Wheeled Locomotion | Introduction
Autonomous Mobile Robots

Paul Furgale

Margarita Chli, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

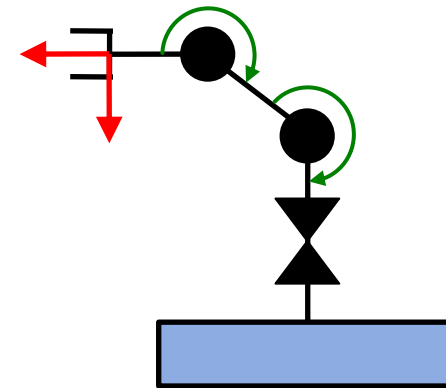Wheeled Locomotion | Introduction | 1

Motivation: Efficiency of Locomotion Types

- Wheeled locomotion
  - Highly efficient on hard surfaces
  - Generally restricted to man-made surfaces
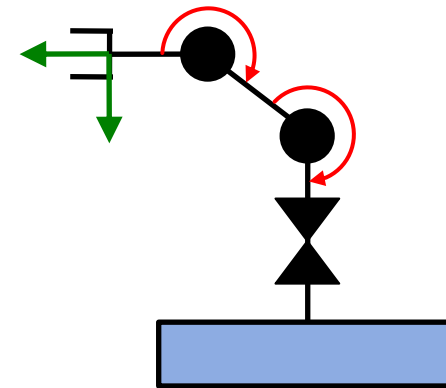 - The de facto standard in mobile robotics

Review: Kinematics

- ## Forward kinematics
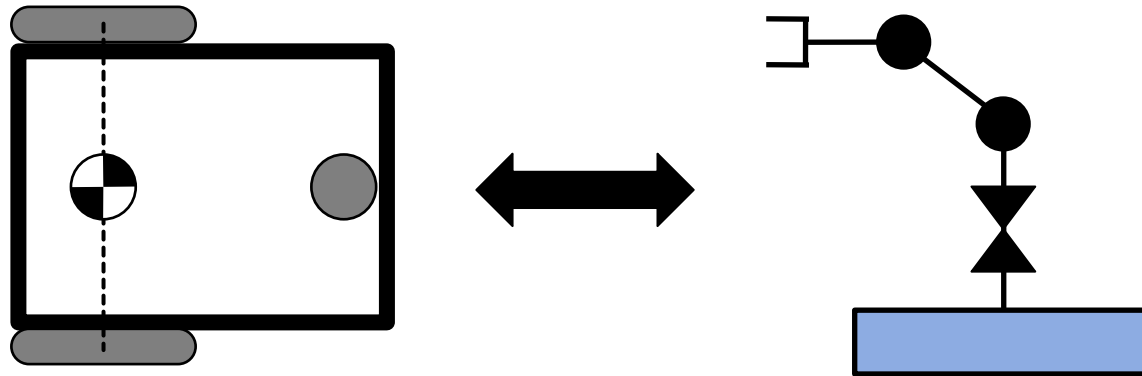 - Given a set of actuator positions, determine the corresponding pose

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 3

Review: Kinematics

- ## Forward kinematics
  - Given a set of actuator positions, determine the corresponding pose

- ## **Inverse kinematics**
  - Given a desired pose, determine the corresponding actuator positions

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 4

Review: Kinematics

- ## Forward kinematics
  - Given a set of actuator positions, determine the corresponding pose

- ## Inverse kinematics
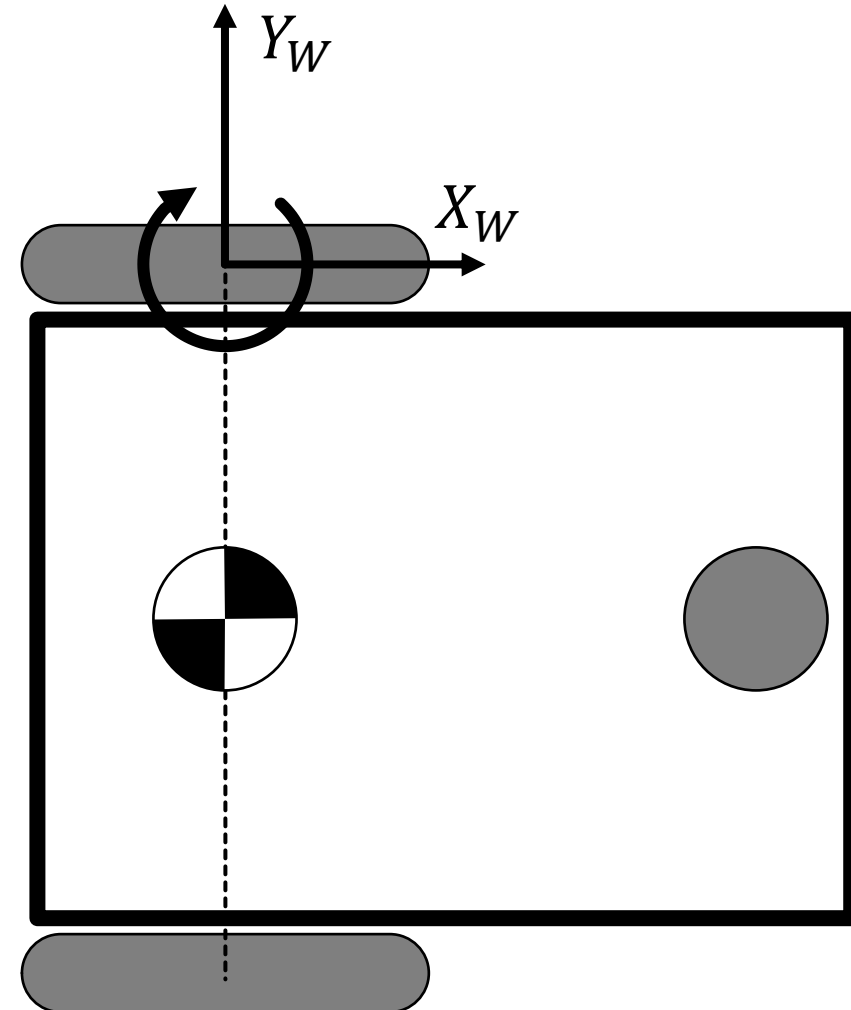 - Given a desired pose, determine the corresponding actuator positions

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 5

Wheeled Kinematics

- Not all degrees of freedom of a wheel can be actuated or have encoders

Y_W

X_W

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

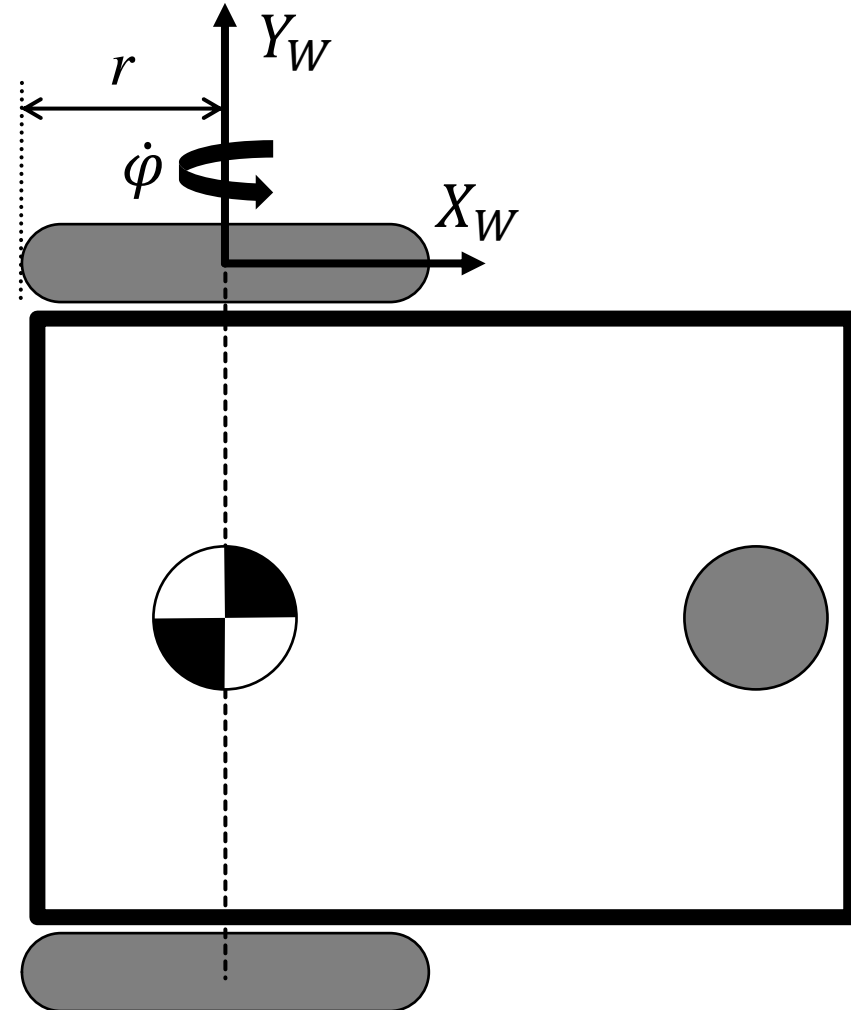Wheeled Locomotion | Introduction | 6

Wheeled Kinematics

- Not all degrees of freedom of a wheel can be actuated or have encoders

- Wheels can impose **differential constraints** that complicate the computation of kinematics

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \dot{\varphi}r \\ 0 \end{bmatrix}$$
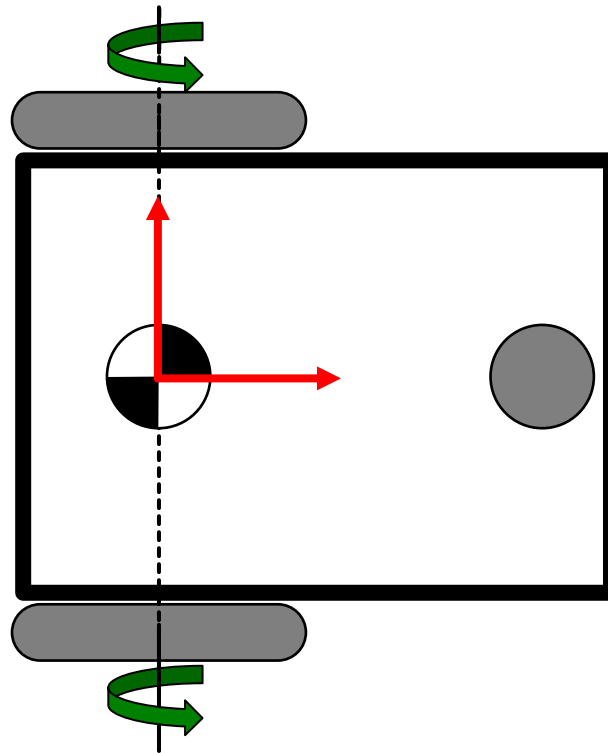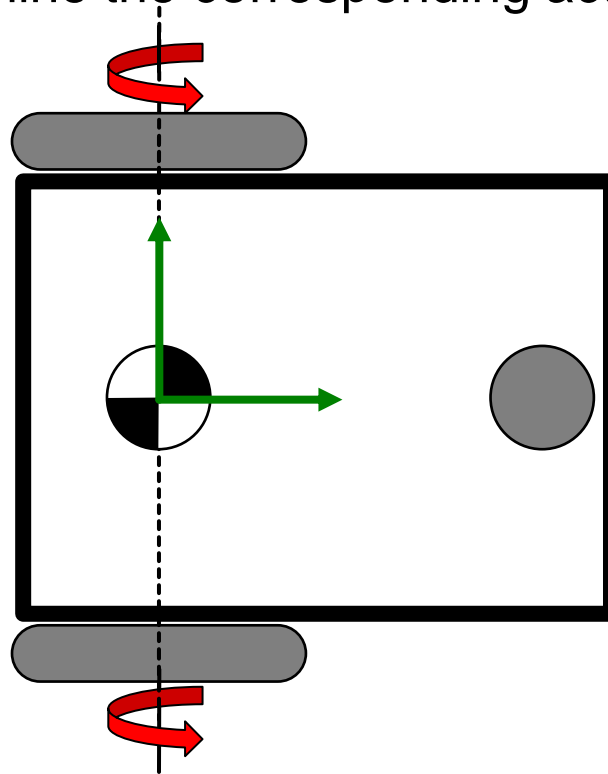
rolling constraint

no-sliding constraint

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 7

Differential Kinematics

- ## Differential forward kinematics
  - Given a set of actuator speeds, determine the corresponding velocity

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

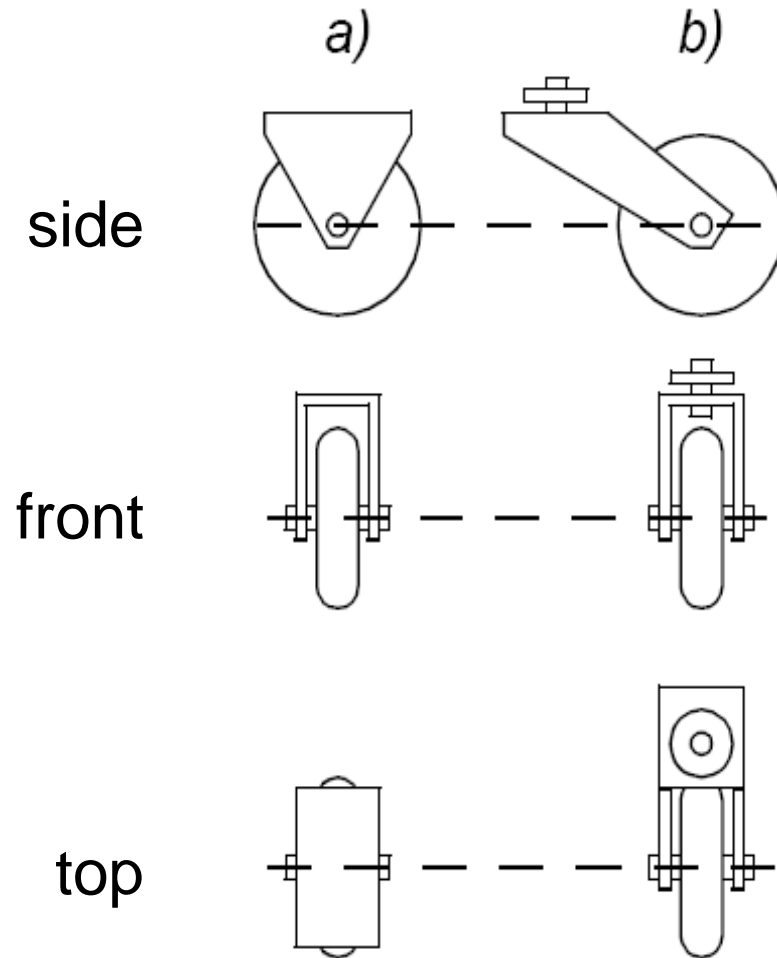Wheeled Locomotion | Introduction | 8

Differential Kinematics

- Differential forward kinematics
  - Given a set of actuator speeds, determine the corresponding velocity
- **Differential inverse kinematics**
  - Given a desired velocity, determine the corresponding actuator speeds

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 9
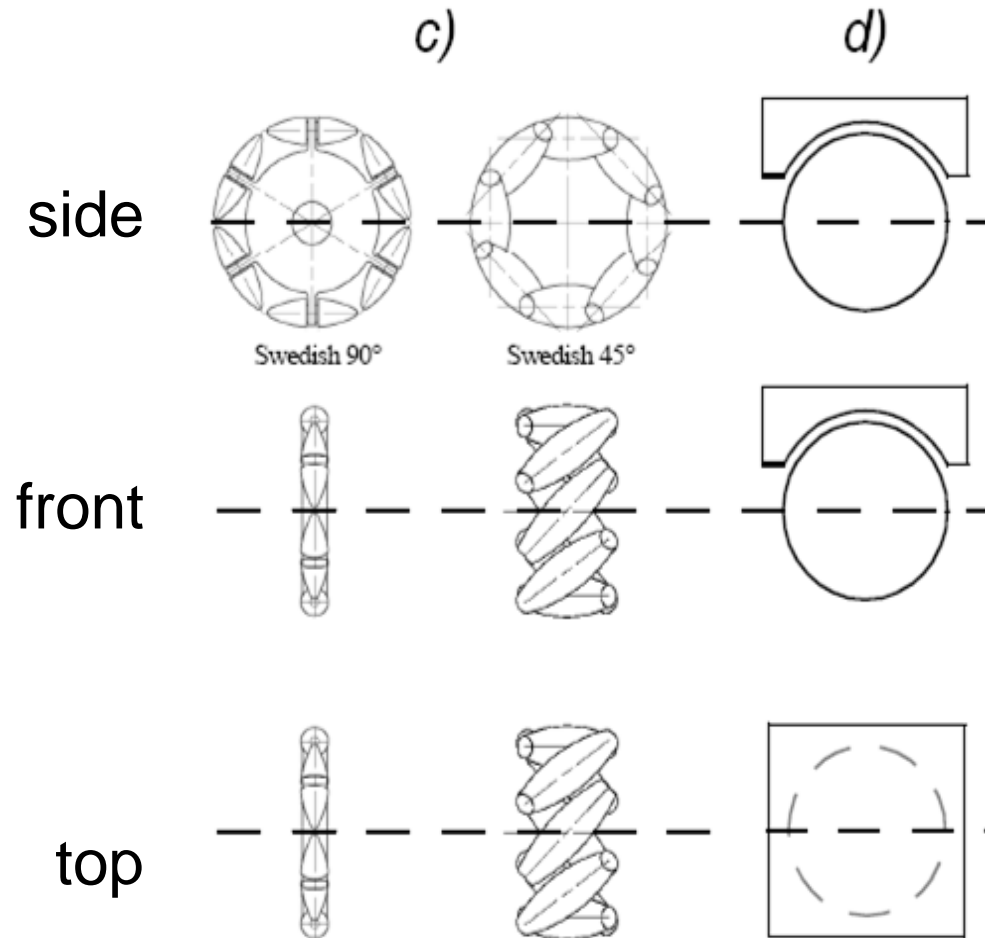
Basic Wheel Types

side

front

top

a) Standard wheel
- Two degrees of freedom
  - Rotation around the wheel axle
  - Free rotation around the contact point
- Can be steered or fixed

b) Castor wheel
- Three degrees of freedom
  - Rotation around the wheel axle
  - Free rotation around the contact point
  - Rotation around the castor axle

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 10

Basic Wheel Types

c)   d)

side   Swedish 90°   Swedish 45°

front

top

c) Swedish wheel
  - Three degrees of freedom
    - Rotation around the wheel axle
    - Free rotation around the rollers
    - Free rotation around the contact point
d) Spherical wheel (ball)
  - Three degrees of freedom

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 11

Basic Wheel Types

By Rotacaster (Rotacaster Wheel Pty Ltd) via Wikimedia Commons

c) Swedish wheel

- Three degrees of freedom
    - Rotation around the wheel axle
    - Free rotation around the rollers
    - Free rotation around the contact point

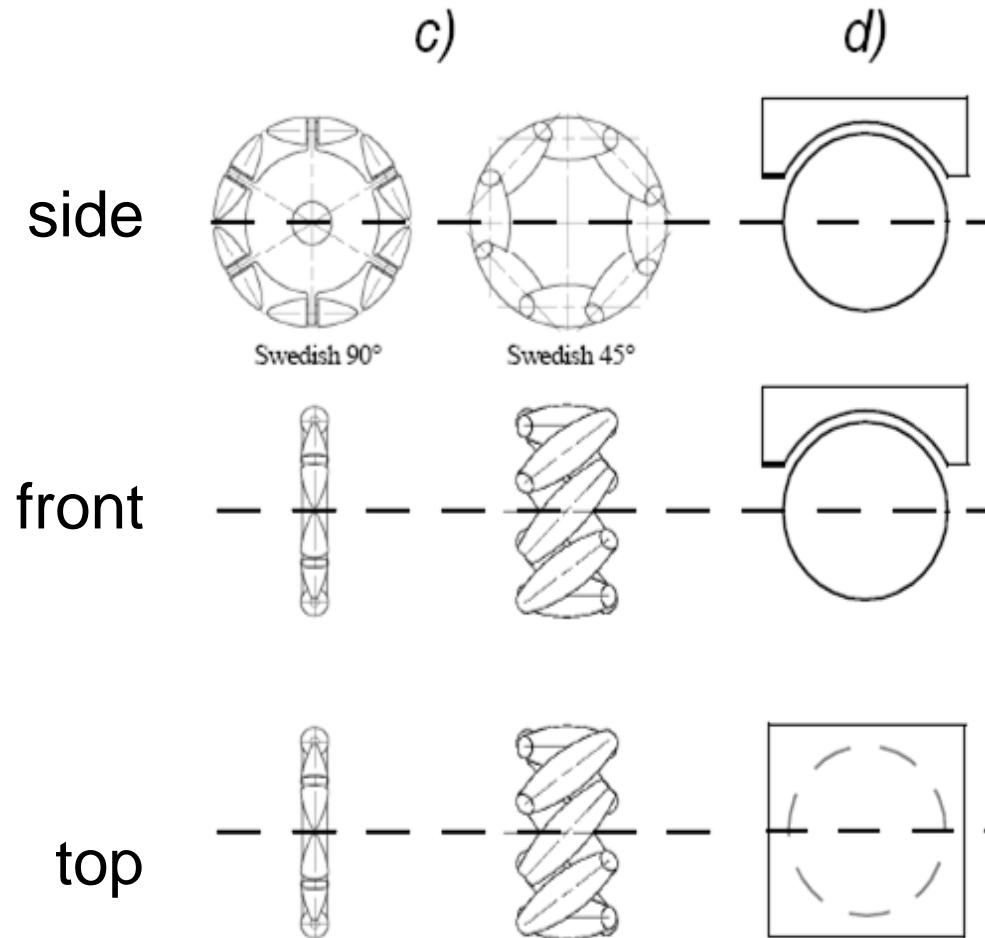d) Spherical wheel (ball)

- Three degrees of freedom

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 12

Basic Wheel Types

c) Swedish 90°   Swedish 45°

side

front

top

c) **Swedish wheel**
  - Three degrees of freedom
    - Rotation around the wheel axle
    - Free rotation around the rollers
   - Free rotation around the contact point

d) **Spherical wheel (ball)**
  - Three degrees of freedom

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 13

Holonomic and Non-Holonomic Systems

- Holonomic systems
  - Differential constrains are **integrable**. They may be expressed as constraints on the robot's pose
  - The robot is able to move instantaneously in any direction in the space of its degrees of freedom

- Non-holonomic systems
  - Differential constraints are **not integrable**. There is no way to express them as constraints on the robot's pose
  - The robot is not able to move instantaneously in every direction in the space of its degrees of freedom

A Holonomic Mobile Robot
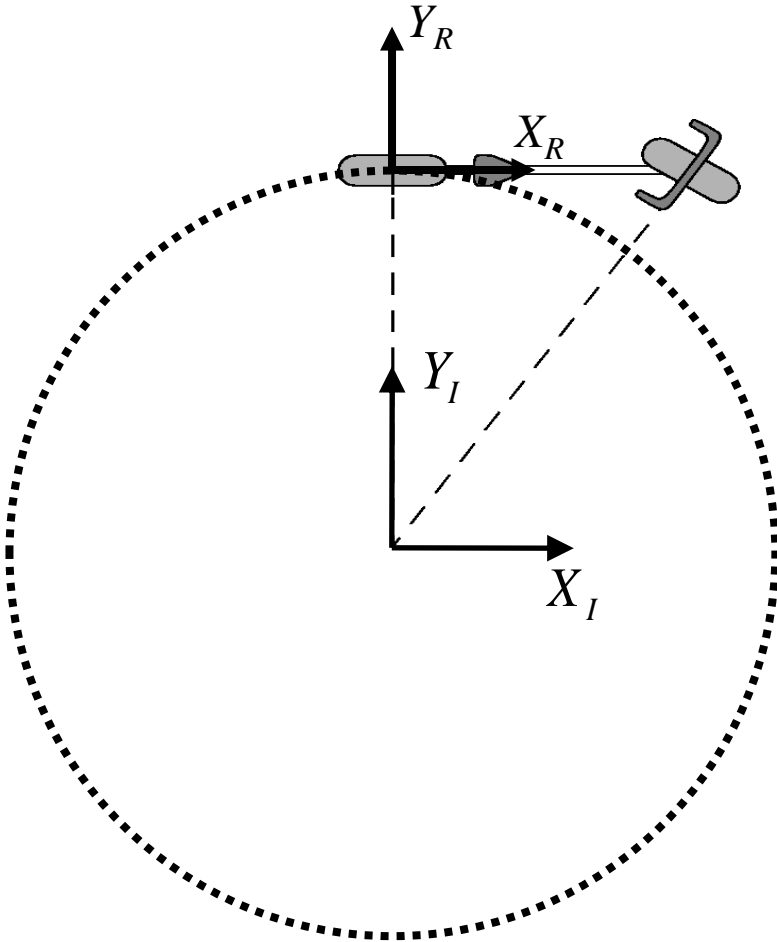
- A bicycle with fixed steering
  - Two non-steerable wheels
  - The workspace of the robot collapses to a single degree of freedom: a circle
  - The differential constraints can be expressed as constraints on position
  - In any configuration, it is possible to choose wheel velocities that move the robot any direction within its workspace

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 15
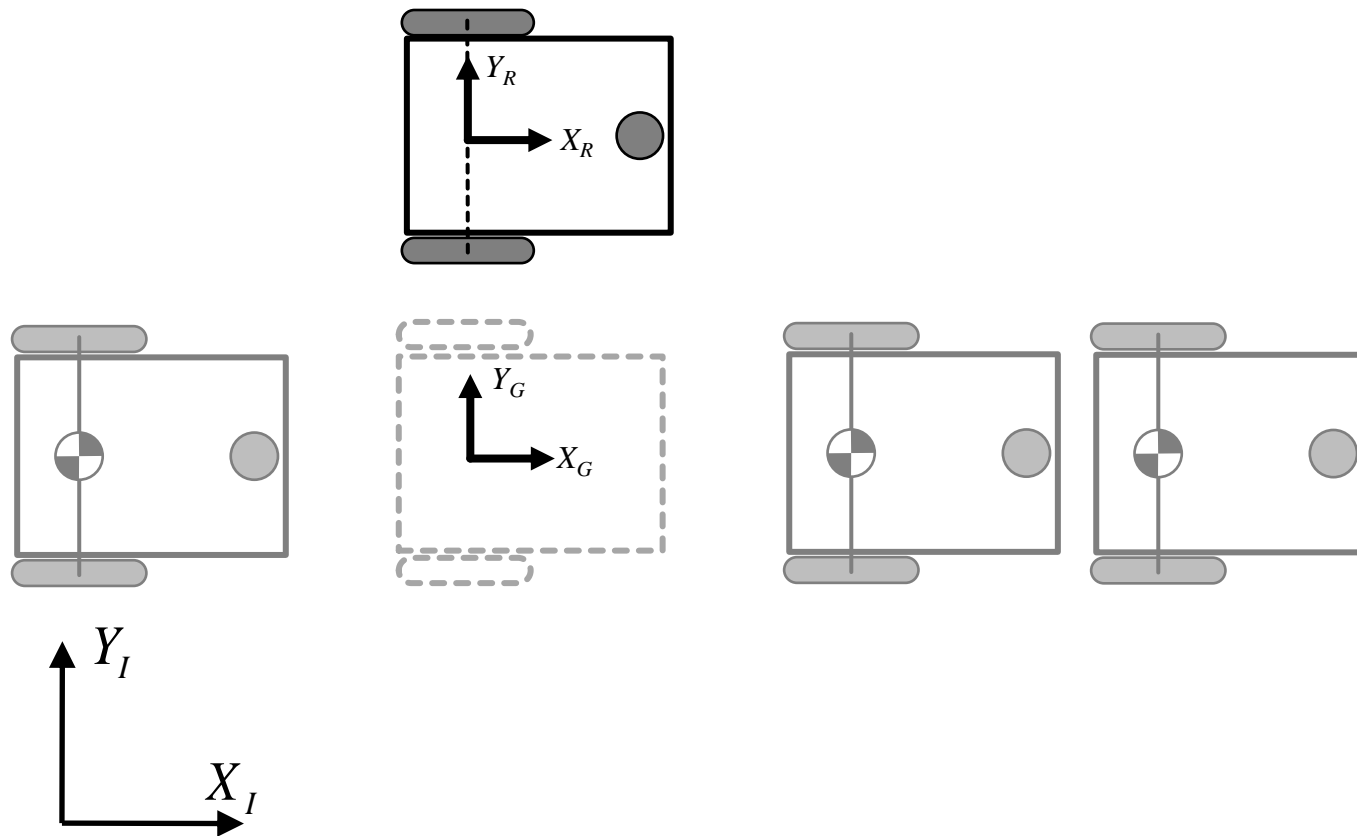
A Holonomic Wheeled Platform

Photographer: Frank C. Müller via Wikimedia Commons

- The office chair
  - Castor wheels impose no differential constraints
  - In any configuration, it is possible to choose wheel velocities that move the robot any with any velocity within the plane

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 16

A Non-Holonomic Mobile Robot

No sliding constraint: $\dot{y}_R = 0$

- A differential drive robot
  - Two non-steerable wheels aligned on a common axis
  - The workspace of the robot encompasses all poses in the plane
  - The no-sliding constraint, $\dot{y}_R = 0$, cannot be expressed as a constraint on position
  - Regardless of the configuration the robot is in, instantaneous motion in the y_R direction is not possible.

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 17

Preview: Deriving the wheel equation

- Next topic: Given a **specific wheel configuration**, what is the relationship between platform speed and wheel constraints

Autonomous Mobile Robots
Margarita Chli, Paul Furgale, Marco Hutter, Martin Rufli, Davide Scaramuzza, Roland Siegwart

Wheeled Locomotion | Introduction | 18