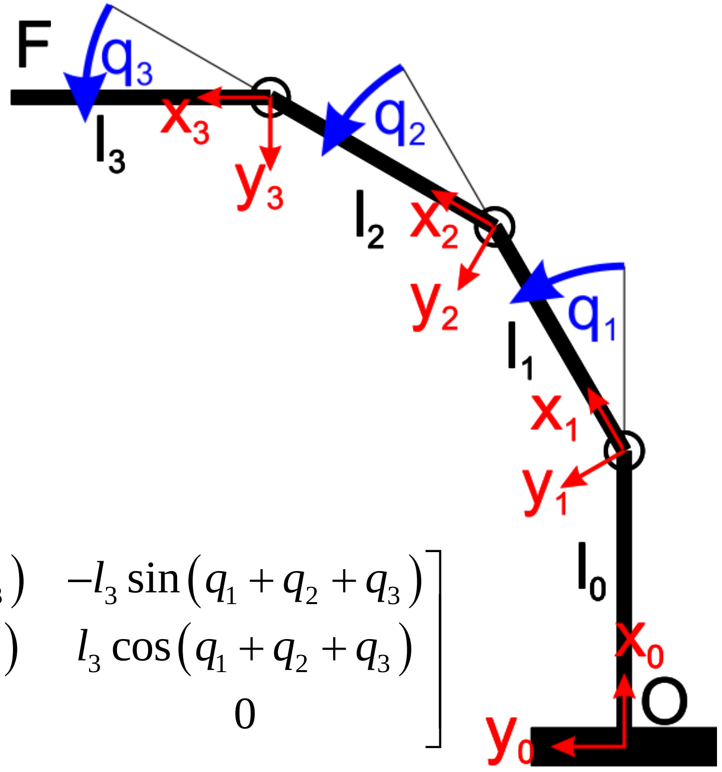
- Determine end-effector Jacobian of a 3-Link arm
  1. Introduce coordinate frames
  2. Introduce generalized coordinates $\mathbf{q} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix}^T$
  3. Determine end-effector position (*see basic example*)

$$ {}_0\mathbf{r}_{OF}(\mathbf{q}) = \begin{pmatrix} l_0 + l_1\cos(q_1) + l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) \\ l_1\sin(q_1) + l_2\sin(q_1+q_2) + l_3\sin(q_1+q_2+q_3) \\ 0 \end{pmatrix} $$

  4. Take the partial derivative

$$ {}_0\mathbf{J}_F(\mathbf{q}) = \frac{\partial\, {}_0\mathbf{r}_{OF}}{\partial \mathbf{q}} $$

$$ = \begin{bmatrix} -l_1\sin(q_1) - l_2\sin(q_1+q_2) - l_3\sin(q_1+q_2+q_3) & -l_2\sin(q_1+q_2) - l_3\sin(q_1+q_2+q_3) & -l_3\sin(q_1+q_2+q_3) \\ l_1\cos(q_1) + l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) & l_2\cos(q_1+q_2) + l_3\cos(q_1+q_2+q_3) & l_3\cos(q_1+q_2+q_3) \\ 0 & 0 & 0 \end{bmatrix} $$



Inverse kinematics

With zero start point

$$\mathbf{q}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

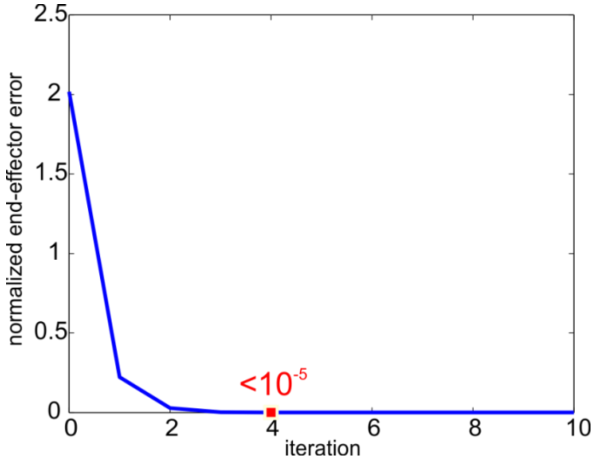
- Find desired configuration
  - Apply inverse kinematics

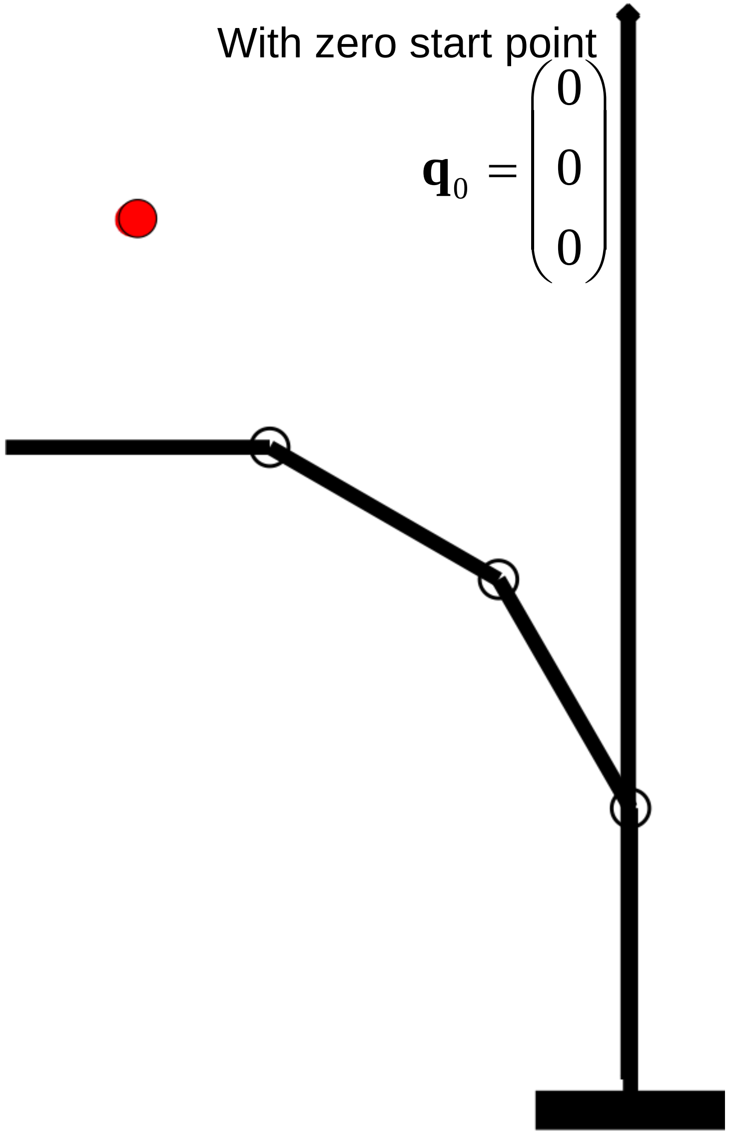
$$\mathbf{r}_F(\mathbf{q}) \Rightarrow \mathbf{q}$$

  - Use iterative numerical solution

$$\mathbf{q}^{i+1} = \mathbf{q}^i + \mathbf{J}^+\left(\mathbf{r}^{goal} - \mathbf{r}^i\right)$$

  - Start value $\mathbf{q}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$





Inverse kinematics

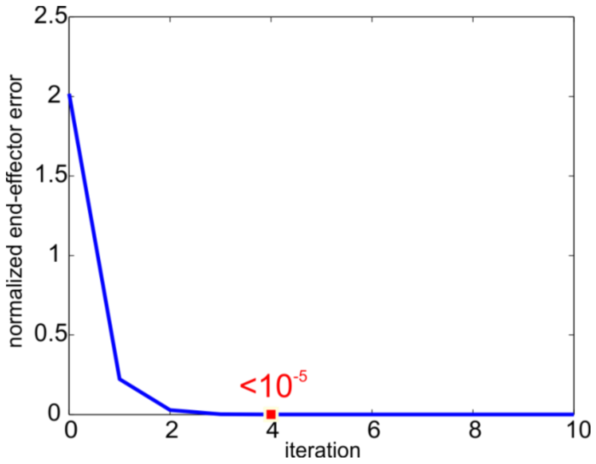
- Find desired configuration
  - Apply inverse kinematics

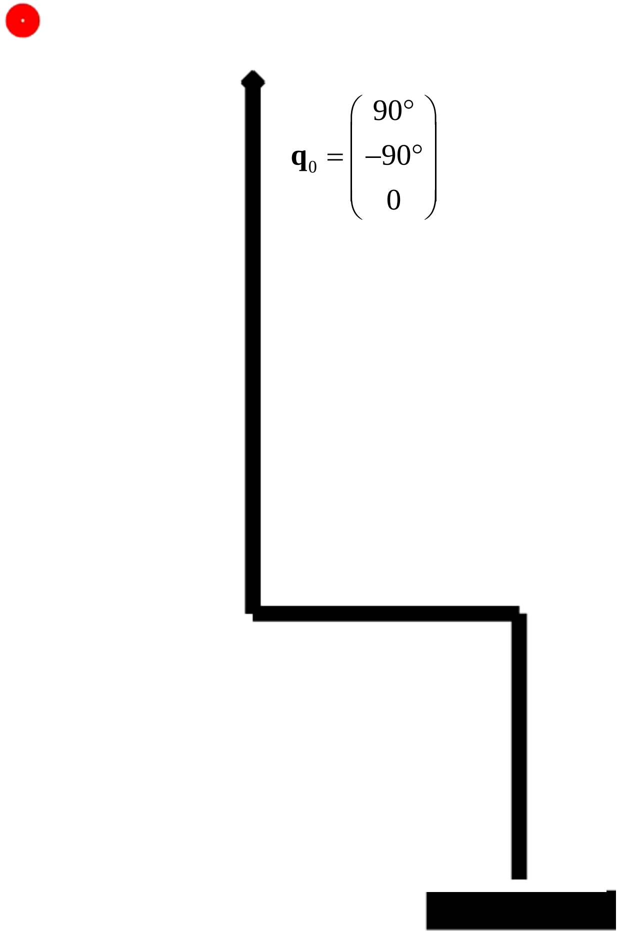
$$\mathbf{r}_F(\mathbf{q}) \Rightarrow \mathbf{q}$$

  - Use iterative numerical solution

$$\mathbf{q}^{i+1} = \mathbf{q}^{i} + \mathbf{J}^{+}\left(\mathbf{r}^{goal} - \mathbf{r}^{i}\right)$$

  - Start value $\mathbf{q}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
  - Multiple solutions possible
    - 2 dimensional problem but 3 joints
    - Depending on start configuration





$$\mathbf{q}_0 = \begin{pmatrix} 90° \\ -90° \\ 0 \end{pmatrix}$$

Inverse differential kinematics

- Move end-effector along trajectory
  - Apply inverse differential kinematics

$$\dot{\mathbf{r}}_F = \mathbf{J}_F \dot{\mathbf{q}} \Rightarrow \dot{\mathbf{q}} = \mathbf{J}_F^+ \dot{\mathbf{r}}_F$$

  - Jacobian $\mathbf{J}_F \in \mathbb{R}^{2\times 3}$ is not invertible
    - Planar problem (2DoF) but 3 actuators
    - Pseudo inverse produces least squares minimal solution $\left\|\dot{\mathbf{q}}\right\|_2$
    - Associated null-space $\mathbf{N}_F = \mathbf{I} - \mathbf{J}_F^+ \mathbf{J}_F$
      - Fulfill other tasks (e.g. obstacle avoidance)

$$\dot{\mathbf{q}} = \mathbf{J}_F^+ \dot{\mathbf{r}}_F + \mathbf{N}_F \dot{\mathbf{q}}_0$$

$$\dot{\mathbf{r}}_F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad q_1(t) = 1 - \cos(t), \quad \dot{q}_1 = \sin(t)$$

$$\dot{\mathbf{q}} = \mathbf{N}_F \left(\mathbf{J}_{q1} \mathbf{N}_F\right)^+ \dot{q}_1, \quad \text{with } \mathbf{J}_{q1} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$