Discrete state space models in survival analysis: a particle filter application

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I cover discrete hazard models with $i = 1, \dots, n_t$ individuals at risk at time $t = 1, \dots, d$ and latent state variables $\vec{\alpha}_1, \dots, \vec{\alpha}_d$. The dynamics are

$$
\begin{aligned}
\vec{\alpha}_t &= \mathbf{F}(\vec{\theta})\vec{\alpha}_{t-1} + \mathbf{R}\vec{\epsilon}_t & \vec{\epsilon}_t &\sim N(\vec{0}, \mathbf{Q}(\vec{\theta})) \\
& & \vec{\alpha}_0 &\sim N(\vec{0}, \mathbf{Q}_0) \\
\eta_{it} &= \vec{\beta}^\top \vec{x}_{it} + \vec{\alpha}_t^\top \mathbf{C}\vec{z}_{it} + o_{it} & y_{it} &\sim g(\eta_{it})
\end{aligned}
$$

where y_{it} is the observed survival or survival length in the period $(t-1, t]$, $\vec{x}_{it}$ and $\vec{z}_{it}$ are observation specific covariates, o_{it} are offsets, and $\vec{\theta}$ and $\vec{\beta}$ are parameters to be estimated. I illustrate an application of particle filtering for estimating the parameters, $(\vec{\beta}, \vec{\theta})$, in an EM algorithm. The pros and cons of the particle filters are highlighted with an emphasis on the computational complexity.

References

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