

Derive eq 1 from article Eberling, B. and Ringwald, A., *Relic Neutrino Absorption Spectroscopy*, 2004. Also see appendix A1, of Lunardini, c., Sabancilar. E., Yang. L., *Ultra High Energy Neutrinos: Absorption, Thermal Effects and Signatures*, 2013.

There are different channels contributing to the process $\nu + \bar{\nu} \rightarrow \text{anything}$. However, the one of main interest is the resonant neutrino-antineutrino annihilation ($\nu\bar{\nu} \rightarrow Z^0 \rightarrow f\bar{f}$) occurring in the s-channel [10]. From its Mandelstam variable $s = (q^\mu + p^\mu)^2$, the resonance energy can be derived using the four momenta of UHE neutrino (q^μ), and background neutrinos (p^μ):

$$q^\mu = \begin{pmatrix} E' \\ \mathbf{q} \end{pmatrix}, \quad p^\mu = \begin{pmatrix} \sqrt{\mathbf{p}^2 + m_j^2} \\ \mathbf{p} \end{pmatrix}$$

For UHE neutrinos $|\mathbf{q}| \gg m_{\nu_j}$, therefore $E' = \sqrt{\mathbf{q}^2 + m^2} \approx |\mathbf{q}| \equiv q$, resulting in:

$$q^\mu = E' \begin{pmatrix} 1 \\ \hat{\mathbf{q}} \end{pmatrix}$$

So the Mandelstam variable, s, in the comoving frame is:

$$\begin{aligned} s &= \{|\mathbf{p}|^2 + m_j^2 - |\mathbf{p}|^2\} + \{E'^2 - E'^2|\hat{\mathbf{q}}|^2\} + \{2(E'\sqrt{\mathbf{p}^2 + m_j^2} - E'|\hat{\mathbf{q}}||\mathbf{p}| \cos(\theta))\} \\ s &= \{m_j^2\} + \{0\} + \{2E'(\sqrt{\mathbf{p}^2 + m_j^2} - p \cos(\theta))\} \\ s &\approx 2E'(\sqrt{\mathbf{p}^2 + m_j^2} - p \cos(\theta)) \end{aligned} \quad (2)$$

Because the Lorentz-invariant quantity \sqrt{s} can be identified as the total energy available in the center-of-mass frame, the resonance occurs at $\sqrt{s} = M_Z$, with $M_Z = 91.1876 \text{ GeV}$ being the Z^0 boson mass. The resonance Energy can be found by solving $\sqrt{s} = M_Z$, and assuming that the background neutrinos are at rest.

$$\begin{aligned} 2E'_{res}(\sqrt{p^2 + m_j^2} - p \cos(\theta)) &= M_Z^2 \\ E'_{res} &= \frac{M_Z^2}{2(\sqrt{\mathbf{p}^2 + m_j^2} - p \cos(\theta))} \\ E'_{res} &= \frac{M_Z^2}{2m_j} \end{aligned} \quad (3)$$