Evaluation and retrofit of the St. Lawrence Parish House in Petrinja that was damaged during the 2020 earthquakes in Croatia

Master Thesis, 2024

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Abstract

Contents

1. Introduction

1.1 Motivation

With the global effort of reducing carbon dioxide emissions to fight against climate change, all industries must find solutions to minimize their carbon footprint. The construction sector is one of the leading emitters of carbon dioxide due to its production and utilization of cement in concrete constructions [1]. These emissions are not only due to the carbon dioxide emitted to provide the necessary energy for production but also because of the release due to the transformation of limestone into clinker, one of the main components of cement [2]. Therefore, it is crucial to reduce the volume of concrete used to build housing and infrastructures to a minimum. One of the most efficient ways to do so is to reuse existing buildings for as long as possible and reinforce them if necessary to be able to exploit them even longer. But what happens if buildings are affected by catastrophic events such as earthquakes? Even though most of the historical buildings in Europe have not been designed to resist such forces it does not necessarily mean that after a seismic event, those buildings cease to be safely exploitable. In the case of massive masonry construction that has been damaged, it is possible to only perform a small set of measures to give them their original strength or even increase their resistance against future earthquakes and thus prolong their service life for as long as possible.

1.2 Context

This thesis focuses on the case of the Parish House of the St. Lawrence church in Petrinja, Croatia that has been heavily damaged in the earthquakes of 2020. The goal of this case study is to predict the behaviour of the structure when subjected to a seismic event and to compare it to the behaviour of the building during the 2020 earthquake. To do this the structure will be modelled as an equivalent frame model (Equivalent frame models (EFMs)) using macro elements such as those proposed by Vanin et. al. [3] implemented in OpenSees software. Furthermore, the analysis done using the OpenSees script will be completed with a set of "hand calculations" of specific, prone-to-happen out-of-plane (Out-of-planes (OOPs)) mechanisms that are analysed using and further developed Jupyter Notebooks created in the Pre Study of this thesis. After analysing the structure, the model will be further used to evaluate chosen reinforcing measures and propose a set of solutions to enhance the seismic performance of the structure.

2. Presentation of the building

2.1 History

The Parish House of the St. Lawrence church (Crkva Sv. Lovre) in Petrinja dates from 1783. During the war in 1990, the church was unfortunately destroyed and was entirely rebuilt, replicating the previous structure as precisely as possible [4]. Not only the church but also the Parish House suffered under the war: the roof of the Parish House had entirely been destroyed, leaving only the two chimneys [5] (the used pages of the source [5] can be found in Appendix A). The images in Figure 2.1 [5] display the restoration and reconstruction in 1996-1997. The main measures were to rebuild the roof structure using a lightweight solution of timber beams and add a fert slab above the first storey including also a reinforced concrete (Reinforced concretes (RCs)) ringbeam around the perimeter of the facades (see Appendix A [5] and Appendix E [6]). Apart from that, no significant changes can be perceived from the pictures. It can be shown (see Figure 2.2) that the building was built with old, massive brick masonry, but the condition of the masonry substance is difficult to determine.

Figure 2.1: Documentation of the reconstruction of the roof [5] (see also Appendix A)

Figure 2.2: Evidence of sampling on the west facade (Image from Zvonko Sigmund)

After the reconstruction, the Parish House served for over 20 years until it was struck by a devastating earthquake. On December 29, 2020, a 6.4 magnitude earthquake struck Croatia's Sisak-Moslavina

County. The earthquake occurred at a depth of 10 km along the Popusko-Petrinja strike-slip fault within the Eurasian plate, having an epicentre at 45.422°N 16.255°E, three kilometres west-southwest of Petrinja. The maximum intensity of the earthquake was VII (severe) on the Modified Mercalli Intensity (MMI) scale and VIII (very damaging) to IX (destructive) on the European Macroseismic Scale (EMS) [4]. The statistical documentation can be found in Appendix B [7].

2.2 Geometry of the building

2.2.1 Facades

Looking at the Parish House from the outside, a bisymmetric facade geometry can be perceived: The west and east facades are almost identical and symmetric. Several small differences can be perceived in the presence of a main entrance door on the west facade and windows of about half the typical size on the south part of the west facade. Because of the main entrance, the central windows on the west facade are slightly further apart compared to the east facade as can be perceived in Figure 2.3. Since the terrain around the building is sloping, the east facade is slightly higher (from the ground) than the west facade and can therefore accommodate three cellar windows on the brown edge at the bottom.

Figure 2.3: The west and east facades of the Parish House

Looking at the south and north facades, the only differences between the two are an additional cellar window on the north facade and a door instead of the bottom west window on the south facade. This can be seen in Figure 2.4.

Figure 2.4: The south and north facades of the Parish House

The plans of the facade can be found in Appendix C. In summary, the facades are about 6 meters high without the bottom edge, and the edge adds up to one meter of height to the facade. The east and west facades are 17.7 meters wide and the north and south facades have a width of 11.9 meters.

The ground-floor windows are 1.4 meters high, whereas the windows on the first floor are 1.7 meters high. The small windows on the south side of the west facade are only 70 centimetres high. However, all the windows have the same width of 1.1 meters.

2.2.2 Ground floor

The inside of the building is divided into a cellar, which will not be studied in this thesis, a ground floor, a first floor and a second floor within the roof structure. The ground floor is covered by a vaulted system that supports the slab of the first floor. The spanning directions of the vaults are indicated in Figure 2.5, where the red arrows point in the direction of the primary curvature and the light-blue arrows point in the direction of the secondary curvature. In the rooms where the vault spans perpendicular to a window or door opening, a local vault allows to access the openings. This can be seen in turquoise in Figure 2.5. It can be noticed that the vaults are mostly barrel vaults (unidirectional) with a few crossed vaults (bidirectional). Figure 2.6 shows two examples: a big barrel vault in the east room spanning from east to west and a smaller crossed vault in the west part of the building spanning in both directions but with the main curvature in the east-west direction. As can be seen in the images the vaults with the big spans differ from the smaller vaults in and around the staircase. The bigger vaults have a rounder shape and start low on the sides to find the maximum height in the centre of the room. The smaller vaults on the other hand are less round and start higher with a smaller difference between their starting point and their maximum height in the centre.

In the plan (Figure 2.5, a big barrel vault can be seen in the east of the building, where the big middle room and the two smaller rooms next to it are all covered by one vault spanning from the staircase in the west to the facade in the east. A similar geometry can be perceived in the west where there is also a common vault in front of the stairs as well as above the two smaller rooms (toilet and storage space for the kitchen) in the south. Here, the vault is interrupted in the north-west room whose vault is perpendicular to it. The vaults in the kitchen (southwest room) and the room in the northwest span from north to south. Furthermore, the vaults continue above the staircase where they are sloping perpendicular to their primary curvature. The exact plan of the ground floor can be found in Appendix C.

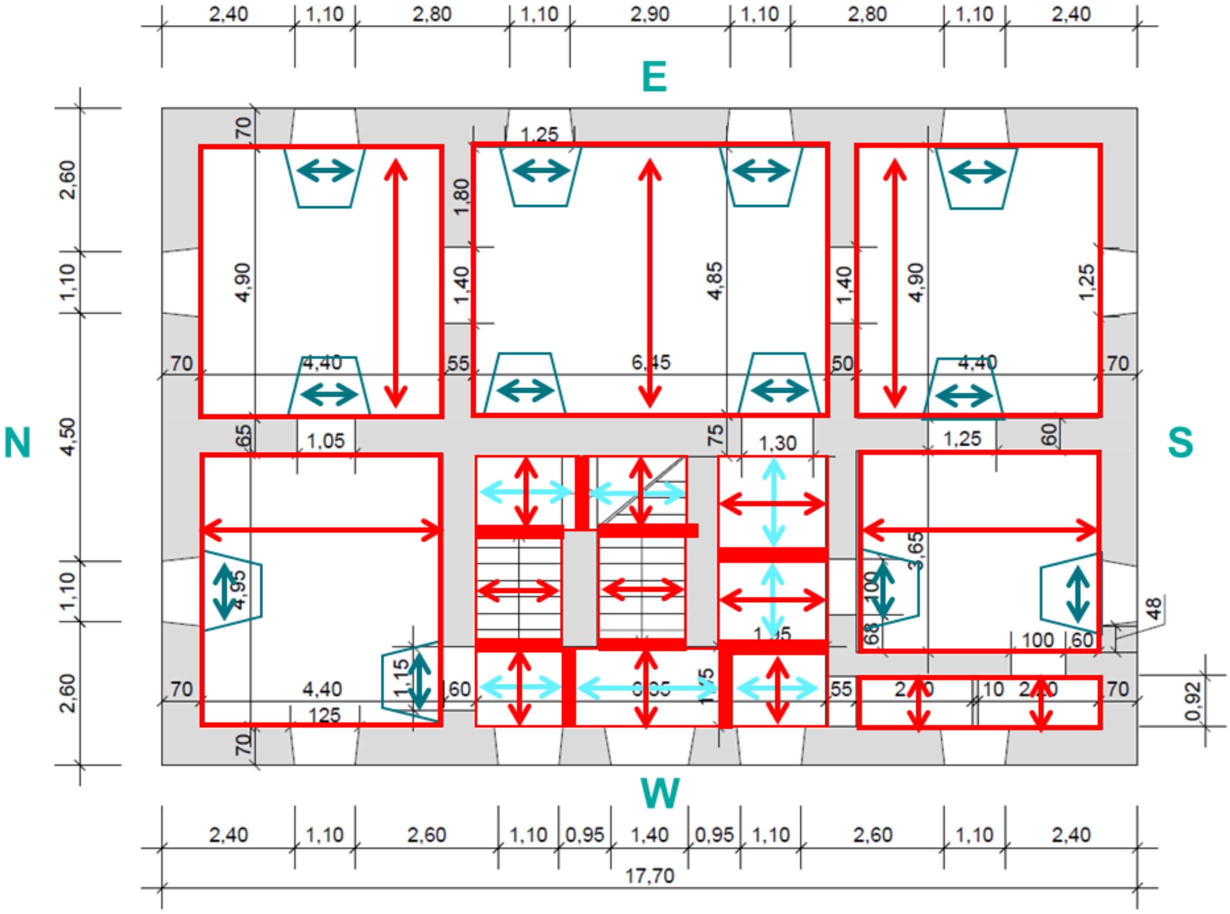


Figure 2.5: Vaulted system in ground floor

Figure 2.6: Vaults examples: East and west

During the interview with the leading engineer of the renovation and reinforcement works after the earthquake [6] (see also AppendixE), the engineer specified that the volume between the outside surface of the vaults, the walls and the lower side of the slab is filled with gravel material and then covered by timber planks. According to Hrvoje, this gravel material has a mass of 1.4 tons per m^3 [6].

2.2.3 First floor

The floor plan of the first floor strongly resembles the one on the ground floor. The measurements indicate however, that the walls on the first floor are slightly thinner than the walls on the ground floor, which is most likely due to the reduced vertical loads and the lack of horizontal thrust (from vaults) in comparison to the ground floor. Furthermore, there are two additional bathrooms on the northeast side between the two rooms, which cannot be found in the plan of the ground floor. At this point, the walls are not supported directly by walls beneath them, but by the vaulted system over the ground floor transmitting the load to the ground floor walls. A detailed plan can be found in Appendix C. These walls seem however to have been built in a later episode and were most likely not part of the original construction meaning that they are non-structural elements and do not need to be modelled as such.

During the interview of 17.11.2023 the leading engineer of the retrofitting project currently being executed [6] revealed that the slab above the first floor had been constructed with the FERT methodology after the damage of the war in the 20th century. A FERT slab is composed of reinforced concrete beams with hollow masonry elements spanning between the beams. The whole composition is covered by a concrete layer to connect the connect elements to each other [6](see also Figure 2.7). Furthermore, an RC ring beam going around the perimeter of the top of the walls at the level of the second floor was introduced during the remodelling in 1997 to connect the walls better and give the building a more box-like behaviour. Later in the interview, Hrvoje [6] also gave up the information that the new roof structure is supported on the outside walls of the building, as well as two columns on the second floor, leading the load down to the walls of the first floor (see also Figure E.2 in Appendix E).

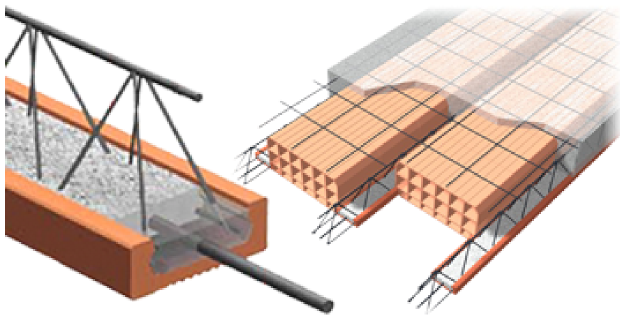


Figure 2.7: FERT slab construction method [8]

2.3 Earthquake damages

During the site visit in April 2023, all the visible damages and cracks on the walls and the vaults were documented, numbered and analyzed. The complete inventory can be found in Appendix D.1 and D.2. In this chapter, only the main observations are summarized.

2.3.1 Facade cracks

The most interesting and conclusive observations are made by looking at the crack patterns of the facades. They reveal diagonal cracks from shear failure, as well as horizontal cracks from out-of-plane (OOP) and flexural failure. In Figures 2.8, 2.9 and 2.10, documenting the cracks in the following chapters, the arrows represent the direction of the seismic force and the cracks of the same colour the corresponding failures.

North and South facades

The cracks on the North facade correspond mainly to shear failures in the spandrels as well as in the piers of the facade (see Figure 2.8). As expected, the diagonal cracks are steeper on the lower part of the facade, corresponding to a failure in shear, and flatter towards the roof, coming from a failure in flexure. Several horizontal cracks at the edges of the facade cannot exactly be identified as a flexural failure, but they may come from a stress concentration or a global OOP mechanism of the West facade.

On the South facade, the crack pattern is very similar to the one on the North facade. A difference can be noted in the cracks in the piers at the outer edges of the facade, where big shear cracks can be found that are not present on the North facade.

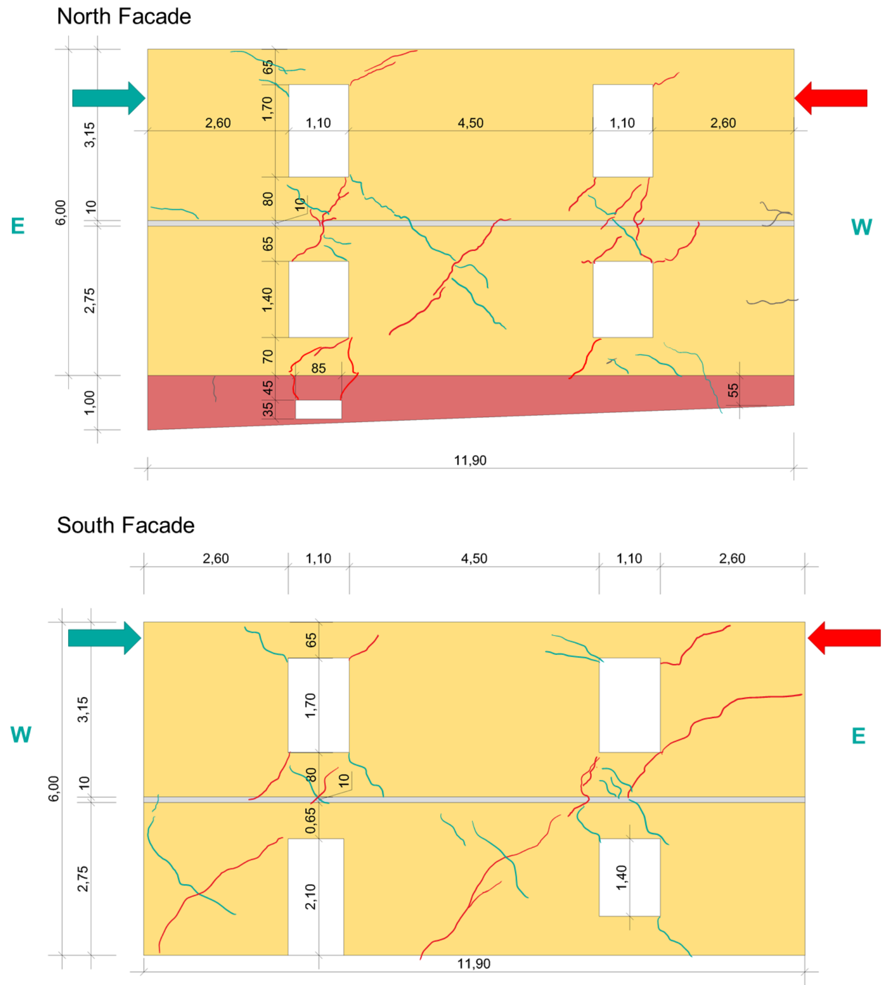


Figure 2.8: Cracking on North and South facade

East and West facades

The East facade shows fewer cracks than the North or South facades. This was to be expected since the seismic load is taken by a bigger wall that has an overall bigger area to resist the stress (see Figure 2.9). The shear cracks observed on this facade are almost exclusively in the spandrels and no shear cracking can be observed in the piers. The flat cracks in the top south and north corners could also come from a detaching of the roof as a rigid block, lifting off and rocking on the building during the earthquake motion. The cracks in grey on the South side and the centre of the facade, however, can be associated with an OOP failure of the East facade described more in detail in chapter 2.4.

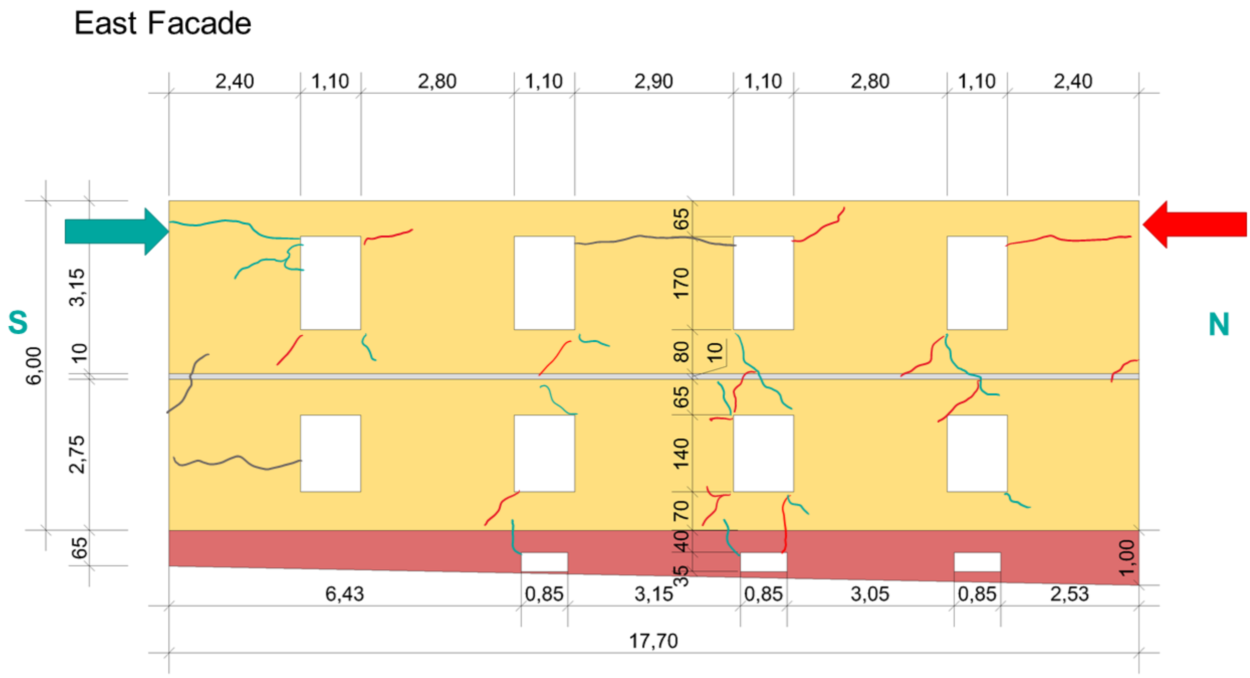


Figure 2.9: Cracking on east facade

On the West facade, there are even fewer shear cracks than on the East facade. However, the most noticeable damage consists of long horizontal cracks above the windows on the ground floor and the windows on the first floor. These are associated with the OOP mechanism of the West facade due to the combination of seismic force and horizontal thrust of the vault (see chapter 2.4).

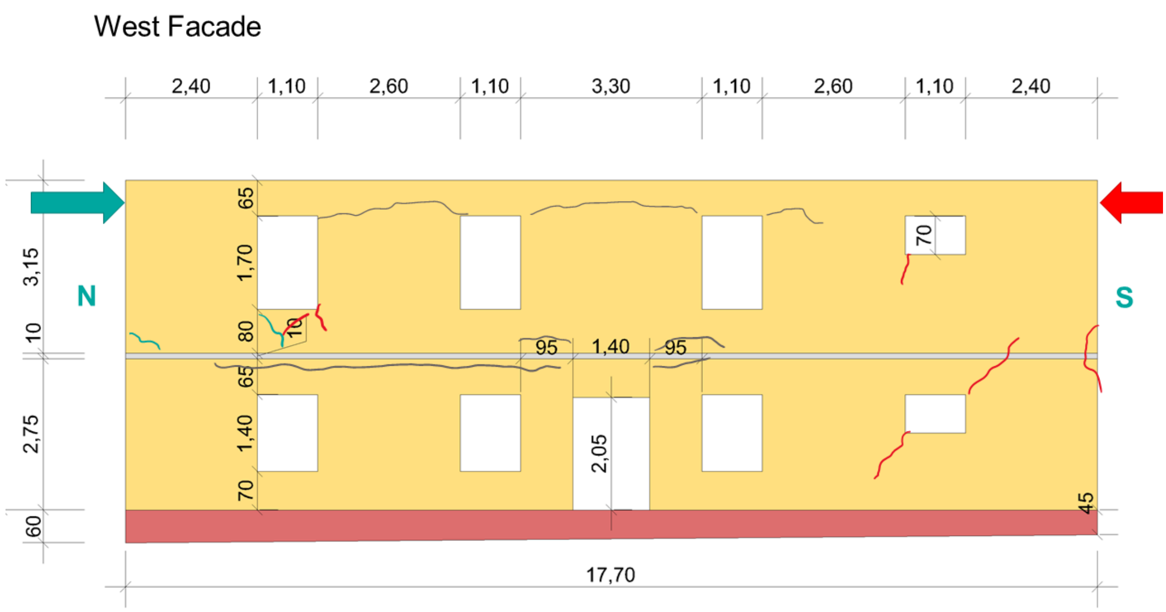


Figure 2.10: Cracking on west facade

2.3.2 Interior cracks

The cracks visible from the inside of the building have been documented in a similar way as the ones on the facades. Note that they have been documented on a plan view of the building and in addition, photos have been added to the documentation to see the vertical geometry of the cracks (See also

Appendix D.1). An example of the documentation can be seen in the figure below showing the cracks of the ground floor.

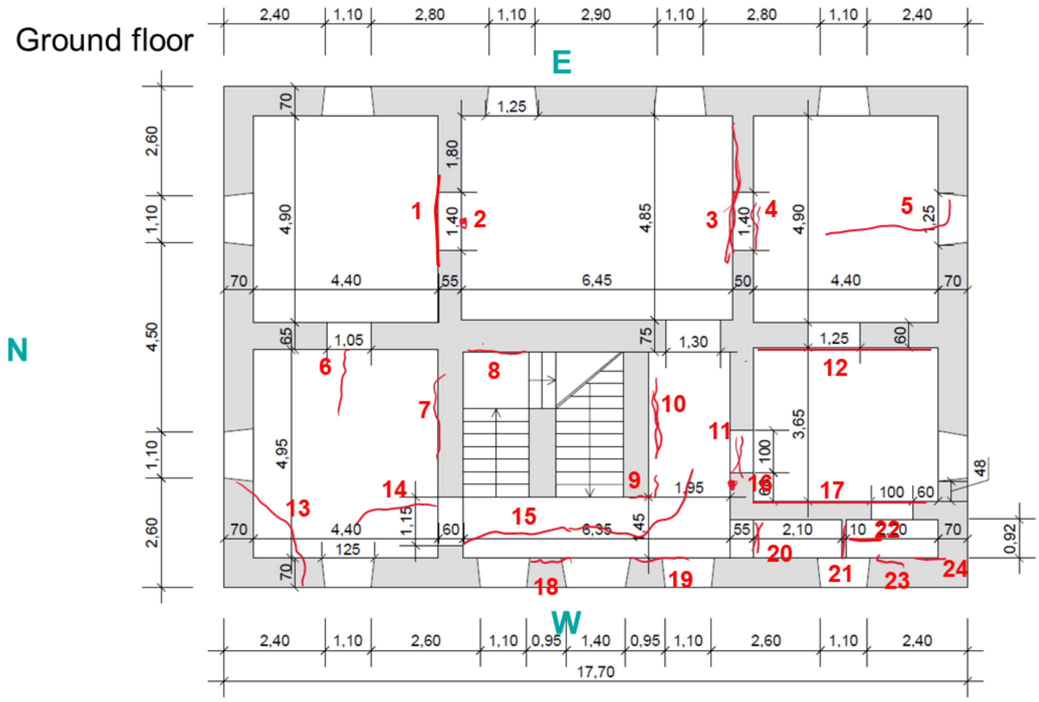


Figure 2.11: Cracking on ground floor

2.4 Observed out-of-plane-mechanisms

When looking at the cracking pattern discussed in chapter 2.3 and documented in Appendix D, two main global mechanisms can be observed. Compared to what D'Ayala proposes, this would correspond to a variant of the F mechanism [9]. The first one concerns the west facade, which is more obvious and the second one concerns the east facade and is less visible. The overall behaviour, however, is the same for both and can be described as follows:

The mechanism consists of five elements moving with respect to each other (see Figure 2.12). A vault that spans perpendicularly between an outside wall/facade and an inside wall that is parallel to the facade. In a first episode, the seismic action moves the facade slightly out of plane. The horizontal thrust that the vault exerts on the facade then helps push the latter out even further. For the facade to be pushed out of its plane, it needs to form three hinges, one on the top where it is restrained by a rigid slab and a ring beam, a second one at its foundation and a third one where it breaks in the middle (assumed to be where the thrust from the vault is applied). The inside structure, on the other hand, also needs to form hinges to be compatible with this global mechanism. Therefore the vault cracks at mid-span as well as at the support on the inside wall. The global mechanism is visible in the following figure:

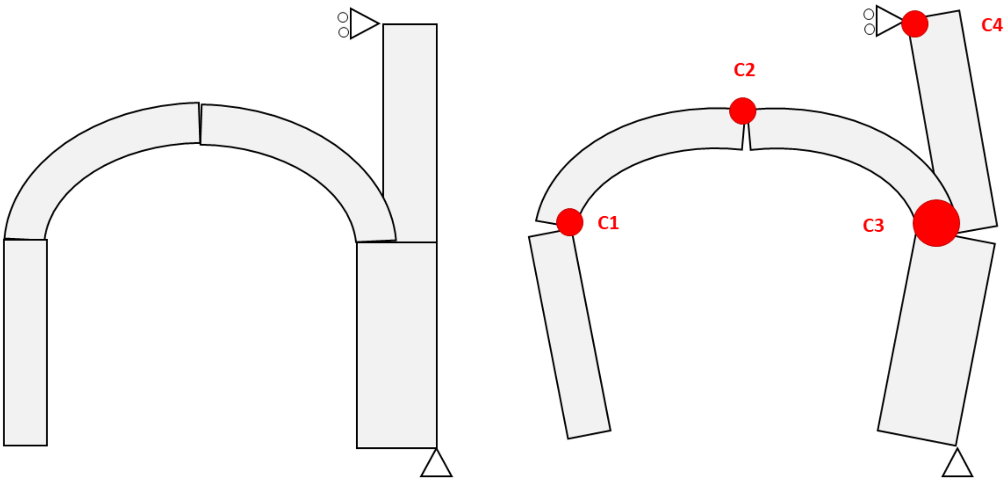


Figure 2.12: Schematic OOP mechanism

2.4.1 West facade

First, the west side of the structure is observed. The concerned area of this mechanism can be seen in Figure 2.13. Several cracks on the facade of the building (see Figure 2.14) can be noticed that indicate the presence of the hinges C3 and C4: Hinge C3 is not very visible but can be assumed to correspond to cracks 3 and 4 of the west facade. Cracks 8, 9 and 10 on the other hand, indicate the presence of hinge C4. Figure 2.15 shows the relevant ground-floor cracks to the mechanism: The cracks number 9 and 17 on the ground floor indicate the presence of hinge C1, whereas cracks 14, 15 and 22 on the ground floor correspond to hinge C2. (All cracks can be also found in Appendix D.1.)

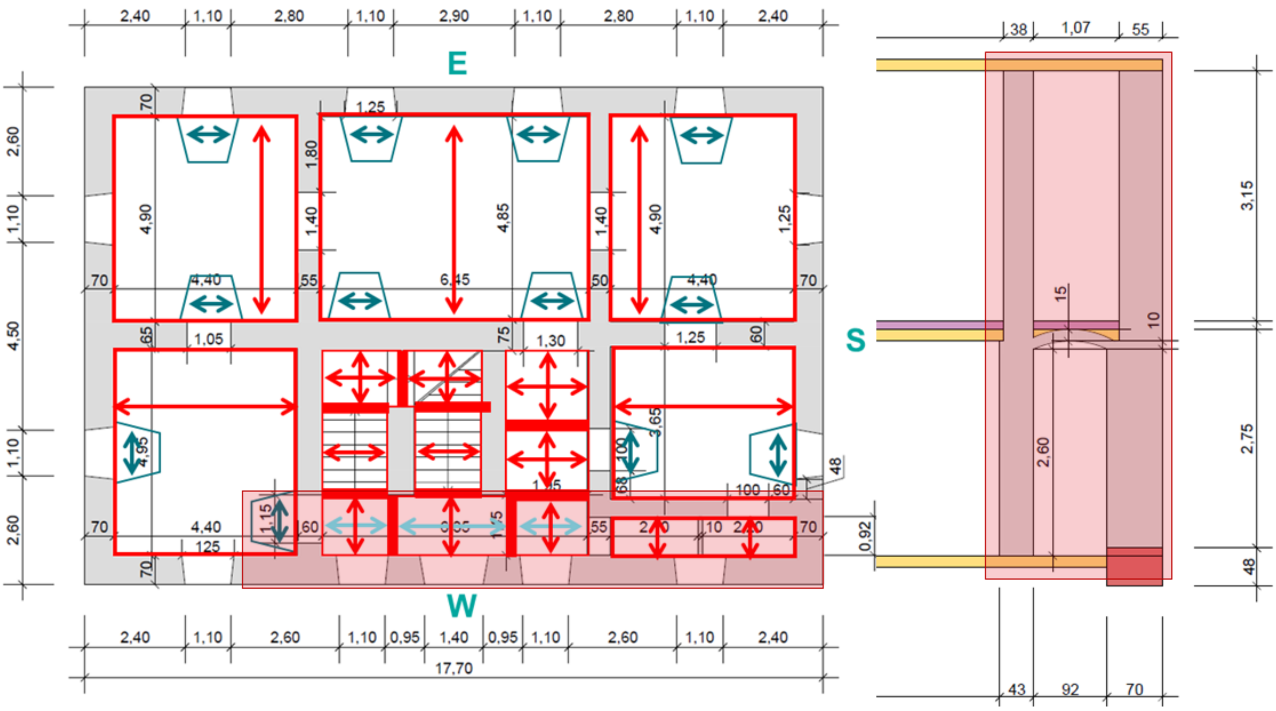


Figure 2.13: OOP mechanism west facade: Plan view and cross-section of the concerned zone

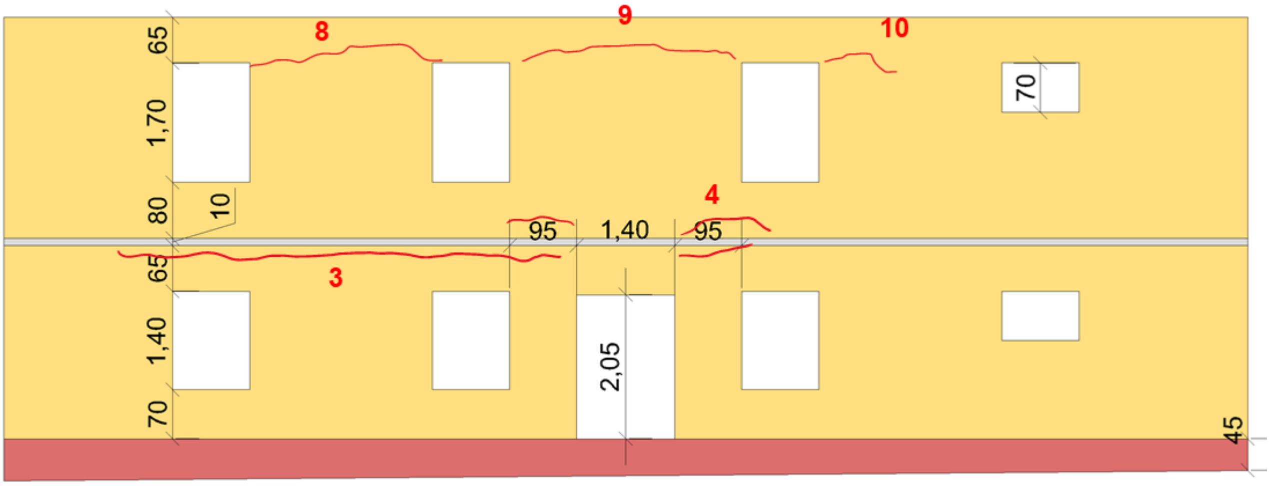


Figure 2.14: OOP mechanism west facade: Relevant cracks that indicate the mechanism, west facade

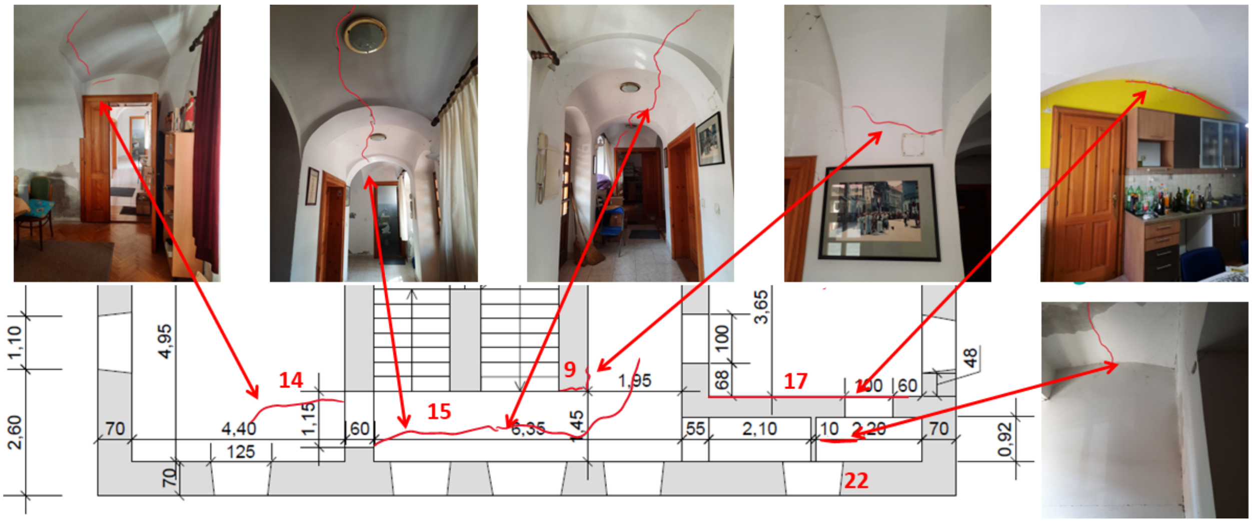


Figure 2.15: OOP mechanism west facade: Relevant cracks that indicate the mechanism, ground-floor

2.4.2 East facade

The second mechanism, on the east facade, is less obvious, and some of the cracks may be too small to see with the naked eye. Nevertheless, some cracks indicate the presence of an F mechanism, proposed by [10]. The concerned part of the building for the east mechanism can be seen in Figure 2.16. First, crack 12 of the ground floor can be the representation of the first hinge (C1) (see Figure 2.17). Despite the fact that crack 5 does not extend across the vault, cracks 2 and 4 indicate that the walls beneath the vault are being torn apart, which could be an indication of the C2 hinge. The C3 hinge is visible in crack 7 of the east facade (Figre 2.18), whereas cracks 1-5 on the east facade indicate the presence of hinge C4. It can be noticed that the mechanism is more pronounced on the south side of the wall (crack 5 ground floor and crack 7 east facade). This could be due to the minimal eccentricity of the stabilizing staircase that is slightly more in the north than the south part of the building, creating a rotational force of the building around its axis. This also makes the south side of the building more flexible, allowing for more deformation and cracking on that side.

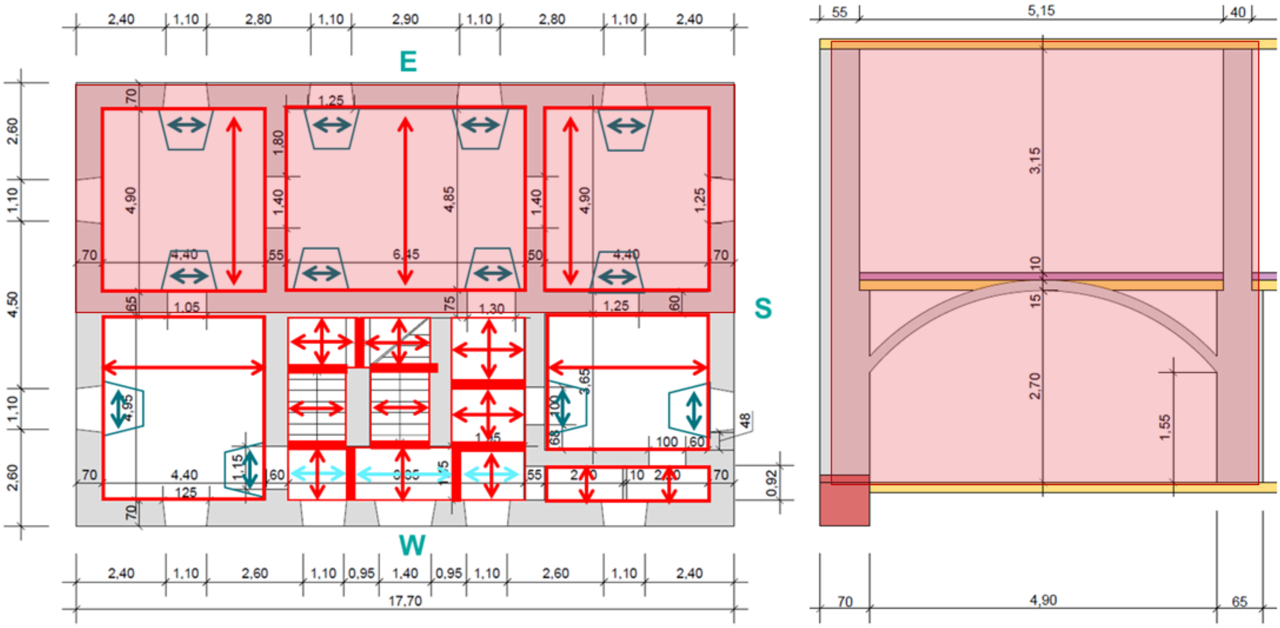


Figure 2.16: OOP mechanism east facade

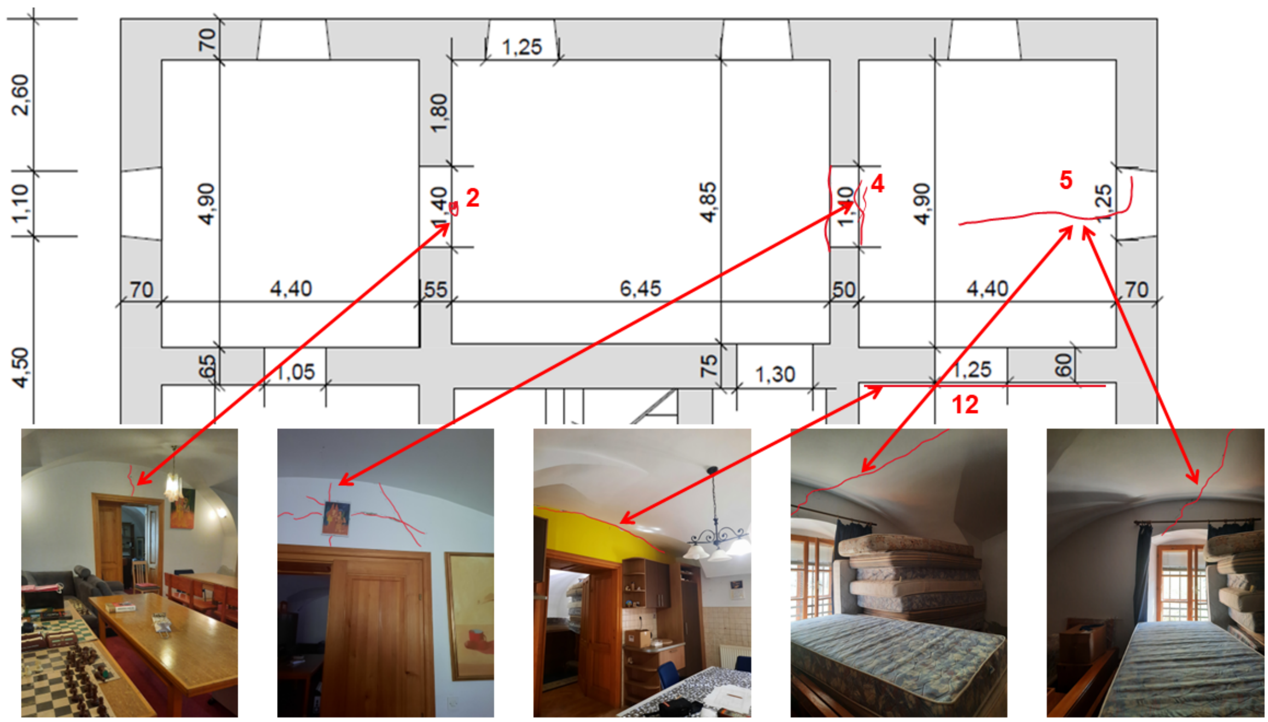


Figure 2.17: OOP mechanism east facade: Relevant cracks that indicate the mechanism, ground-floor

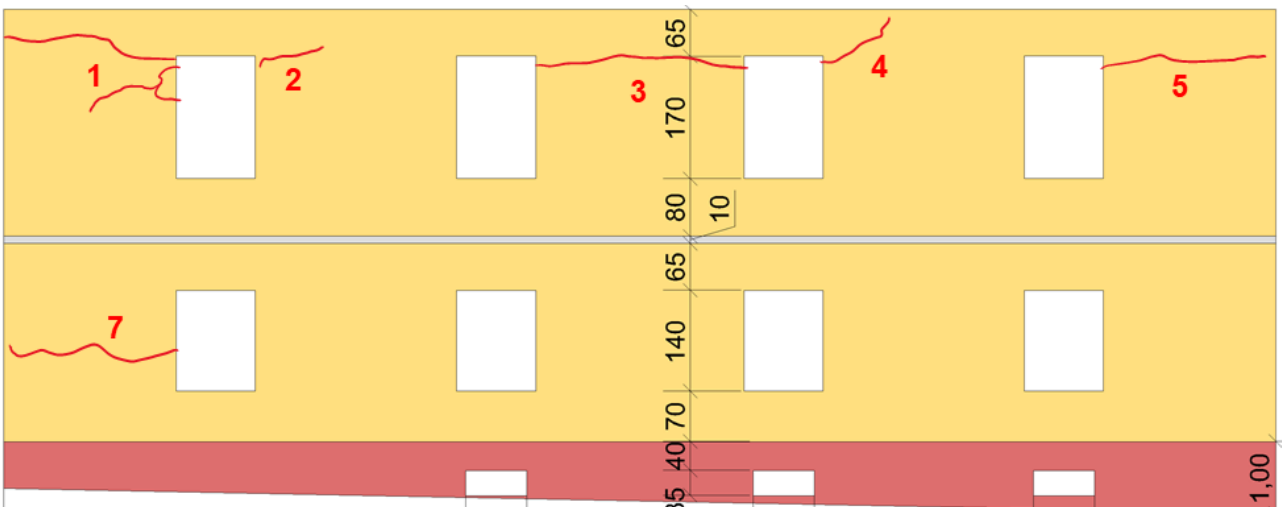


Figure 2.18: OOP mechanism east facade: Relevant cracks that indicate the mechanism, east facade

3. Methodology

This chapter aims to highlight the design and verification bases and the methodology of the analyses to be performed on the Parish House of Petrinja. It has been decided to apply the newest version of the eurocodes and to follow the methodology proposed by Eurocode 1998 as much as possible within the scope of this project [11][12].

3.1 Eurocodes

The codes and regulations used for the analysis of the Parish House are listed below. The relevant code used is the Eurocode. At the moment the current version (first generation) is under revision. However, due to the implication of my supervisor and mentor Prof. K. Beyer in the editorial on the new generation of Eurocode 6 and 8, the drafted version of the second generation of EC 1996 as well as 1998 parts 1-1 and 3 will be used in this thesis. The relevant codes are the following:

- EN 1990 Structural safety, serviceability and durability (Version 2002) [13]
- EN 1991 Actions on structures, part 1-1 Densities, self-weight, imposed loads for buildings (Version 2009) [14]
- EN 1992 Design of concrete structures - Part 1-1: General rules and rules for buildings (Version 2011) [15]
- prEN 1996 Design of masonry structures, part 1-1 General rules for reinforced and unreinforced masonry structures (Draft 2021) [16]
- prEN 1998 Design of structures for earthquake resistance, part 1-1: General rules and seismic action (Draft 2022) [11]
- EN 1998 Design of structures for earthquake resistance, part 3 Assessment and retrofitting of buildings and bridges (Draft 2023) [12]

3.2 Software

The software used in the elaboration of this thesis is the following:

- **AutoCAD 2023/2024**: For the production of the plans and the mesh drawings in Chapters
- **Jupyter Notebooks**: For the elaboration of pushover curves for different wall geometries in the Chapter 5 [17]
- **Microsoft Excel**: For the calculation of model properties [18]
- **MATLAB**: To develop scripts for pre- and postprocessing of the analysis data and to run the OpenSees model [19]
- **OpenSees**: The finite element software used for the dynamic analysis of the equivalent frame model [20]

3.3 Analysis procedure

Since the Parish House is an existing structure, the rules of EN 1998-3 "Assessment and retrofitting of buildings and bridges" will be applied for the analysis of the resistance of the building [12]. In masonry buildings, one of the most common reasons for the failure is the low resistance of the masonry walls against acceleration or force in the direction perpendicular to their plane, i.e. out-of-plane (OOPs). Therefore, it is not only important to analyse and verify the global behaviour of the structure but also to evaluate localized failure mechanisms, herein called OOP mechanisms.

3.3.1 Global analysis

For the global analysis of the Parish House, the choice is to model the structure with an equivalent frame model (EFMs) and to perform a dynamic response history analysis (Response history analysiss (RHAs)) as proposed in paragraph "§11.5.1.6 Verification through non-linear response-history analysis" EN 1998-3 [12]. Although Eurocode 1998-1-1 requires to use a minimum of seven different ground motions (§6.6 Response-history analysis[11]), it is decided to only use one to remain within a reasonable scope for this thesis. The used ground motion is described in Chapter 4.2.1. The response of the building, i.e. the "design value of action effects", due to the action of the ground acceleration corresponds to the maximum displacement of the top nodes. This effect will be amplified by a factor of $\gamma_{Sd} = 1.15$ according to statement (5) of"§4.2.2 Verification rules"[12]. To verify the structure, the design value of the action effects should not exceed the resistance limit. In the case of non-linear response history analysis, the resistance corresponds to the displacement of the top node at which the limit state of one element of the structure is reached in a non-linear pushover curve (see EN 1998-1-1 §6.5 [11]). Since the model in OpenSees, used for the dynamic RHAs would need a lot of calibration to achieve accurate non-linear static analysis results, this part will be done using a simplified method proposed in [21]. The detailed methodology can be found in Chapter 7.1.

3.3.2 Out-of-plane analysis

For the evaluation of the stability of specific walls to their resistance against local or global OOPs mechanisms, chapter "§6.5 Non-linear static analysis" [11] will be applied. The method described in §6.5.4 [11], also known as the N2-method is used to evaluate the OOPs resistance of the walls using a pushover curve that is transformed into an equivalent single degree of freedom system (Single-degree-of-freedoms (SDOFs) system). When applying a non-linear kinematic analysis as described in "§11.3.3.2 Non-linear kinematic analysis (displacement capacity of the mechanism)" of EN 1998-3 [12], the displacement capacity according to the statement (2) d) is $d_u = 40\%\ d_0$ and $d_{u2} = 60\%\ d_0$. The d_0 marks the displacement where the seismic multiplier α reaches zero, meaning there is no lateral resistance. For the verification of the mechanisms d_u serves as the limit for the Damage-limitations (DLs) limit state and d_{u2} is the limit for the Near-collapses (NCs) limit state.

3.3.3 Partial factors

To account for uncertainties in the modelling, the material and the resistance parameters of the verification it is of important to consider safety factors on the characteristic values.

Model based uncertainty

Eurocode 1998-3 proposes a specific approach to this based on the "Knowledge levels" explained in "§5 Information for structural assessment" [12]. Since the knowledge of the structure about geometry, construction details, as well as material properties is fairly small and no further investigations are made within this project, the knowledge level is set at 1 for all the categories. The safety factor on deformation capacities is determined according to Table 11.7 [12] and is assumed to be $\gamma_{Rd} = 1.85$.

For the shear resistance of the members, the partial factor depends on the mode of resistance. For the resistance in flexure, the partial factor can be taken as $\gamma_{Rd} = 2.15$ (of Table 11.4 EN 1998-3 [12]). For the resistance of the member against shear sliding the partial factor is set as $\gamma_{Rd} = 1.65$ (Table 11.5 EN 1998-3 [12]) and for the resistance of the member against diagonal shear cracking the factor is set as $\gamma_{Rd} = 1.55$ (Table 11.6 EN 1998-3 [12]). Note that those are the values for piers, the ones for spandrels are slightly different. However, since those resistances only play a role in the elaboration of the pushover curve in Chapter 7.1, where only the resistance of the masonry piers is considered, the partial factors for spandrels are not of importance.

For the verification of the OOPs resistance only partial factors are applied on the deformation capacity as described in Chapter 3.3.2

Material uncertainties

Only the material properties of the masonry are of relevance for the verification of the structure. The partial factor is determined as in Table 4.1 1996-1 [16] and is $\gamma_M = 2.5$

Load uncertainties

Load and mass uncertainties are accounted for as prescribed in Eurocode 1990 [13]. More explanation can be found in Chapter 4.2.

4. Characteristics of the structure

4.1 Material properties

During the site visit in April 2023, a lot of information about the building could be retrieved such as general dimensions and some construction history about the Parish House (see also chapter 2.1). Nevertheless, there are still a lot of insecurities about other parameters such as the material parameters of the masonry and the mortar or the concrete of the FERT slab. Therefore, it is important to make realistic assumptions about the material to model the structure as close to reality as possible and receive reasonable results. By clause (1) "§5.5 Representative values of material properties", EN1998-3 [12] the following rule applies:

"For existing materials, design values of material properties Xd for calculating resistances to be used in local verifications, should be taken as the mean. Mean values should be obtained from testing and additional sources of information, and different mean values may be considered in different areas of the structure, as appropriate, based on test results"

Since no testing is possible, the mean value of reference values from the literature can be taken. The resistance values of the masonry will need to be divided by the partial factor γ_M for material properties (as per EN 1996-1 Table 4.1 [16])

4.1.1 Masonry properties

From photos from drone recordings provided by Zvonko Sigmund, in the zones of sampling, the bricks seem to be a reddish to brown colour (see Figure 2.2). Otherwise, there is unfortunately not more information.
The Italian code [22] suggests the following values:

Table 1. Reference values of the mechanical parameters and average specific weights for selected types of masonry (extract from Table C8A.2.2. of Circ. NTC08, 2009).

Masonry typology	f_m (N/mm^2)	τ_o (N/mm^2)	E (N/mm^2)	G (N/mm^2)	W (kN/m^3)
	min-max	min-max	min-max	min-max	
Irregular stone masonry (pebbles, erratic, irregular stone)	1.0 1.8	0.020 0.032	690 1050	230 350	19
Uncut stone masonry with facing walls of limited thickness and infill core	2.0 3.0	0.035 0.051	1020 1440	340 480	20
Cut stone with good bonding	2.6 3.8	0.056 0.074	1500 1980	500 660	21
Soft stone masonry (tuff, limestone, etc.)	1.4 2.4	0.028 0.042	900 1260	300 420	16
Dressed rectangular (ashlar) stone masonry	6.0 8.0	0.090 0.120	2400 3200	780 940	22
Solid brick masonry with lime mortar	2.4 4.0	0.060 0.090	1200 1800	400 600	18

Figure 4.1: Masonry properties as suggested by the Italian code [22][23]

Therefore, it is reasonable to take the average of the suggested properties for solid brick masonry with lime mortar, which is supposedly the material used in the Parish House. The following characteristic properties are assumed:

- Compressive strength of the masonry: $f_M = 3.2\ N/mm^2$

- Young's modulus: $E = 1500\ N/mm^2$
- Shear modulus: $G = 500\ N/mm^2$

The friction coefficient as well as the Poisson's ratio can be taken as suggested by Lourenço et. al. [24]: As initial estimation the cohesion is estimated to $c = 0.18\,MPa$ and the coefficient of friction is estimated to $\mu = 0.65$. To estimate the density of the masonry the Annex of the SIA 261 (Swiss code) can be used as it gives a good estimations of material densities [25]. The suggested volumetric load is of $\rho_M = 18\,kN/m^3$ ("Maçonnerie montée sans crépi, briques de terre cuite pleines"). Applying the partial factor $\gamma_M = 2.5$ on the resistance values, the following properties are retained:

- Compressive strength of the masonry: $f_{M,d} = f_M/\gamma_m = 3.2/2.5 = 1.28\ N/mm^2$
- Young's modulus: $E = 1500\ N/mm^2$
- Shear modulus: $G = 500\ N/mm^2$
- Cohesion: $c_d = c/\gamma_M = 0.18/2.5 = 0.072\ N/mm^2$
- Coefficient of friction: $\mu = 0.65$
- Volumetric load: $\rho_M = 18\ kN/m^3$

The initial shear strength of the masonry can be determined using table 5.4 of EN 1996-1-1 [16]: since the compressive strength of the mortar is assumed to of about $f_m = 1.7\ N/mm^2$, the initial shear strength is about $f_{vk0} = 0.1\ N/mm^2$. The mean value can be derived using Appendix K 3.2.2 of EN 1996-1-1 [16]. $f_{vm0} = 1.3 * f_{vk0} = 0.13\ N/mm^2$. These values will be of interest in the calculation of the push-over curve in Chapter 7.1.

Drift model

The model used with OpenSees [20] requires the introduction of a drift model to describe the in-plane behaviour of the masonry piers and spandrels. Therefore, the model proposed by Eurocode 1998-3 "§11 Specific rules for masonry buildings" [12] is used. Figure 4.2 below shows the general shape of the force-drift curve.

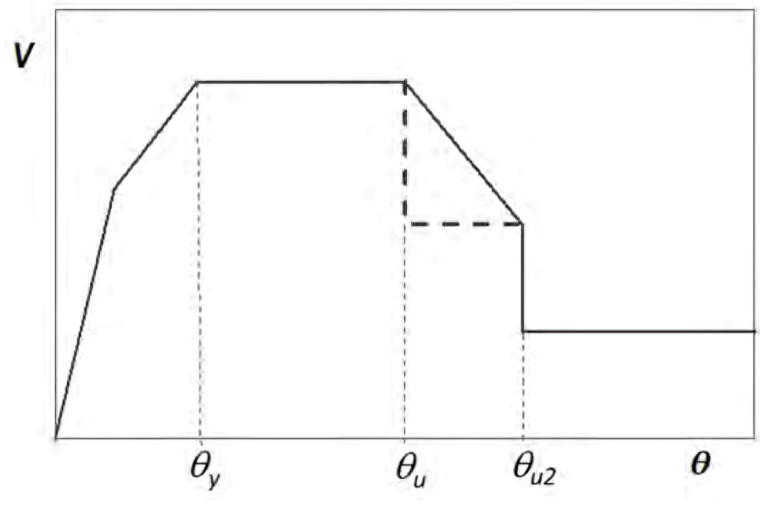


Figure 4.2: Force-deformation relationship for masonry members [12]

The curve is characterized by the following values:

- θ_y is the drift at the moment the masonry member starts yielding
- θ_u is the ultimate member drift starting from which the shear resistance starts to drop.

- θ_{u2} is the member drift at which the shear resistance has dropped by 20%.
- V_d is the maximum shear resistance of the member.

These values depend on the mode of failure of the member. If the member fails in flexure the values are:

- $\theta_{f,ud} = \theta_{f,u}/\gamma_{Rd} = 0.012/1.85 = 0.0065$
- $\theta_{f,u2d} = \theta_{f,ud} * 4/3 = 0.0086$

For shear sliding failure the values are:

- $\theta_{s,ud} = \theta_{s,u}/\gamma_{Rd} = 0.008/1.85 = 0.0043$
- $\theta_{s,u2d} = \theta_{s,ud} * 4/3 = 0.0058$

And for diagonal shear failure, the values are:

- $\theta_{d,ud} = \theta_{d,u}/\gamma_{Rd} = 0.006/1.85 = 0.0032$
- $\theta_{d,u2d} = \theta_{d,ud} * 4/3 = 0.0043$

4.1.2 Timber properties

The new roof, built in the late 20th century is probably made out of coniferous wood, a lighter kind of wood with a weight of $\gamma_{timber,mod.} = 5\,kN/m^3$. The other material characteristics of the roof structure are not of importance because the roof is considered to behave as a rigid block due to the concrete slab that was added during the reconstruction in the late 20th century (see also chapter 2.1). The planks constituting the surface of the first-floor slab (above the vault) are assumed to be made out of local timber essences. According to Lourenço et. al. [24] the weight of the material can be assumed to be $\gamma_{timber,hist.} = 8.1\;kN/m^3$. The other material properties of timber are not of importance since the timber elements do not act as structural members in the case of the Parish House.

4.1.3 Concrete properties

During the renovation and reconstruction works in 1996-1997 the roof, as well as the slab above the first floor, were rebuilt (see chapter 2.1). The reconstruction measures also contained the addition of a FERT slab above the first floor to replace the existing one, as well as an RC ring beam (see Appendix E [6]). Other than that, no information about the construction materials of the roof is available. Therefore, the material properties are roughly estimated using the given values in EC 1992 [15]:

- Young's modulus for concrete: $E = 30\,GPa$
- Poisson's ratio: $\nu = 0.2$
- Shear modulus: $G = E/(2(1+\nu)) = 12.5\,GPa$

The engineer explained that in practice the FERT slab can be modelled as a concrete slab of 12 cm thickness, which will also be done in this case.

4.2 Loads and loadcases

Since this analysis is done according to Eurocode standards, the load cases corresponding to a seismic case should correspond to the ones provided in Eurocode 0 [14] and Eurocode 1 [13]. According to EN 1990, the general format of the effects of actions should be:

$E_d = E\left\{G_{k,j}; P; A_{Ed}; \psi_{2,i}Q_{k,i}\right\} \quad j \geq 1; i \geq 1$

Where:

- $G_{k,j}$: Characteristic value of permanent action j, i.e. the self-weight
- P: Relevant representative value of a prestressing action (not present in this case).
- A_{Ed}: Design value of seismic action
- $Q_{k,i}$: Characteristic value of the accompanying variable action I, such as live loads in the building or snow. $\psi_{2,i}$: Factor for quasi-permanent value of a variable action i

The actions that can be considered on the structure are:

- Self-weight
- Live load (from people living in the building)
- Load on the roof for maintenance
- Snow load
- Wind action

Keep in mind, that only the seismic case will be analyzed in this thesis, the ultimate limit state for non-seismic load cases and the serviceability are admitted to be verified.

4.2.1 Seismic load

To verify the structure against the combined action of the gravity loads as well as the actions of a seismic event, a dynamic analysis will be performed using the software OpenSees. Since there were no measurement stations nearby Petrinja that had recorded the earthquake in December 2020 a different approach had to be chosen.
The first step is to extract the reference peak ground acceleration (PGA) of Petrinja. The Department of Geophysics at the University of Zagreb offers a website, where the PGA corresponding to different return periods can be looked up [26] (see screenshot of the website in Appendix F. For a return period of 95 years (Damage limitation case) an acceleration of $a_{gR} = 0.074\ g$ and for a return period of 475 years (Non-Collapse case) an acceleration $a_{gR} = 0.152\ g$ is given. The next step is to determine the soil class of Petrinja. The geological map of the region Sisak, to which belongs Petrinja, shows that Petrinka lies within the region of category "a1" [27] (see map in Appendix G). This corresponds to a "Sedimentiterasa: siltovi, pijesci, sijunci" (Croatian) which means it is a soil made of silt and clay that indicates a soil category E according to Eurocode 1998-1-1 [11].

Finally, the spectra for the Near-Collapse (NCs) state and Damage Limitation (DLs) state as mentioned in Eurocode 1998-3 can be calculated. If they are plotted against the record of the Montenegro earthquake in 1979, measured at the Albatros station in the south of Montenegro (Latitude: 41.919° and Longitude: 19.221°) [28], factorized by 1.0 and 0.5 respectively, it can be seen that the resemblance of the spectra is high. The earthquake was one of the most damaging ones of the 20th century with its moment magnitude of 6.9 measured during the man shock on 15th of April 1979. The ground acceleration record can be seen in Figure 4.5 below for the east-west, as well as for the north-south excitation of the earthquake. The resemblance of the spectrum to the Eurocode spectrum visible in Figures 4.3 and 4.4 make it perfectly fitting to use as the ground motion for the response history analysis.

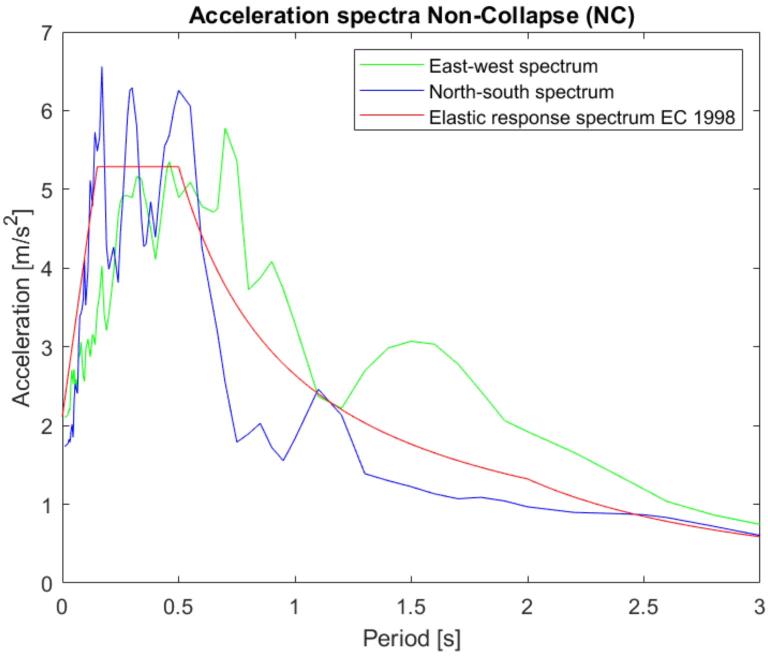


Figure 4.3: Near-Collapse spectrum against spectra of the Montenegro 1997 earthquake

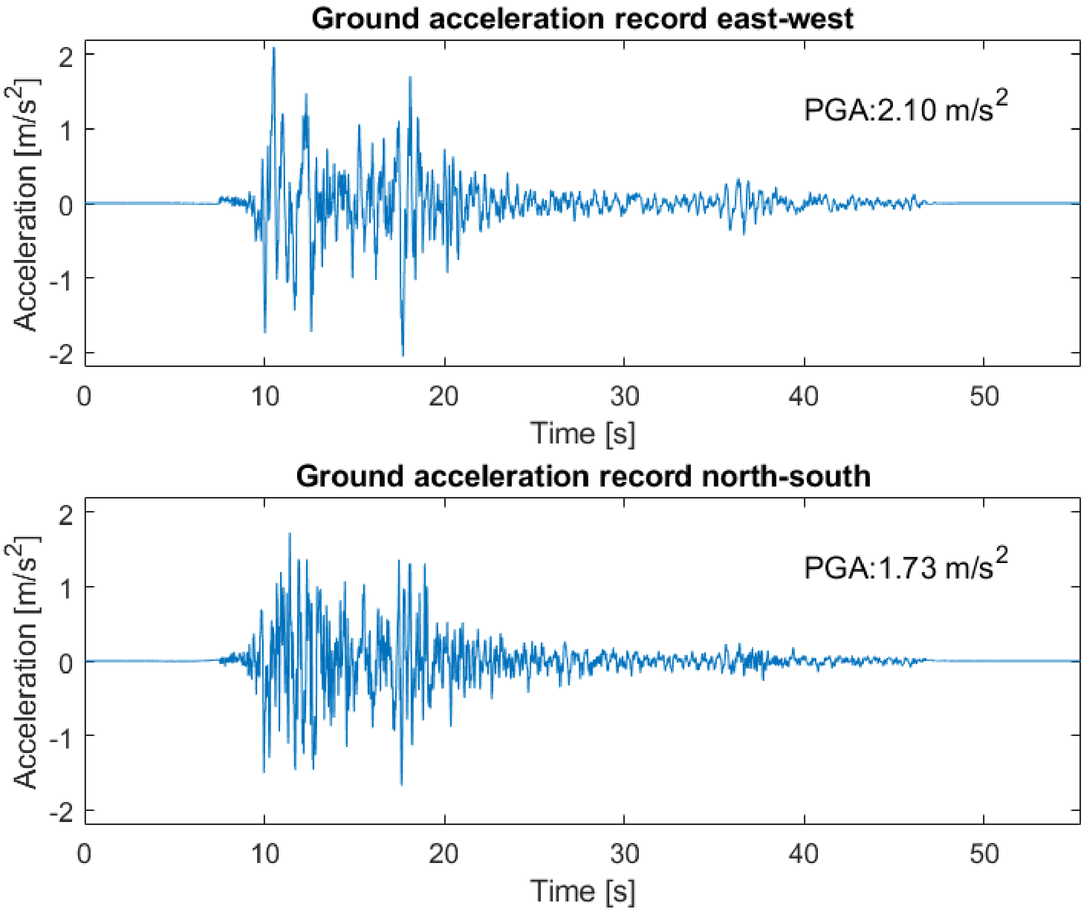


Figure 4.5: Ground acceleration records of the Montenegro earthquake 1979, Albatros station [28]

4.2.2 Self-weight

The self-weight comes from the construction materials, as well as the non-structural parts of the building such as roofing, tiles etc. The materials considered for all further calculations are the following:

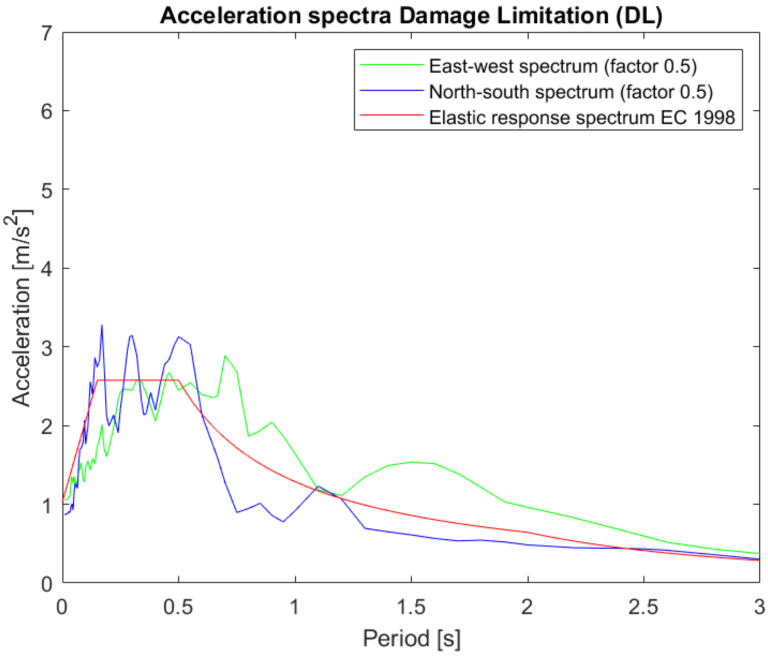


Figure 4.4: Damage Limitation spectrum against spectra of the Montenegro 1997 earthquake

- Masonry for construction: $\gamma_{masonry} = 18\ kN/m^3$ [25]
- Modern timber for construction: $\gamma_{timber,mod.} = 5\ kN/m^3$ [25]
- Historical timber for construction: $\gamma_{timber,hist.} = 8.1\ kN/m^3$ [24]
- Timber as covering material of the floors: $\gamma_{timber,covering} = 8\ kN/m^3$[29]
- Plasterboards: $\gamma_{plaster} = 12\ kN/m^3$ [25]
- Concrete: $\gamma_{concrete} = 25\ kN/m^3$ [25]

The following surface loads or densities can be considered:

- Roof:
  - Roof tiles and lath (proposed by the Swiss code SIA 261 Annex A [25])
    $g_k = 0.75 kN/m^2$
  - Roof isolation (since it is a new roof, see for example Swisspor [30])
    $g_k = 0.25\ kN/m^3 \times 0.2\ m = 0.05\ kN/m^2$
  - Timber members of the roof structure (a mean thickness of $0.2\ m$ is admitted)
    $g_k = 5\ kN/m^3 \times 0.2\ = 1\ kN/m^2$
  - Plasterboards for finishing the roof (bottom visible in rooms, proposed by the Swiss code SIA 261 Annex A [25])
    $g_k = 0.01\ m \times 12\ kN/m^3 = 0.12\ kN/m^2$
- Floor layers of the floor above the first level:
  - Parquet flooring (a mean thickness of $0.015\ m$ is admitted)
    $g_k = 0.8\ kN/m^3 \times 15\ mm = 0.1\ kN/m^2$
  - Fert slab (can be assumed to be equivalent to a concrete slab with a thickness of $0.10\ m$ [6] in Appendix E)
    $g_k = 25\ kN/m^3 \times 0.15\ m$
- Floor layers of the floor above the ground floor:

- Parquet flooring (a mean thickness of $0.015\,m$ is admitted)
  $g_k = 8.1\,kN/m^3 \times 0.015\,m = 0.1\,kN/m^2$
- Timber planks (a mean thickness of $0.03\,m$ is admitted, see [6] in Appendix E)
  $g_k = 8.1\,kN/m^3 \times 0.1\ = 0.8\,kN/m^2$
- Filling material above vault (14 tons per m^3, see [6] in Appendix E)
  $g_k = 14'000\,kg/m^3 = 1.4\,kN/m^3$ (volume directly calculated in Excel file)
- Masonry from vaults (see masonry of the walls, thickness of vaults is admitted to $0.15\,m$)
  $g_k = 18\,kN/m^3 \times 0.15\,m = 2.7\,kN/m^2$

• Masonry of the walls $g_{k,10} = 18\,kN/m^3$ (proposed by the swiss code SIA 261 Annex A [25])

4.2.3 Live load

The live load in the Parish house comes from its use as a residential building. According to EC 1 [13] this corresponds to a category A which implies a characteristic surface load of $q_k = 2.00\,kN/m^2$ (same for floors or stairs). As mentioned above, this load will need to be multiplied by a factor ψ_2 to consider only the quasi-static part of the load. For category A this corresponds to a factor of $\psi_2 = 0.3$ (see Table A1.1 EC 0 [14]).

4.2.4 Other loads and forces

A load applied on the roof for maintenance is considered of category H (EC 1 [13]) but according to Eurocode 0, its factor for the seismic load case is equal to $\psi_2 = 0.0$ (see Table A1.1 EC 0 [14]). Therefore, this load can be neglected. The same goes for the snow and wind action: although their effect is non-negligible, the load factor for the seismic load case is $\psi_2 = 0.0$ (see Table A1.1 EC 0 [14]) for both of them and those loads can therefore be neglected.

5. Analysis of out-of-plane mechanisms

In addition to the two observed OOP F-mechanisms, described in Chapter 2.4, this chapter describes which other failure mechanisms were prone to happen. Furthermore, the observed as well as two possible prone-to-happen OOP mechanisms will be analysed. To do so, an approach based on the moment equilibrium of the wall or an energy based approach is used.
Within the scope of this Master's Thesis and in collaboration with Caroline Heitmann, a set of Jupyter Notebooks accompanied by a joint report was elaborated. Those files form an interactive interface for users to discover the OOP resistance of walls for several simple mechanisms that can be personalized to their needs. A full description of those mechanisms can be found in the mentioned co-written report [17]. The specific Jupyter Notebooks, which have been adapted specifically for the mechanisms analyzed in this chapter, can be found in Appendix H. The other Jupyter Notebooks of the collaboration are referenced under [17] and can be downloaded from a GitHub repository.

5.1 Possible out-of-plane mechanisms that were not observed

Looking at the possible mechanisms presented in the paper "An Integrated Procedure for the Assessment of Seismic Vulnerability of Historic Buildings" by D'Ayala and Speranza [10] (see Figure 5.1), there are several mechanisms that were imaginable but were not observed in the case of the Parish House. It is nevertheless interesting to discuss them from a qualitative perspective and to analyse some of them in a quantitative way to evaluate why they did not appear and what seismic action would have been necessary to trigger the mechanism.

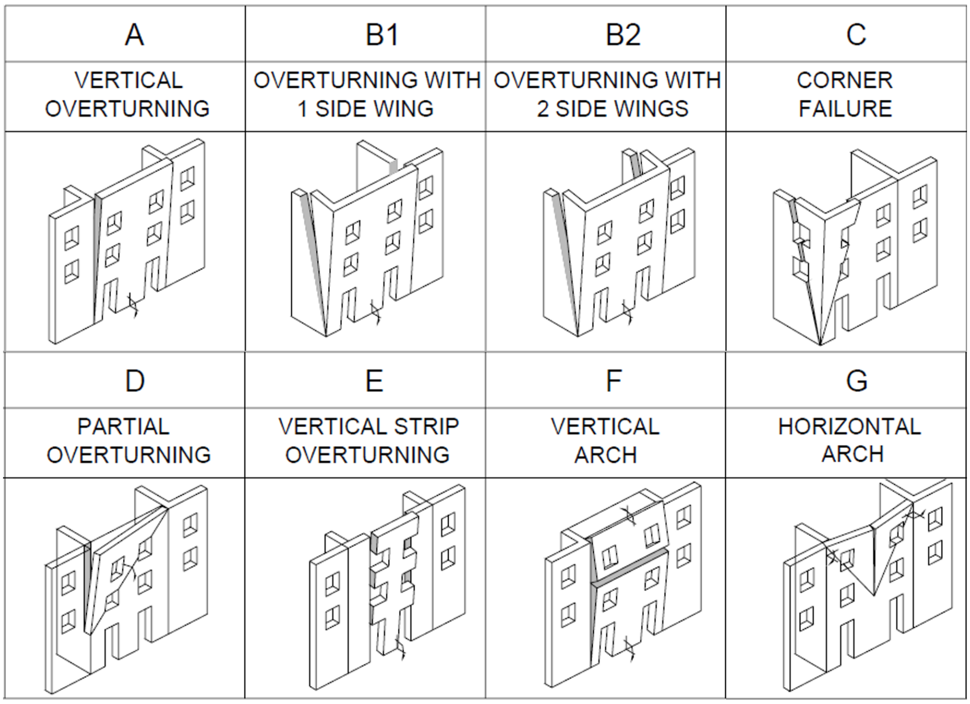


Figure 5.1: Possible mechanisms [10]

Since no vertical cracks at the edges of the facades have been observed (see also Chapter 2.3 and Appendix D), it can be assumed that there is a good interlocking of the bricks in the edges of the facades and the perpendicular walls are in general well connected. Therefore, mechanisms with vertical cracks along the edges of the facade or vertical cracks at the section of connection to a perpendicular wall behind the facade, like mechanisms A, B1, D and E are complicated to trigger and can thus be excluded. Mechanisms B2 and C would be possible especially when the walls are already cracked diagonally from the in-plane action of the earthquake. However, since the top slab is a FERT slab, with the addition of a ring beam around the top perimeter of the facades, the required uplifting of the top slab is almost entirely restricted. Furthermore, the slab restraining the wall at mid-height increases the resistance to those failure mechanisms even further. This makes the mechanisms improbable and is a good reason to exclude them from the quantitative analysis. Therefore, only mechanism G remains and will be analyzed through the means of a quantitative analysis.

5.1.1 G-Mechanism

The G-mechanism, as described by D'Ayala and Speranza [9][10], consists of a wall "horizontally arching" out-of-plane. This means that the fractures are mainly vertical (or diagonal) and the separated wall blocks not only turn around a horizontal axis of rotation but also about a vertical or a diagonal axis. This overturning movement is especially critical for masonry walls that already present vertical/diagonal cracking from previous or the same earthquake action and will eventually start to move out of the plane. If the facade is connected to walls perpendicular to their plane, the interlocking of the well-executed wall-to-wall connection restrains this mechanism.

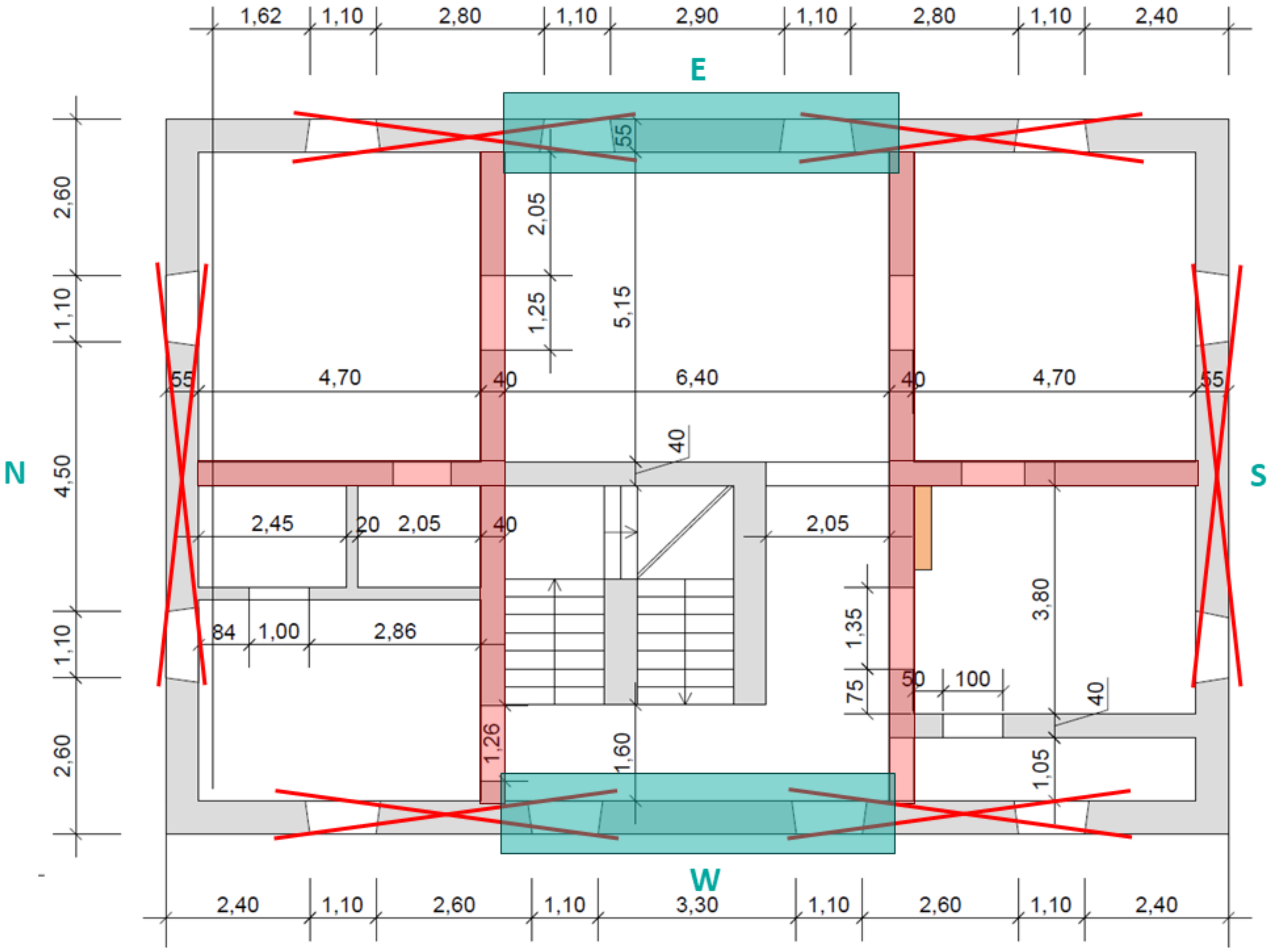
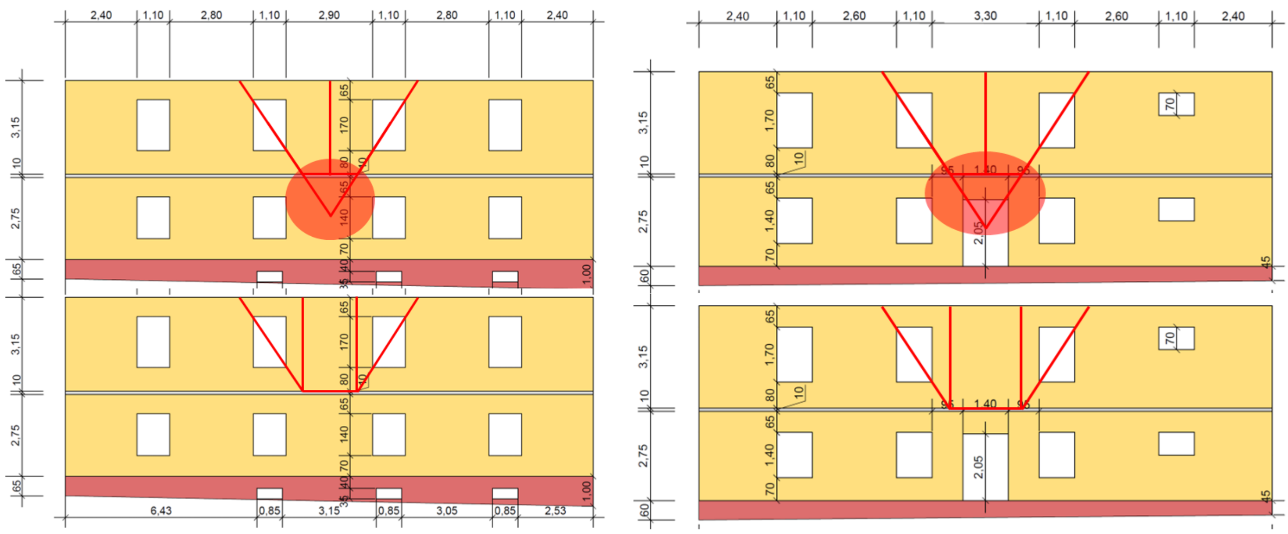


Figure 5.2: Compatible places for G-mechanism

As mentioned before, this mechanism could not be observed in the Parish House. Therefore, the first step, to analyse the resistance against such mechanisms, is to find zones within the geometry of the structure where compatibility is assured and a mechanism could have been triggered. Since windows already create an opening in the wall that only needs to be prolonged to separate the wall into blocks, these are the most vulnerable places for diagonal cracks. Therefore, the most likely scenario is a mechanism including two windows, i.e. with cracks going diagonally "through" the windows to create the triangular block. Looking at the geometry of the Parish House, the ground-floor walls are excluded as they are well connected to the adjacent vaults on the inside of the building that restrain the OOP movement. On the first floor, there are only two places where there are two neighbouring windows where the facade is not restrained by a perpendicular wall in between the windows. Figure 5.2 shows the window pairs that are excluded for the mechanism marked with a red cross and the zones that are possible locations for the G-mechanism marked in turquoise.

The next step consists of finding the geometry of the blocks moving during the mechanism. By drawing the original mechanism schema proposed by D'Ayala and Speranza [9][10] onto the east and west facades it can be seen that this classical shape, with two triangular blocks, is not compatible with the placing of the slab: The slab is assumed to retain the wall for any movement OOP. Therefore, the diagonal cracks will be interrupted at the slab level. Furthermore, the now remaining trapezoidal shapes would need to turn on their support surface on the bottom to fall out, requiring a lot of energy to overcome the friction on the support surface. As a result, an adapted variant with a rectangular middle block is far more likely (see Figure 5.3). These mechanisms will thus be examined in the following subchapter.



(a) East facade

(b) West facade

Figure 5.3: Compatible G-mechanisms, top images: incompatible version, bottom: compatible version

5.2 Analysis of the out-of-plane resistance

In the next phase the before-described mechanisms, observed or potential, are analysed for their stability and resistance against horizontal acceleration in their respective OOP direction. To recapitulate, four mechanisms are analysed in more detail:

1. A global F-mechanism of the west facade, considering the thrust of the vaulted system on the inside of the building
2. A similar F-mechanism in the east facade
3. A G-mechanism in the upper west facade
4. A G-mechanism in the upper east facade

5.2.1 Assumtions for the out-of-plane mechanisms

The F-mechanism proposed by [10] is further treated by Griffith et. al. [31], there the authors model the mechanism as a wall separated into two blocks that move out of their plane by rotating around the top and the base crack while staying connected at the mid crack. By calculating the moment equilibrium about the base point and the mid-point of the wall, the remaining lateral resistance to seismic action for a given deformation can be obtained and a pushover curve can be specified. The G-mechanism on the other hand needs to be analysed by a virtual work / energy-based approach, considering the displacement and rotation of each block. More details can be found in D'Ayala et al. (2003)[9].
For the analysis, the following assumptions are made:

- Rigid diaphragms at the top and bottom of the F-mechanism, meaning there is no differential displacement within the wall.
- Friction between the slab and the wall for the G-mechanism only acts up to a limit displacement $\delta_{max} = t/2$, after that, the friction drops to zero because the connection is lost.
- Only small displacements are considered, i.e. a linearization of the displacements and lever-arms of the forces is done (see also the report co-written with Caroline Heitmann [17] for more background).
- The blocks of the wall remain rigid, meaning that there is no elastic deformation within the moving blocks.
- The added stiffness to the F-mechanisms from the wall perpendicular to the OOP mechanism is neglected.
- The seismic acceleration on the wall is assumed to be constant over the height of the wall.
- It is assumed that the slab of the first floor is directly supported by the vault and does not sit on the walls. Therefore, no friction force of the slab on the wall is considered, hence no restraining force.
- The horizontal thrust of the vault increases with the displacement δ because it is proportional to the height, which is, in turn, in relation to the shape of the vault (see also Chapter 5.2.2).
- The thrust of the vault is lost, once the vault yields, i.e. if its highest point (at mid-span) is lower than its supports. Then the loads coming from the vault turn 0 since it is assumed to have failed.
- The connection between the top and bottom wall is considered to be broken once the displacement (δ) exceeds the thickness of the lower wall. At this point, the loads coming from the upper block are taken directly at the top of the lower block (in reality the wall is considered to be broken at this point).

5.2.2 Modelling the forces from the vault

As explained in Chapter 2.4, the particular type of F-mechanism analyzed for the Parish House strongly depends on the thrust of the vault on the inside of the building. To model the thrust in the algorithm calculating the pushover curve, the following general model is assumed to estimate the horizontal thrust of the vault:

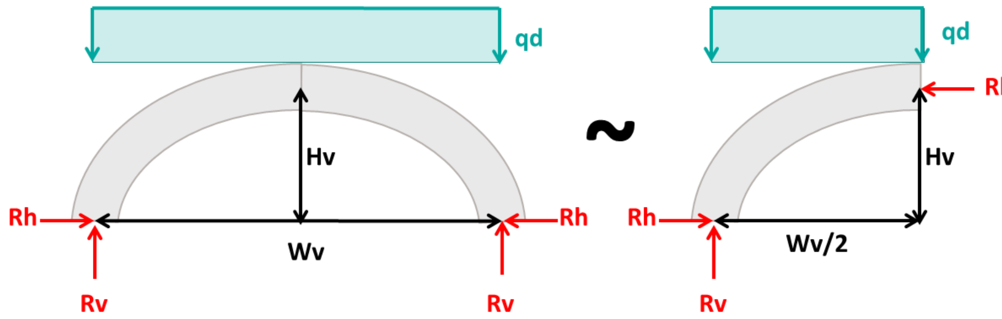


Figure 5.4: Vault model for calculation of thrust

It is assumed that the masonry vault is already cracked at midspan. Then the moment equilibrium of the half-vault (see right side of Figure 5.4) can be calculated as:

$$\sum M_O = R_h \cdot H_v - q_d \cdot W_v/2 * W_v/4 = 0 \tag{5.1}$$

Thus, the reaction forces of the vault can be calculated as follows:

$$R_v = q_d \cdot W_v/2 \tag{5.2}$$

$$R_h = \frac{q_d \cdot W_v^2}{8 \cdot H_v} \tag{5.3}$$

As seen in the equation the horizontal thrust is proportional to the span of the vault squared and inversely proportional to the height of the vault. When considering that the width of the vault increases with the displacement of the wall, meaning that $W_{v,\delta} = W_v + \delta$, and that the vault pieces behave as a rigid block, meaning that $\sqrt{(W_v/2)^2 + H_v^2} = \sqrt{(W_{v,\delta}/2)^2 + H_{v,\delta}^2}$ must remain constant, the horizontal and vertical thrust can be calculated as a function of δ:

$$R_v(\delta) = q_d \cdot (W_v + \delta)/2 \tag{5.4}$$

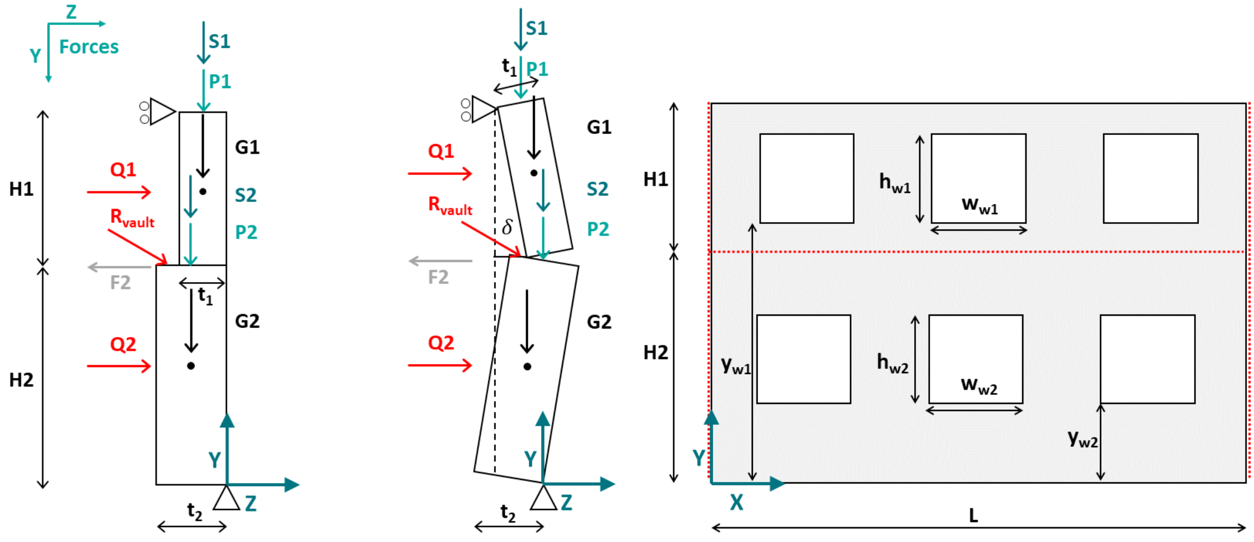
$$R_h(\delta) = \frac{q_d \cdot W_{v,\delta}^2}{8 \cdot H_{v,\delta}} = \frac{q_d \cdot (W_v + \delta)^2}{8 \cdot \sqrt{(W_v/2)^2 + H_v^2 - (\frac{W_v+\delta}{2})^2}} \tag{5.5}$$

5.2.3 Equilibrium formulation

F-Mechanism

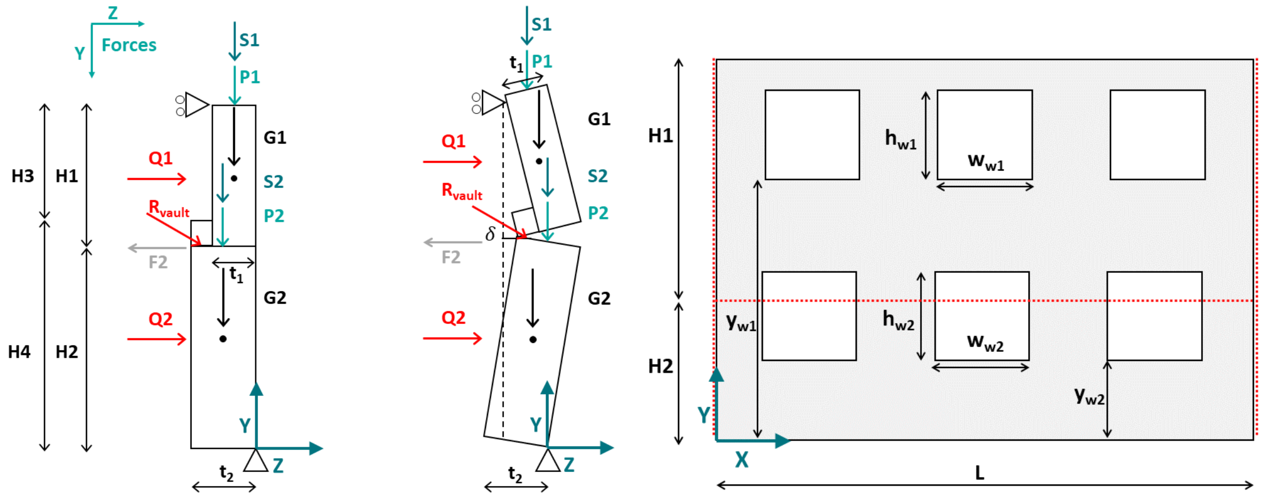
As mentioned before, the pushover behaviour during the F-mechanism can be described using a moment equilibrium about the base point of rotation in addition to an equilibrium formulation about the upper point of rotation [31]. Two cases have to be considered: The first case applies to the mechanism of the west facade, where the crack line of the C3 hinge is clearly above the window opening. The second case applies to the east facade, where the crack of the wall goes through the window opening. The Figures 5.5 and 5.6 show the geometry of the two variants of the mechanism.

Whatever the exact mechanism, the structure can be separated into two moving wall blocks, and the equilibrium formulation can be written as follows:



(a) Cross-section (b) Elevation

Figure 5.5: F-mechanism west side



(a) Cross-section (b) Elevation

Figure 5.6: F-mechanism east side

Global equilibrium about the bottom support:

$$\sum M_{bottom} = \overline{V} \cdot \overline{z}_{la} + \overline{H} \cdot \overline{y}_{la} = \begin{bmatrix} G_1 \\ S_1 \\ P_1 \\ S_2 \\ P_2 \\ R_v \end{bmatrix} \cdot \begin{bmatrix} \delta_{G1} - t_1/2t_1/2 \\ t_1/2 \\ \delta_{G2} - t_2/2 \\ \delta - t_2/2 \\ \delta - t_2/2 \\ \delta - t_2/2 \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \\ F_1 \\ F_2 \\ R_h \end{bmatrix} \cdot \begin{bmatrix} y_{G1} \\ y_{G2} \\ H_1 + H_2 \\ H_2 \\ H_2 \end{bmatrix} = 0 \quad (5.6)$$

Equilibrium of the top block about its lower rotation axis:

$$\sum M_{top} = \overline{V}_{UB} \cdot \overline{z}_{la,UB} + \overline{H}_{UB} \cdot \overline{y}_{la,UB} = \begin{bmatrix} G_1 \\ S_1 \\ P_1 \end{bmatrix} \cdot \begin{bmatrix} \delta_{G1} + t_1/2 - \delta \\ t_1/2 - \delta \\ t_1/2 - \delta \end{bmatrix} + \begin{bmatrix} Q_1 \\ F_1 \end{bmatrix} \cdot \begin{bmatrix} y_{G1} - H_2 \\ H_1 \end{bmatrix} = 0 \quad (5.7)$$

Where:

- $\overline{V}$ and $\overline{H}$ are the vertical and horizontal forces acting on the wall and $\overline{z}_{la}$ and $\overline{y}_{la}$ their respective lever arms.

- $\overline{V}_{UB}$ and $\overline{H}_{UB}$ are the vertical and horizontal forces acting on the upper block of the wall and $\overline{z}_{la,UB}$ and $\overline{y}_{la,UB}$ their respective lever arms.
- G_1 and G_2 are the self-weight loads of each block
- Q_1 and Q_2 are the seismic forces on each block applied to the centre of gravity of each block respectively, corresponding to the self-weight load times the acceleration factor for each block (y_{G1} and y_{G2}, measured from the bottom)
- S_1 and S_2 are the slab loads on each block (creating friction)
- P_1 and P_2 are other different vertical loads applied to the top centre of each block
- F_1 and F_2 are the horizontal restraining forces on each block, F_2 depends on S_2 but F_1 is in this case unknown until the equations are solved.
- R_h and R_v are the horizontal and the vertical force coming from the vault (see Chapter 5.2.2)
- δ is the displacement at the separation of the two blocks and δ_{G1} and δ_{G2} are the displacements at the centres of gravity of each block

G-Mechanism

To derive the push-over curve of the G-mechanism an energy-based approach is used similar to the one proposed by D'Ayala [9]. The wall is separated into three blocks, two of which are triangular and the third centre block of a rectangular shape. The crack line diagonally crosses the windows, separating them into two parts diagonally. The geometry can be seen in Figure 5.7 below.

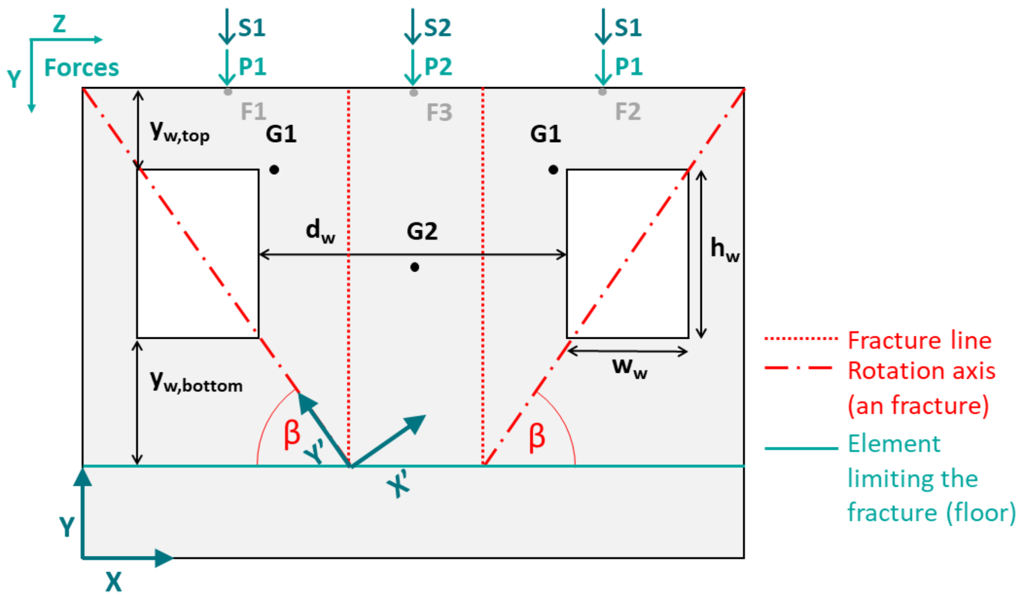


Figure 5.7: G-mechanism elevation

For the energy-based approach, the equilibrium is formulated by looking at a deformed state and adding to it an incremental deformation. The energy of each force/load is then calculated and the sum of the external energy is set equal to the sum of the internal work.
Equilibrium equation:

$$\begin{bmatrix} 2\,G_1 \\ 2\,S_1 \\ 2\,P_1 \\ S_2 \\ P_2 \end{bmatrix} \cdot \begin{bmatrix} cos(\beta)\;(x'_{G1}/x'_\delta * \delta - t/2)/x'_\delta \\ cos(\beta)\;(x'_{top,triang}/x'_\delta * \delta - t/2)/x'_\delta \\ cos(\beta)\;(x'_{top,triang}/x'_\delta * \delta - t/2)/x'_\delta \\ (\delta - t/2)/H \\ (\delta - t/2)/H \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \\ F_1 \\ F_2 \end{bmatrix} \cdot \begin{bmatrix} x'_{G1}/x'_\delta \\ 1/2 \\ x'_{top,triang}/x'_\delta \\ 1 \end{bmatrix} = 0 \tag{5.8}$$

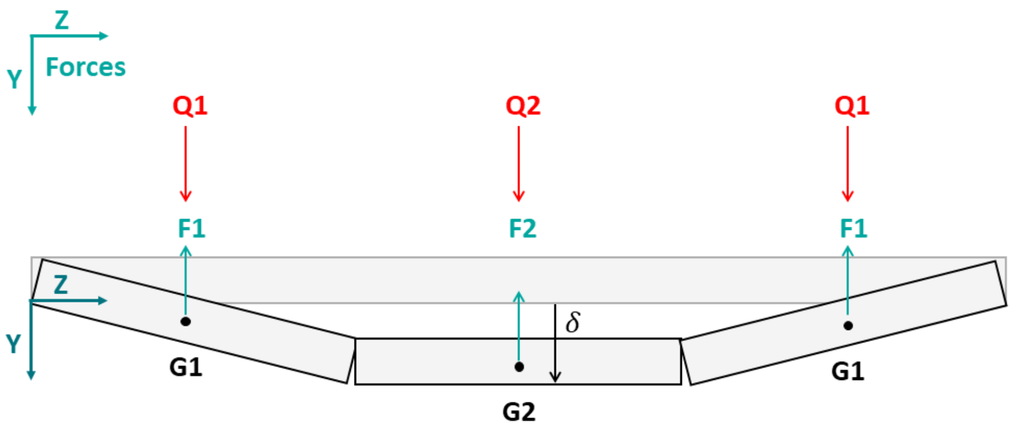


Figure 5.8: G-mechanism plan view

- G_1 and G_2 are the self-weight loads of each block
- Q_1 and Q_2 are the seismic forces on each block applied to the centre of gravity of each block respectively, corresponding to the self-weight load times the acceleration factor for each block
- S_1 and S_2 are the slab loads on each block (creating friction)
- P_1 and P_2 are other different vertical loads applied to the top centre of each block
- F_1 and F_2 are the horizontal restraining forces on each block due to friction with the slab
- x'_δ is the distance between the top corner of the triangular element (where the displacement is δ and the diagonal rotation axis, x'_{G1} is the distance between the centre of gravity and the diagonal rotation axis and $x'_{top,triang}$ is the distance between the centre of the top of the triangular block and the diagonal rotation axis.
- H is the height of the mechanism and t is the wall thickness

5.2.4 Transformation to an equivalent single degree of freedom system and N2 verification

After the pushover curves have been established, the results of this multi-degree-of-freedom (MDOF) systems need to be transformed into equivalent single-degree-of-freedom (SDOF) systems to be able to execute the N2 method. Following the methodology of Eurocode 1998-1-1 [11] §6.5.3 (1) the general procedure is described here:

1. Define the mass vector of the mechanism:

$$m = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} * \rho_{masonry} \tag{5.9}$$

   $m = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} * \rho_{masonry}$
   Where V_n is the volume of one of the blocks n moving in the mechanism. For the F-mechanism, this would correspond to two blocks, for the G-mechanism there are three blocks involved.

2. Define the modal shape vector of the mechanism:

$$\phi = \begin{bmatrix} \phi_1 & \phi_2 & \ldots & \phi_n \end{bmatrix} \tag{5.10}$$

   Where ϕ_n is the shape coefficient of bock n at its centre of gravity. This vector is normalized to one of the blocks of the mechanism, i.e. one of the ϕ values is equal to 1.

3. Calculate the equivalent mass as:

$$m^* = \sum m_i \phi_i \tag{5.11}$$

   This is the mass of the equivalent SDOF system representing the mechanism.

4. Calculate the transformation factor as:

$$\Gamma = \frac{m^*}{\sum m_i \phi_i^2} \tag{5.12}$$

This will be needed later.

5. Transform the forces of the pushover curve into equivalent forces:

$$F^* = \frac{F_b}{\Gamma} \tag{5.13}$$

F_b corresponds to the total force acting on the whole system (all blocks).

6. Use the displacement at the centre of gravity of one block to calculate the equivalent displacement:

$$d^* = \frac{d_n}{\Gamma \phi_n} \tag{5.14}$$

d_n corresponds to the displacement at node (centre of gravity of block n) and ϕ_n corresponds to the corresponding value of the modal vector.

7. The equivalent acceleration of the system can then be calculated as

$$a^* = \frac{F^*}{m^*} \tag{5.15}$$

This will then be used to plot the capacity curve on the same plot as the acceleration-displacement response spectrum (ADRS).

After having determined those values, the pushover curve of the equivalent SDOF system is simplified into a tri-linear approximation as proposed by Griffith et al. [31]. And finally, the tri-linear equivalent pushover curve of the SDOF system is plotted against the ADRS curve. In this combined plot, the initial linear part of the pushover curve is continued until reaching an intersection with the ADRS. The process is shown in Figure 5.9 below.

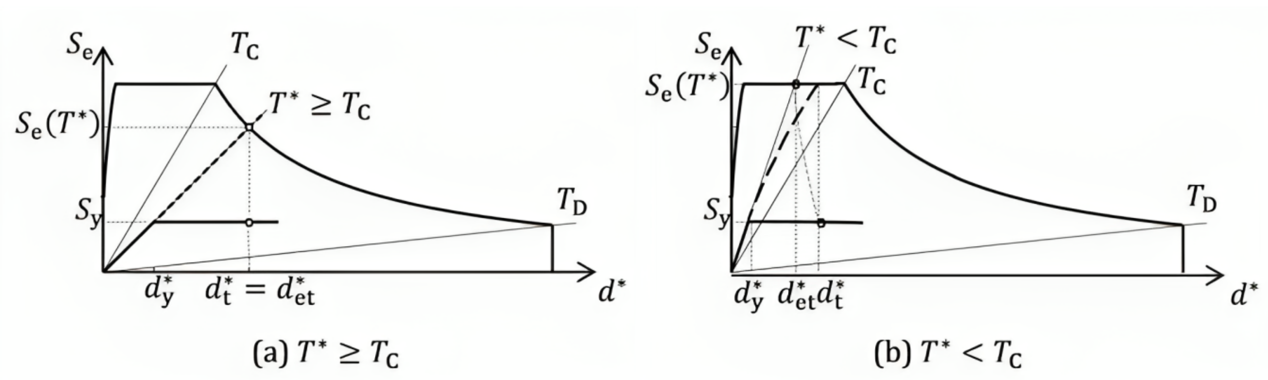


Figure 5.9: Determination of the target displacement for the equivalent SDOF system (hardening neglected)[11]

For cases where $T^* > T_C$ the target displacement can be determined by reading the displacement at the point of intersection of the continued linear part and the ADRS curve. This target displacement can then be compared to the limit displacement as explained in Chapter 3.3.2.

5.2.5 Analysed cases

F-Mechanism west facade

The first case analysed numerically is the F-mechanism of the west facade shown in Chapter 2.4. The calculations are provided in the Jupyter Notebook for the OOP calculations (see Appendix H). The following geometric properties are obtained from the plans (Appendix C) and the loads are calculated in the modelling Excel file (see Appendix xyz) and then introduced into the code:

Wall geometry	H_1	H_2	L	t_1	t_2
[m]	3.25	2.75	14.2	0.55	0.70
Window geometry, upper wall	h_{w1}	w_{w1}	y_w1	n_{w1}	
[m]	1.70	1.10	3.65	4	
Window geometry, lower wall	h_{w2}	w_{w2}	y_{w2}	n_{w2}	
[m]	1.40	1.10	0.7	4	
Vault geometry	h_v	w_v	t_v		
[m]	0.1	1.75*	0.15		
Deformation limitation	δ_{max}				
[m]	$t_2/2$				

Table 5.1: Geometric characteristics F-mechanism west facade

*For the width of the vault a part of the wall thickness is added.

Loads on the upper body	Roof load (P_1)	Slab load (S_1)
[N]	71326	67209
Loads on the lower body	Load (P_2)**	Slab load (S_2)***
[N]	26825	0
Load on the vault	q_{vault}	
[N/m]	37769	

Table 5.2: Loads F-mechanism west facade

** This corresponds to the filling material above the vault.
*** The assumption is made that the slab of the first floor is uniquely supported by the vaults and does not connect directly to the wall. The vertical load on the lower block therefore only comes from the vault. This means the friction between the slab of the first floor and the wall is neglected.

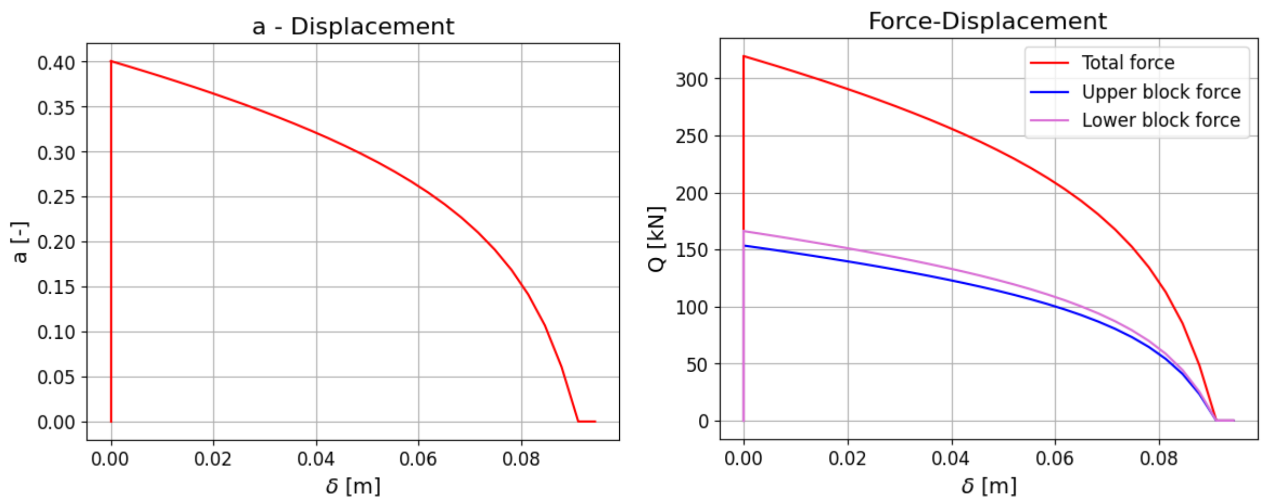
After introducing the characteristics of the mechanism and the loads into the Jupyter Notebook the following output is obtained (see also Figure 5.10):

	δ [m]	α [-]	Q_1 [kN]	Q_2 [kN]
Start of rocking	0	0.401	153.4	163.0
Loss of lateral resistance	0.091	0	0	0

Table 5.3: Characteristic values of the pushover curve of the west facade, F-mechanism

Two points can be noticed in Figure 5.10:

1. The wall starts rocking at an α-value of 0.401
2. At a displacement of only around nine centimetres the wall loses its resistance entirely because the horizontal thrust of the vault becomes too large.

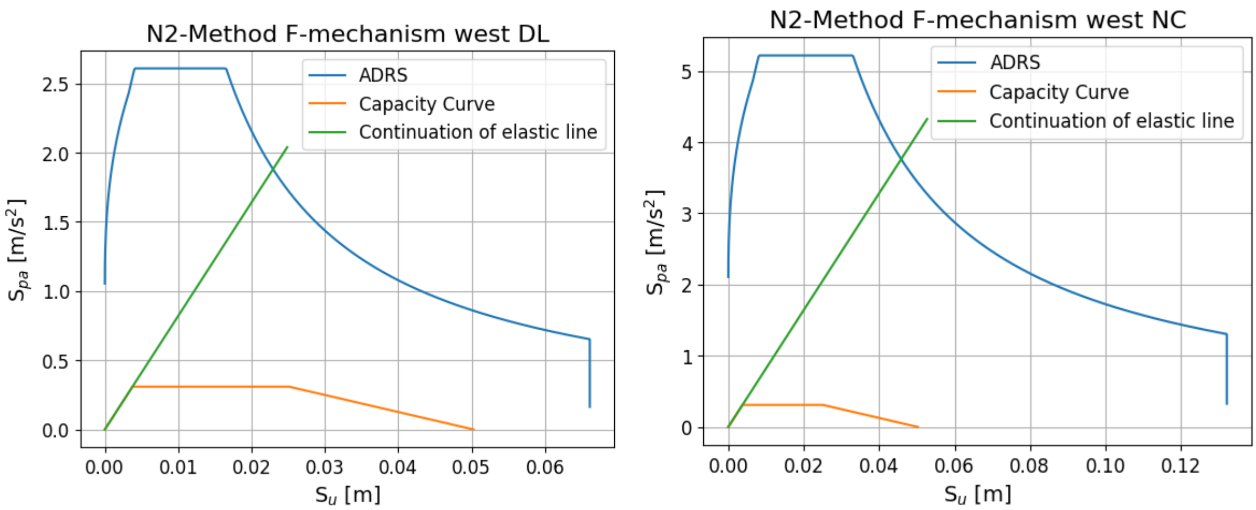


(a) α-Factor describing the seismic load

(b) Force-Displacement curve

Figure 5.10: F-mechanism west side results

It is interesting to note that the required acceleration to start the wall rocking is rather low at $a = 3.93m/s2$. When that limit is reached, the displacement capacity however is quite high at about 10 centimetres. Although the vault in this case is rather shallow and would exercise a bigger thrust than a higher vault, the load on it is rather low, which is the main reason for the thrust to stay low. This does however not mean that the mechanism did not produce. On the contrary, the acceleration to trigger the mechanism is relatively low which means that the mechanism was possibly triggered but did not reach failure due to the somewhat big displacement capacity.



(a) Verification DL limit state

(b) Verification NC limit state

Figure 5.11: Verification of the resistance against the F-mechanism west facade with the N2 method

Looking at the results for the N2 verification method it is noticeable that neither the NC limit state nor the DL limit state are verified. The limit value for the NC limit state lies at $d_{u2} = 0.03\,m$ whereas the target displacement derived from the ADRS Graph lies at $d_{NC} = 0.046\,m$. Since $d_{u2} < d_{NC}$ this limit state is not verified. Similarly, the limit value for the DL limit state is $d_{u1} = 0.02\,m$ whereas the target displacement is of $d_{DL} = 0.023\,m$. In the same way, $d_{u1} < d_{DL}$ shows that the limit state is not verified. The details of the calculation can be found in Appendix H
Even though the analysis seeks to describe the stability of the OOP mechanism as exactly as possible, several factors could not have been taken into account. The negligence of the perpendicular walls

supporting the vault, as well as the friction between masonry bricks at the edges of the mechanism, are elements that would increase the resistance and displacement capacity of the system. However, these elements are complex to model and therefore it has been decided to not consider them and aim for the safer side. Although there is only a small non-conformance in the NC and DL states it is necessary to reinforce the building against this mechanism. This mechanism forms a point of risk and weakness, even more so now that the structure already has residual deformation from the earthquake, and it is essential to reinforce the vault (with cables) to counter the push and keep the building intact during the next earthquake.

F-mechanism east facade

The same analysis has been done for the second F-mechanism, located on the east side of the building. This mechanism varies slightly because the wall does not break at the interface of the storeys but at the level of the support of the vault (as shown in Figure 5.6). Therefore, the code was slightly adapted to take the different geometry into account (see Jupyter Notebook in Appendix H). The geometric input data is summarized in the table below:

Wall geometry	H_1	H_2	H_3	H_4	L	t_1	t_2
[m]	4.55	1.55	2.7	3.4	17	0.55	0.7
Window geometry, upper wall	h_{w1}	w_{w1}	y_{w1}	n_{w1}			
[m]	1.7	1.1	3.65	4			
Window geometry, lower wall	h_{w2}	w_{w2}	y_{w2}	n_{w2}			
[m]	1.4	1.1	0.7	4			
Vault geometry	h_v	w_v	t_v				
[m]	1.25	5.6	0.15				
Deformation limitation	δ_{max}						
[m]	$t_2/2$						

Table 5.4: Geometric characteristics F-mechanism east facade

The loads calculated to act on top of the wall as well as on the vault are the following:

Loads on the upper body	Roof load (P_1)	Slab load (S_1)
[N]	78288	155075
Loads on the lower body	Load(P_2)	Slab load(S_2)**
[N]	248250	0
Load on the vault	q_{vault}	
[N/m]	63000	

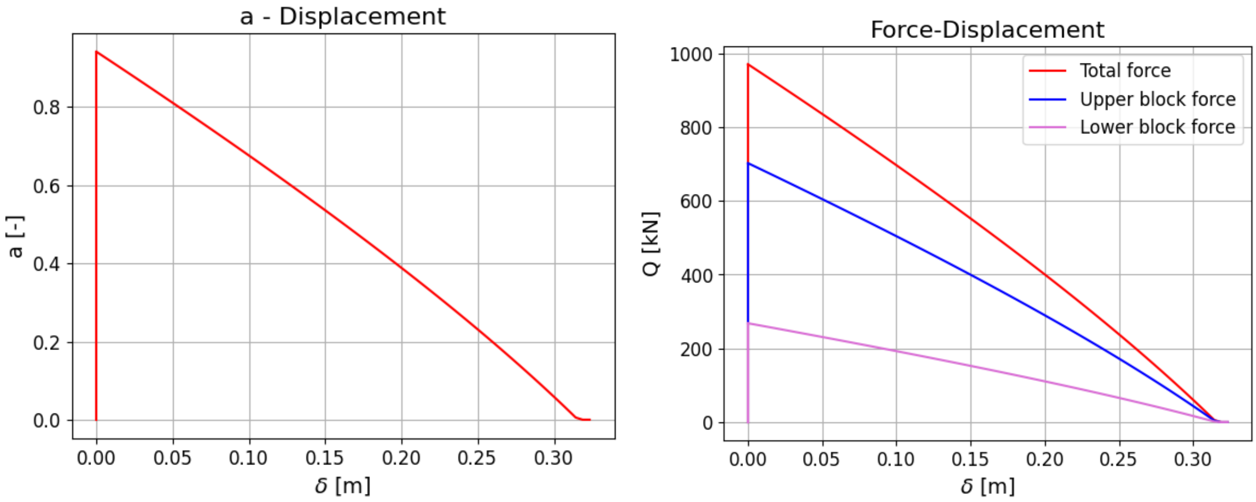
Table 5.5: Loads F-mechanism east facade

After introducing the characteristics of the mechanism and the loads into the Jupyter Notebook the following output is obtained:

	δ [m]	α [-]	Q_1 [kN]	Q_2 [kN]
Start of rocking	0	0.941	702.1	268.1
Loss of lateral resistance	0.319	0	0	0

Table 5.6: Characteristic values of the pushover curve of the east facade, F-mechanism

The pushover curve can be seen in Figure 5.12 below.

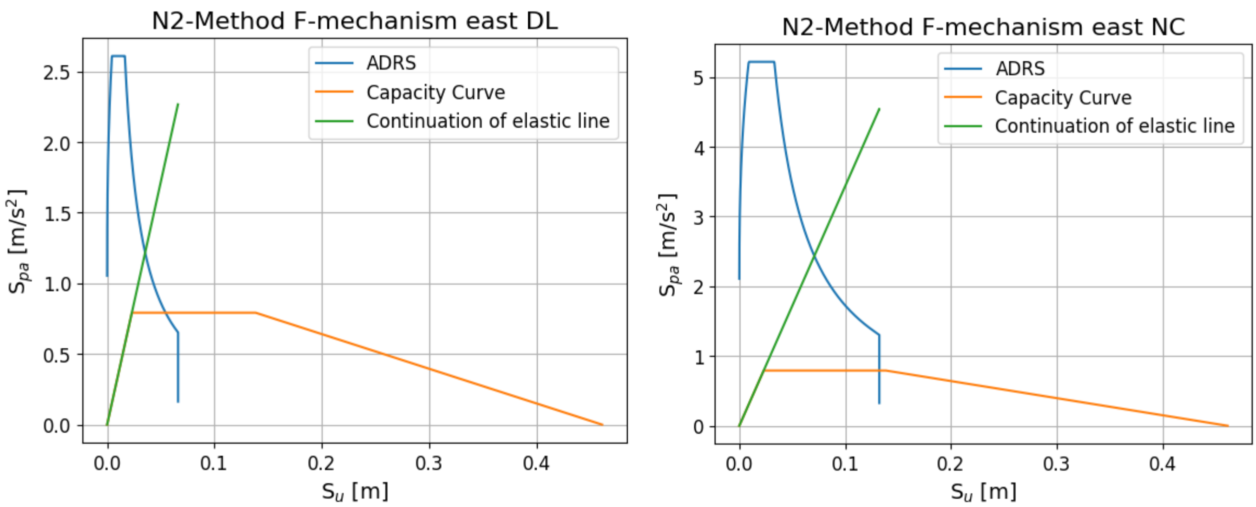


(a) α-Factor describing the seismic load

(b) Force-Displacement curve

Figure 5.12: F-mechanism east side results

The same key points as for the est mechanism can also be seen in Figure 5.12. The difference here is that the phase before the vault breaks is remarkably longer and the pushover curve is therefore less curved. The larger size (width and height) of the vault entails two things: the supports of the vault need to be moved further away, i.e. a larger OOP movement is required, before the vault breaks and due to the bigger height, the horizontal thrust of the vault is smaller in comparison to the vault of the west facade.
Comparing those results to the ones from the mechanism observed on the West facade it can be noticed that the acceleration to start the rocking of the facade of the East side is roughly twice as large. It is also worth noting that the displacement capacity is roughly four times that of the west facade. It makes, therefore, sense that fewer cracks corresponding to this OOP mechanism could be observed within the building than for the west OOP mechanism. This mechanism is also less prone to happen due to its larger acceleration to start the rocking in comparison to the west mechanism. In general, this indicates that the larger vaults, where the horizontal component of the support load is smaller due to the higher inclination of the vaults at their supports, are more stable than the small vaults around the main entrance and the stairs.



(a) Verification DL limit state

(b) Verification NC limit state

Figure 5.13: Verification of the resistance against the F-mechanism east facade with the N2 method

Looking at the N2 verification the following can be adopted:
The limit value for the NC limit state lies at $d_{u2} = 0.277\ m$ whereas the target displacement derived from the ADRS Graph lies at $d_{NC} = 0.074\ m$. Since $d_{u2} > d_{NC}$ this limit state is verified. Similarly, the limit value for the DL limit state is $d_{u1} = 0.184\ m$ whereas the target displacement is of $d_{DL} = 0.035\ m$. In the same way, $d_{u1} > d_{DL}$ shows that the limit state is verified. The details of the calculation can be found in Appendix H. This side of the building therefore does not require reinforcing of the vaults to lessen the thrust.

G-mechanism west facade

After having looked at the mechanism that was observed after the earthquake of 2020 (see also Chapter 2.4) we can now consider two alternative mechanisms that did not occur: The two G-mechanisms on the west and the east facades. The characteristic dimensions assumed for the analysis are measured in the plans (see Appendix C) and can be found in Table 5.7. The loads on the body are calculated in the same way as for the F-mechanism and can be found in Table 5.8 and a friction coefficient between the top slab and the masonry of $\mu = 0.6$ was admitted. Using a second type of Jupyter Notebook (see Appendix H), specially developed for these kinds of mechanisms, the results can be seen in Table 5.9 as well as in Figure 5.14.

Wall geometry	$y_{w,top}$	$y_{w,bottom}$	d_w	h_w	w_w	t
[m]	0.65	0.90	3.30	1.70	1.10	0.55
Deformation limitation	δ_{max}					
[m]	t/2					

Table 5.7: Geometric characteristics G-mechanism west facade

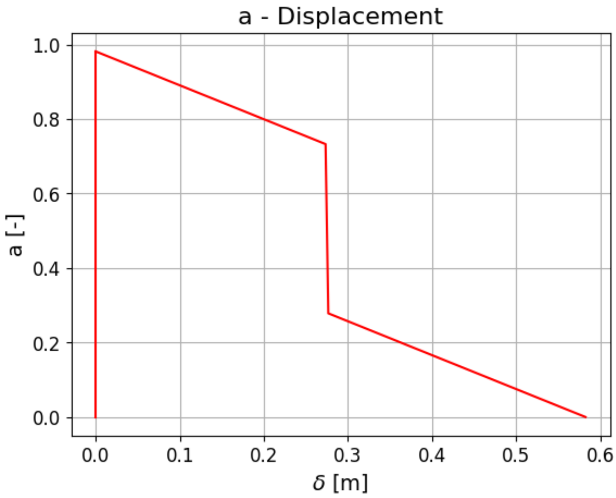
Loads on the triangular body	Roof load (P_1)	Slab load (S_1)
[N]	11305	10700
Loads on the rectangular body	Roof load(P_2)	Slab load(S_2)
[N]	11479	10865

Table 5.8: Loads G-mechanism west facade

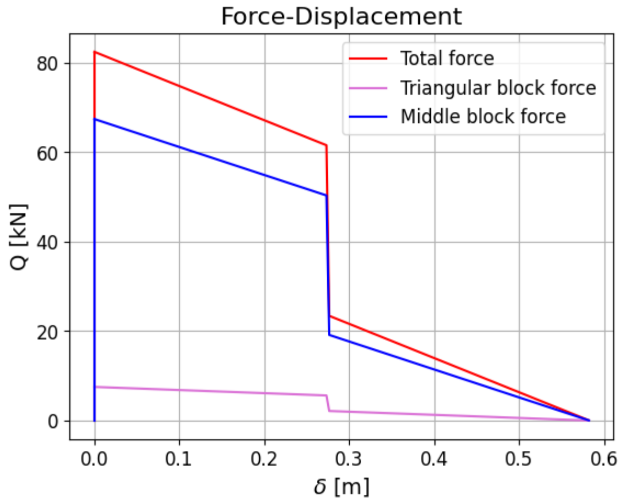
	δ [m]	α [-]	Q_1 [kN]	Q_2 [kN]
Start of rocking	0	0.981	7.5	67.4
Loss of lateral resistance	0.579	0	0	0

Table 5.9: Characteristic values of the pushover curve of the west facade, G-mechanism

By looking at the characteristic values of the pushover curve summarized in the table above, it can be noticed that the acceleration to start the rocking of the two blocks is about the same as the acceleration needed for the F-mechanism on the east side. The initial acceleration to start the mechanism, however, is dependent on the friction coefficient between the wall and the slab above. If we assume that the slab is a FERT slab with an RC ring beam surrounding it, this connection might even be more resistant which would make the mechanism even unlikelier. In addition to that, the displacement capacity of the G-mechanism of the west facade is roughly six times the capacity of the F-mechanism on the west facade. This is mainly because the F-mechanism is enhanced by the thrust of the vaults and thus makes the wall less stable and because of the ring beam holding the wall of the G-mechanism back.



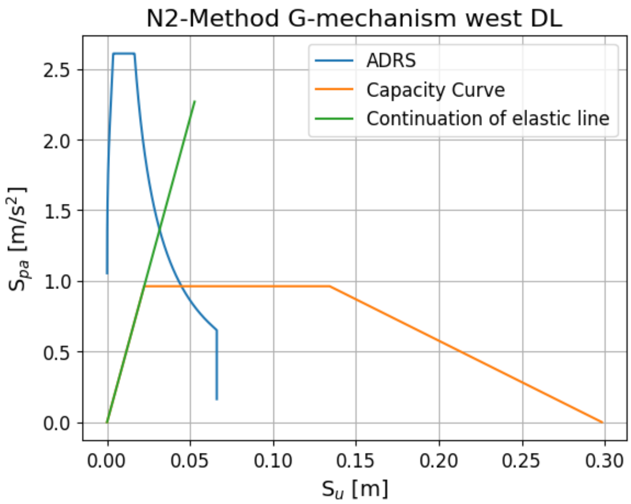
(a) α-Factor describing the seismic load



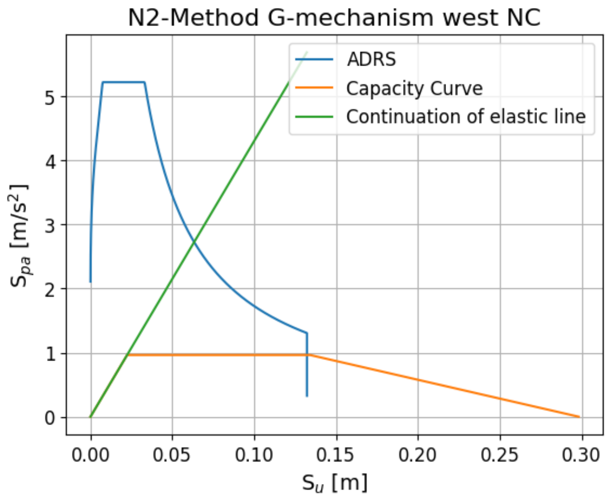
(b) Force-Displacement curve

Figure 5.14: G-mechanism west side results

When looking at the pushover curve, one of the main differences from the F-mechanism to be noticed is the shape: The curve follows a linear line with a sudden drop at δ_{max} where the slab loses connection to the wall and does not restrain it any longer. Therefore, we can see that the friction restraining the block from falling out, only influences the initial acceleration to start the mechanism, but not on the displacement capacity.



(a) Verification DL limit state



(b) Verification NC limit state

Figure 5.15: Verification of the resistance against the G-mechanism west facade with the N2 method

The verification is performed as follows: The target displacement for the DL limit state is $d_{DL} = 0.032\,m$ which is smaller than the limit displacement $d_{u1} = 0.119\,m$. Therefore, the DL limit state is verified. In the same way, the target displacement for the NC limit state is $d_{NC} = 0.063\,m$ which is smaller than the limit displacement $d_{u2} = 0.179\,m$. Therefore, the NC limit state is verified.

G-mechanism east facade

Looking at the east side, the mechanism analysed is the same as the one on the west side. The main differences consist of a more compact geometry, i.e. windows closer, and heavier loads on top of the blocks due to the bigger span of the slab. The details can be seen in Tables 5.10 and 5.11.

Wall geometry	$y_{w,top}$	$y_{w,bottom}$	d_w	h_w	w_w	t
[m]	0.65	0.90	2.90	1.70	1.10	0.55
Deformation limitation	δ_{max}					
[m]	t/2					

Table 5.10: Geometric characteristics G-mechanism east facade

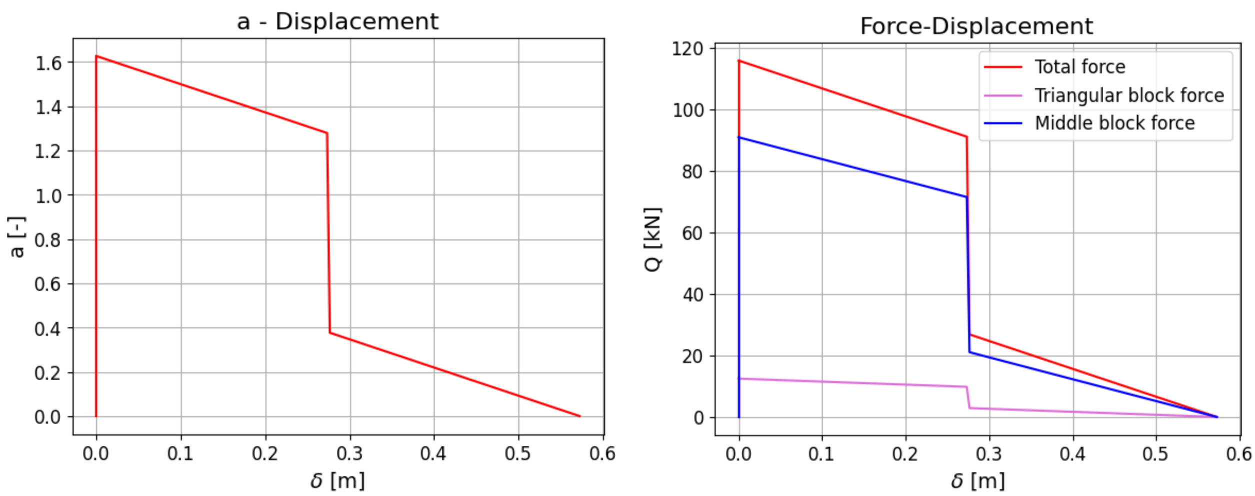
Loads on the triangular body	Roof load (P_1)	Slab load (S_1)
[N]	11305	19000
Loads on the rectangular body	Roof load(P_2)	Slab load(S_2)
[N]	9329	15678

Table 5.11: Loads G-mechanism east facade

The main results of the G-mechanism can be perceived in Table 5.12 below: The acceleration needed to trigger the mechanism is even higher than for the west G-mechanism. This is mainly due to the higher load of the slab and the smaller block size, making it more resistant to OOP action. The displacement capacity however is approximately the same as for the previous mechanism. As already mentioned above, the friction on top of the wall only has an influence on the acceleration needed to trigger the rocking but not on the displacement capacity. Since the geometry of this second G-mechanism is fairly similar to the one on the west side, it only makes sense to also have approximately the same displacement capacity.

	δ [m]	α [-]	Q_1 [kN]	Q_2 [kN]
Start of rocking	0	0.981	7.5	67.4
Loss of lateral resistance	0.579	0	0	0

Table 5.12: Characteristic values of the pushover curve of the east facade, G-mechanism

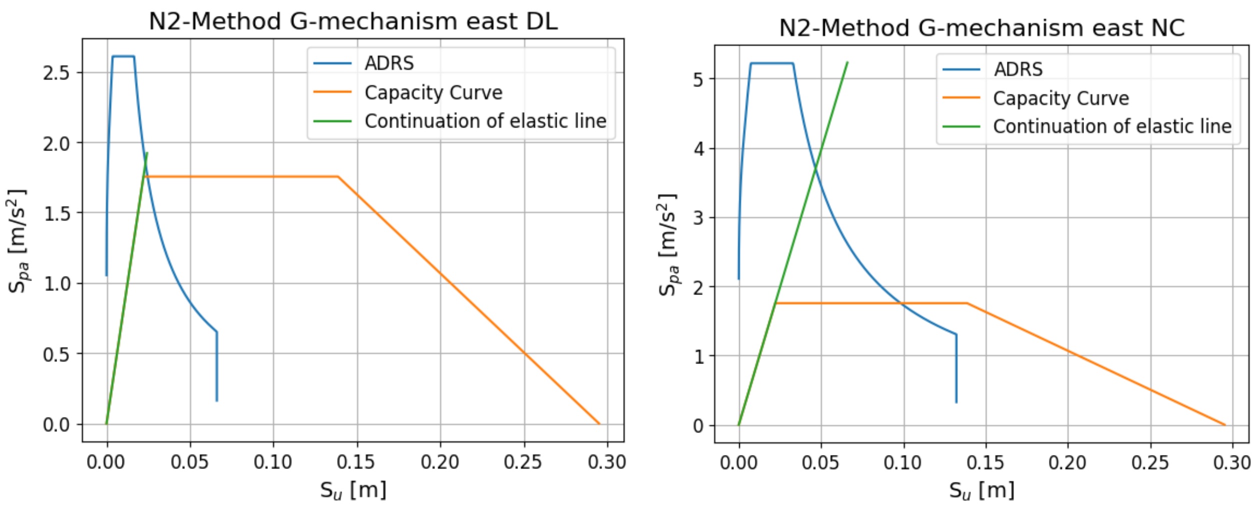


(a) α-Factor describing the seismic load

(b) Force-Displacement curve

Figure 5.16: G-mechanism east side results

Looking at the pushover curve of this second mechanism that was not observed and its characteristic values, we can see why this mechanism is even unlikelier to occur than the first G-mechanism on the west facade.



(a) Verification DL limit state

(b) Verification NC limit state

Figure 5.17: Verification of the resistance against the G-mechanism east facade with the N2 method

The verification procedure is the same as for the other three mechanisms: The target displacement for the DL limit state is $d_{DL} = 0.023\ m$ which is smaller than the limit displacement $d_{u1} = 0.118\ m$. Therefore, the DL limit state is verified. In the same way, the target displacement for the NC limit state is $d_{NC} = 0.047\ m$ which is smaller than the limit displacement $d_{u2} = 0.177\ m$. Therefore, the NC limit state is verified.

5.2.6 Conclusion and discussion of results

In conclusion, it is safe to say that there is a significant difference in the stability of the facades of the Parish House to different mechanisms. Although the vaults can have a good influence in rigidifying the structure, their effect on the OOP resistance of adjacent walls is critical. They even enhance the mechanism if their curvature is low. This makes them more critical for the mechanism on the west facade than on the south facade. The analysis of the G-mechanisms shows a significant difference between the acceleration needed to trigger a G mechanism compared to an F-mechanism. The G-mechanisms are much less prone to happen due to the good connection of the top of the wall to the RC ringbeam, which tends to hold them in their plane. Therefore, the accelerations to trigger the G-mechanisms need to be significantly higher than the one to trigger the G-mechanisms. Furthermore, since all the connections at the corners of the building, i.e. the interlocking were well executed, the risk of all other global mechanisms with a whole facade falling out was low to none. Likewise, the connections between the vaults and the walls or the slabs and the walls seem to be well executed. The ring beam could even add to that quality of interlocking which makes the other more local mechanisms unlikely to happen.

Learinings for the retrofitting

The table below summarizes the results of the verifications of the stability against the OOP failure in one of the previously described mechanisms. It can be noticed that the target displacement of three mechanisms is significantly lower than the limit displacement of the corresponding limit state. Only the F-mechanism on the west facade does not meet the necessary resistance and deformation capacity to verify the NC and the DL limit states. Therefore, it is required to reinforce the structure against this mechanism. One of the possible solutions are steel ties above the vaults, spanning between the two walls supporting the vault, to counter the thrust of the vault. The exact solution will be discussed in Chapter 8.

Mechanism	F west	F east	G west	G east
Limit displacement DL [m]	0.020	0.184	0.119	0.118
Target displacement DL [m]	0.023	0.035	0.032	0.023
Verification DL	Not verified	Verified	Verified	Verified
Limit displacement NC [m]	0.030	0.277	0.179	0.177
Target displacement NC [m]	0.046	0.074	0.063	0.047
Verification NC	Not verified	Verified	Verified	Verified

Table 5.13: Summary of the verification against OOP failure

For the retrofitting of the structure, the main learnings can be summarized as follows:

The most vulnerable item of the construction is the vaults. To prevent OOP mechanisms from happening in the next earthquake they must be stiffened (west facade) or at least repaired and restored to their original state (east OOP). Another important step will be the regular maintenance of all wall-to-wall and all slab/vault-to-wall connections to ensure good interlocking in the future. It can however be concluded that only one out of four OOP scenarios is truly prone to happen, whereas the other three remain rather improbable considering their target displacements that are significantly smaller than the respective limits.

6. Modelling with OpenSees

The modelling approach chosen to analyse the structure is an equivalent frame model (EFM) using macro-elements as proposed by Vanin et al.[3] and performing non-linear dynamic response history analysis (RHA) using the software OpenSees [20].

6.1 Walls with macroelements

To model the walls of the structure, the first step is to divide the walls into piers, spandrels and nodes. The elements are distinguished by their different locations: Piers are the blocks of masonry next to or horizontally in between windows, spandrels are the masonry blocks above or vertically in between windows, nodes connect the piers and spandrels, and finally, shell elements modelling the slabs will be introduced as proposed in the next chapter.

The spandrels and piers are modelled as three-node, three-dimensional macroelements as proposed by Vanin et. al. [3]. Those three-dimensional three-node macroelements for modelling masonry panels introduce several differences from previous approaches, making the approach more accurate. Firstly, it extends the methodology of the previously developed macroelement designed to uniquely simulate the in-plane response of masonry panels. In contrast to the previously used macroelements by Lagomarsino et. al. [32], the updated version integrates both the in-plane and out-of-plane behaviour of masonry walls. In addition, it incorporates second-order geometric effects through a P-Δ formulation, providing a more realistic representation of the structural behaviour, especially for the nonlinear behaviour of structures. It couples shear behaviour with axial load and flexural behaviour without the need for iterations, ensuring the accurate application of equilibrium conditions at the three flexural interfaces. Moreover, the proposed macroelement accommodates the use of complex material models, allowing for more flexibility in capturing the nonlinear behaviour displayed by masonry panels. This novel macroelement has been integrated into the OpenSees software, an open-source platform for structural and earthquake engineering simulations, facilitating convenient access and utilization by the research community. [3]

The macroelement is defined as a one-dimensional element in three-dimensional space with two nodes at the element ends and one node in the middle. It is made up of a set of two panels that are separated by three nonlinear sections that account for axial strain deformations and are subjected to the same average shear deformation. The Figure 6.1 below shows the three deformation modes the macroelement can model:

1. In-plane flexure
2. In-plane flexure and shear
3. Out-of-plane (OOP) response (particularly important for masonry)

[3]

6.1.1 Choice of mesh

Since the structure is mostly regular with some irregularities on the west and the south facade and on the inside walls, the meshing approach described by Bracchi et al. with a minimum clear height is considered [33]. This approach is in this particular case preferred over the average opening height or even the 30° rule because of its simple application [34]. As an example, the division of the west facade into elements is shown in Figure xy:

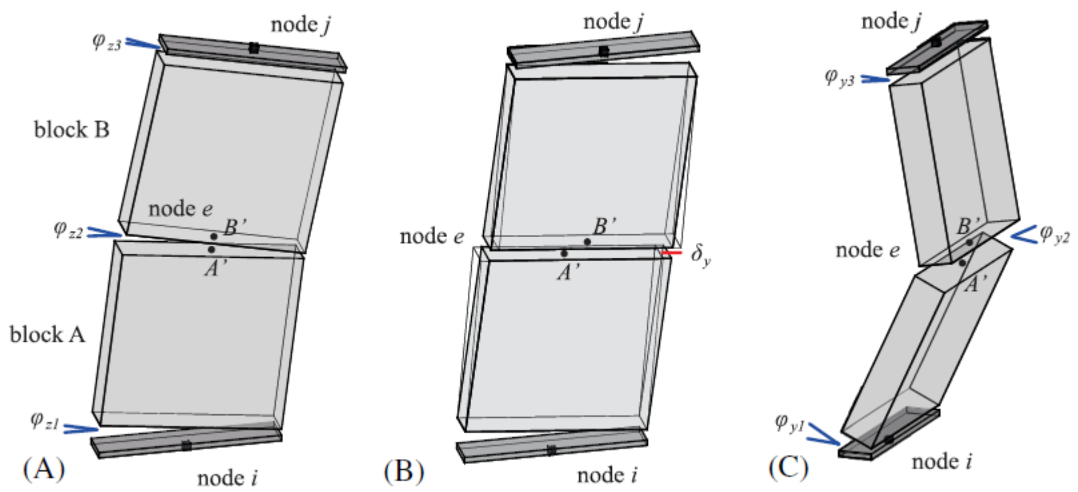
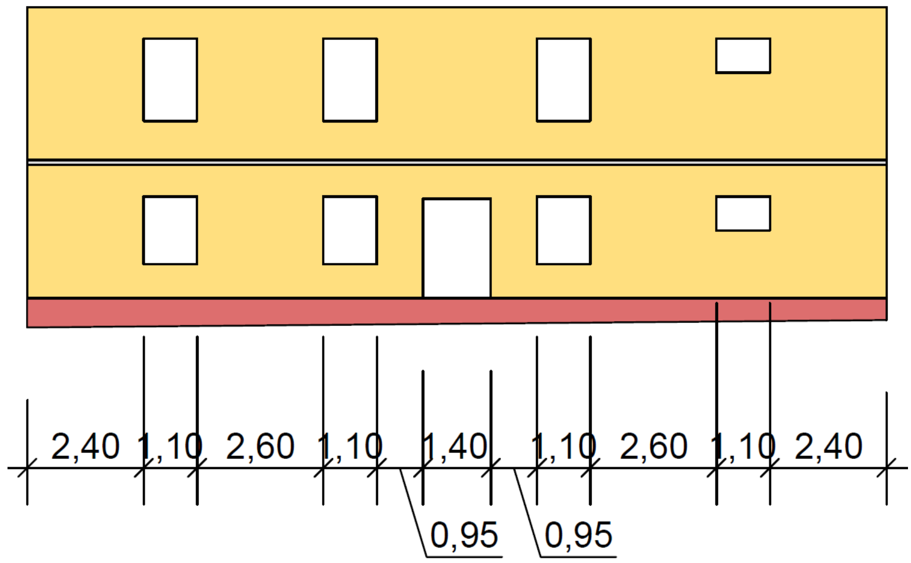
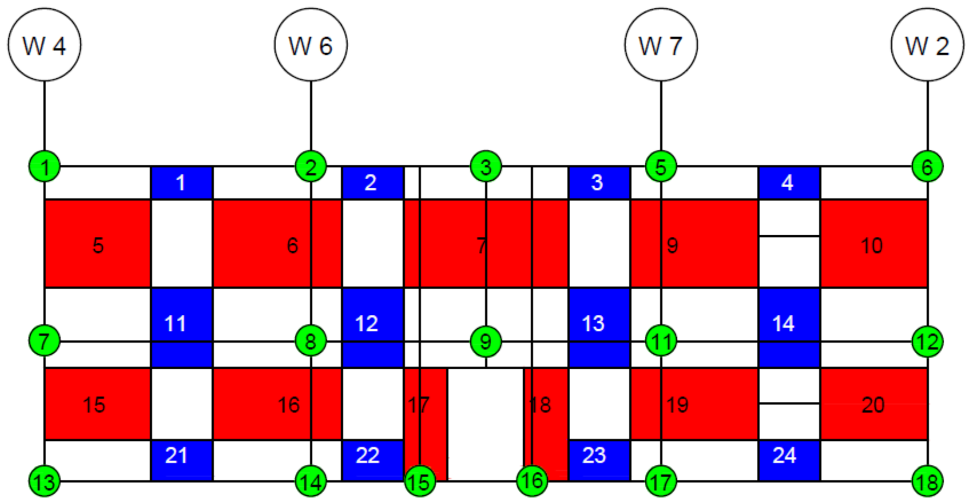


Figure 6.1: Deformation modes of the macroelement: (A) in-plane flexure only, (B) in-plane flexure and shear and (C) OOP response [3])



(a) West facade geometry



(b) West facade macroelements

Figure 6.2: Division into macro elements of the west facade

6.2 Slabs and vaults

6.2.1 Vaults

As established in Chapter 2.2.2, the slab above the ground floor is supported by a complex system of vaults spanning in both directions. The bigger vaults as seen in Figure 2.5 are barrel vaults, meaning that they have one main curvature and one spanning direction, whereas the smaller vaults are cross vaults that span in both directions but have a main curvature. To model these vaults in the EFM, they are introduced as equivalent shells with the same structural behaviour. Since the model can only account for 2D elements, the vaults are transformed into shell elements with an orthotopic membrane section using the method proposed by Cattari et. al. in their paper "Modelling of vaults as equivalent diaphragms in 3D seismic analysis of masonry buildings". Their work consisted of developing analytical expressions to determine Young's modulus of an equivalent slab having the same behaviour as the vault. Their reference were vaults modelled using finite element models (FEM) to validate their hypothesis.

Barrel vaults

To determine the equivalent stiffness of a slab to represent a barrel vault, the vault needs to be looked at in two directions: the direction of the main span (parallel to the curvature) and the longitudinal direction (where the cross-section is arched). Figure 6.3 illustrates a barrel vault and its characteristic dimensions. The vaults are defined using the internal width and height. Assuming material parameters of E (Young's modulus) and G (shear modulus) the stiffness of the vault in the spanning direction can be described using the formulas proposed by Cattari [35].

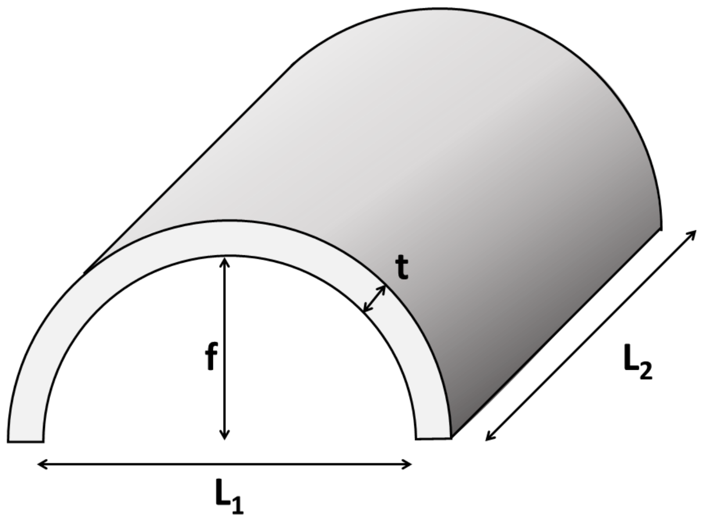


Figure 6.3: Dimensions of a general barrel vault

$$\frac{E_{v,1}}{E} = \left(a_1 \frac{t}{L_1} + a_2\right)\left(\frac{f}{L_1}\right)^{a_3 \frac{t}{L_1} + a_4} \tag{6.1}$$

$$\frac{G_{v,1}}{G} = \left(b_1 \frac{t}{L} + b_2\right)\left(\frac{f}{L}\right) + b_3 \frac{t}{L} + b_4 \tag{6.2}$$

Where:

- E is the Young's modulus of the plane element
- E_v is the Young's modulus of the slab representing the vault
- t is the thickness of the vault
- L_1 is the span of the vault
- a_1, a_2, a_3 and a_4 are factors that were determined during the research of Cattari [35] and are the following:
  $a_1 = 0.05$, $a_2 = -0.0011$, $a_3 = 3.1767$ and $a_4 = 0$, for fixed support conditions of the vault.

- In the same way b_1, b_2, b_3 and b_4 are factors that were calibrated and are the following: $b_1 = 1.6433$, $b_2 = -1.8191$, $b_3 = 0.1133$ and $b_4 = 1.1189$

In the other direction, the longitudinal way, the stiffness needs to be transformed to an equivalent one corresponding to a simple slab as well. The methodology used for this is the following:

$$\frac{E_{v,2}}{E} = \frac{A_v}{A_s} \tag{6.3}$$

Where:

- A_v is the cross-sectional area of the vault.
- A_s is the area of the cross-section of the equivalent slab.

The two parameters can be defined as follows: A_v is calculated as the area of an ellipse minus the area of a smaller ellipse.

$$A_v = ((L_1/2 + t)(f + t) - L_1/2 \cdot f)\ \pi \tag{6.4}$$

Cross vaults

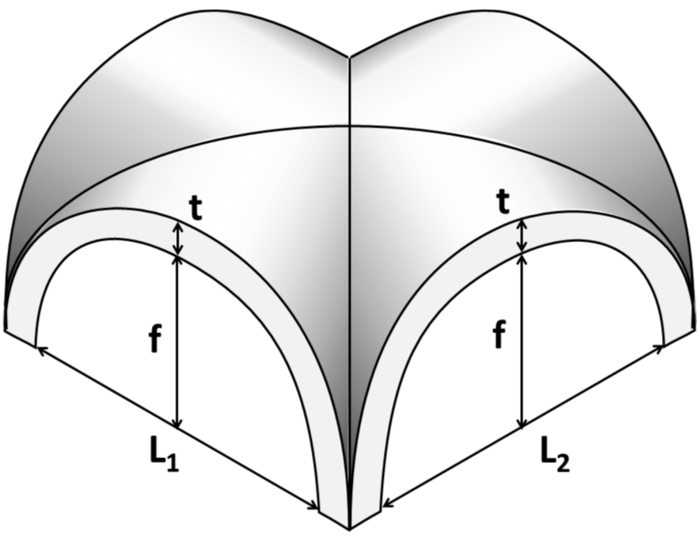


Figure 6.4: Dimensions of a general cross vault

The equivalent stiffness of a cross vault is calculated similarly to how it was calculated for the barrel vault. The main difference is that the vault has two curvatures in the two main directions. Therefore, the stiffness and shear modulus are calculated using similar formulas to the ones for the barrel vault but for both directions. For one direction the formulas are as shown below. To calculate the equivalent values for the other direction only the length L_1 needs to be replaced by L_2.

$$\frac{E_{v,1}}{E} = \left(a_5\,\frac{t}{L_1} + a_6\right) ln\left(\frac{f}{L_1}\right) + a_7\,\frac{t}{L_1} + a_8 \tag{6.5}$$

$$\frac{G_{v,1}}{G} = \left(b_5\frac{s}{L} + b_6\right)\left(\frac{f}{L}\right)^2 + \left(b_1\frac{s}{L} + b_2\right)\left(\frac{f}{L}\right) + b_3\frac{s}{L} + b_4 \tag{6.6}$$

The main dimensions from the formula are indicated in Figure 6.4. The a and b factors are the following:

- a_5, a_6, a_7 and a_8 are the calibrated factors of Cattari [35] and are the following: $a_5 = -0.91$, $a_6 = -0.1567$, $a_7 = 1.9133$ and $a_8 = 0.0721$
- In the same way b_1, b_2, b_3, b_4, b_5 and b_6 are factors that were calibrated and are the following: $b_1 = -1.12$, $b_2 = -4.1766$, $b_3 = 8.9033$, $b_4 = 0.7273$, $b_5 = -34.323$ and $b_6 = 5.6202$

6.2.2 FERT Slabs

The slabs on the first floor were built using the FERT-method. This means that the main elements are reinforced concrete beams holding up the secondary elements which are perforated clay bricks (see also Appendix E). The whole system is then connected with a layer of concrete covering the whole surface. In the interview with the construction engineer currently working on the project, they revealed that the usual way to model such slabs is to model them as a concrete slab with a thickness between 10 and 12 centimetres. In contrast to the usual concrete slabs, however, a main static span remains in which the loads are transferred to only two opposite walls. Since this slab was added to the structure in the late 20th century, the construction methods were not the same as during the original constructions. This means that the spanning directions of the slabs above the first floor probably do not correspond to the spanning directions of the vaults. It is more likely, since the concrete elements are likely to be prefabricated, that the spans were chosen in a way to have the same spans for every slab. For the Parish house, this would mean that on the east side of the building, the slabs span in an east-west direction (as does the vault), but it is more likely that on the west side of the building, the spanning direction is the same rather than in the north-south direction as the vaults.

6.2.3 Staircase vaults

As seen in Figure 2.5 of chapter 2 the staircase leading to the first and second floor is situated in the centre of the building. It has also been mentioned that the stairs going from the ground floor to the first floor are covered by a system of vaults that act at the same time as support for the stairs to the second floor. They consist of inclined barrel vaults at the entrance and exit of the stairs and inclined cross-vaults where the stairs take a turn. Since they are situated on a level between the rest of the floors of the structure and are furthermore inclined, it is decided not to model them in a detailed way but to simplify the model with respect to the real structure. Figure 6.5 below shows the simplification procedure: On the first image, the concerned walls are shown. It is to be noted that only the relevant parts for the staircase of walls 5 and 6 are shown (the plan view of all the walls with their numbers can be found in ...). In the centre image, a qualitative drawing of the vaults is provided where the previously described vaults are shown. Since these vaults are situated in between floors, it has been decided to represent them with equivalent slabs, using the previously described method (see the Chapter 6.2.1), with half of them situated at the first-floor level and the other half located on the second-floor level. This modeling strategy is visible in the right-hand drawing of Figure 6.5
After having modelled the staircase vaults as equivalent slabs it was important to fix the orthotropic membrane sections with all degrees of freedom (DOFs) fixed to the relevant floor nodes and likewise fix all the DOFs of the floor nodes to the equivalent wall nodes (also the torsional DOFs). Especially the ones connecting the torsional DOF of the staircase walls to the slabs were important to stop the walls from turning.

6.3 Reinforced concrete ring beam

As discovered in the discussion with the engineer working on the project [6], the addition of a ring beam made out of reinforced concrete, during the remodelling in 1997, added a remarkable amount of overall stiffness to the building and thus increased the box behaviour of the latter. Therefore, this ring beam must be modelled in OpenSees. The beam is assumed to have a square cross-section with the following properties:

- Height $h = 0.3\,m$ and width $b = 0.5\,m$
- Cross-section area $A = h \cdot b = 0.15\,m^2$
- Sectional moment of inertia around vertical axis $I_y = \frac{h \cdot b^3}{12} = 0.003125\,m^4$
- Sectional moment of inertia around horizontal axis $I_z = \frac{b \cdot h^3}{12} = 0.001125\,m^4$

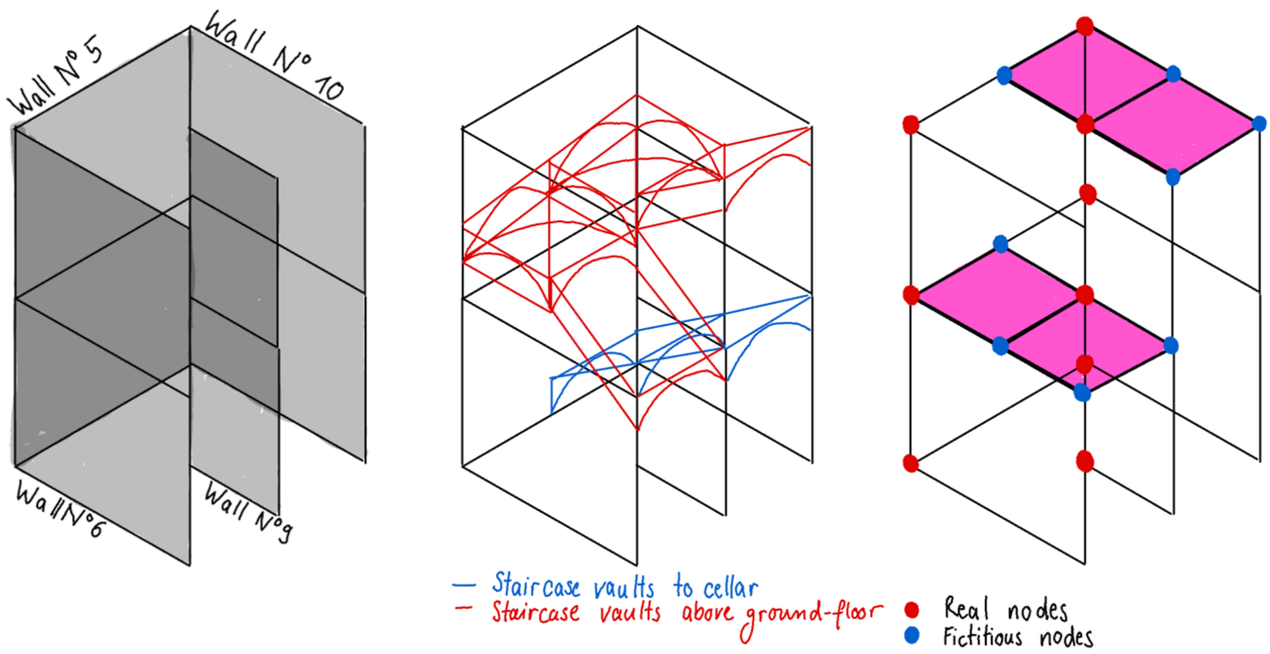


Figure 6.5: Modelling procedure of staircase vaults

- Torsional moment of inertia $J = \frac{b \cdot h \cdot (b^2+h^2)}{12} = 0.00425\, m^4$

The same properties defined in chapter 4.1.3 are also applied for the ring beam. The ring beam is modelled in a way to connect all the top nodes of the facades with each other. It is assumed that the connection between the masonry and the concrete is well-established and in good condition.

6.4 Supports

6.5 Connections

6.5.1 Wall-to-wall connections

6.5.2 Floor-to-wall connections

7. Analysis of the global behaviour

7.1 Pushover analysis (Resistance)

7.1.1 Methdology

The elaboration of the pushover curve tightly follows the procedure proposed by Lestuzzi et al. described in the book "Evaluation parasismique des constructions existantes" [21]. The first step is to identify the walls, i.e. the piers, that provide resistance against horizontal forces. For this calculation, the OOPs resistance of the walls is neglected. The piers defined in Chapter 6 are taken including the nodal panel, resulting in piers of full height in between windows. The resistance of the spandrels is neglected in this approach, they influence however the coupling of the walls. This is reflected in the choice of height between the base of the wall and the point of inflexion of the moment. For no coupling at all, this height corresponds to the total height of the wall for full frame effect this will correspond to half the height of the piers. In this case, the height to the point of inflexion is taken as 2/3 of the height of the wall, meaning there is some coupling but not a full frame behaviour.

Next, it is necessary to determine the normal load acting on each of the piers. This is done by calculating the slab load, as well as the load from the roof and the self-weight of the walls and applying them to the walls. The calculation can be found in the Excel file in Appendix xyz. The shear resistance of each pier can then be determined using the formulas proposed in [21] that are based on the Eurocode 1998-3 §11.4.1.1 [12]. Members failing in flexure have a resistance of:

$$V_{f,d} = \frac{V_{f,k}}{\gamma_{Rd}} = \frac{D\,N}{2\,H_0\,\gamma_{Rd}}(1 - \frac{N}{D\,t : f_{M,m}\,0.85}) \tag{7.1}$$

Similarly, members failing in shear have a resistance of*:

*It is expected that shear sliding failure is dominant over shear by diagonal cracking failure.

$$V_{sd} = \frac{V_{sk}}{\gamma Rd} = \frac{D'\,t}{\gamma_{Rd}}\,(f_{v0} + \frac{\mu\,N}{D'\,t}) \tag{7.2}$$

To determine D' the following formulas are used:

$$D' = 3\,D \cdot (\frac{1}{2} - \frac{M_u}{N\,D}) \tag{7.3}$$

$$M_u = \frac{\sigma_0\,D^2\,t}{2} \cdot (1 - \frac{\sigma_0}{0.85\,f_{Md}}) \tag{7.4}$$

$$\sigma_0 = \frac{N}{D\,t} \tag{7.5}$$

Where:

- D is the in-plane horizontal dimension of the wall (length of the piers)
- t is the thickness of the wall.
- D' is the depth of the compressed area at the end section of the pier.
- H_0 is the distance between the section where the flexural resistance is attained and the contra flexure point.
- N is the axial load at the end section.

- $f_{M,m}$ is the mean compressive strength of the masonry, i.e. the value of the compressive strength elaborated in Chapter 4.1.1 before applying the partial factor.
- μ is the masonry friction coefficient, which may be assumed equal to 0.5.
- f_{v0} is the shear strength in the absence of vertical load and equal to $f_{vm0} = 0.13\ N/mm^2$ as calculated in Chaper 4.1.1
- γ_{Rd} is the partial factor applied to the shear resistance as defined in Chapter 3.3.3. In the case of shear resistance in flexure $\gamma_{Rd} = 2.15$

Next, the yield drift can be calculated for each pier using formula (5.8) in [21]:

$$\delta_y = V_{Rd}(\frac{H \cdot (3\,H_0 - H)}{6 \cdot EI_{eff}} + \frac{6/5}{GA_eff}) \tag{7.6}$$

Considering also the drift limits defined in Chapter 4.1.1 the pushover curve for each pier can be established. Adding up the pushover curves of each floor's piers, a separate general pushover curve for each storey can be determined.

To determine the global, combined behaviour of the first floor on the ground floor when being pushed over together the following steps need to be applied:

1. In the first phase two loads are applied on the "equivalent wall" (see Figure 7.1) representing the behaviour of the building when pushed over. In this elastic phase, the forces pushing the wall over are proportional to the seismic mass of each floor, i.e. $F_1 = \alpha \cdot M_1/M_{tot}$ and $F_2 = \alpha \cdot M_2/M_{tot}$.
2. As soon as one wall yields this wall will continue to be pushed over without further increasing the forces F_1 and F_2, i.e. at a constant multiplier α. This means that only the yielding wall will continue to deform, whereas the other will remain at the same deformation.
3. When the yielding wall reaches its limit displacement the multiplier α decreases such that the shear in the yielding wall follows the pushover curve of that floor. This means that at the same time, the force on the wall that is not yielding will decrease, which in turn will decrease the deformation of that wall.
4. Finally the curve is continued until reaching a factor $\alpha = 0$.

It is to note that with this method any possible eccentricity of the centre of stiffness to the centre of mass of the building is neglected. Since the structure is fairly symmetric, this is a valid hypothesis.

7.1.2 Results

North-south direction

Applying the described procedure to the piers of the walls resisting in the north-south direction of the building, the pushover curve shown in Figure 7.2 can be established.

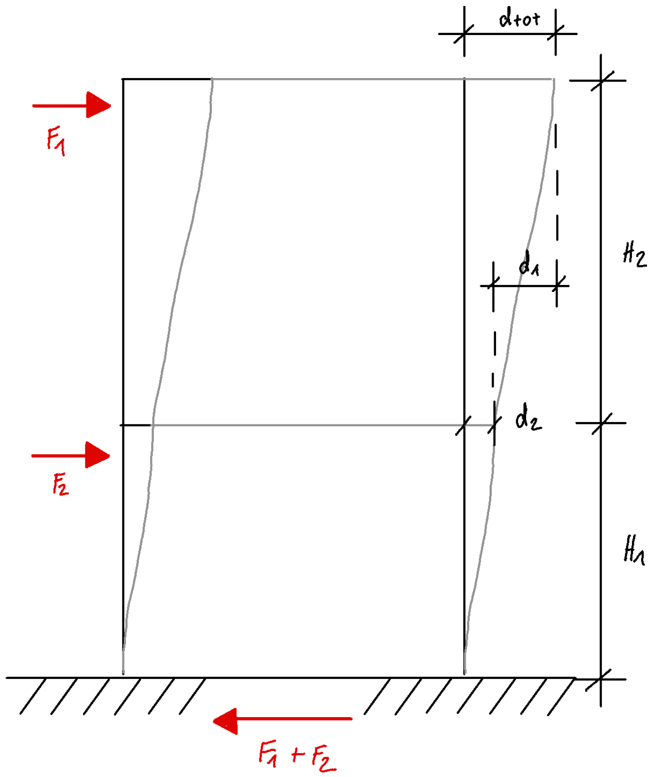


Figure 7.1: Representation of the pushover of the building as an equivalent wall

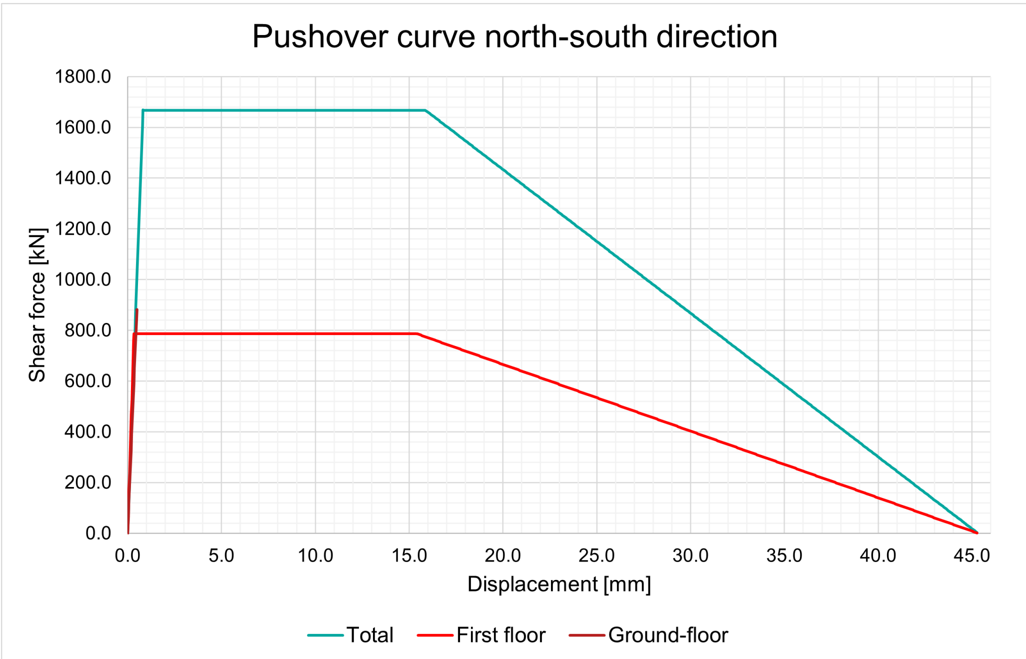


Figure 7.2: Pushover curve north-south direction

The following characteristic values for the global pushover behaviour of the wall can be determined from the calculation of the global pushover curve:

- The total shear resistance in north-south direction is $V_{Rd,ns} = 1669.5\,kN$.
- The elastic displacement when reaching the maximum shear resistance is $d_{y,ns} = 0.8\,mm$.
- The maximum displacement before the resistance starts to degrade is $d_{u,ns} = 15.9\,mm$.
- At 80 % resistance drop of at least one wall the shear resistance remains at $0.8 \cdot V_{Rd,ns} = 1335.6\,kN$.

- The displacement corresponding to a drop of 80 % in the resistance of at least one pier is of $d_{u2,ns} = 21.7\ mm$.

East-west diretion

Similar results can be obtained when applying the procedure to the piers in the east-west direction. The Pushover curve can be found in Figure 7.3

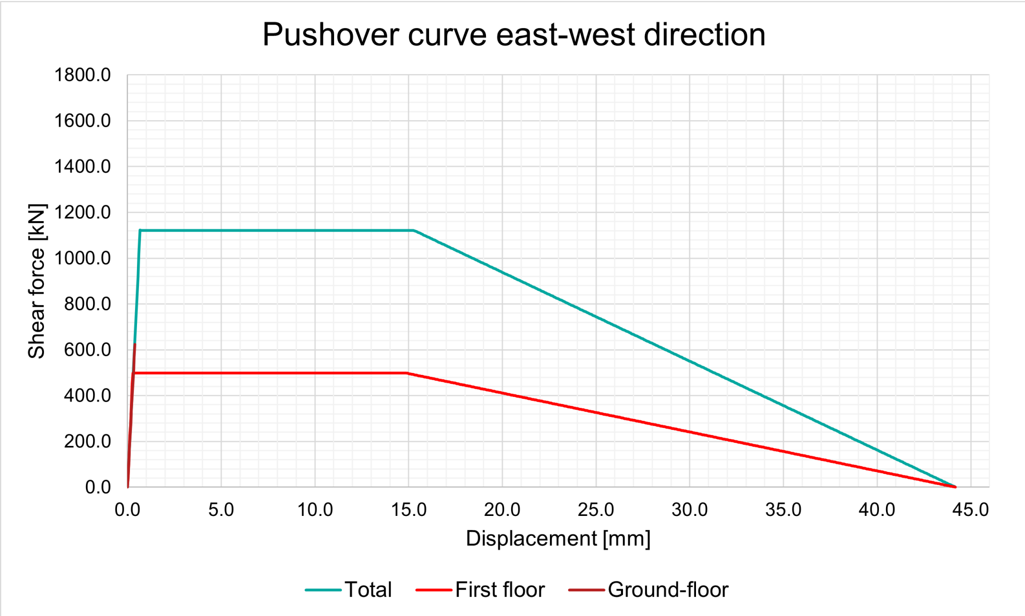


Figure 7.3: Pushover curve east-west direction

The following characteristic values for the global pushover behaviour of the wall can be determined from the calculation of the global pushover curve:

- The total shear resistance in north-south direction is $V_{Rd,ew} = 1122.2\ kN$.
- The elastic displacement when reaching the maximum shear resistance is $d_{y,ew} = 0.7\ mm$.
- The maximum displacement before the resistance starts to degrade is $d_{u,ew} = 15.3\ mm$.
- At 80 % resistance drop of at least one wall the shear resistance remains at $0.8 \cdot V_{Rd,ew} = 897.8\ kN$.
- The displacement corresponding to a drop of 80 % in the resistance of at least one pier is of $d_{u2,ew} = 21.0\ mm$.

7.1.3 Design limit for response history analysis

Having established the pushover curves in each direction of the building, the limit displacements for the DLs limit state can be calculated using formula 11.38 of EN 1998-3 [12]:

$$d_{DL} = d_y/\gamma_{Rd} \tag{7.7}$$

Where: $\gamma_{Rd} = 2.0$ as suggested by Table 11.9 EN 1998-3 [12].

For the verification of the NCs limit state the limit deformation is calculated using formula 11.37 EN 1998-3 [12]:

$$d_{NC} = d_{u2}/\gamma_{Rd} \tag{7.8}$$

Where: $\gamma_{Rd} = 1.9$ as suggested by Table 11.8 EN 1998-3 [12]. This results in the following limit values for each direction of solicitation and limit state:

Limit state / Direction	**North-south**	**East-west**
Damage limitation (DLs)	$d_{DL,ns}2 = 0.4\,mm$	$d_{DL,ew} = 0.4\,mm$
Near collapse (NCs)	$d_{NC,ns} = 11.4\,mm$	$d_{NC,ew} = 11.1\,mm$

Table 7.1: Limit displacements of the roof nodes

7.2 Response-histoy analysis (Solicitation)

7.3 Verification

8. Retrofitting

In the last chapter of this thesis, special attention is given to the future of the building. After the damages due to the earthquake in 2020, the building was classified as not safe to use anymore. Therefore, the community decided to use it as storage space for items not frequently needed, for which they did not need to enter the building frequently. However, before the seismic event, the building had been an important part of the community of St. Lawrence church and home to the church priest. It is therefore important to reestablish the safety of the building so that it can be used again and reinforce the structure so that during future earthquakes the building remains mostly undamaged or only endures small damages that are repairable.

8.1 Methodology

For the development of retrofitting procedures, a systematic approach as presented in the Seismic Engineering lecture at EPFL by Dr. Savvas Saloustros [36] based on the ISCARSAH Guidelines [37] is used. The procedure follows the same steps as modern medicine:

1. Anamnesis
2. Diagnosis
3. Therapy
4. Control

The initial step, anamnesis, entails recalling the historical context, past damage, construction history, structural alterations, and the structure's performance in previous events. This phase provides a comprehensive understanding of the structure's evolution and vulnerabilities.

Following anamnesis, the diagnosis phase involves assessing the current condition and pinpointing the root of the problem. This is achieved through in-situ inspection, material characterization, and structural analysis. Understanding the current state is crucial for formulating effective retrofitting strategies.

Once the issues are identified, the therapy phase comes into play. This step aims to address the root of the problem through the thoughtful design and application of interventions. This may include structural modifications, material reinforcements, or other engineering solutions tailored to the specific challenges revealed during the diagnosis.

Finally, control is an ongoing aspect of retrofitting. Monitoring the efficiency of the intervention is essential to ensure the good behaviour of the chosen retrofitting solutions. This involves employing various techniques to track changes in structural behaviour over time, ensuring that the implemented retrofitting measures remain effective and the structure remains resilient to potential risks.[37]

8.2 Important criteria

The ISCARSAH Principles [38] provide a set of criteria/rules to apply to ensure compatibility, effectiveness and historical preservation. The most important criteria that need to be fulfilled for a retrofitting procedure are as follows:

1. Each retrofitting procedure of a historical building is a multidisciplinary approach. Engineers, as well as architects, conservators, material scientists etc. are needed to understand every aspect of the structure and to tailor the measures to the project.

2. The retrofitting procedure should aim to safeguard the cultural and historical significance of the building as a whole and preserve its authenticity in a cultural context.
3. The short- and long-term effects of any intervention should be evaluated before its application. It is a good idea that interventions can be removed or undone in case there will be more appropriate methods in the future.
4. A scientific methodology, as proposed in chapter 8.1 should be applied to get an objective understanding of the problem and apply the right remedies.

8.3 Anamnesis

In the past, the Parish House underwent various challenges, notably during the 20th-century war and the earthquake in 2020. War-induced damage primarily affected the roof and possibly the second-floor slab, as documented in Chapter 2.1. The lightweight roof acted as a shield during bombings, protecting the main structure from the impacts. Therefore, only the roof was damaged and needed replacement. Reconstruction efforts after the war, finished in 1997, included the addition of a reinforced concrete (RC) ring beam and a fert slab (see also in chapter 2.1), to replace the damaged slab and enhance the resistance of the building. Furthermore, the entire roof structure needed to be rebuilt, which was done using timber beams to create a lightweight solution and non-structural masonry walls to separate the rooms. Finally, some new, non-structural walls were added on the second-floor level to create more bathrooms, but they are assumed to be non-structural walls. It is assumed that adding those non-structural elements did not change the structural system and behaviour of the building.

The 2020 earthquake caused severe damage to the structural elements, but the main structure remains mostly intact and without collapse. The seismic action caused structural damage, particularly to walls and vaults. Diagonal shear cracks have mainly been observed in the north and south facades and on inside walls with the same orientation. This phenomenon can be explained by the lower stiffness of the building in the east-west direction (due to the shorter walls), compared to the north-south direction. Some shear cracking can however also be observed in the spandrels of the east facade. In the east and west facades the main damages come however from horizontal cracks due to some global OOP failures as described in Chapter 2.4. All in all, the building behaved very robustly and the damage caused does not pose an immediate threat to the integrity of the building.

The FERT slabs, as well as the ring beam, show no signs of decay or damage, meaning they behaved well under the 2020 earthquake. It can be assumed that the FERT slab is well linked to the ring beam and acts as a rather stiff diaphragm holding the top of the walls together. Their addition certainly increased the box behaviour of the building and could have been one of the main reasons for the robust behaviour of the building.

8.4 Diagnosis

As mentioned in the previous chapter the building has undergone severe damage due to the seismic action but the structure remains intact under gravity loading. A lot of shear cracks, as well as horizontal cracks, could be observed that alter the rigidity and horizontal resistance of the walls. Those damages are likely to lead to problems during the occurrence of another seismic event: Due to the reduced stiffness and resistance to lateral loads, new in-plane loads will potentially be redistributed to other undamaged parts of the building, possibly damaging those as well. Furthermore, the cracks lead to a separation of the wall into blocks, enabling all sorts of OOP mechanisms that were previously not possible. Damaged vaults now only require the reactivation of the already-formed pins during the next earthquake, which needs significantly less energy. Another problem lies in the previously activated OOP facade mechanisms: The cracking pattern shown in Chapter 2.4 shows that the mechanism did to some degree happen and move the walls out of their respective planes. This means that those walls probably have a slight residual OOP displacement. Due to the residual displacement, the thrust of the vaults will already be increased, knowing that it is proportional to the square of the span and inversely proportional to the vault's height (see Chapter 2.2.2). Therefore, this increased vault span makes the walls more vulnerable to (re-)activating the OOP mechanisms in the next earthquake.

8.5 Therapy

8.5.1 "Scuci-cuci"

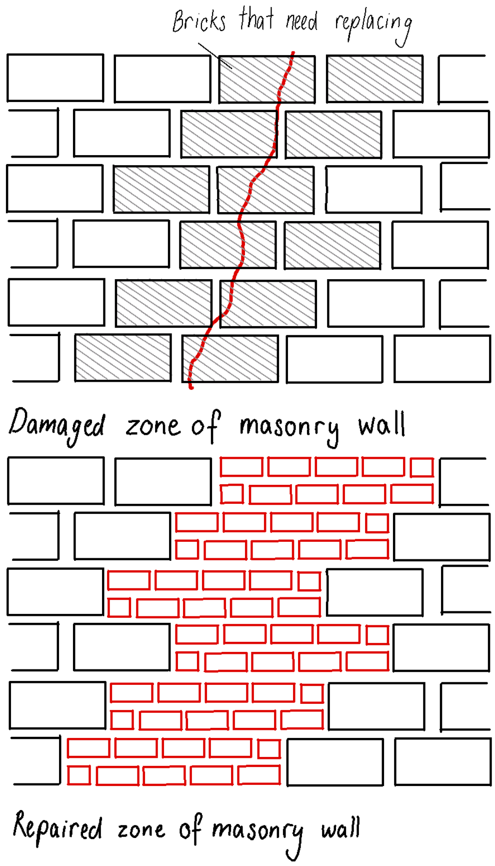


Figure 8.1: "Scuci-cuci" method

The so-called "scuci-cuci" method, i.e. "unstitch-stitch", aims to repair a damaged wall by replacing the cracked elements with new bricks and mortar. This method is used only locally to recover heavy damages, i.e. cracks that pass through the bricks themselves. The purpose is not to strengthen the masonry and increase its stiffness but to reestablish continuity and connection between the bricks and to restore the wall to its initial state. To be effective and compatible with the rest of the structure, those materials should be similar to the original ones in terms of shape, dimensions, stiffness and strength. The best is to reuse the removed material if some of it is still intact (i.e. bricks that are uncracked). If this is not possible an extensive analysis of the mechanic, chemical and physical characteristics of the masonry is needed to select compatible substitution materials. In walls where only one leaf is damaged and the crack does not pass through to the other side, it is also possible to only replace the damaged parts. The procedure consists of the following steps: Before starting the reparation work, it is essential to prop up the structure in order to redirect the downward movement of the loads to relieve the area to be replaced. Then, the construction workers will start scraping out the mortar between the bricks that need replacing. Once the bricks are "freed" it is usually possible to pull them out manually. If the thickness of the wall or the loads are more elevated, this step can be done employing hydraulic jacks welded to steel sections. After this step, the new bricks and mortar are introduced from the bottom up, striving for the most penetration as possible. Before carefully removing the props, the curing process should be checked. If the mortar is not solid enough this could lead to settlements in the whole structure otherwise. After the phase of settling and adjusting of the wall the area between the new and old masonry is sealed with more mortar. [39]

Regarding the ISCARSAH Principles mentioned in Chapter 8.2 it would be discouraged to intervene in such an invasive and irreversible manner. However, after considering alternative approaches, it was determined during the research procedure that these interventions would be even less effective at returning the structure to its original state of stiffness and strength while also being much more invasive (see for example jacketing or other methods proposed in [39]).

In the case of the Parish house, most of the cracked zones need to be repaired using this method: Zones pervaded by cracks in multiple directions will need to be replaced entirely (as seen in Figure 8.2), whereas walls that are only damaged by specific isolated cracks can profit of local substitution only in the zone of the crack (for example Figure 8.2).

(a) Specific single crack (b) Entirely cracked zone

Figure 8.2: Two different zones of cracking

8.5.2 Repointing

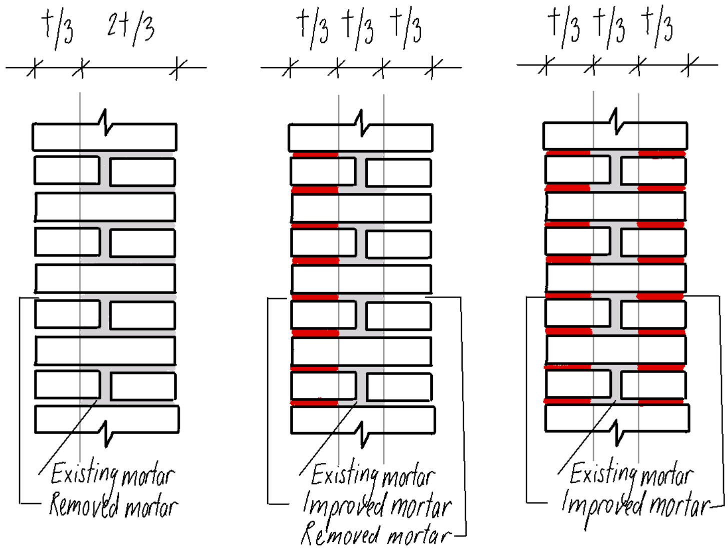


Figure 8.3: Repointing method

One method that is frequently used in various types of masonry is deep repointing. This procedure should be used if the deterioration is limited to the mortar. It entails partially replacing the mortar joints with higher-quality mortar in order to improve the mechanical properties of the masonry. It is most effective when applied to both sides of a masonry wall. The masonry resistance to both vertical and horizontal loads can be strengthened by this method, but only by a considerable amount if a significant percentage of the initial weak mortar can be replaced. The best results, however, are shown in terms of rigidity, which increases substantially due to the confinement effect of the joints. The fact that deep repointing's effectiveness is highly dependent on the quality of the on-site application could be a drawback. Since the mechanical properties of the mortar are strongly dependent on the ratio of the mixture, they might vary considerably from batch to batch if the ratio of water to powder is not exactly the same. Furthermore, it is crucial to replace the entire volume of the ablated mortar to ensure a good connection between the single bricks. [39]

The treatment works especially well on walls that are less than 60 centimetres thick [39]. Although the wall thicknesses of the Parish House are frequently more than 60 centimetres, this does not automatically rule out the technique. In this instance, the technique is applied more to repair than to strengthen the structural element. Particular cases are shown in Figure 8.4 below. It should be applied as a main repair method to cracked walls where the cracks remain within the mortar part of the masonry and not damage the bricks (where cracks remain horizontal or follow the mortar line). In zones where cracks pass through bricks, the method should be applied in addition to the replacement of the damaged bricks ("Scuci-cuci" see Chapter 8.5.1

(a) Crack GF 21 (b) Crack FF 23

Figure 8.4: Zones where repointing could be efficient

8.5.3 Steel ties against vault thrust

9. Further investigations

10. Conclusion

11. Use of AI and data availability

12. Acknowledgements

Bibliography

[1] Robbie Andrew. Global CO2 emissions from cement production, September 2022.

[2] Zaid Faridi. The Chemistry Behind Concrete.

[3] Francesco Vanin, Andrea Penna, and Katrin Beyer. A three-dimensional macroelement for modelling the in-plane and out-of-plane response of masonry walls. Earthquake Engineering & Structural Dynamics, 49(14):1365–1387, November 2020.

[4] Eduardo Miranda, Svetlana Brzev, Nenad Bijelic, Željko Arbanas, Marko Bartolac, Vedran Jagodnik, Damir Lazarević, Snježana Mihalić Arbanas, Sonja Zlatović, Andrés Acosta, Jorge Archbold, James Bantis, Jovana Borozan, Ivana Božulić, Nikola Blagojević, Cristian Cruz, Héctor Dávalos, Erica Fischer, Selim Gunay, Marijana Hadzima-Nyarko, Pablo Heresi, Dimitrios Lignos, Ting Lin, Marko Marinković, Armando Messina, Sebastian Miranda, Alan Poulos, Giulia Scagliotti, Ingrid Tomac, Igor Tomic, Katerina Ziotopoulou, Željko Žugić, and Ian Robertson. Petrinja, Croatia December 29, 2020, Mw 6.4 Earthquake Joint Reconnaissance Report (JRR). Report, ETH Zurich, January 2021. Accepted: 2021-01-25T06:19:22Z Publication Title: Joint Reconnaissance Report Volume: PRJ-2959.

[5] Ivan Rizmaul. Obnova sakralnik objekata zupe svetog Lovre u Petrinji 1995 -2015.

[6] Martina Vujasinović and Hrvoje. Meeting with the engineer working on the project of the parish house in Petrinja, November 2023.

[7] USGS. M 6.4 - 2 km WSW of Petrinja, Croatia, March 2021.

[8] Eurozox. Warehouse spuž | Eurozox.

[9] Dina D'Ayala and Elena Speranza. Definition of Collapse Mechanisms and Seismic Vulnerability of Historic Masonry Buildings. 2003.

[10] Dina D'Ayala and Elena Speranza. An Integrated Procedure for the Assessment of Seismic Vulnerability of Historic Buildings. January 2002.

[11] European comitee for Standardization (CEN). Eurocode 1998: Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings, 2022.

[12] European comitee for Standardization (CEN). Eurocode 1998: Design of structures for earthquake resistance - Part 3: Assessment and retrofitting of buildings and bridges (Draft), 2023.

[13] European comitee for Standardization (CEN). Eurocode 1990: Basis of structural design, 2002.

[14] European comitee for Standardization (CEN). Eurocode 1991: Actions on structures - Part 1-1: General actions - Densities, self-weight, imposed loads for buildings, 2002.

[15] European comitee for Standardization (CEN). Eurocode 1992: Design of concrete structures - Part 1-1: General rules and rules for buildings, 2011.

[16] European comitee for Standardization (CEN). Eurocode 1996: Design of masonry structures - Part 1-1: General rules for reinforced and unreinforced masonry structures (Final draft), 2021.

[17] Aline Bönzli and Caroline Heitmann. Jupyter Notebook for OOP calculations, June 2023.

[18] Microsoft Corporation. Microsoft Excel, November 2023.

[19] Inc The MathWorks. MATLAB, August 2023.

[20] OpenSees - Open System For Earthquake Engineering Simulation, September 2023.

[21] Pierino Lestuzzi and Marc Badoux. Évaluation parasismique des constructions existantes: bâtiments en maçonnerie et en béton armé. Presses polytechniques et universitaires romandes, Lausanne, 2013.

[22] NTC 2008 e Circolare - Norme sismiche per le costruzioni.

[23] Guido Magenes and Andrea Penna. EXISTING MASONRY BUILDINGS: GENERAL CODE ISSUES AND METHODS OF ANALYSIS AND ASSESSMENT. Eurocode 8 Perspectives from the Italiand Standpoint Workshop, pages 185–198, 2009.

[24] Paulo B. Lourenço and Angelo Gaetani. Finite element analysis for building assessment: advanced use and practical recommendations. Routledge, Abingdon New York, NY, 2022.

[25] SIA 261. SIA 261 Actions sur les structures porteuses, January 2014.

[26] University of Zagreb Department of Geophysics, Faculty of science, Allegretti I, Herak D., Ivancic I., Kuk V., Maric K., Markusic S., and Sovic I. Maps of seismic areas of the Republic of Croatia.

[27] Osnovna geološka karta Republike Hrvatske, 2012.

[28] Lucia Luzi, Giovanni Lanzano, Chiara Felicetta, Maria Clara D'Amico, Emiliano Russo, Sara Sgobba, Francesca Pacor, and ORFEUS Working Group 5. Engineering Strong Motion Database (ESM), version 2.0. July 2020. Medium: text/html,application/json,image/jpeg,application/vnd.fdsn.mseed,text/plain,application/zip,text/plain,text/x Publisher: Istituto Nazionale di Geofisica e Vulcanologia (INGV) Version Number: 2.0.

[29] Bodenbeläge - Parkett | Gebhardt Holz-Zentrum GmbH.

[30] swisspor. EPS Roof Toiture avec pente intégrée standard. Publisher: MASSIVE ART WebServices GmbH.

[31] M. Griffith, Guido Magenes, GIAMMICHELE MELIS, and LUIGINO PICCHI. Evaluation of out-of-plane stability of unreinforced masonry walls subjected to seismic excitation. JOURNAL OF EARTHQUAKE ENGINEERING, 7:141–169, January 2003.

[32] Sergio Lagomarsino, Andrea Penna, Alessandro Galasco, and Serena Cattari. TREMURI program: An equivalent frame model for the nonlinear seismic analysis of masonry buildings. Engineering Structures, 56:1787–1799, November 2013.

[33] S. Bracchi, M. Rota, A. Penna, and G. Magenes. Consideration of modelling uncertainties in the seismic assessment of masonry buildings by equivalent-frame approach. Bulletin of Earthquake Engineering, 13(11):3423–3448, November 2015.

[34] E. Quagliarini, G. Maracchini, and F. Clementi. Uses and limits of the Equivalent Frame Model on existing unreinforced masonry buildings for assessing their seismic risk: A review. Journal of Building Engineering, 10:166–182, March 2017.

[35] S. Cattari and S. Resemini & S. Lagomarsino. Modelling of vaults as equivalent diaphragms in 3D seismic analysis of masonry buildings. In Structural Analysis of Historic Construction: Preserving Safety and Significance, Two Volume Set. CRC Press, 2008. Num Pages: 8.

[36] Katrin Beyer and Savvas Saloustros. CIVIL 522 - Seismic Engineering, in the Master classes of Civil Engineering at EPFL, 2023.

[37] ISCARSAH Guidelines. Technical report, 2005.

[38] ISCARSAH Principles. Technical report, 2003.

[39] Deliverable 3.2 - Critical review of retrofitting and reinforcement techniques related to possible failure. WORKPACKAGE 3: Damage based selection of technologies, 2010.

[40] Beam and block floor slab system, May 2020. Section: Revit Structure Forum.

Glossary

ADRS acceleration-displacement response spectrum. 36

DL damage-limitation. 19, 24, 38–42, 44, 55, 56

EFM equivalent frame model. 5, 19, 46, 48

FEM finite element model. 48

MDOF multi-degree-of-freedom. 35

NC near-collapse. 19, 24, 38–42, 44, 55, 56

OOP out-of-plane. 5, 19, 20, 28, 30, 31, 36, 38, 40, 43–47, 52

RC reinforced concrete. 6, 41, 44

RHA response history analysis. 19, 46

SDOF single-degree-of-freedom. 19, 35, 36

A. Parish documentation

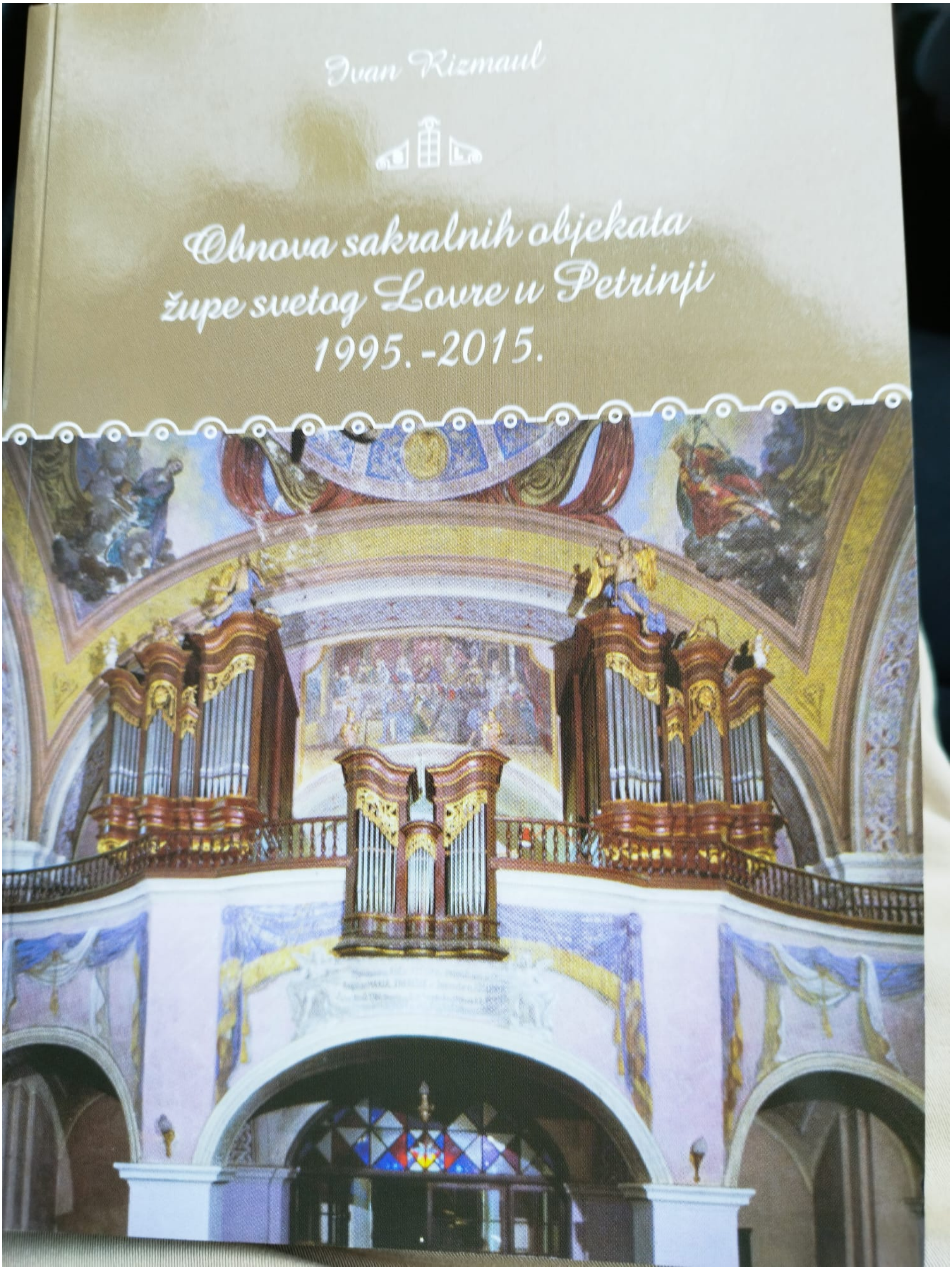


Figure A.1: Title Image

grijanja u župnom dvoru. Instalacije za centralno grijanje izveo je majstor Jakov Damjanović. Ta novčana sredstva nisu bila dovoljna za vanjsko uređenje *fasade* pa su skupljani dobrovoljni prilozi župljana i konačno je obnova završena 1986. Iste godine, šest godina nakon podnošenja zamolbe i godinu dana nakon uplate, u župni dvor je uveden i telefon. Tadašnji petrinjski župnik Valent Posavec je, pored brojnih zaslužnih ljudi i dobrotvora prigodom obnove *fasade* posebno zahvalio: Dragi Panjkretu, Alojziju Rimcu, Mati Damjanoviću, Josi Damjanoviću, Marku Josiću, Roku Vrbancu, Ivanu Beloševiću, Nikoli Jurkoviću, Dragi Draškoviću i Ilji Orlovcu. Nakon obnovljene župne crkve središte Petrinje bilo je uljepšano novim izgledom župnog dvora. Župni dvor je oštećen tijekom neprijateljske okupacije (1991.-1995.).

Postavljanje deke na župni dvor, 19. srpnja 1996.

Župni dvor, 8. kolovoza 1996.

Postavljanje betonske deke na župnom dvoru.

Postavljanje krovišta na župnom dvoru, rujan 1996.

Župni dvor, veljača 1997.

Pročelje crkve Sv. Lovre Petrinja u izgradnji, srpanj 1997.

123

Figure A.2: Page about Parish house

B. Earthquake documentation USGS

USGS
science for a changing world

Earthquake Shaking **Orange Alert**

USAID FROM THE AMERICAN PEOPLE
GSN ANSS

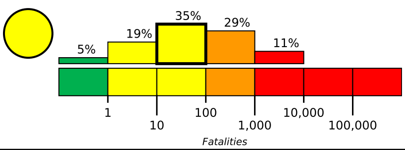
PAGER Version 10

M 6.4, 2 km WSW of Petrinja, Croatia

Origin Time: 2020-12-29 11:19:54 UTC (Tue 12:19:54 local)
Location: 45.4244° N 16.2573° E Depth: 10.0 km

Created: 9 weeks, 3 days after earthquake

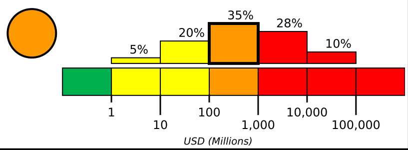
Estimated Fatalities



Orange alert for economic losses. Significant damage is likely and the disaster is potentially widespread. Estimated economic losses are 0-3% GDP of Croatia. Past events with this alert level have required a regional or national level response.

Yellow alert for shaking-related fatalities. Some casualties are possible.

Estimated Economic Losses



Estimated Population Exposed to Earthquake Shaking

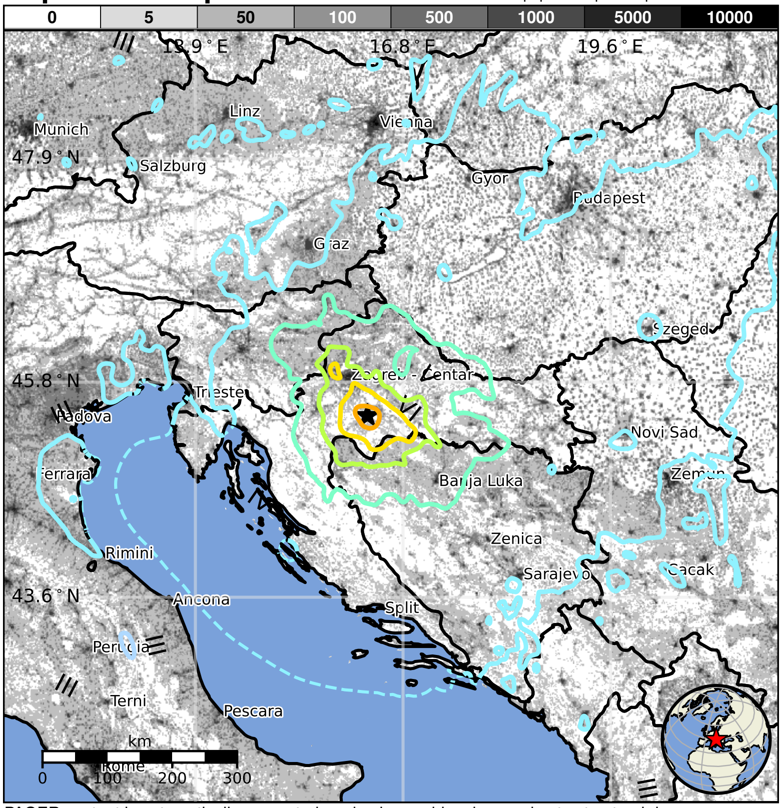
ESTIMATED POPULATION EXPOSURE (k=x1000)		–*	44,046k*	22,187k	2,135k	1,064k	474k	54k	0	0
ESTIMATED MODIFIED MERCALLI INTENSITY		I	II-III	IV	V	VI	VII	VIII	IX	X+
PERCEIVED SHAKING		Not felt	Weak	Light	Moderate	Strong	Very Strong	Severe	Violent	Extreme
POTENTIAL DAMAGE	Resistant Structures	None	None	None	V. Light	Light	Moderate	Mod./Heavy	Heavy	V. Heavy
	Vulnerable Structures	None	None	None	Light	Moderate	Mod./Heavy	Heavy	V. Heavy	V. Heavy

*Estimated exposure only includes population within the map area.

Population Exposure

population per 1 sq. km from Landscan

0	5	50	100	500	1000	5000	10000



PAGER content is automatically generated, and only considers losses due to structural damage.
Limitations of input data, shaking estimates, and loss models may add uncertainty.
https://earthquake.usgs.gov/earthquakes/eventpage/us6000d3zh#pager

Structures

Overall, the population in this region resides in structures that are vulnerable to earthquake shaking, though resistant structures exist. The predominant vulnerable building types are mud wall with wood and unreinforced brick with mud construction.

Historical Earthquakes

Date (UTC)	Dist. (km)	Mag.	Max MMI(#)	Shaking Deaths
1998-04-12	222	5.6	V(9k)	0
1997-09-26	379	5.7	VII(2k)	14
1976-05-06	254	6.5	IX(55k)	965

Recent earthquakes in this area have caused secondary hazards such as landslides that might have contributed to losses.

Selected City Exposure

from GeoNames.org

MMI	City	Population
VIII	Petrinja	14k
VIII	Sisak	36k
VII	Budasevo	2k
VII	Oresje	1k
VII	Pokupsko	<1k
VII	Zapresic	18k
III	**Munich**	**1,260k**
III	**Rome**	**2,319k**
III	Belgrade	1,274k
III	**Budapest**	**1,741k**
III	**Vienna**	**1,691k**

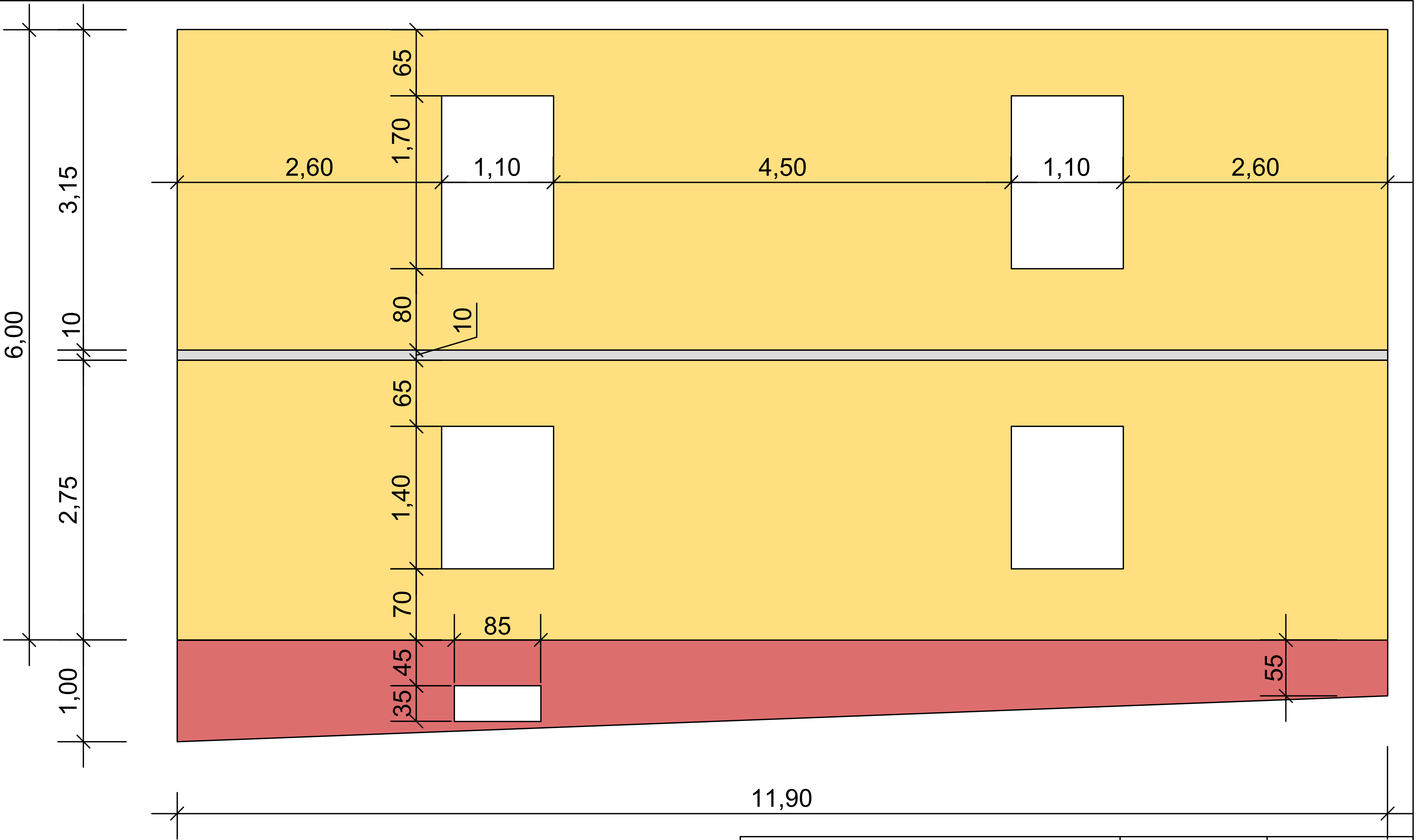
bold cities appear on map. (k = x1000)

Event ID: us6000d3zh

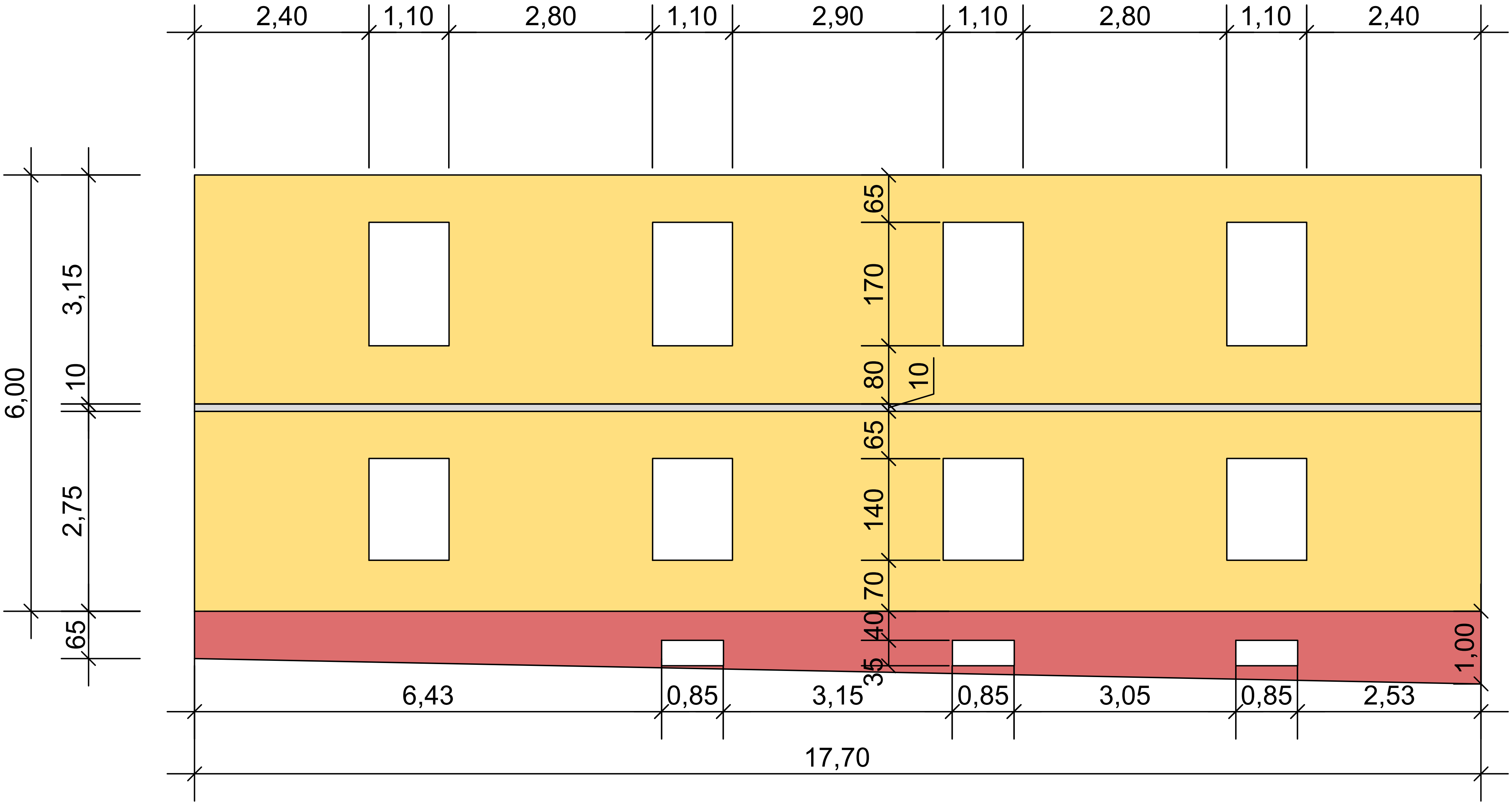
Figure B.1: USGS Earthquake documentation Petrinja 29.12.2020 [7]

C. Plans

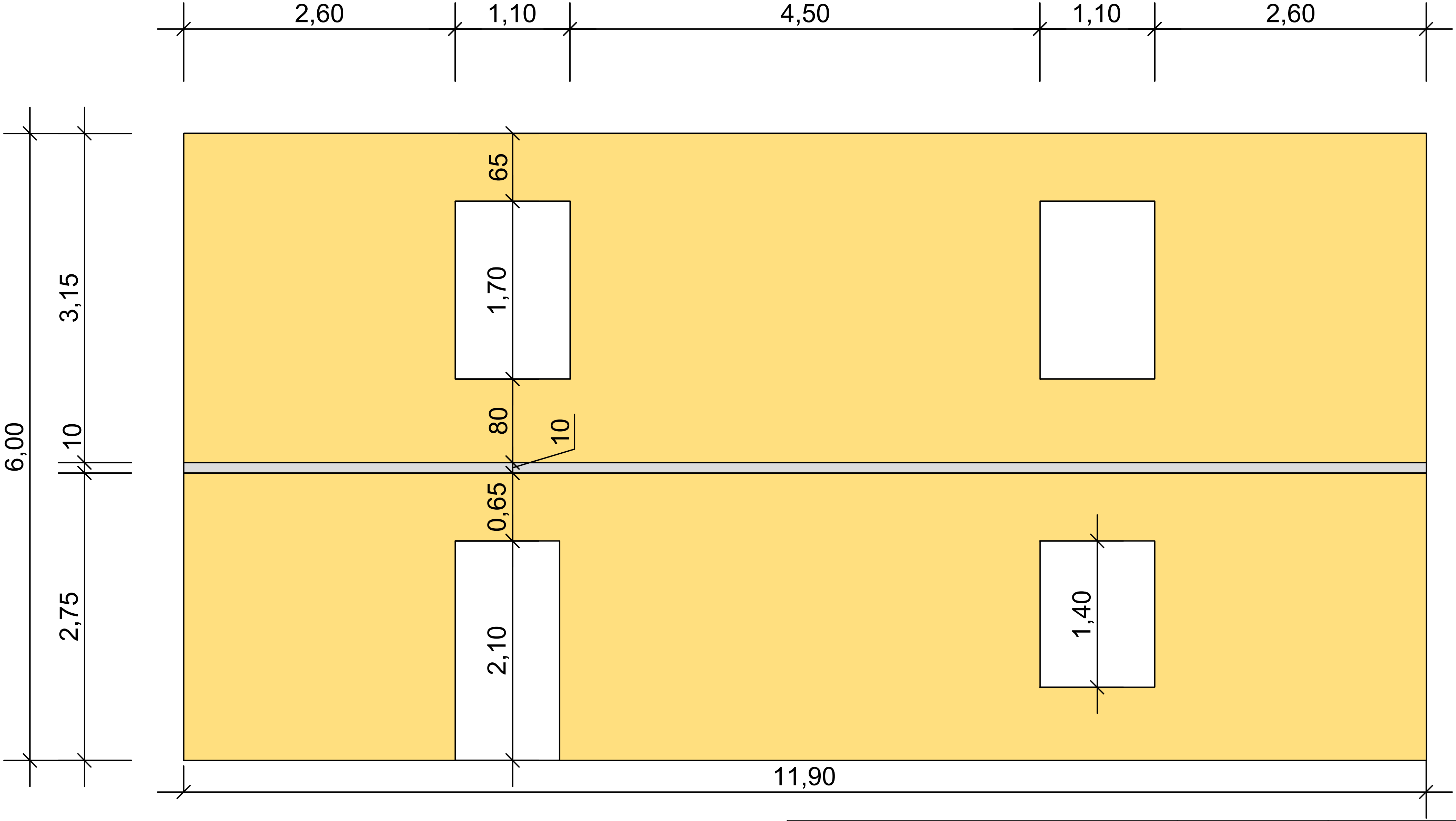
The main task during the site visit in April 2023 was to measure and document the geometry of the building itself. Since those measurements were not done by a professional the plans will therefore have an exactitude of +/- 5 centimeters. This should however be exact enough to give a good indication about the seismic behaviour of the building.



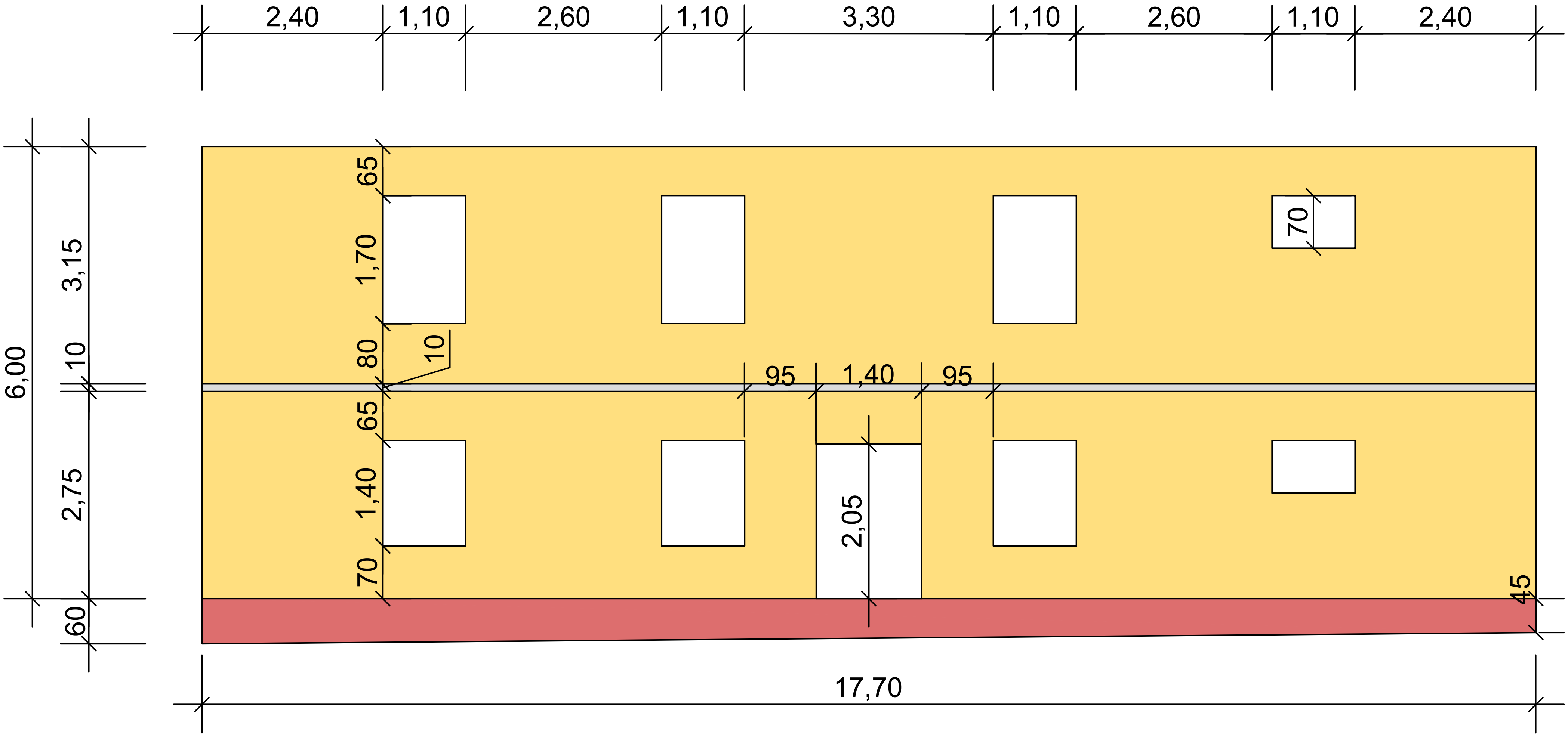
Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
	Crkva sv. Lovre	Country :	Croatia
EESD	Norh Facade	Date :	20.05.2023
		Scale :	1:50



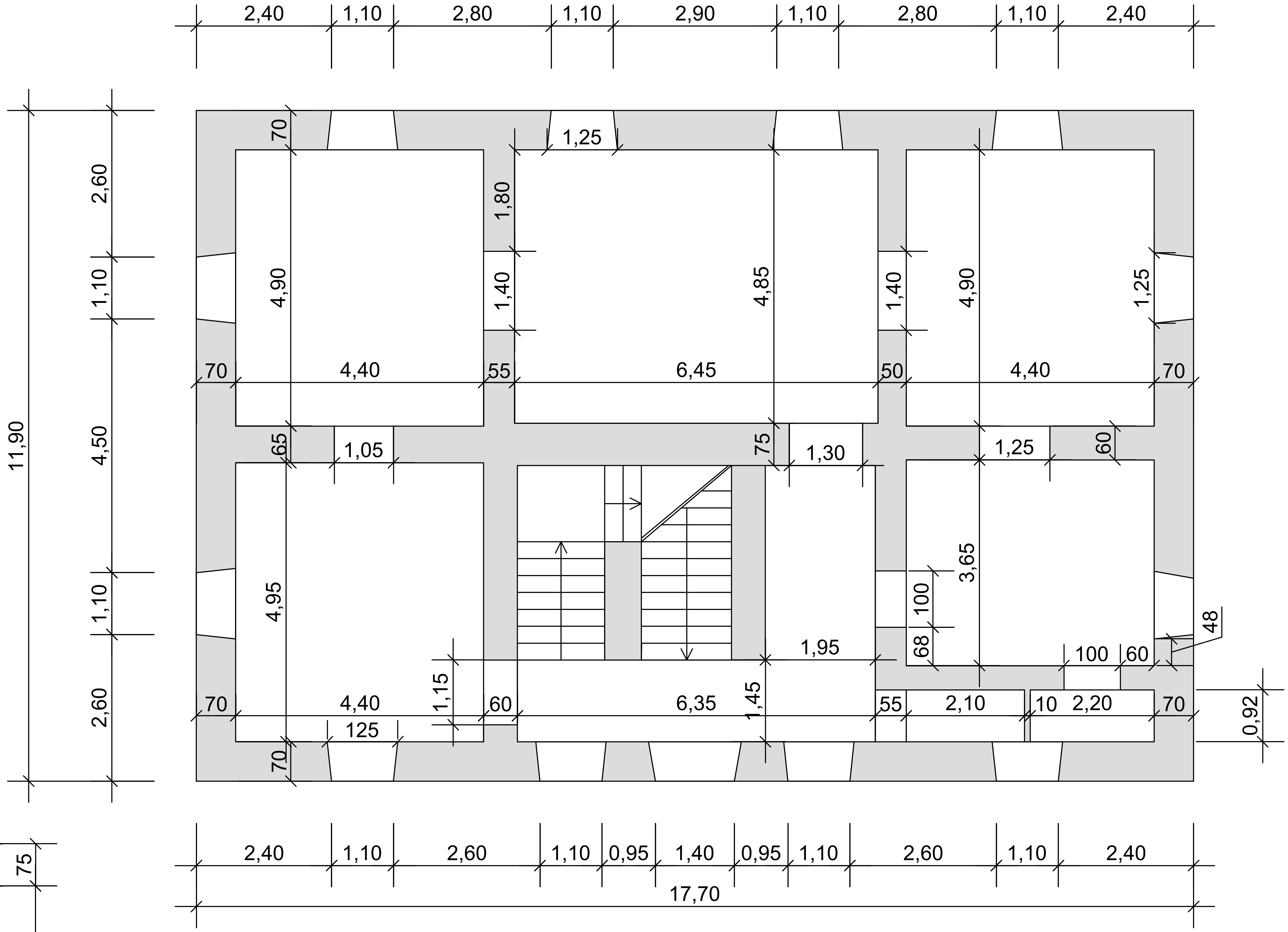
Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
	Crkva sv. Lovre	Country :	Croatia
EESD	East Facade	Date :	20.05.2023
		Scale :	1:75



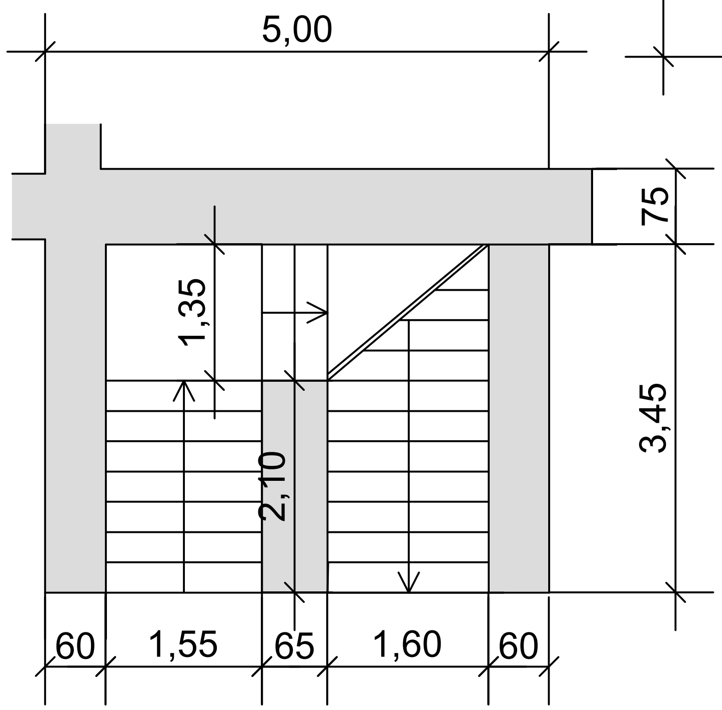
Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
	Crkva sv. Lovre	Country :	Croatia
EESD	South Facade	Date :	20.05.2023
		Scale :	1:50



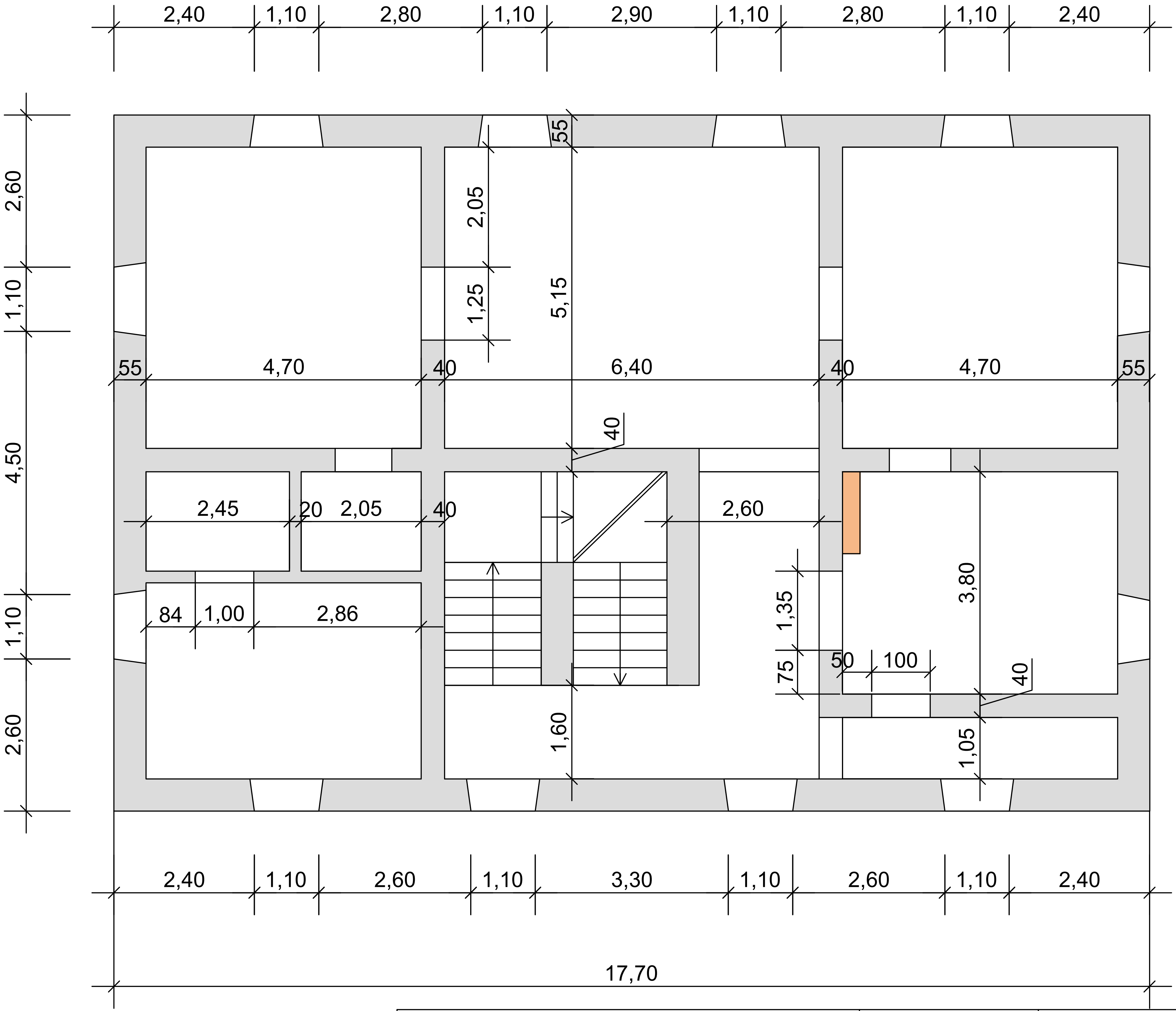
Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
	Crkva sv. Lovre	Country :	Croatia
	West Facade	Date :	20.05.2023
		Scale :	1:75



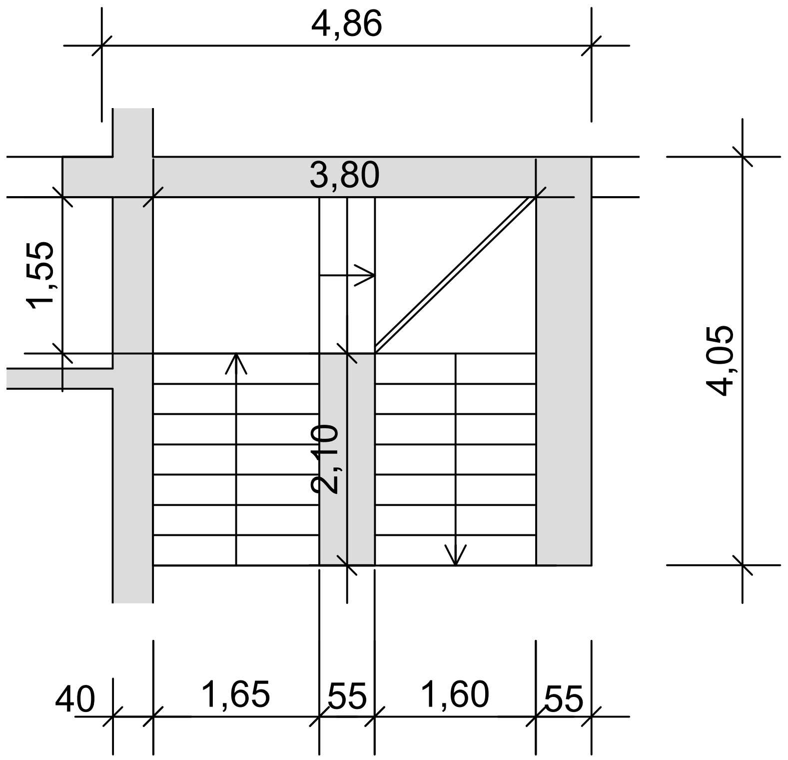
Stair case detail (S 1:75)



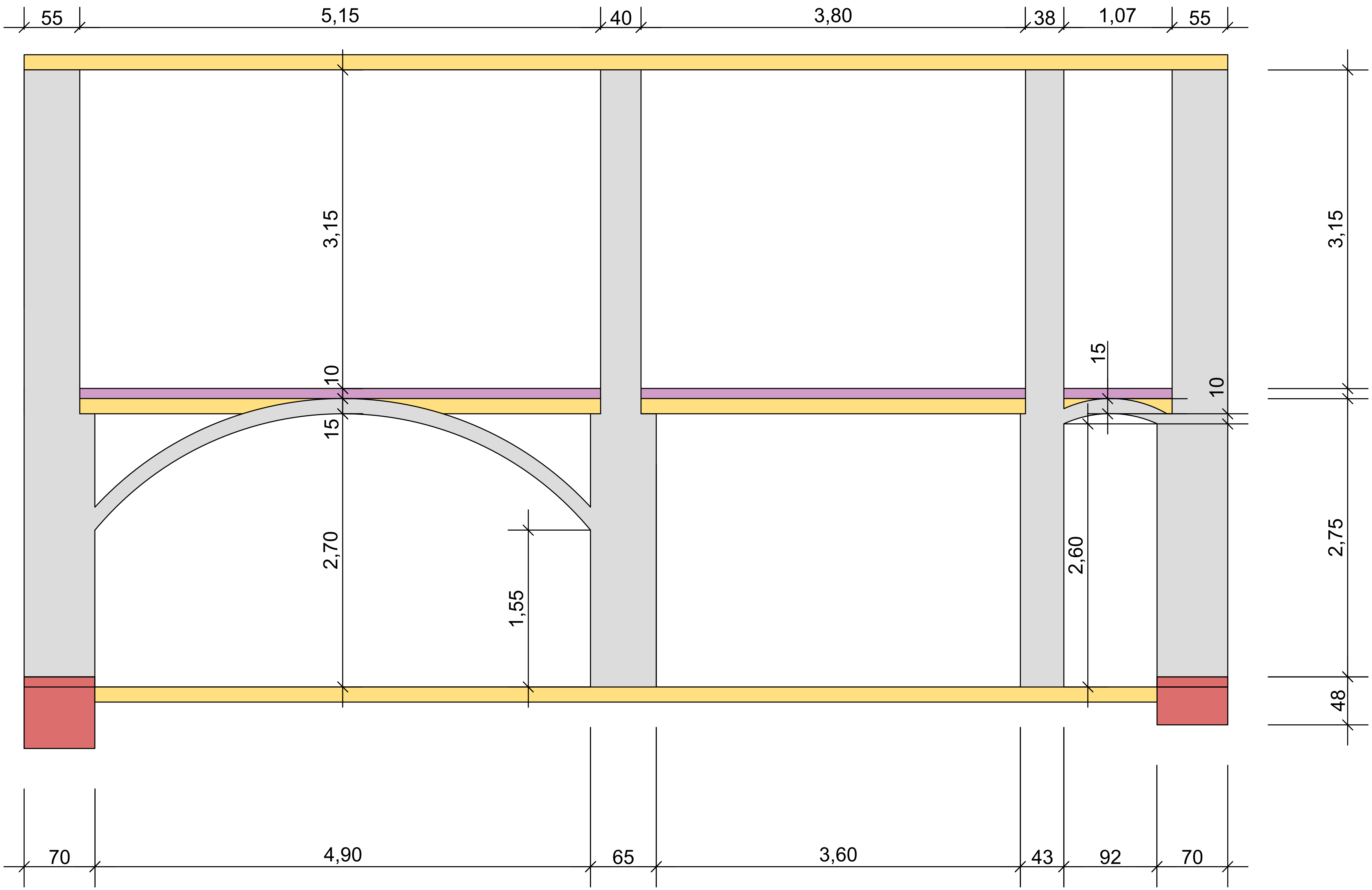
Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
EESD	Crkva sv. Lovre	Country :	Croatia
	Ground Floor, Plan view	Date :	25.05.2023
		Scale :	1 : 75



Stair case detail (S 1:75)



Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
EESD	Crkva sv. Lovre	Country :	Croatia
	Ground Floor, Plan view	Date :	25.05.2023
		Scale :	1:75



Parish House		Author :	Aline Bönzli
		Municipality :	Petrinja
	Crkva sv. Lovre	Country :	Croatia
EESD	E-W (South),	Date :	01.06.2023
	Cross-section	Scale :	1:40

D. Crack inventory

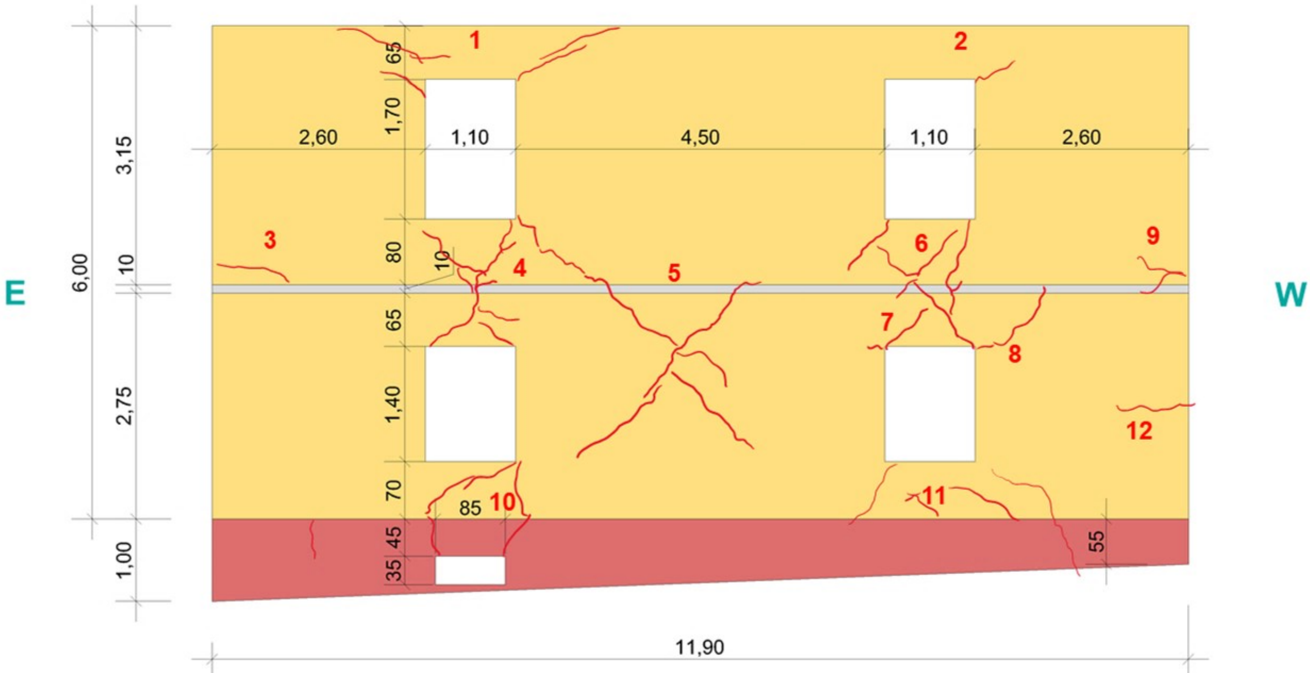
During the site visit in April 2023, the building was thoroughly documented through photos and scans (using the MatterPort system on a smartphone and a tripod). Using these images, the phases of the crack documentation were the following:

1. Draw all visible cracks of the facade or interior wall on a picture.
2. Transfer the cracks to AutoCAD plans and elevations.
3. Number the cracks.
4. Describe the cracks and location and their associated mechanism in an Excel sheet.
5. Give the crack a distinct label (add an acronym before the number to distinguish between different facades and floors, e.g. GF = ground floor, FF = first floor etc.).
6. Highlight the mechanisms by connecting the cracks that are the result of the same failure.

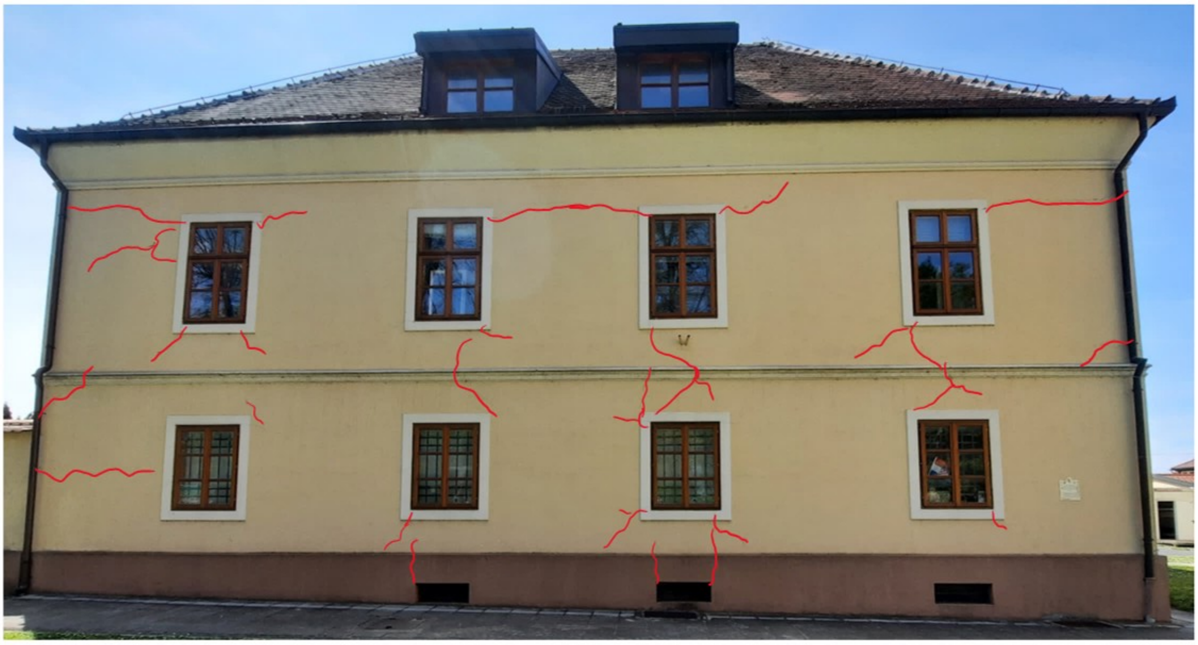
D.1 Visual documentation

North Facade

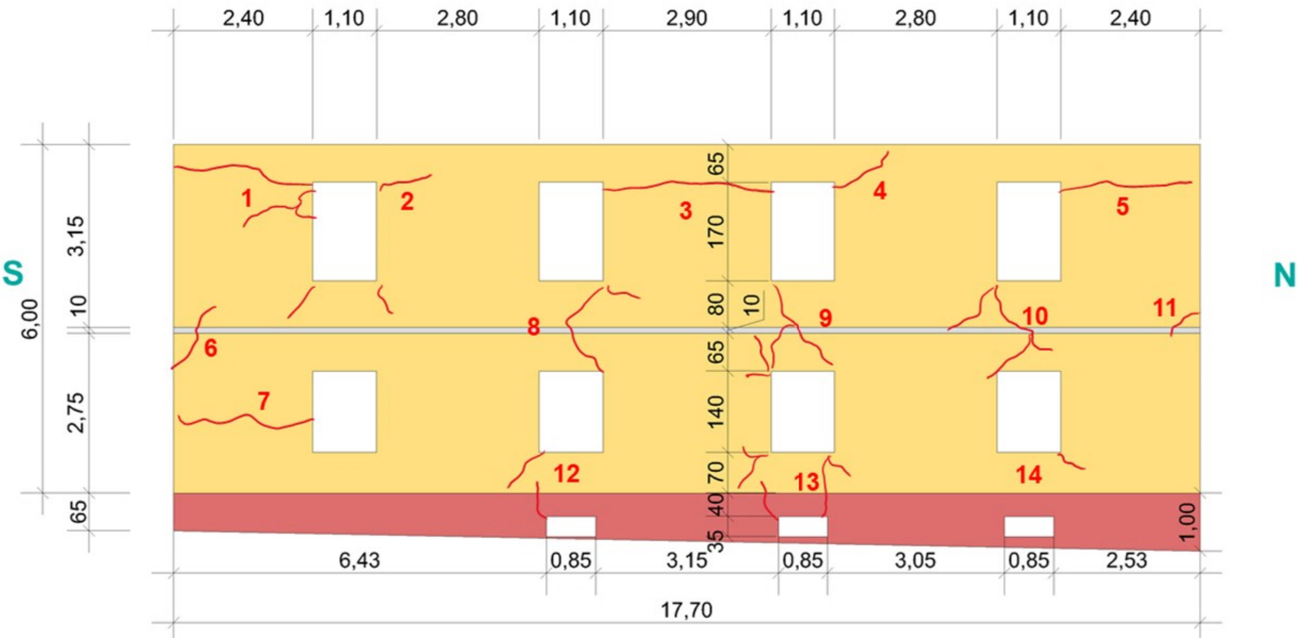
North Facade



East Facade

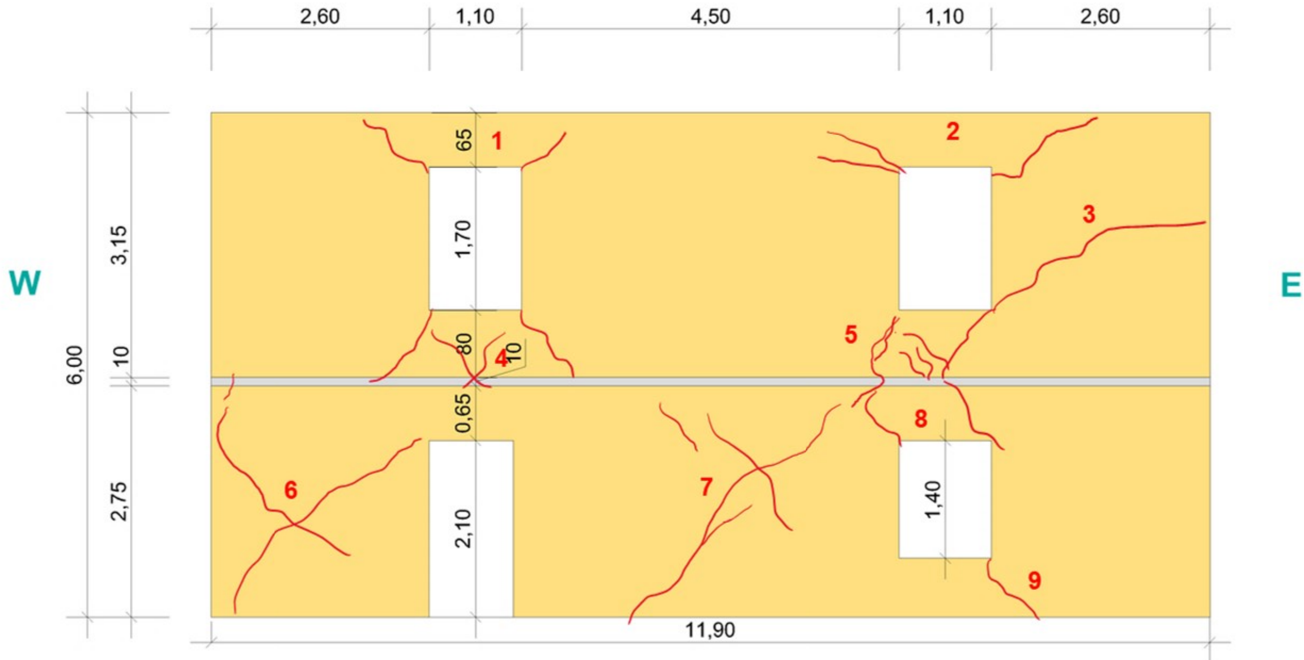


East Facade



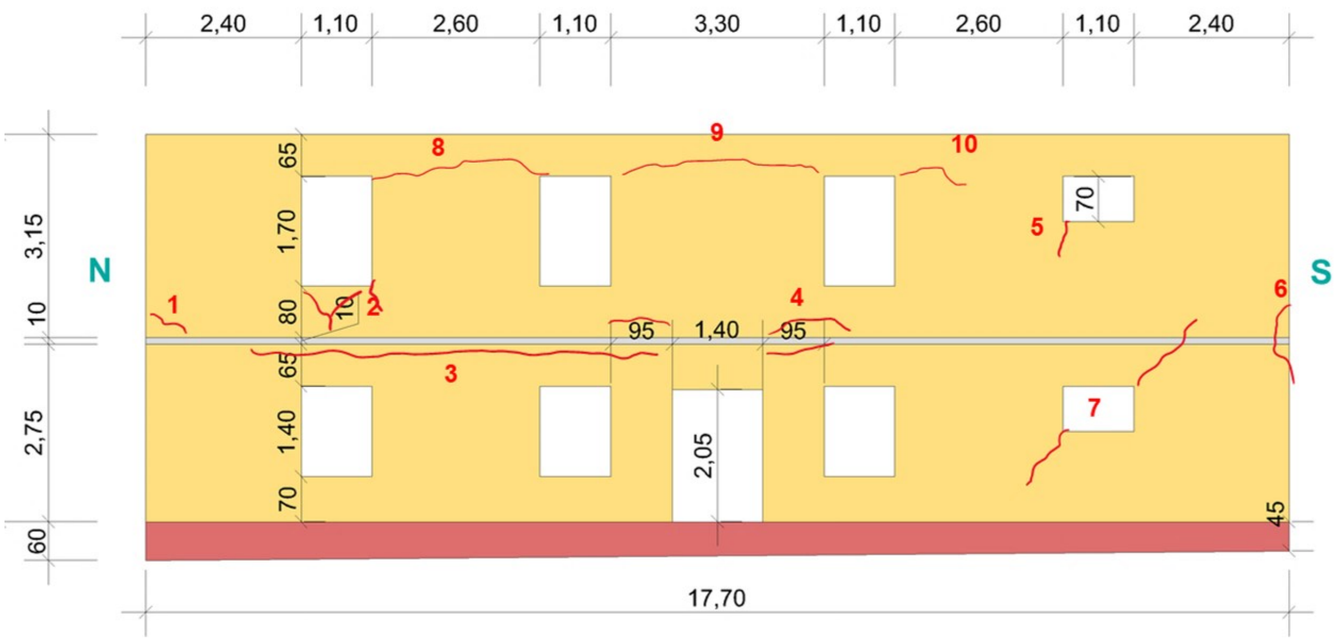
South Facade

South Facade

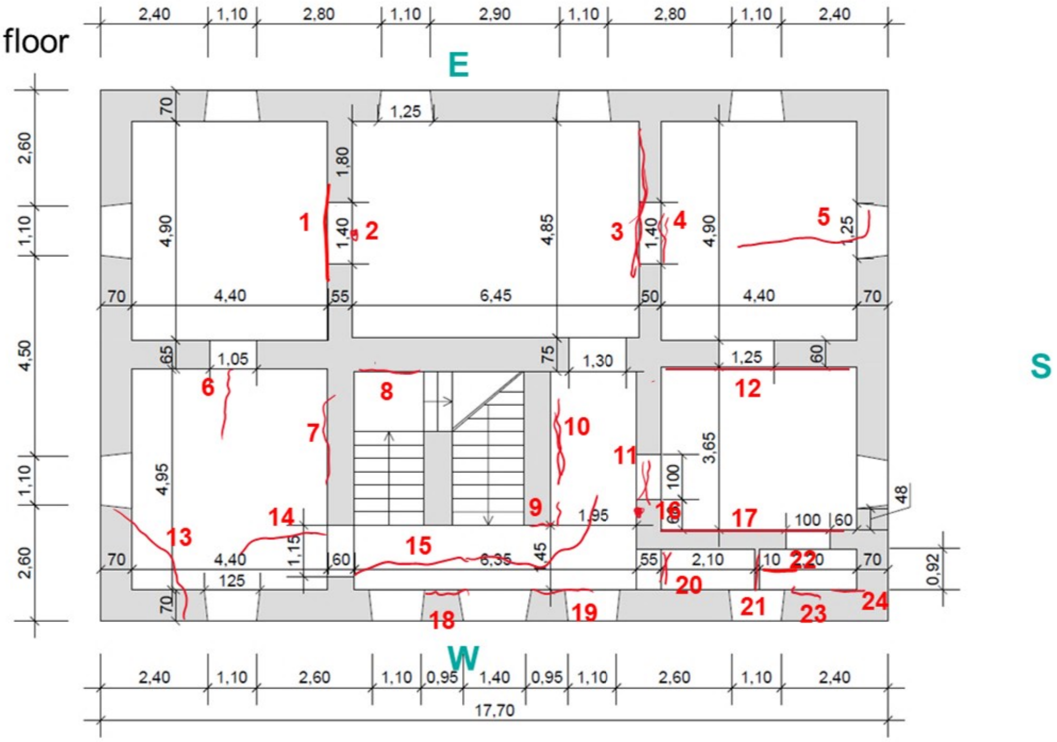


West Facade

West Facade

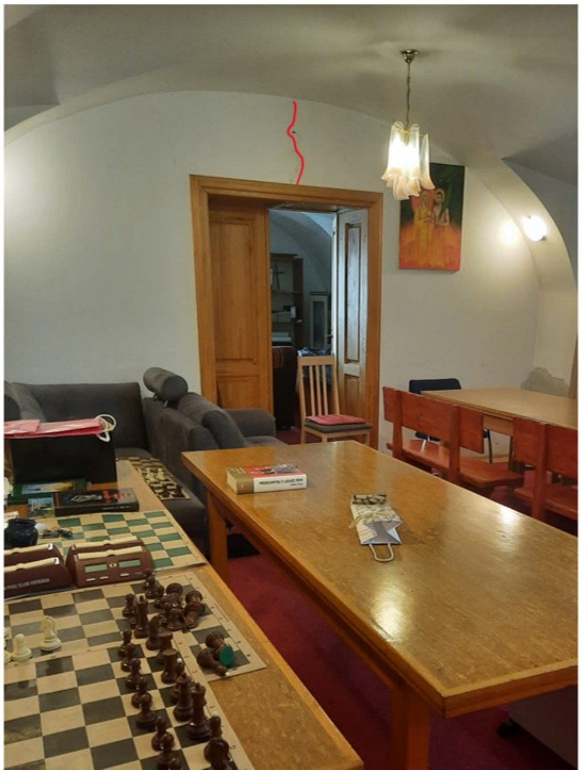


Ground floor



N° 1

N° 2



N° 3

N° 4

N° 5

N° 6

N° 7

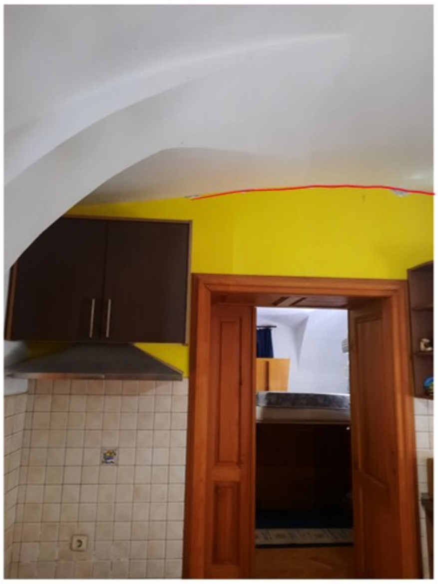
N° 8

N° 9

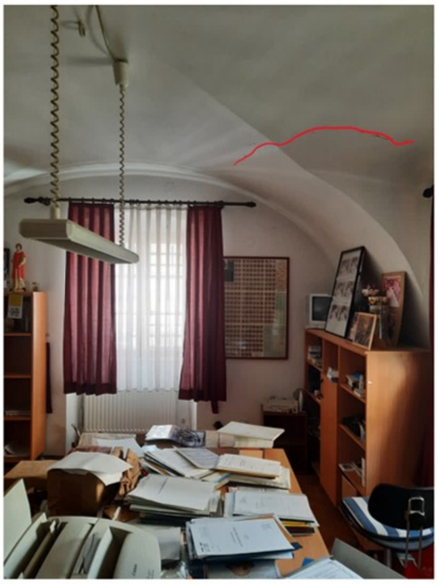
N° 10

N° 11

N° 12



N° 13



N° 14

N° 15

N° 16

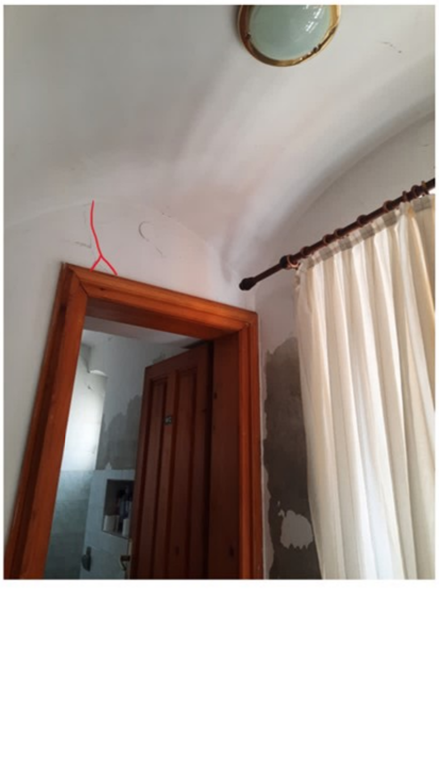
N° 17

N° 18

N° 19 Top

N° 19
Bottom

N° 20



N° 21

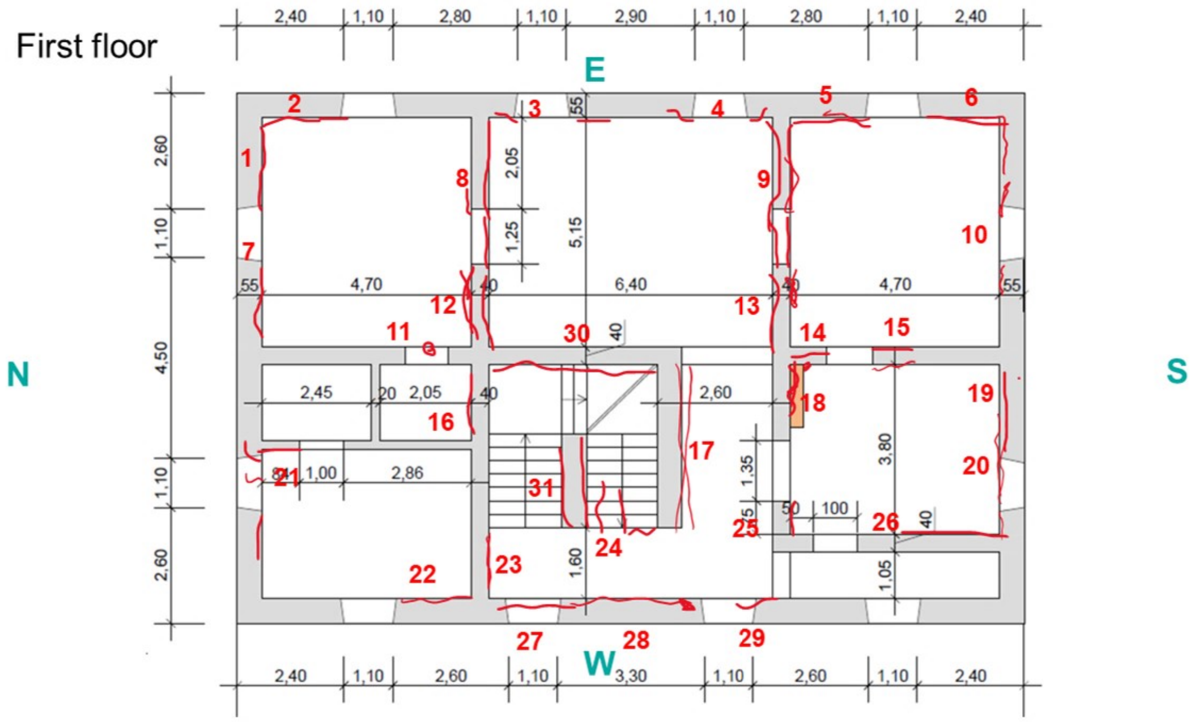


N° 22

N° 23

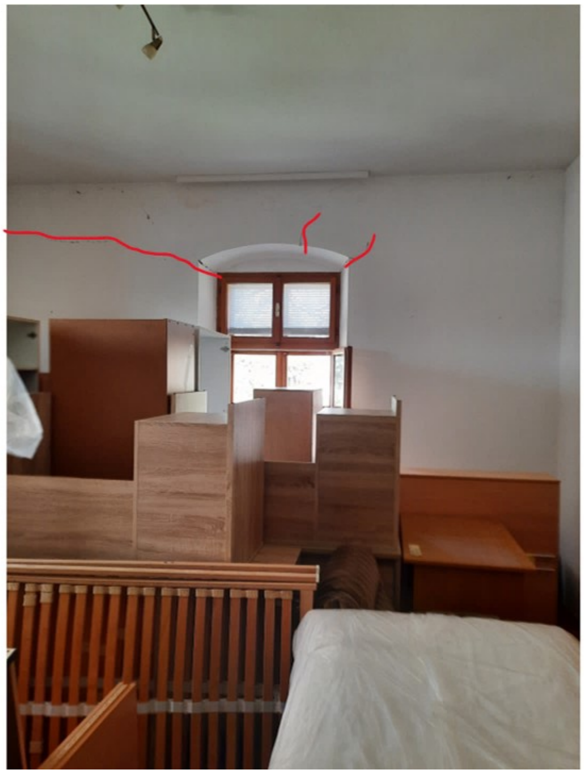
N° 24

First floor



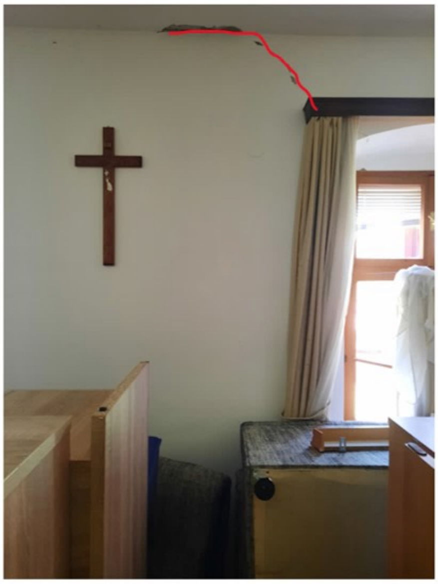
N° 1

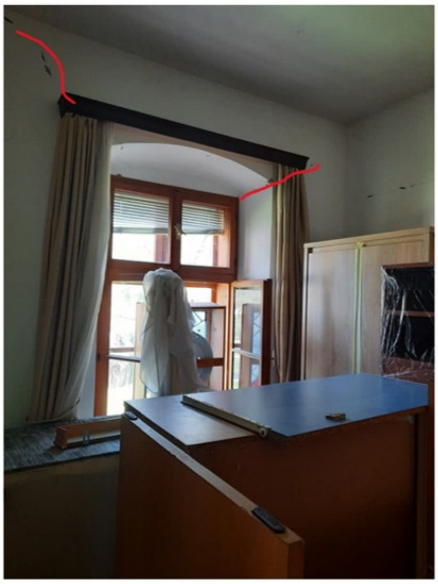
N° 2



N° 3

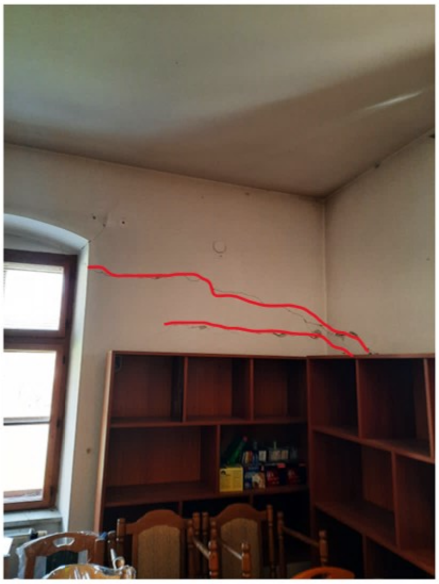
N° 4





N° 5

N° 6

N° 7

N° 8

North side

South side

N° 9

North side

South side

N° 10

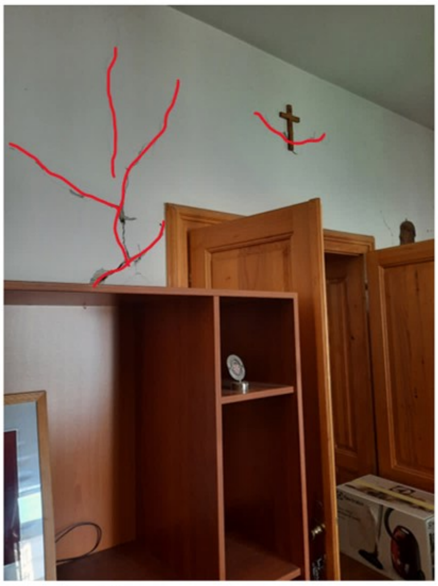
N° 11

N° 12

North side

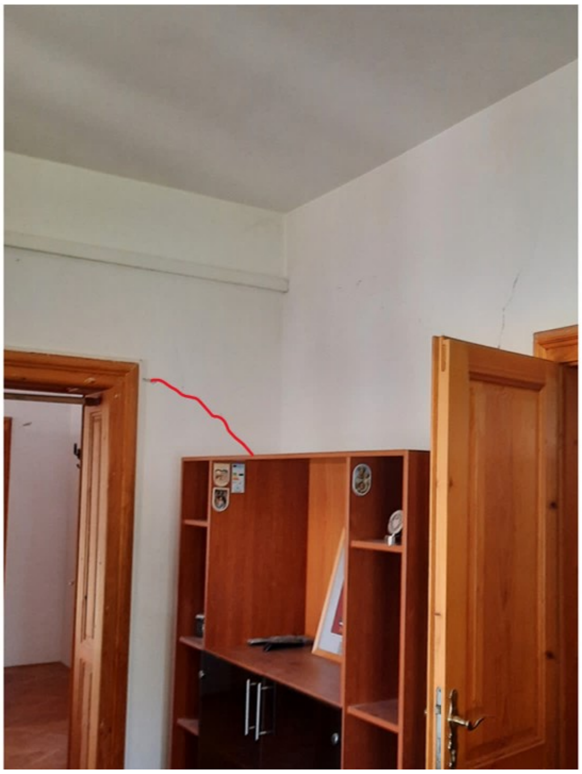
N° 13

North side



South side

N° 14



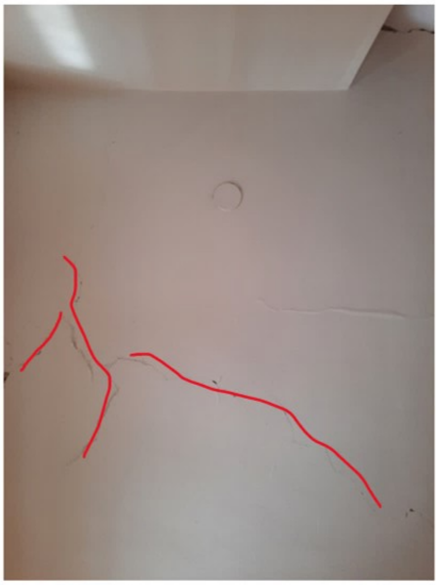
N° 15

East side

West side

N° 16

North side



Sourth side

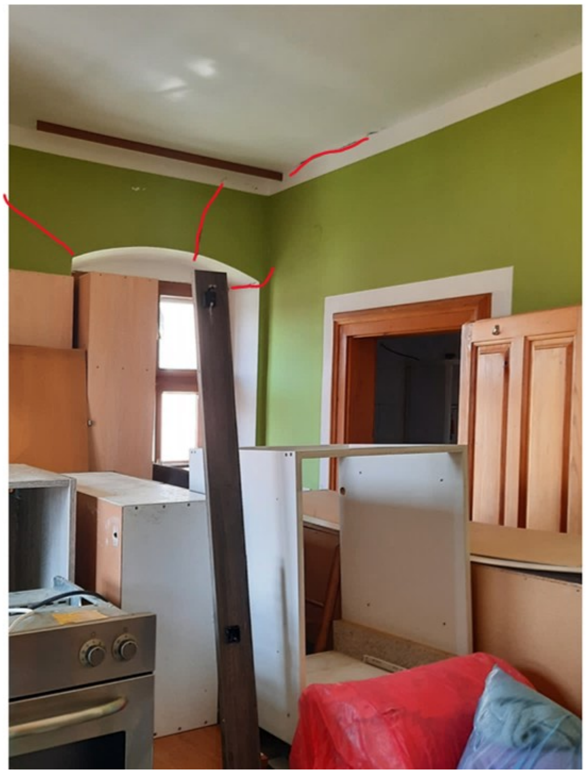
N° 17

N° 18

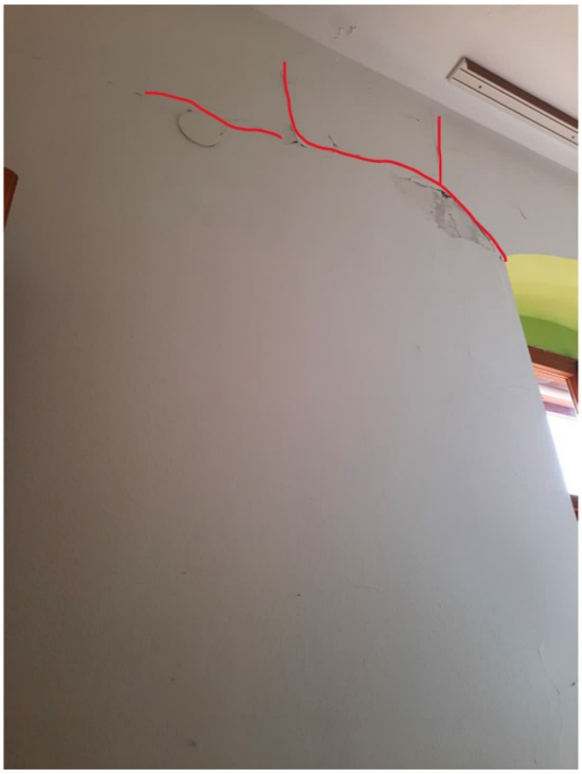
N° 19

N° 20

N° 21



N° 22



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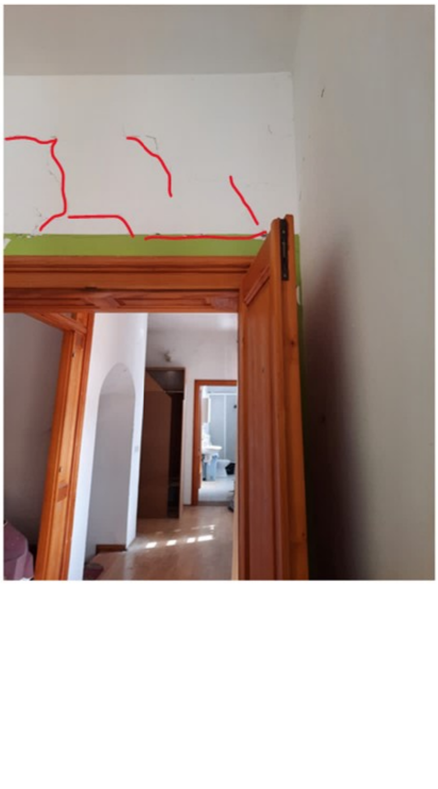

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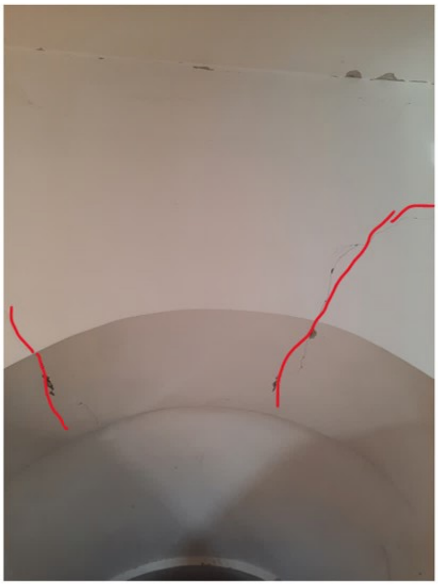
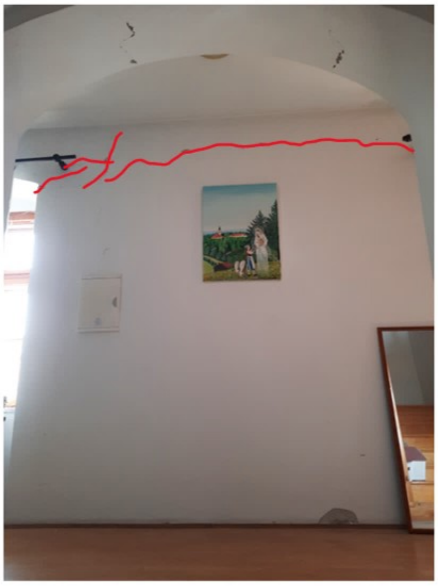


N° 25

N° 26

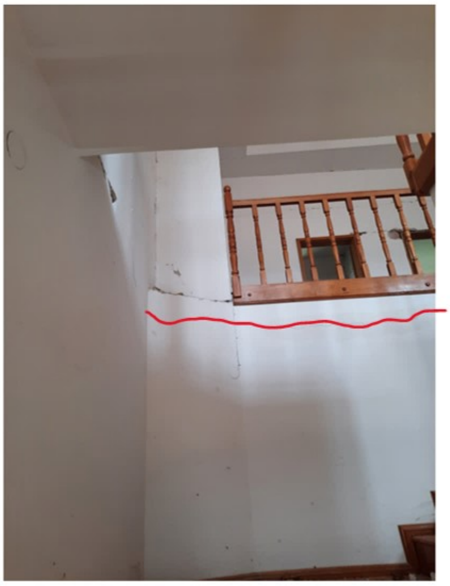
N° 27

N° 28



N° 29

N° 30

N° 31

D.2 Table crack documentation

Aline F. Bönzli
06.06.2023

Damage inventory:

Location: Parish House, Petrinja, Croatia
Date: 11./12.04.2023
Item: North Facade

Number	Description	Localization	Associated mechanism	Label	Connected cracks
1	Diagonal cracks	Starting from either top corner of the top east window	Cracking due to flexural mechanism with lifting off of the roof	NF1	FF7
2	Diagonal crack	Starting from the top west corner of the west top window and bottom east corner of top west window	Flexural crack due to force from south	NF2	FF21
3	Horizontal crack	Starting at the east extremity of the facade, at mid-height	Flexural crack from a global mechanism bending the whole facade or shear crack from a stress concentration in the corner of the building together with crack n° 11 on east facade	NF3	EF11
4	Diagonal spandrel cracks	Between the east side windows	Shear cracks / spandrel cracks	NF4	NF5 / NF6 / NF7
5	Diagonal cracks through the	Center of the facade	Shear crack in between the pier and node zone -> might be difficult to achieve in simulations?	NF5	NF4 / NF6 / NF7
6	Vertical/Diagonal cracks	Starting from the bottom corners of the top west window	Cracking due to a possible OOP mechanism in the spandrel below the top window. Due to the lower vertical load and no restraining slab on top this part of the wall is sensible to OOP bending.	NF6	NF4 / NF5 / NF7
7	Diagonal cracks	Above bottom west window	Shear crack in spandrel	NF7	NF4 / NF5 / NF6
8	Diagonal crack	Starting from the top west and bottom east corner of bottom west window	Flexural or shear crack due to seismic excitation towards the the east	NF8	
9	Horizontal crack	Starting at the west extremity of the facade, at mid-height	Flexural crack from a global mechanism bending the whole facade or shear crack from a stress concentration in the corner of the building together with crack n° 1 on west facade	NF9	WF1
10	Diagonal cracks	Between the corners of the bottom east window and the cellar window	Spandrel failure in shear	NF10	
11	Diagonal cracks	Starting from the bottom corners of the bottom west window	Shear cracks in spandrel	NF11	
12	Horizontal crack	Starting at the west extremity of the facade, at quarter-height (middle of bottom storey wall)	Corner stress concentration, might be in relation with ground floor crack 13 to cope with the out of plane failures GF14, GF15, GF18, GF19	NF12	GF14 / GF15 / GF18 / GF19 / WF3 / WF4

Aline F. Bönzli
06.06.2023

Damage inventory:

Location: Parish House, Petrinja, Croatia
Date: 11./12.04.2023
Item: East Facade

Number	Description	Localization	Associated mechanism	Label	Connected cracks
1	Horizontal/Diagonal crack	Starting from top south corner of top south window	Flexural crack- in relation with n° 3 south facade	EF1	SF3
2	Diagonal cracks	Starting from all corners of the top most south window	Shear crack of seismic excitation in both directions	EF2	
3	Horizontal crack	Connecting the inner corners of the top middle windows	Crack from possible OOP failure or lift off of roof together with cracks 1,2,4,5?	EF3	EF1 / EF2 / EF4 / EF5
4	Diagonal crack	Starting from the top north corner of the top middle north window	Shear crack from seismic excitation from south (north)	EF4	
5	Horizontal crack	Starting at the north extremity of the facade, at the level of the top corner of the top window	Flexural crack from a global bending of the wall in relation with crack n° 1 on north facade??	EF5	NF1 / FF2 / FF1
6	Diagonal crack	Starting at the south extremity of the facade, at mid height	Stress concentration in corner?	EF6	??
7	Horizontal crack	Starting at the south extremity of the facade, at mid height of the bottom window	Correspondig to the outer hinge of the vault mechanism associated with crack n°5 in the ground floor vault	EF7	GF5
8	Diagonal crack	Between the middle south windows, starting at bottom north corner of top window and top north corner of bottom window	Shear crack in spandril between the windows	EF8	EF9 / EF10
9	Diagonal crack	Between the middle north windows, starting from top corners of bottom windows and bottom south corner of top window	Shear crack in spandril between the windows	EF9	EF8 / EF10
10	Diagonal crack	Between the north windows, starting from top south corner of bottom windows and bottom south corner of top window	Shear crack in spandril between the windows	EF10	EF8 / EF9
11	Horizontal crack	Starting at the north extremity of the facade, at mid-height	Flexural crack from a global mechanism bending the whole facade or shear crack from a stress concentration in the corner of the building together with crack n° 3 on east north	EF11	NF3
12	Vertical/Diagonal crack	Middle south window at bottom south corner and cellar window beneath it	Cracking due to combination of spandril shear crack and a possible OOP mechanism in the spandril below the bottom window. Due to the lower vertical load and no restraining slab on top this part of the wall is sensible to OOP bending.	EF12	
13	Vertical/Diagonal crack	Middle north bottom window, between bottom corners and top corners of cellar window beneath it	Cracking due to combination of spandril shear crack and a possible OOP mechanism in the spandril below the bottom window. Due to the lower vertical load and no restraining slab on top this part of the wall is sensible to OOP bending.	EF13	
14	Diagonal crack	Bottom north corner of bottom north window	Flexural crack from seismic excitation from north (south)	EF14	

Aline F. Bönzli
06.06.2023

Damage inventory:

Location: Parish House, Petrinja, Croatia
Date: 11./12.04.2023
Item: South Facade

Number	Description	Localization	Associated mechanism	Label	Connected cracks
1	Diagonal cracks	Top and bottom corners of top west window	Flexural crack from seismic excitation in both directions, maybe in relation with global lift off mechanism of roof	SF1	SF1/2 / FF10/19/20
2	Diagonal cracks	Top corners of top east window	Flexural crack from seismic excitation in both directions, maybe in relation with global lift off mechanism of roof	SF2	SF1/2 / FF10/19/20
3	Horizontal /Diagonal crack	east (east) extremity of the facade ad mid height of the top storey connecting to the bottom east corner of the top east window	Flexural crack coming from global mechanism, in relation with crack n°1 on east facade	SF3	
4	Diagonal cracks	Spandrel cracks below top west window	Shear crack in spandril from seismic excitation in both directions, possibly in combination of OOP failure below the window.	SF4	
5	Diagonal cracks	Spandrel cracks below top east window	Shear crack in spandril/OOP failure mode below window -> in combination with crack n°8	SF5	SF8
6	Diagonal cracks	Crossed shear cracks in bottom west pier	Shear crack in pier from seismic excitation in both directions	SF6	
7	Diagonal cracks	Crossed shear cracks in bottom center pier	Shear crack in pier and node from seismic excitation in both directions -> could be difficult to reproduce in model	SF7	
8	Diagonal cracks	Spandrel cracks above bottom east window	Shear crack /maybe in relation with an OOP mechanism related to crack n° 5?	SF8	SF5
9	Diagonal cracks	Starting from bottom east corner of bottom east window	Flexural/shear crack due to seismic excitation to the east (west)	SF9	

Aline F. Bönzli
06.06.2023

Damage inventory:

Location: Parish House, Petrinja, Croatia
Date: 11./12.04.2023
Item: West Facade

Number	Description	Localization	Associated mechanism	Label	Connected cracks
1	Horizontal crack	Starting at the north extremity of the facade, at mid-height	Flexural crack of a global bending of the facade and/or a stress concentration in the corner, causin sliding between the bricks ->in relation with n°8 of the north facade	WF1	NF3
2	Diagonal crack	Spandrel crack between the bottom of the north most top window and the top of the north most bottom windwow	Shear crack in spandril zone in combination with eventual OOP failure of wall below window moving out of its plane due to lower vertical load and absence od restraining force	WF2	
3	Horizontal crack	Ad midheight of the facade above the bottom south two windows	OOP Failure from the vault pushing outside	WF3	WF 4, WF 8, WF 9, WF 10, GF 9, GF 13, GF 14, GF 15, GF 17, GF 22
4	Horizontal crack	At mid height of the facade in the middle above the door and the bottom window next to it	OOP Failure from the vault pushing outside -> continuation of crack n°3	WF4	WF 3, WF 8, WF 9, WF 10, GF 9, GF 13, GF 14, GF 15, GF 17, GF 22
5	Vertical crack	Starting at bottom north corner of top south most window	???	WF5	
6	Vertical / Diagonal crack	Starting at the south extremity of the facade, at mid-height	???	WF6	
7	Diagonal crack	Starting at the top south corner and at the bottom north corner of the bottom south most window	Shear crack from a seismic excitation to the north (norht)	WF7	
8	Horizontal crack	Connecting the tom corners of the two top north windows	Flexural crack from a global lift off mechanism of the roof? Mayve also from and OOP failure of the west facade turning about its middle axis being pushed out by the vaulting system in the ground floor	WF8	WF 3, WF 4, WF 9, WF 10, GF 9, GF 13, GF 14, GF 15, GF 17, GF 22
9	Horizontal crack	Connecting the tom corners of the two middle top windows	Flexural crack from a global lift off mechanism of the roof? Mayve also from and OOP failure of the west facade turning about its middle axis being pushed out by the vaulting system in the ground floor	WF9	WF 3, WF 4, WF 8, WF 10, GF 9, GF 13, GF 14, GF 15, GF 17, GF 22
10	Horizontal crack	On the top south corner of the top middle south window	Flexural crack from a global lift off mechanism of the roof? Mayve also from and OOP failure of the west facade turning about its middle axis being pushed out by the vaulting system in the ground floor	WF10	WF 3, WF 4, WF 8, WF 9, GF 9, GF 13, GF 14, GF 15, GF 17, GF 22

Aline F. Bönzli
06.06.2023

Damage inventory:

Location: Parish House, Petrinja, Croatia
Date: 11./12.04.2023
Item: Ground floor

Number	Description	Localization	Associated mechanism	Label	Connected cracks
1	Vertical and diagonal cracks	Above north-east door connecting the eastern smaller room to the big room in the east	Shear cracks, maybe OOP failure of the wall beneath the vault	GF1	GF1/2/3/4/5/12, EF3/4/7, FF3/4
2	Vertical cack	Above north-east door connecting the big room in the east to the eastern smaller room, on the other side of the wall as crack n°1	Another vault mechanism that is not visible forming in in the vault of the big east room	GF2	GF1/2/3/4/5/12, EF3/4/7, FF3/4
3	Diagonal crack crossing the	On the wall between the big eastern room and the smaller south eastern room. Starting at the bottom left corner and ending in the middle above the door.	Shear failure of the wall stabilizing the vault in the big room in the east.	GF3	GF1/2/3/4/5/12, EF3/4/7, FF3/4
4	Diagonal crack (partly vertic	On the wall between the smaller south-eastern room and the big eastern room, above the door on the other side of wall as crack n°3	Shear failure of the wall stabilizing the vault in the big room in the east. Maybe connected to an OOP failure of the wall above the door.	GF4	GF1/2/3/4/5/12, EF3/4/7, FF3/4
5	Longitudinal vault crack	Starting from the south window of the sout-eastern room and continuing in the longitudinal direction of the vault	OOP failure of the walls connected to the vault creating the first hinge in the middle of the vault. Due to seismic excitation in the east-west direction. Outer hinge crack can be associated to n°7 on east facade?	GF5	GF1/2/3/4/5/12, EF3/4/7, FF3/4
6	Longitudinal vault crack	Starting above the east door connecting the norht-west room to the north-east room	OOP failure of the walls connected to the vault creating the first hinge in the middle of the vault. Due to seismic excitation in the north-south direction. Outside hinge forming on the wall to the kitchen associated to crack n°12	GF6	GF13
7	Vertical /diagonal crack	On the south wall of the north-west room (between room and staircase).	Shear cracks taking seismic excitation in the east west direction	GF7	GF10
8	Diagonal crack	On the back wall of the staircase	Shear cracks taking seismic excitation in the north-south direction	GF8	
9	Horizontal cracks	Around the top and bottom of inside column/pier in the corner of the staircase leading down.	Cracks can be associated to the vault cracks n°15, constituting the hinges of the OOP movement of the support of the vaults. Seismic excitation in all directions	GF9	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
10	Diagonal cracks	On the south wall between staircase and hallway	Mirrored cracks, symmetric to n°7 coming from seismic excitation in the east-west direction. Cracks going through the enitre wall, visible from the stairs as well as from the hall way.	GF10	
11	Diagonal cracks	Above door leading to the kitchen.	Spandril cracking from east-west seismic excitation	GF11	
12	Horizontal cracks	On the interface of east wall of the kitchen and the vaulted ceiling	Cracking coming from vault failure associated to crack n°5 forming the outside hinge or coming from differential movement between wall and vault	GF12	GF1/2/3/4/5/12, EF3/4/7, FF3/4
13	Horizontal crack	Around vault corner in north west corner of the north-west room	Could be associated to crack n°12 on north facade->out of compatibility of the global OOP failure of the West facade	GF13	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
14	Longitudinal vault crack	Crack at middle of local door vault of the sout entrance to the north-west room	Continuation of crack n° 15 corresponding to a vault failure leading to and OOP mechanism of the west facade, associated with crack n°3 on the west facade forming the outside hinge	GF14	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
15	Longitudinal vault crack	Crack at mid span of vault in the entrance in the north-south direction of the building	Continuation of crack n° 14 corresponding to a vault failure leading to and OOP mechanism of the west facade, associated with crack n°3 on the west facade forming the outside hinge	GF15	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
16	Vertical crack	Crack on the right side of the north entry to the kitchen	Differential movement of the walls?	GF16	
17	Horizontal crack	On the interface of west wall of the kitchen and the vaulted ceiling	Cracking coming from vault failure associated to crack n°22 forming the outside hinge or coming from differential movement between wall and vault	GF17	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
18	Horizontal crack	On the north side of the main entrance, below the end of an arch stabilizing the vaulted system in the hallway	Could be associated to crack n°15 from crushing on the inside of the low hinge of the vault mechanism, similar as crack n°19	GF18	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
19 top	Horizontal crack	On the south side of the main entrance, below the end of an arch stabilizing the vaulted system in the hallway	Could be associated to crack n°15 from crushing on the inside of the low hinge of the vault mechanism, similar as crack n°18	GF19	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
19 bottom	Horizontal & diagonal cracks	On the pier on the south side of the main entrance	1. Horizontal crack at the bottom of the pier corresponding from a felxural/OOP failure in relation with the vault mechanism n°15 pushing the top of the wall out creating a hing at the inside of the pier base 2. Crossed diagonal crack corresponding to a shear failure of the pier in the middle	GF20	
20	Diagonal cracks	Above door leading to the toilet.	Spandril cracking from east-west seismic excitation	GF21	
21	Horizontal crack	At mid height on the wall separating the toilet from the storage room	Shear mechanism creating sliding between the upper and the lower wall parts?	GF22	WF 3/4/8/ 9/10, GF 9/13/14/15/17/18/19/ 22, FF22/27/28/29
22	Longitudinal vault crack	On the vault of the storage room starting at the wall against the toilet	Vault mechanism connected to crack n°17 in the kitchen forming the outside hinge.	GF23	
23	Diagonal crack	Starting from the south corner of the window in the storage room.	Associated to shear crack around the window in n° 7 of the west facade cracks	GF24	
24	Horizontal crack	In the south-west corner of the storage space ->associated to local vault failure n°22	??	GF25	

Damage inventory:

Location: Parish House, Petrinja, Croatia
Date: 11./12.04.2023
Item: First Floor

Number	Description	Localization	Associated mechanism	Label	Connected cracks
1	Horizontal crack	Starting at the east upper corner of the east window on the north facade/ crack 1 on north facade	Flexural cracking from a seismic excitation in the east-west direction, or uplifting roof from a seismic excitation in the north-south direction. Or maybe even outward pushing of the purlin roof beams.	FF1	NF1
2	Horizontal crack	Starting at north corner of north window on east facade / crack 5 on east facade	Flexural cracking from a seismic excitation in the north-south direction, or uplifting roof from a seismic excitation in the east-west direction. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor. Or maybe even outward pushing of the purlin roof beams.	FF2	EF5
3	Horizontal cracks	Starting on either top corner of the middle-north window on the east facade / cracks 3 and 4 of east facade	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF3	GF1/2/3/4/5/12, EF3/4/7, FF3/4
4	Horizontal cracks	Starting on either top corner of the middle-south window on the east facade / crack 3 east facade	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF4	GF1/2/3/4/5/12, EF3/4/7, FF3/4
5	Horizontal crack	Starting from top north corner of south window on east facade/related to crack 2 on east facade	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF5	EF2
6	Horizontal crack	Starting from top south corner of south window on east facade and continuing to the sout facade/related to crack 1 on east facade and 3 on south facade	Shear crack from east-west excitation?	FF6	EF1 / SF3
7	Horizontal crack	Starting at the upper corners of the east window on the north facade/ crack 1 on north facade	Flexural cracking from a seismic excitation in the east-west direction, or uplifting roof from a seismic excitation in the north-south direction. Or maybe even outward pushing of the purlin roof beams.	FF7	NF1
8	Diagonal cracks	On the east side of the door connecting the big central eastern room to the north east room	Shear and flexural cracks from a seismic excitation in the east-west direction, similar to crack 9,12 and 13	FF8	FF8/9/12/13
9	Diagonal cracks	On the east side of the door connecting the big central eastern room to the south east room	Shear and flexural cracks from a seismic excitation in the east-west direction, similar to crack 8, 12 and 13	FF9	FF8/9/12/13
10	Diagonal/horizontal cracks	On either side of the window on the south facade of the room in the south east/ crack 2 south facade	Flexural cracking from a seismic excitation in the east-west direction, or uplifting roof from a seismic excitation in the north-south direction. Or maybe even outward pushing of the purlin roof beams.	FF10	SF1/2 / FF10/19/20
11	Vertical crack	On the north upper corner of the door to the bathroom of the north east room	??	FF11	
12	Diagonal cracks	On the west side of the door connecting the big central eastern room to the north east room	Shear and flexural cracks from a seismic excitation in the east-west direction, similar to crack 8, 9 and 13	FF12	FF8/9/12/13
13	Diagonal/horizontal cracks	On the west side of the door connecting the big central eastern room to the south east room	Shear and flexural cracks from a seismic excitation in the east-west direction, similar to crack 8, 9 and 12	FF13	FF8/9/12/13
14	Diagonal cracks	On the north side of the door connecting the south east to the south west room	Shear crack from east-west excitation?	FF14	FF15
15	Diagonal cracks	On the south side of the door connecting the south east to the south west room	Shear crack from east-west excitation?	FF15	FF14
16	Diagonal cracks	On the north wall of the staircase	Shear cracks due to a seismic excitation in the east-west direction, together with cracks 17	FF16	FF17
17	Diagonal cracks	On the south wall of the staircase	Shear cracks due to a seismic excitation in the east-west direction, together with cracks 16	FF17	FF16
18	Diagonal/vertical cracks	On the east side of the north access door of the south-west room	??	FF18	
19	Horizontal crack	On the east side of the window of the sout west room (south facade)/crack 1 south facade?	Flexural cracking from a seismic excitation in the east-west direction, or uplifting roof from a seismic excitation in the north-south direction. Or maybe even outward pushing of the purlin roof beams.	FF19	SF1/2 / FF10/19/20
20	Horizontal crack	On either side of the window on the south facade of the room in the south west/ crack 1 south facade	Flexural cracking from a seismic excitation in the east-west direction, or uplifting roof from a seismic excitation in the north-south direction. Or maybe even outward pushing of the purlin roof beams.	FF20	SF1/2 / FF10/19/20
21	Diagonal/vertical cracks	around window of the north west room on the north facade/ crack 2 north facade	Flexural cracking from a seismic excitation in the east-west direction, or uplifting roof from a seismic excitation in the north-south direction.	FF21	
22	Horizontal crack	Starting from south corner of west window of north west room	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF22	FF22/27/28/29
23	Horizontal crack	Above door to north west room	Flexural cracking to accomodate movements: uplift of of roof structure or OOP failure of west facade starting from out push of the vaults in the ground floor	FF23	
24	Horizontal crack	At middle of vault of the staris coming up to the first floor	Local failure from staircase vault pushing outward and creating the first hinge in the middle	FF24	
25	Diagonal/vertical crack	On the west side of the north door to the south-west room	Maybe shear cracking due to seismic excitation in the east-west direction	FF25	
26	Horizontal/diagonal crack	on west walll of south west room starting on east corner of the door to the bathroom	Flexural cracking from a seismic excitation in the north-south direction, or uplifting roof from a seismic excitation in the east-west direction.	FF26	
27	Horizontal cracks	On either side of the middle-north window on the west facade	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF27	FF22 / FF28 / FF29 / WF8 / WF9 / WF10
28	Horizontal crack	Connecting cracks 27 and 29	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF28	FF22 / FF27 / FF29 / WF8 / WF9 / WF10
29	Horizontal crack	On either side of the middle-south window on the west facade	Uplifting of the rigid roof structure due to a seismic excitation in the east west direction or pushing outwards of the purlin roof beams. Possible hinge for global OOP failure starting with the outpush of the vault in the ground-floor.	FF29	FF22 / FF27 / FF28 / WF8 / WF9 / WF10
30	Horizontal crack	On the east wall of the staircase	Flexural crack or uplifting of the roof, but both seem improbable…	FF30	
31	Horizontal/diagonal crack	On the middle wall of the staircase	Shear/flexural cracking due to seismic excitation in the east-west direction	FF31	

E. Meeting with the engineers working on the project of the parish house in Petrinja

Name: Hrvoje & Martina Vujasinović, INTRADOS-PROJEKT d.o.o., Zagreb, Croatia [6]
The interview was led by Aline Bönzli in English with translations to Croatian by Dr. Igor Tomić, post-doctoral researcher at the EESD Laboratory of EPFL.
Question: Do you have any knowledge of the materials used for construction? Did you perform tests to get some material characteristics?
Answer: No material tests were performed on the material of the Parish house. However, by opening parts of the façade, it could be confirmed that the whole walls are made out of brick masonry. Question: What do you know about the vaults that form the support system for the slab above the ground floor?
Answer: The vaults are formed by one layer of bricks, meaning they have a thickness of 15 cm. The volume between the curved outer surface of the vaults, the walls and the slab supporting the living surface of the above floor has been filled with some sort of gravel/sand with a density of 14 kN per m^3 The top surface is formed by timber planks supporting the parquet flooring.
Question: The slab above the first floor had been exposed after the damages of the homeland war in the late 20th century. There have been some renovation and remodelling works on that floor and the roof of the building. What do you know about the structural system and the materials that were used?
Answer: Floor above first floor: The structural system of the slab above the first floor is a Fert system. The slab is made out of concrete beams with clay bricks in between (see figure below).

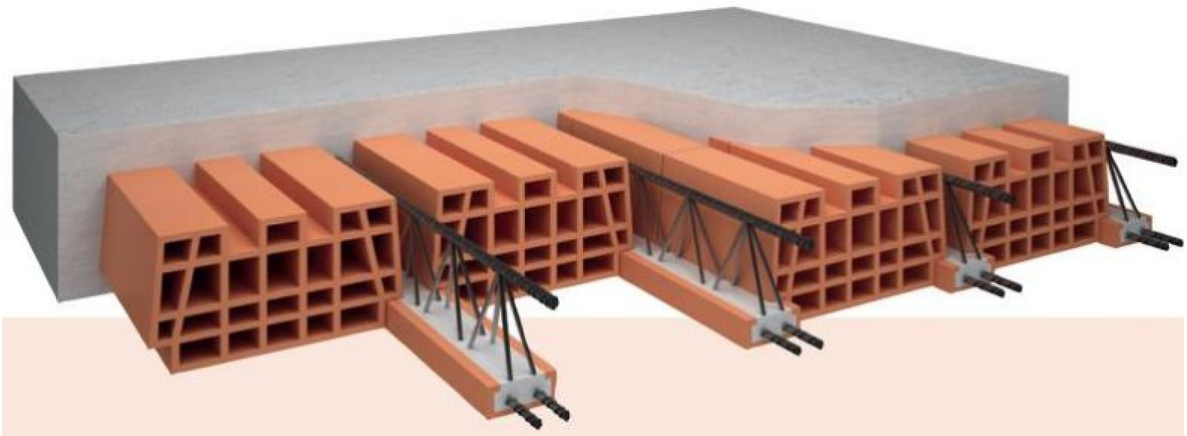


Figure E.1: Fert slab [40]

Those slabs support the load between two parallel walls and can be modelled by an equivalent concrete slab of thickness 10-12 cm. It has also been found that a reinforced concrete ring beam was added to the building during the remodelling process. The ring beam goes around the whole building and is well connected to the concrete beams of the Fert slab. This system adds stiffness to the whole building. Roof: The roof was completely reconstructed with timber elements. The roof structure is supported by the ring beam as well as by two columns bringing the load down to the walls of the first floor.

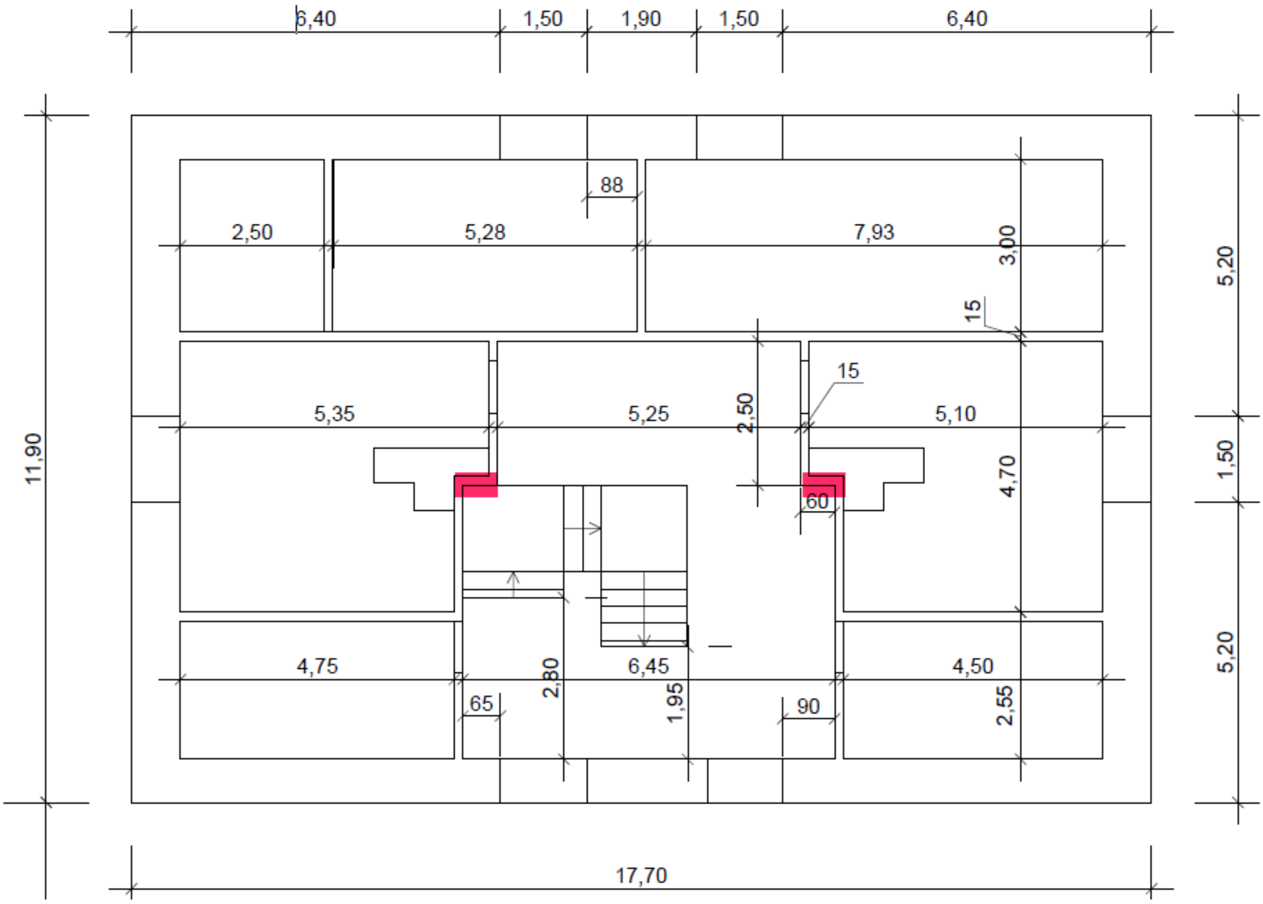
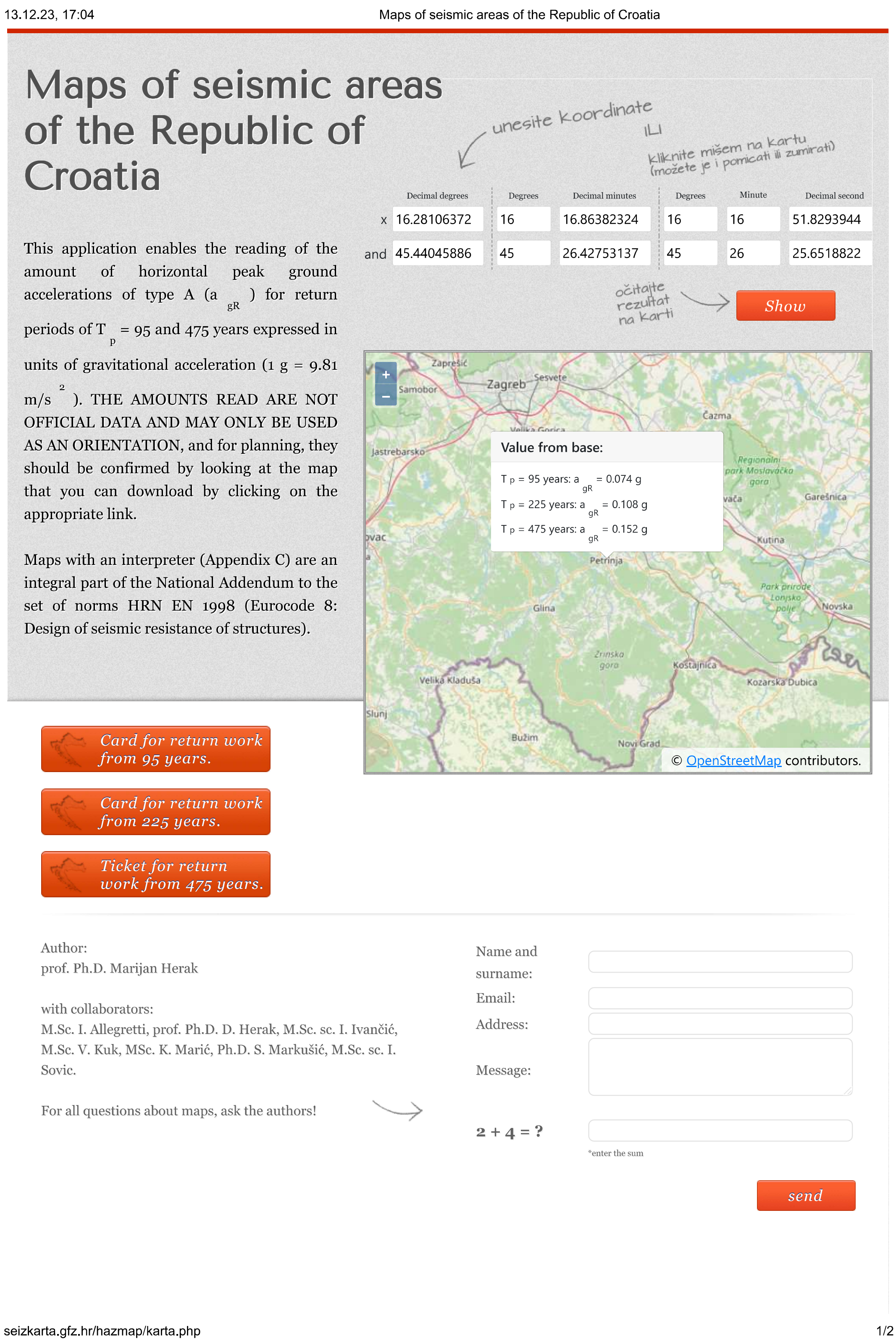


Figure E.2: Columns supporting the Roof structure

F. Reference peak ground acceleration Petrinja



13.12.23, 17:04 Maps of seismic areas of the Republic of Croatia

Maps of seismic areas of the Republic of Croatia

unesite koordinate
ILI
kliknite mišem na kartu
(možete je i pomicati ili zumirati)

	Decimal degrees	Degrees	Decimal minutes	Degrees	Minute	Decimal second
x	16.28106372	16	16.86382324	16	16	51.8293944
and	45.44045886	45	26.42753137	45	26	25.6518822

očitajte rezultat na karti

Show

This application enables the reading of the amount of horizontal peak ground accelerations of type A (a_{gR}) for return periods of T_p = 95 and 475 years expressed in units of gravitational acceleration (1 g = 9.81 m/s^2). THE AMOUNTS READ ARE NOT OFFICIAL DATA AND MAY ONLY BE USED AS AN ORIENTATION, and for planning, they should be confirmed by looking at the map that you can download by clicking on the appropriate link.

Maps with an interpreter (Appendix C) are an integral part of the National Addendum to the set of norms HRN EN 1998 (Eurocode 8: Design of seismic resistance of structures).

Card for return work from 95 years.

Card for return work from 225 years.

Ticket for return work from 475 years.

Author:
prof. Ph.D. Marijan Herak

with collaborators:
M.Sc. I. Allegretti, prof. Ph.D. D. Herak, M.Sc. sc. I. Ivančić, M.Sc. V. Kuk, MSc. K. Marić, Ph.D. S. Markušić, M.Sc. sc. I. Sovic.

For all questions about maps, ask the authors!

Name and surname:

Email:

Address:

Message:

2 + 4 = ?

*enter the sum

send

seizkarta.gfz.hr/hazmap/karta.php 1/2

Figure F.1: Reference peak ground acceleration (PGA) of Petrinja [26]

G. Geological map Sisak region, Croatia

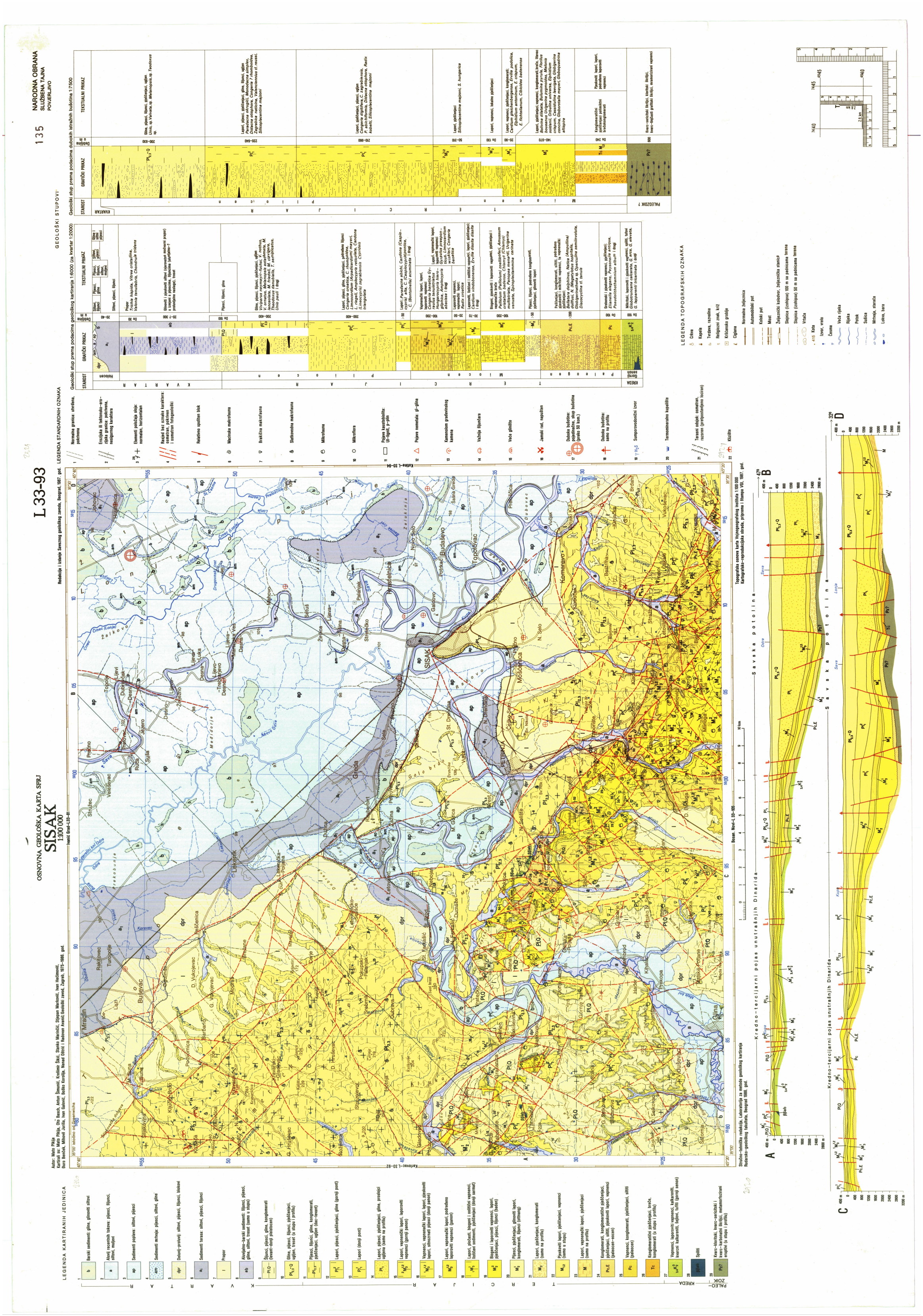


Figure G.1: Geological map Sisak [27]

H. Code for OOP failures

Out-of-plane failure calculations

F-Mechanism Parish House West Facade

Author: Aline F. Bönzli

Master Thesis 2023/2024

Supervisors: Prof. Dr. Katrin Beyer, Dr. Igor Tomic

Co-author: Caroline L. Heitmann

Caroline and I intensely collaborated on the elaboration of various notebooks for diffeent OOP mechanisms, generic and specifically applied to our respective buildings treated in our theses.

Introduction

This notebook can be used to calculate the push-over curve for an out-of-plane mechanisms of a masonry structure subject to seismic action. The mechanism in question is a F-mechanism (D'Ayala, 2002) and the calculations were performed for the west facade of the Parish House at St. Lawrence Church in Petrinja, Croatia.

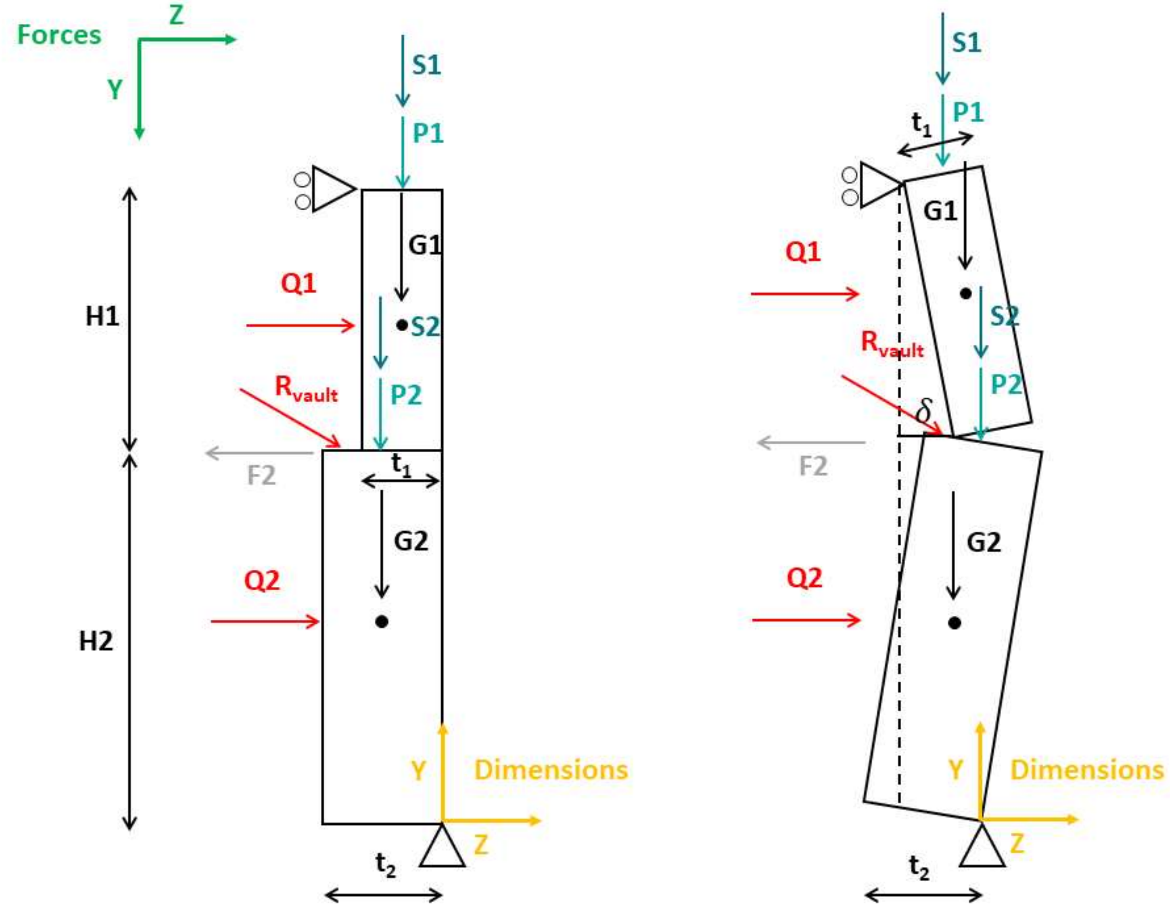
First, the geometric and material characteristics will be defined, then depending on these inputs, the displacement capacity for out-of-plane accelerations will be returned in form of a push-over graph.

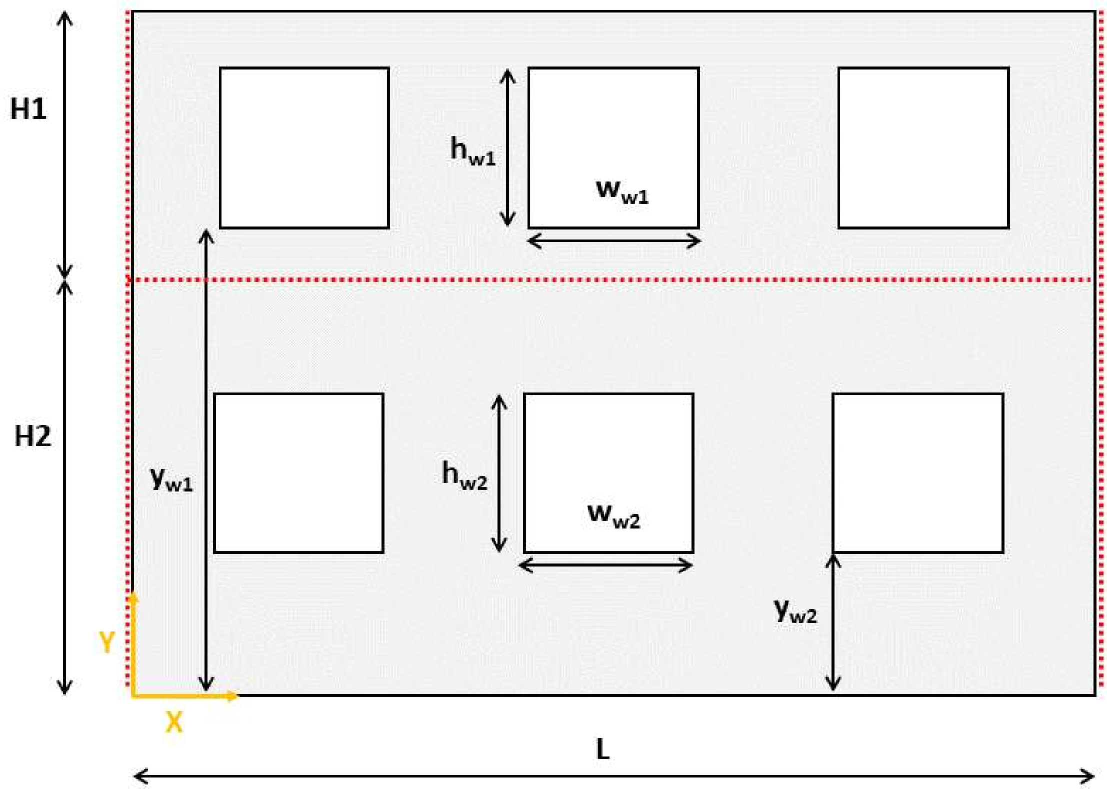
In [1]:
```
# Import the basic packages for the notebook
import numpy as np                      # Numerical library
import pandas as pd                     # Data analysis library
import matplotlib.pyplot as plt         # Plotting library
import matplotlib.ticker as maticker # Tick locating and formatting library
import matplotlib.colors as mcolors  # Colours
from sympy.solvers import solvers    # Library to solve equations
from sympy import Symbol, Eq, nsolve # For solving equations
import math
```

1. Define the wall geometry

The chosen example for this code is a wall with two rows of windows and two thicknesses moving out of plane around a mid axis where the wall changes thickness. A sketch with the main dimensions is shown below. At the height where the wall changes thickness, a vault is supported which will add a vertical load, as well a horizontal thrust.

In [5]:
```
from IPython.display import Image, display
display(Image(filename='Mec_F_3.png'))
display(Image(filename='Mec_F_2.png'))
```





This block is used to define the geometry of the wall and the windows within.

In [6]:
```
## Wall geometry
H1 = 3.25           # Height of the upper part of the wall,    y-direction [m]
H2 = 2.75           # Height of the lower part of the wall,    y-direction [m]
L  = 14.2           # Total length of wall,                    x-direction [m]
t1 = 0.55           # Thickness of the upper part of the wall, z-direction [m]
```

```
t2 = 0.7            # Thickness of the lower part of the wall, z-direction [m]


## Window geometry, upper row
h_w1 = 1.7          # Height of window on upper wall [m]
w_w1 = 1.1          # Width of window on upper wall[m]
y_w1 = 3.65         # Vertical distance from y = 0 to bottom corner of window on up
n_w1 = 4            # Number of windows on upper wall [-]

## Window geometry, lower row
h_w2 = 1.4          # Height of window on lower wall [m]
w_w2 = 1.1          # Width of window on lower wall[m]
y_w2 = 0.7          # Vertical distance from y = 0 to bottom corner of window on lo
n_w2 = 4            # Number of windows on lower wall [-]

## Vault geometry
h_v = 0.3           # Height of the vault at mid-span with respect to its support c
w_v = 1.75          # Main span width of the vault [m]
t_v = 0.15          # Thickness of the vault [m]


## Slab information
max_delta = t2/2    # Maximum displacement, when the slab and wall lose connection
                    # restraining the wall [m]
```

2. Define material properties

This block is used to define the material properties of the wall and the connecting slab. In this case the friction coefficient is set to zero since the slab is directly supported by the vault, which is in return in a monolithical connection to the wall.

In [7]:
```
rho_masonry = 18000         # Density of the masonry [N/m^3]
mu_f = 0.0                  # Friction coefficient between slab and masonry wall
```

3. Define the loads and forces

In this paragraph, the loads and forces acting on the wall are defined. The following block defines the vertical load on top of the wall coming from a slab (S) or a superimposed wall (P). Note that the S-load will be used to directly calculate the restraining force due to friction between the wall and the slab. If you want to introduce a vertical load that does not produce any friction, please use the P-load.

In [8]:
```
P1 = 71326                  # vertical load on top of the upper wall [N]
S1 = 67209                  # slab load acting on top of the upper wall [N]
P2 = 26825                  # vertical load on top of the lower wall [N]
S2 = 0                      # slab load acting on top of the lower wall [N]
q_vault = 37769             # surface load acting on vault [N/m]
```

4. Calculation of overturning moment

This paragraph calculates the pushover curve for the given case.

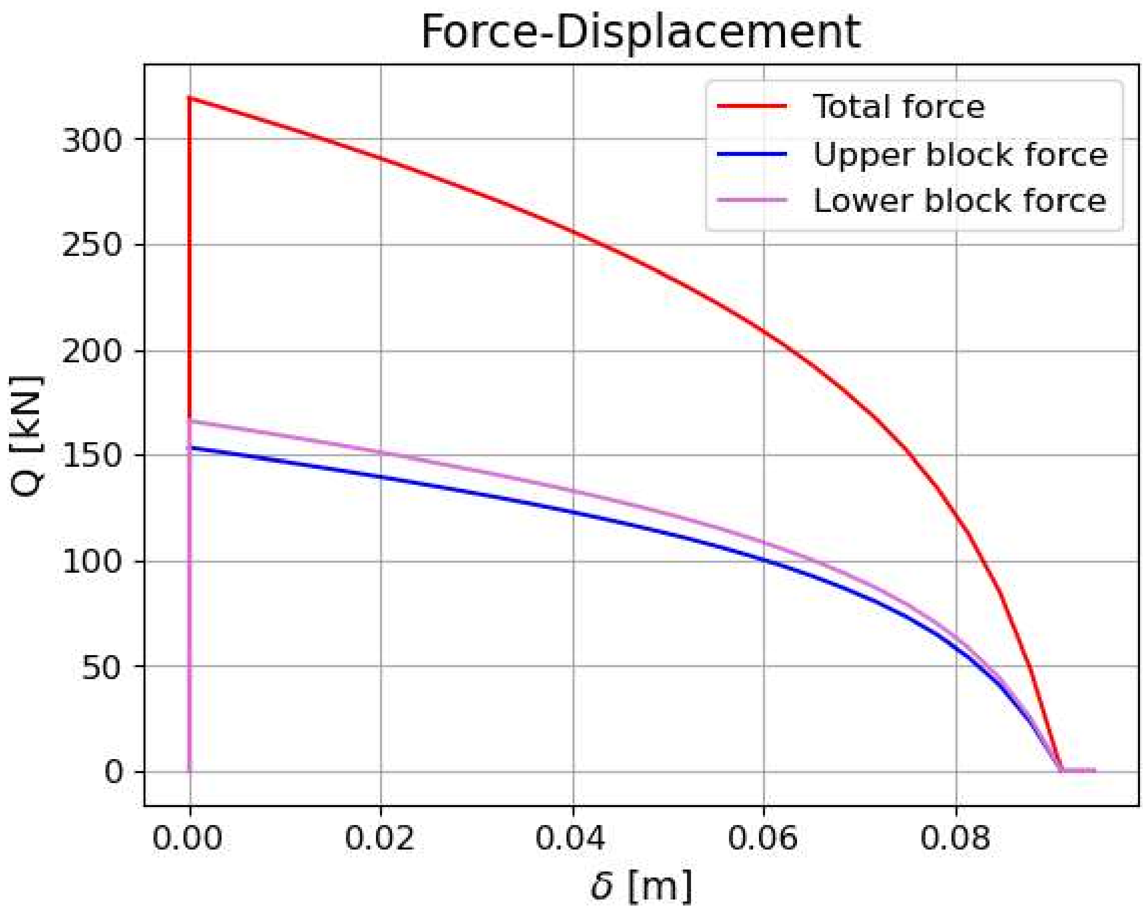
In [18]:
```
# This function calculates the center of gravity of the wall, y-direction using the
def cog_fun(n_w,h_w,y_w,w_w,H,L,y_W):
    # n_w = number of windows in the wall block
```

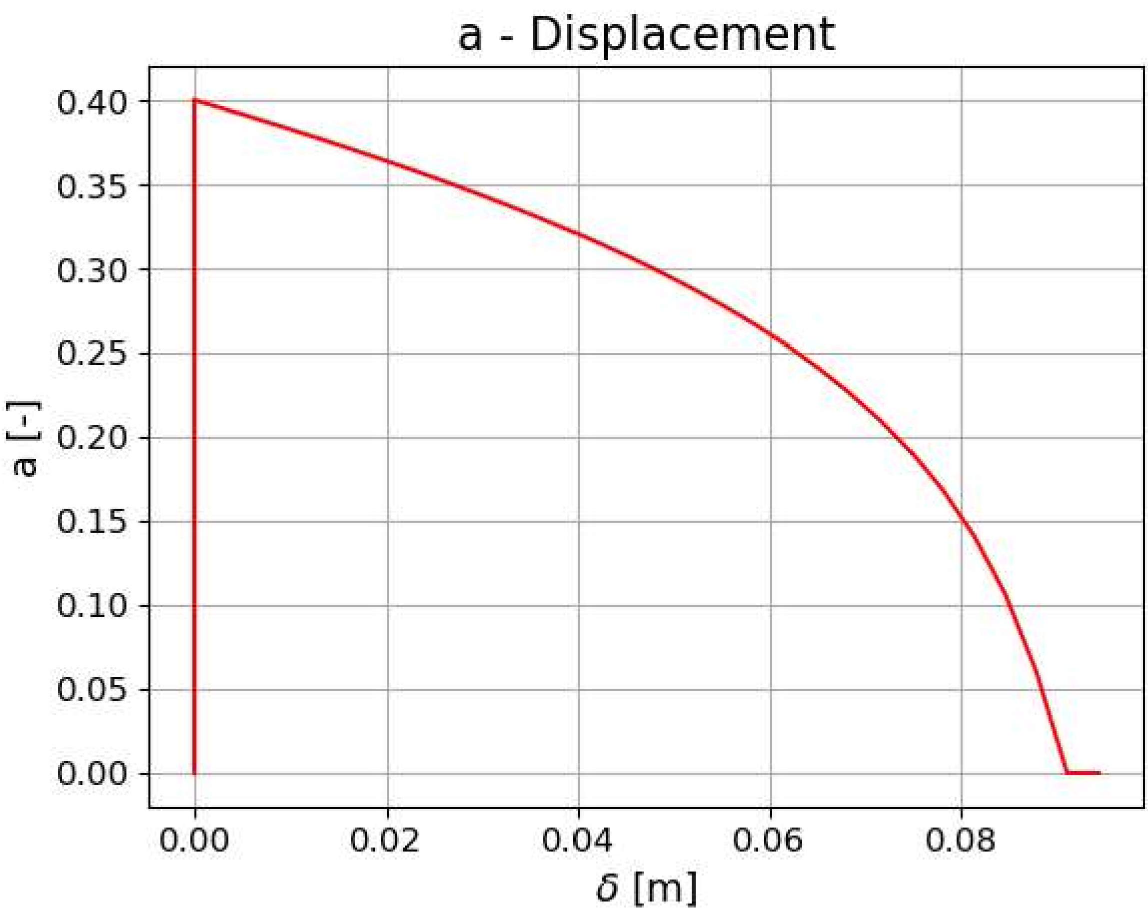
```python
    # h_w = height of the windows [m]
    # y_w = distance of ground to lower corner of the windows [m]
    # w_w = width of the windows [m]
    # H   = height of the wall block [m]
    # L   = length of the wall block [m]
    # y_W = height of the lower corner of the wall block above ground [m]
    A_w = n_w*h_w*w_w          # total area of windows [m^2]
    A = H*L                    # area of wall [m^2]
    ybar = H/2                 # centroid of wall with respect to its lower corner,
                               # rectangular profile with constant thickness [m]
    ybar_w = y_w+h_w/2-y_W     # centroid of wall with respect to lower corner of wo
                               # rectangular profile with constant thickness [m]

    COG_y = ((A*ybar)-(A_w*ybar_w))/(A-A_w)+y_W

    return COG_y

# This function calculates the horizontal thrust and the vertical load from the vau
def vault_fun(delta):
 # delta = displacement of the wall OOP [m]

    # defining displaced geometry
    w_v_new = w_v + delta                     # new width of the vault [m]
    l_v = math.sqrt((w_v/2)**2 + h_v**2)      # length of the diagonal connecting the
                                              # mid-span of the vault [m]
    if w_v_new >= l_v*2:                      # check if the vault is still in place
        return np.array([0,0])
    else:
        h_v_new = math.sqrt(l_v**2-(w_v_new/2)**2) # new height of the vault [m]

     # calculate support loads
        R_h = (q_vault * w_v_new**2 /          # horizontal thrust [N]
              (8 * h_v_new))
        R_v = q_vault * w_v_new /2             # vertical support load [N]

    return np.array([R_v,R_h])

# In this function we find the alpha value corresponding to the part of the self we
# wall, representing the seismic load (Q1=alpha*G1 and Q2=alpha*G2), for an equilib
# at a given displacement delta
def alpha_fun(delta):

    delta_G1 = delta*(H1+H2-COG_1)/H1                         # Displacement at th
    delta_G2 = delta*COG_2/H2                                 # Displacement at th
    F2_it = F2
    if delta > H1:
        return [0,0]
    # Verify if the slab at cracking level is still in contact
    if delta > max_delta:
        F2_it=0
    # Calculate the thrust of the vault
    [R_v,R_h] = vault_fun(delta)
    if [R_v,R_h] == [0,0]:
        return [0,0]

    a = Symbol('a')
    F1 = Symbol('F1')
    # Equilibrium about the lower support point
    equation1 = Eq( -F1*Htot                                   # Moment from upper
                    -F2_it*H2                                  # Moment from middl
                    +a*G1*COG_1                                # Moment from seism
                    +a*G2*COG_2                                # Moment from seism
                    -(P1+S1)*t1/2                              # Moment from verti
                    +(P2+S2+R_v)*(delta-(t2/2))                # Moment from verti
```

```python
                    +G1*(delta_G1-(t1/2))                        # Moment from selfw
                    +G2*(delta_G2-(t2/2))                        # Moment from selfw
                    +R_h*H2                                      # Moment from the t
                , 0)

    # Equilibrium of the upper body about its rotation corner
    equation2 = Eq( -F1*H1                                       # Moment from upper
                    +a*G1*(COG_1-H2)                             # Moment from seism
                    +(P1+S1)*(t1/2-delta)                        # Moment from verti
                    +G1*(delta_G1+(t1/2)-delta)                  # Moment from selfw
                    ,0)

    # Initial guesses to solve the equation (this is important to converge fast)
    a_guess = 1
    F1_guess = G1+G2

    # Solve the equation for alpha (a) and the upper reaction force (F1)
    [a,F1]=nsolve((equation1, equation2), (a, F1), (a_guess, F1_guess))

    return [a,F1]

## Derived wall properties
Htot=H1+H2          # total height of the wall [m]

# Calculating the net section of the wall (section without area of windows) [m^2]
A_net1 = H1*L-n_w1*h_w1*w_w1    #Net area of upper wall / Area of upper wall without
A_net2 = H2*L-n_w2*h_w2*w_w2    #Net area of lower wall / Area of lower wall without

# Calculating the volume of the two wall blocks
V_net1 = A_net1*t1              #Net volume of upper wall / Volume of upper wall wit
V_net2 = A_net2*t2              #Net volume of lower wall / Volume of lower wall wit

#Calculating the centers of gravity for both bodies with respect to the ground [m]
COG_1 = cog_fun(n_w1,h_w1,y_w1,w_w1,H1,L,H2)
COG_2 = cog_fun(n_w2,h_w2,y_w2,w_w2,H2,L,0)

## Derived loads and forces
G1 = rho_masonry*V_net1         # gravity load of the upepr wall (self-weight) [N]
G2 = rho_masonry*V_net2         # gravity load of the lower wall (self-weight) [N]
F2 = mu_f*S2                    # friction force from slab acting on top of the lowe

## Plot force-displacement relationship of the overturning of the wall
delta_vals = np.linspace(0,H1,1000)
alpha_vals = np.zeros((np.size(delta_vals)))
F1_vals = np.zeros((np.size(delta_vals)))

for i in range(np.size(delta_vals)): # Calculates the corresponding alpha and F1 fo
    [alpha_vals[i],F1_vals[i]]=alpha_fun(delta_vals[i])

# Transformation to only display values up to failure
alpha_vals = np.append(alpha_vals[alpha_vals>0],[0,0],axis=0)

delta_vals = delta_vals[: np.size(alpha_vals)]
F1_vals= F1_vals[:np.size(alpha_vals)]

q_tot_vals = alpha_vals * (G1+G2)/1000  # Maximum total seismic load [kN]
q_1_vals = alpha_vals * G1/1000         # Maximum seismic load on upper body[kN]
q_2_vals = alpha_vals * G2/1000         # Maximum seismic load on lower body[kN]

# Plot force-displacement relationship of the overturning of the wall
plt.figure()
plt.grid()
plt.plot([0,0],[0,q_tot_vals[0]],color='red')
plt.plot(delta_vals,q_tot_vals,label='Total force',color='red')
```

```
plt.plot([0,0],[0,q_1_vals[0]],color='blue')
plt.plot(delta_vals,q_1_vals,label='Upper block force',color='blue')

plt.plot([0,0],[0,q_2_vals[0]],color='orchid')
plt.plot(delta_vals,q_2_vals,label='Lower block force',color='orchid')

plt.legend(fontsize=12)
plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('Q [kN]', fontsize=14)
plt.title('Force-Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)



# Plot acceleration-displacement relationship of the overturning of the wall
plt.figure()
plt.grid()
plt.plot([0,0],[0,alpha_vals[0]],color='red')
plt.plot(delta_vals,alpha_vals,color='red')

plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('a [-]', fontsize=14)
plt.title('a - Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

## Maximum earthquake force to start rocking
Q_tot_max = q_tot_vals[0]/1000          # Maximum total seismic load [kN]
Q_1_max = alpha_vals[0] * G1/1000       # Maximum seismic load on upper body[kN]
Q_2_max = alpha_vals[0] * G2/1000       # Maximum seismic load on lower body[kN]

a_max = alpha_vals[0]                   # Maximum alpha value [-]
acc_max = a_max*9.81                    # Maximm acceleration on upper body [-]
delta_max = delta_vals[-2]              # Maximum displacemet [m]

print(f"The maximum forces on each block are: Q1= {round(Q_1_max,2)}kN Q2={round(Q_
print('After reaching that limit, the wall will start rocking out of plane.')
print('This corresponds to acelerations of acc= '+str(round(acc_max,2))+' m/s2')
print(f"The a-factor is a={round(a_max,3)}")
print('The limit displacement corresponds to '+str(round(delta_max,3))+' m. After r
```

```
The maximum forces on each block are: Q1= 153.37kN Q2=166.03kN
After reaching that limit, the wall will start rocking out of plane.
This corresponds to acelerations of acc= 3.93 m/s2
The a-factor is a=0.401
The limit displacement corresponds to 0.091 m. After reaching this displacement th
e wall loses all resistance.
```





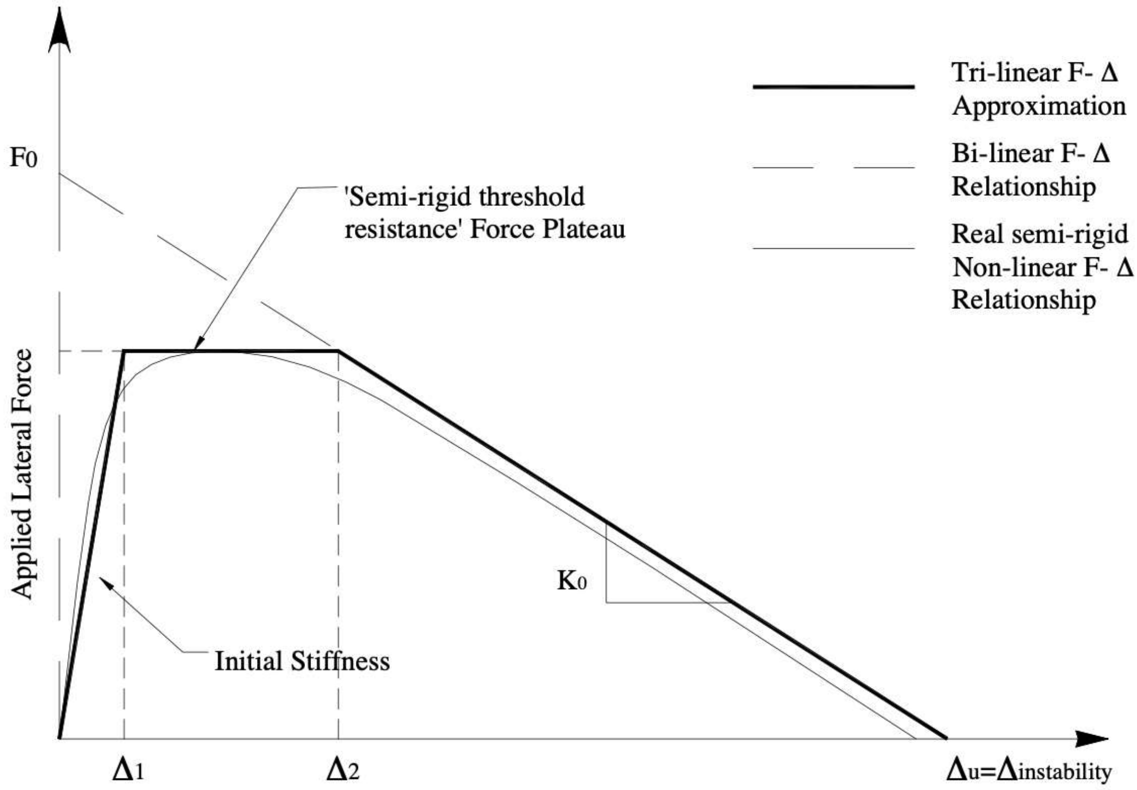
5. Computation of capacity curve

This section allows the analysis of the structural response in the context of the seismic spectrum. The goal is to compare the capacity curve to the ADRS response spectrum, which will allow the behaviour of the wall to be interpreted (e.g., indicating whether it would meet its design requirements).

The main input is the spectral acceleration (for a 475-year return period) for the wall in its given location, as well as its period. For users of the Swiss code, SIA 261 chapter 16.2.1.2 provides the horizontal soil acceleration for each seismic zone; otherwise it can be found in the relevant National Annex of the Eurocode.

As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, Δ_1 and Δ_2, and the maximum displacement capacity, Δ_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, Δ_u is approximated to be equivalent to Δ_o in this code.

In [11]:
```
display(Image(filename='Griffith_Trilinear.png'))
```



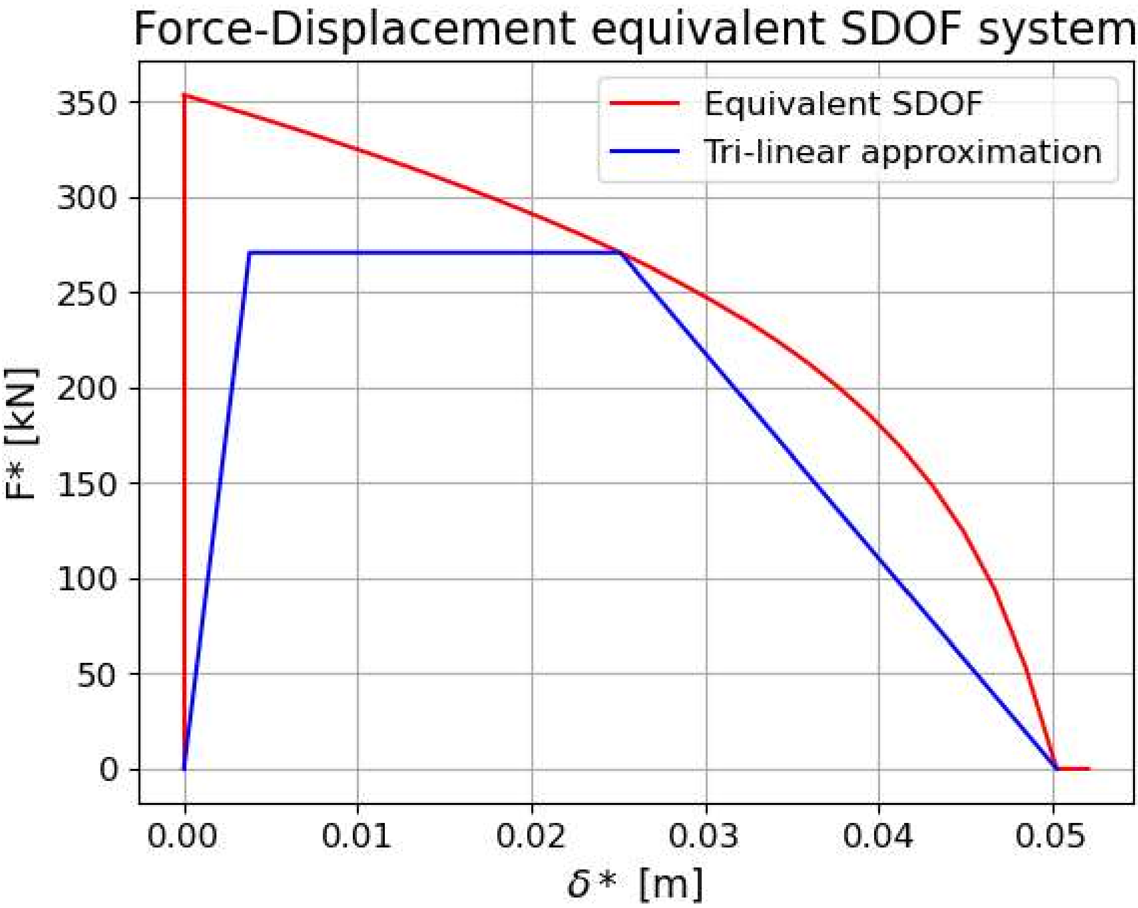
In [12]:
```
agr = 0.152            # Horizontal ground acceleration [g] for a return period of 4

# Choose the coefficients for the trilinear model per Griffith et al. (2003)
d1 = 0.075             # Range from 0.05 to 0.20
d2 = 0.5               # Range from 0.25 to 0.50
```

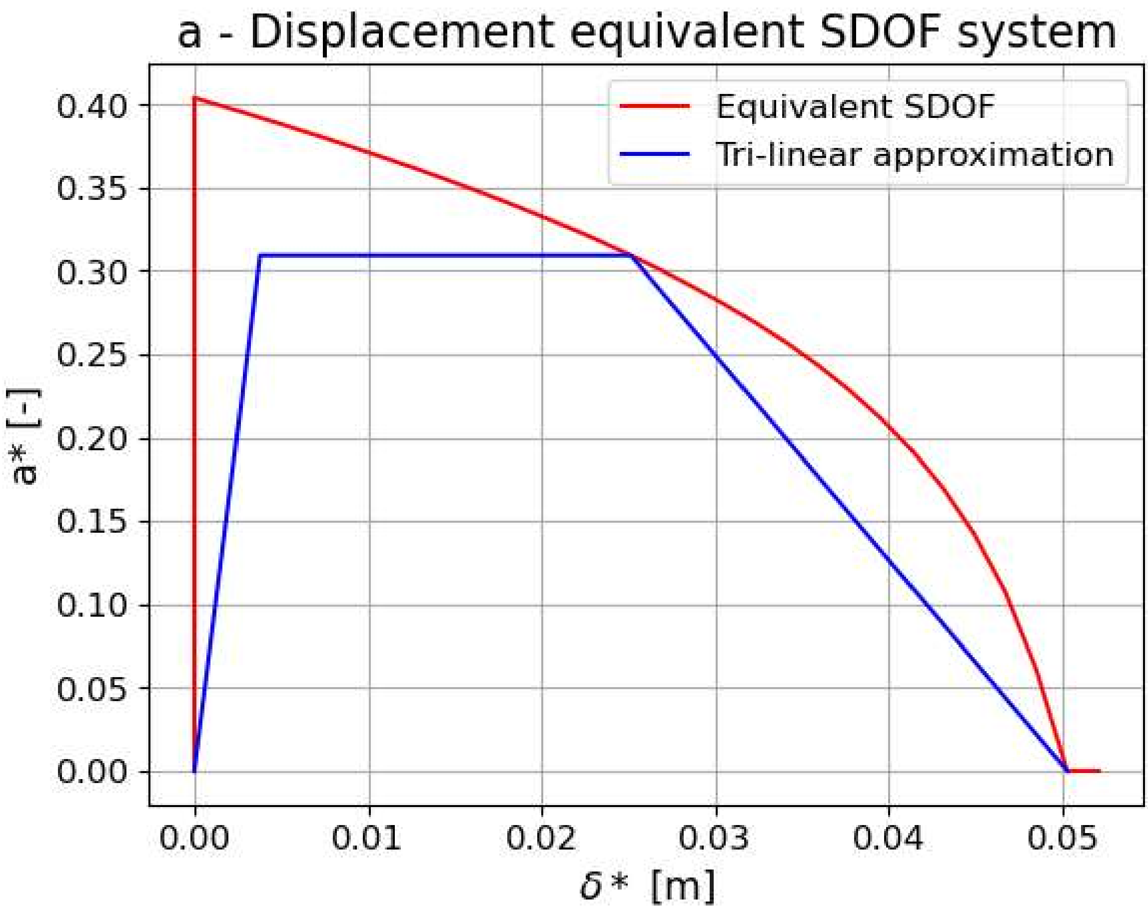
Response spectra are defined for single degree of freedom (DOF) systems. In this case, the mechanism involves more than one DOF. Thus, the capacity curve needs to be transformed into a capaciti curve for an equivalent SDOF system. Furthermore, after the transformation the pushover curve is aproximated as a tri-linear curve as proposed above (Griffith et al., 2003).

In [19]:
```
m = np.array([V_net1,V_net2])*rho_masonry                   # mass of wall [kg]
g = 9.81                                                    # gravitational constant
Qs = np.sum(m)*agr*g                                        # seismic force acting o
phi = np.divide([(Htot-COG_1)/(Htot-H1), COG_2/H2],(COG_2/H2))       # modal shape

# Effective properties of the SDOF system
m_star = np.dot(m,phi)                                      # effective mass (SDOF)
Gamma = m_star / np.dot(m,phi**2)                           # participation factor [
Fb = q_tot_vals * 1000                                      # total shear force [N]
F_star = Fb/Gamma                                           # equivalent shear force
d_2 = delta_vals/H2*COG_2                                   # displacement at COG of
phi2 = phi[1]                                               # modal value at COG of
d_star = d_2/(Gamma*phi2)                                   # effective displacement
a_star = F_star/m_star                                      # effective acceleration

# Tri-linear approximation of the pushover curve according to  Griffith et al. (200
[a_1,F1]=alpha_fun(d1*delta_max)
[a_2,F1]=alpha_fun(d2*delta_max)
F_star1 = a_1*(G1+G2)/Gamma
F_star2 = a_2*(G1+G2)/Gamma
a_star1 = F_star1/m_star
a_star2 = F_star2/m_star
d_star1 = (d1*delta_max)/H2*COG_2/(Gamma*phi2)
d_star2 = (d2*delta_max)/H2*COG_2/(Gamma*phi2)
d_star_max = d_star[-2]
F_star_max = F_star2
F_star_app = [0,F_star_max, F_star_max,0]
d_star_app = [0,d_star1,d_star2,d_star_max]
a_star_app = F_star_app/m_star
a_star_max = F_star_max/m_star

# Displacement capacities according to EN 1998-3
d_u1 = 0.4*d_star_max
d_u2 = 0.6*d_star_max

# Pushover curve of the equivalent SDOF Force-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,F_star[0]/1000],color='red')
plt.plot(d_star,np.divide(F_star,1000),label='Equivalent SDOF',color='red')
plt.plot(d_star_app,np.divide(F_star_app,1000),label='Tri-linear approximation',col

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('F* [kN]', fontsize=14)
plt.title('Force-Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

# Pushover curve of the equivalent SDOF Acceleration-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,a_star[0]],color='red')
plt.plot(d_star,a_star,label='Equivalent SDOF',color='red')
plt.plot(d_star_app,a_star_app,label='Tri-linear approximation',color='blue')
plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('a* [-]', fontsize=14)
plt.title('a - Displacement equivalent SDOF system', fontsize=16)
```

```
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
```

Out[19]:
```
(array([-0.05,  0.  ,  0.05,  0.1 ,  0.15,  0.2 ,  0.25,  0.3 ,  0.35,
         0.4 ,  0.45]),
 [Text(0, -0.05, '−0.05'),
  Text(0, 0.0, '0.00'),
  Text(0, 0.05, '0.05'),
  Text(0, 0.10000000000000002, '0.10'),
  Text(0, 0.15000000000000002, '0.15'),
  Text(0, 0.2, '0.20'),
  Text(0, 0.25000000000000006, '0.25'),
  Text(0, 0.30000000000000004, '0.30'),
  Text(0, 0.35000000000000003, '0.35'),
  Text(0, 0.4, '0.40'),
  Text(0, 0.45, '0.45')])
```





To compute the response spectrum, it is necessary to specify the soil type per SIA 261 Tableau 24 or the Eurocode National Annex for a particular country.

Alternatively, sometimes the response spectra are defined according to the limit state for analysis. The available limit states are: 'serviceability', 'damage limitation', 'significant damage' or 'near collapse. More information can be found in EC 8 chapter 4.2.3.

After the computation of the response spectrum, the capacity curve of the equivalent SDOF system can be plotted alongside the acceleration-displacement response spectrum. This provides a direct comparison between the specific behavior of the elements in question (in this case, the wall) and the spectrum of the desired limit state (either serviceability state, damage limitation, significant damage, or near collapse), and it can quickly give the spectral acceleration for a certain spectral displacement.

This displacement-based evaluation utilizes the equal displacement "rule" for periods smaller than T_c and the equal energy "rule" for periods larger than T_c. Thus, to interpret the results, extend the initial positive linear part of the capacity curve until it intersects the ADRS, and draw a vertical line at this intersection. If $T < T_c$, the actual displacement will be slightly larger than the displacement at the intersection of the capacity curve and the vertical line; if $T > T_c$, the actual displacement will be equivalent to the displacement at the intersection of the capacity curve and the vertical line. The displacement capacity is exceeded if the determined displacement is larger than the end of the capacity curve.

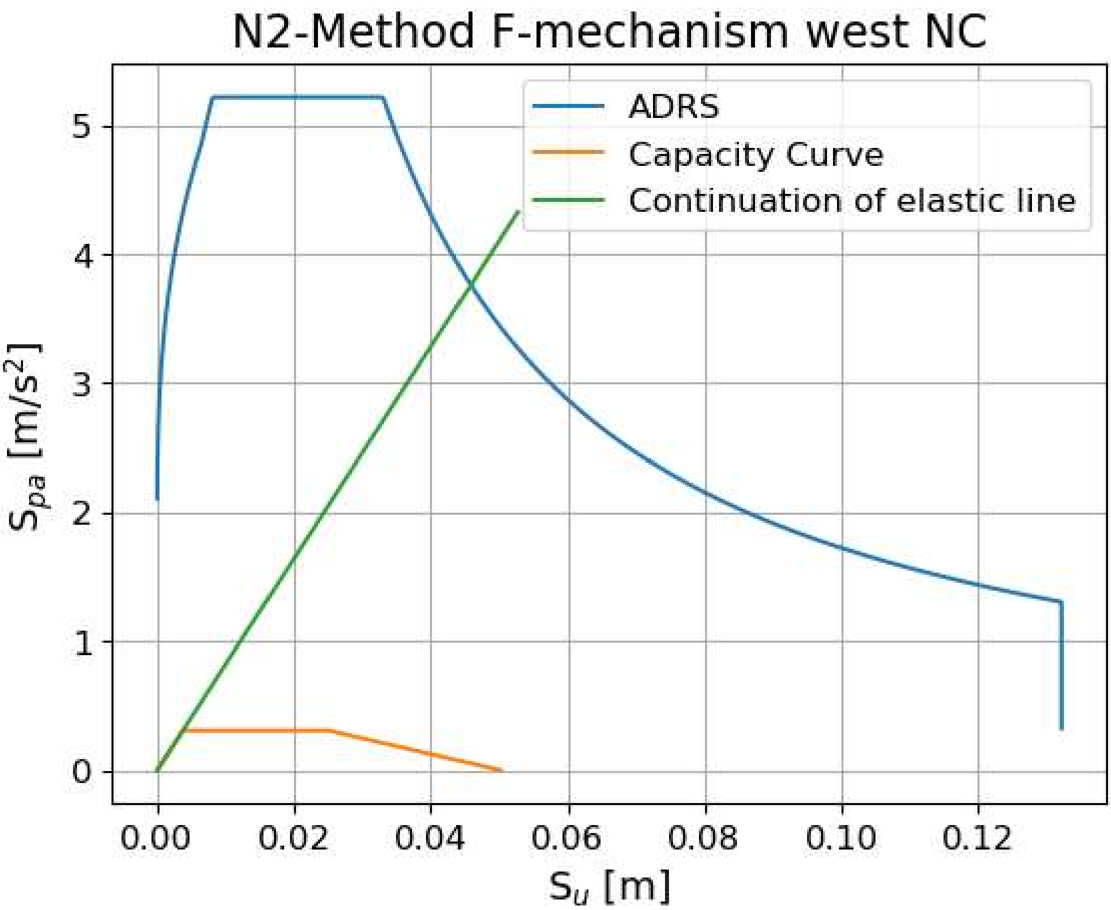
In [14]:
```
soil_type = 'E'          # specify the soil type (ranging from A to E)
```

In [15]:
```
## Determine the periods that define the spectrum according to the soil type, with
def spectrum_vals(soil_type):
    if soil_type == 'A':
        S = 1.00                                        # [-]
        Tb  = 0.15                                      # [s]
        Tc  = 0.40                                      # [s]
        Td  = 2.00                                      # [s]
    elif soil_type == 'B':
        S = 1.20                                        # [-]
        Tb  = 0.15                                      # [s]
        Tc  = 0.50                                      # [s]
        Td  = 2.00                                      # [s]
    elif soil_type == 'C':
        S = 1.15                                        # [-]
        Tb  = 0.20                                      # [s]
        Tc  = 0.60                                      # [s]
        Td  = 2.00                                      # [s]
    elif soil_type == 'D':
        S = 1.35                                        # [-]
        Tb  = 0.20                                      # [s]
        Tc  = 0.80                                      # [s]
        Td  = 2.00                                      # [s]
    elif soil_type == 'E':
        S = 1.40                                        # [-]
        Tb  = 0.15                                      # [s]
        Tc  = 0.50                                      # [s]
        Td  = 2.00                                      # [s]

    return S,Tb,Tc,Td

## Determine the spectral acceleration according to the structure's fundamental per
def RS(T,agr,S,Tb,Tc,Td):

    # determine spectral ground acceleration for dimensionning
    g = 9.81                                            # [m/s^2]
    agd = agr*g                                         # [m/s^2]
    zeta = 0.05                                         # assume damping ratio to be 5% [-
    eta = max(np.sqrt(1/(0.5+10*zeta)),0.55)            # reduction coefficent [-]

    # compute spectrum according to period
    if T<Tb:
        Se = agd*S*(1+(T/Tb)*(eta*2.5-1))
    elif T>=Tb and T<Tc:
        Se = agd*S*eta*2.5
    elif T>=Tc and T<Td:
        Se = agd*S*eta*2.5*(Tc/T)
    else:
        Se = agd*S*eta*2.5*(Tc*Td/T**2)

    return Se
```

In [16]:
```
## Compute ADRS
# Initialize vectors
T_vals = np.array(np.linspace(0.001,4,1000))
Su_vals = np.zeros((len(T_vals),1))
Se_vals = np.zeros((len(T_vals),1))
[S,Tb,Tc,Td] = spectrum_vals(soil_type)

for i in range(0,1000):
    omega = 2*np.pi/T_vals[i]                                           # natural f
    Se_vals[i] = RS(T_vals[i],agr,S,Tb,Tc,Td)                           # spectral
    if T_vals[i] < Tc:
        Ry = max(Se_vals[i]*m_star/Qs,1)                                # R relatic
```

```python
        wd_int = ((Ry-1)*Tc/T_vals[i] + 1)*Se_vals[i]/((omega**2)*Ry)   # displacem
        Su_vals[i] = min(wd_int,3*Se_vals[i]/omega**2)                   # spectral
    else:
        Su_vals[i] = Se_vals[i]/(omega**2)                                # spectral

## Plot capacity curve NC
# plot results
plt.figure()
plt.grid()
plt.plot(Su_vals,Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/5)],Su_vals[:round(len(Su_vals)/5)]/d_star1*a_
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method F-mechanism west NC', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

def find_closest_pair(d1, f1, d2, f2):

    data_set1 = np.column_stack((d1, f1))
    data_set2 = np.column_stack((d2, f2))

    min_distance = float('inf')
    closest_pair = None

    for point1 in data_set1:
        for point2 in data_set2:
            dist2d =point1 - point2
            distance = (dist2d[0]**2+dist2d[1]**2)**0.5
            if distance < min_distance:
                min_distance = distance
                closest_pair = (point1[0], point1[1], point2[0], point2[1])

    return closest_pair


closest = find_closest_pair(Su_vals, Se_vals, Su_vals[:round(len(Su_vals)/5)],Su_va
target_disp_NC = (closest[0]+closest[2])/2

print(f"The target displacement NC is of d={round(target_disp_NC,3)} m")
print(f"The NC limit displacement is d_u2={round(d_u2,3)} m")
if target_disp_NC<=d_u2:
    print('Therefore the NC limit state is verified')
else:
    print('Therefore the NC limit state is NOT verified!!!!')
```

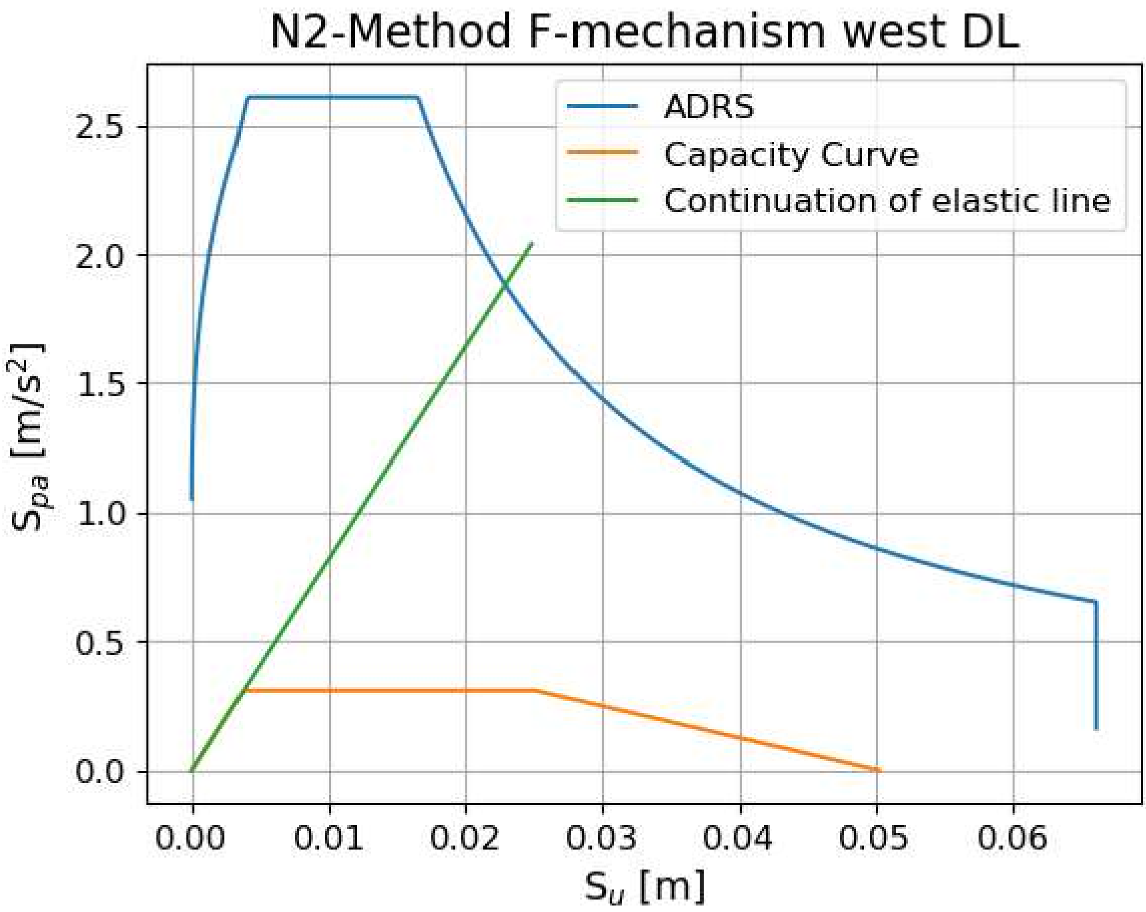
```
The target displacement NC is of d=0.046 m
The NC limit displacement is d_u2=0.03 m
Therefore the NC limit state is NOT verified!!!!
```



In [17]:

```
## Plot capacity curve DL
# plot results
plt.figure()
plt.grid()
plt.plot(0.5*Su_vals,0.5*Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/10)],Su_vals[:round(len(Su_vals)/10)]/d_star1*
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method F-mechanism west DL', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

closest = find_closest_pair(0.5*Su_vals, 0.5*Se_vals, Su_vals[:round(len(Su_vals)/1
target_disp_DL = (closest[0]+closest[2])/2

print(f"The target displacement DL is of d={round(target_disp_DL,3)} m")
print(f"The DL limit displacement is d_u1={round(d_u1,3)} m")
if target_disp_DL<=d_u1:
    print('Therefore the DL limit state is verified')
else:
    print('Therefore the DL limit state is NOT verified!!!!')
```

```
The target displacement DL is of d=0.023 m
The DL limit displacement is d_u1=0.02 m
Therefore the DL limit state is NOT verified!!!!
```


N2-Method F-mechanism west DL
ADRS
Capacity Curve
Continuation of elastic line
S_{pa} [m/s^2]
S_u [m]
0.0
0.5
1.0
1.5
2.0
2.5
0.00
0.01
0.02
0.03
0.04
0.05
0.06

Out-of-plane failure calculations

F-Mechanism Parish House East Facade

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Master Thesis 2023/2024

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Introduction

This notebook can be used to calculate the push-over curve for an out-of-plane mechanisms of a masonry structure subject to seismic action. The mechanism in question is a F-mechanism (D'Ayala, 2002) and the calculations were performed for the east facade of the Parish House at St. Lawrence Church in Petrinja, Croatia.

First, the geometric and material characteristics will be defined, then depending on these inputs, the displacement capacity for out-of-plane accelerations will be returned in form of a push-over graph.

In [1]:

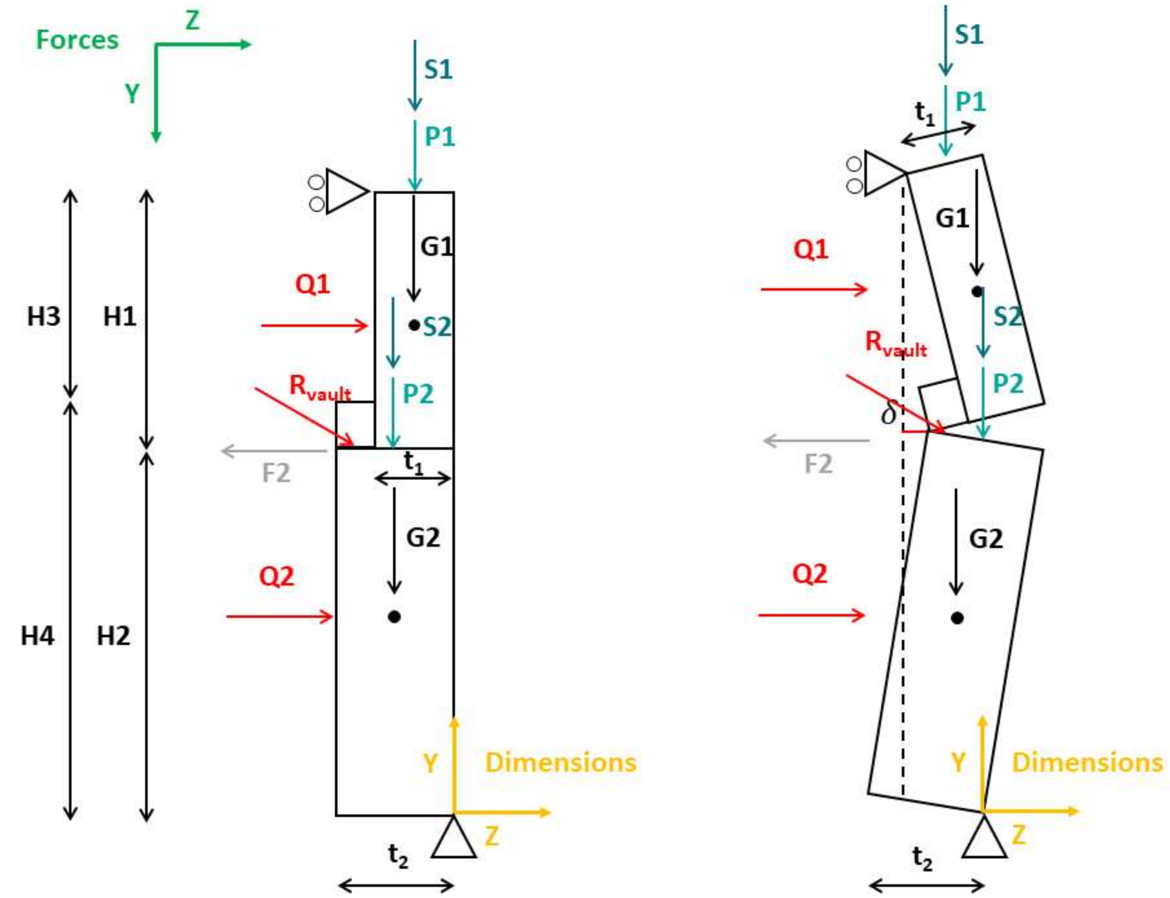
```
# Import the basic packages for the notebook
import numpy as np                      # Numerical library
import pandas as pd                     # Data analysis library
import matplotlib.pyplot as plt         # Plotting library
import matplotlib.ticker as maticker # Tick locating and formatting library
import matplotlib.colors as mcolors   # Colours
from sympy.solvers import solvers      # Library to solve equations
from sympy import Symbol, Eq, nsolve # For solving equations
import math
```

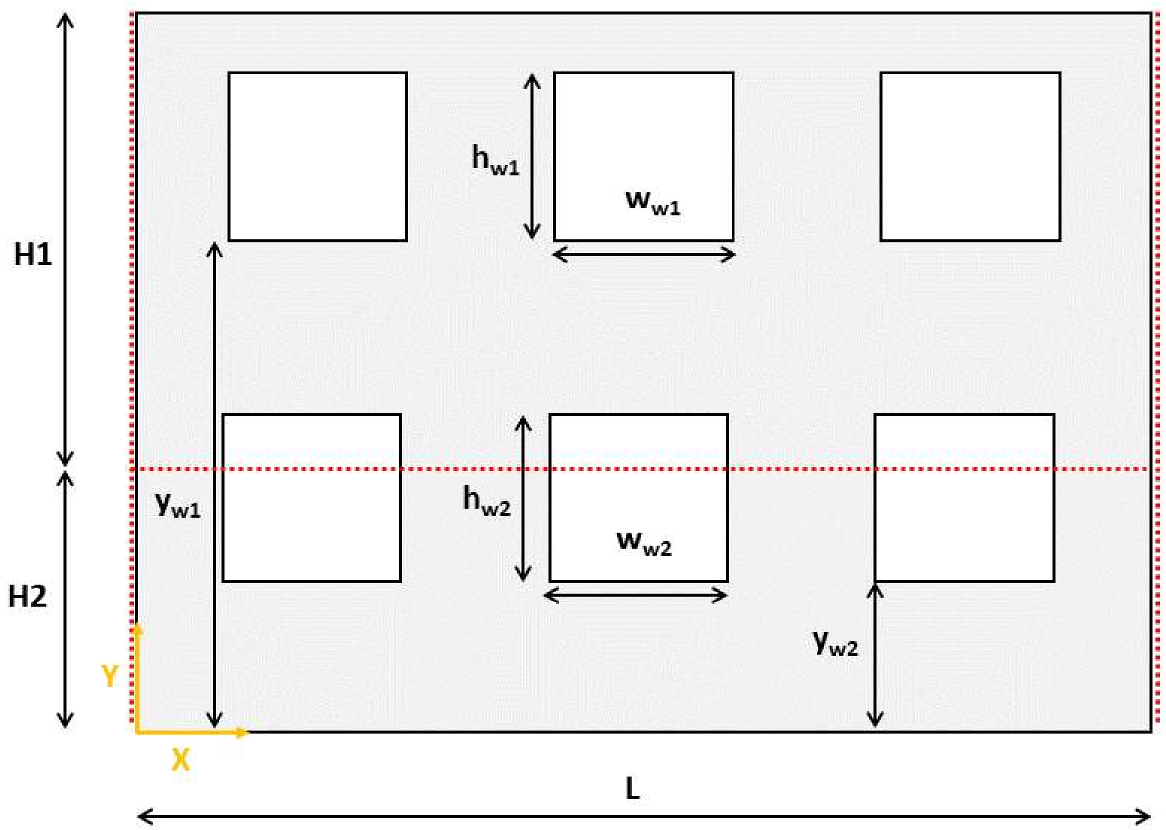
1. Define the wall geometry

The chosen example for this code is a wall with two rows of windows and two thicknesses moving out of plane around a mid axis where the wall changes thickness. A sketch with the main dimensions is shown below. At the height where the wall changes thickness, a vault is supported which will add a vertical load, as well a horizontal thrust.

In [18]:

```
from IPython.display import Image, display
display(Image(filename='Mec_4_3.png'))
display(Image(filename='Mec_4_2.png'))
```





This block is used to define the geometry of the wall and the windows within.

In [4]:
```
## Wall geometry
H1 = 4.55           # Height of the upper part of the wall,     y-direction [m]
H2 = 1.55           # Height of the lower part of the wall,     y-direction [m]
H3 = 2.70           # Height of the wall with thickness t1,     y-direction [m]
H4 = 3.40           # Height of the wall with thickness t2,     y-direction [m]
```

```
L = 17.00           # Total length of wall,                     x-direction [m]
t1 = 0.55           # Thickness of the upper part of the wall, z-direction [m]
t2 = 0.70           # Thickness of the lower part of the wall, z-direction [m]


## Window geometry, upper row
h_w1 = 1.7          # Height of window on upper wall [m]
w_w1 = 1.1          # Width of window on upper wall[m]
y_w1 = 3.65         # Vertical distance from y = 0 to bottom corner of window on up
n_w1 = 4            # Number of windows on upper wall [-]

## Window geometry, lower row
h_w2 = 1.4          # Height of window on lower wall [m]
w_w2 = 1.1          # Width of window on lower wall[m]
y_w2 = 0.7          # Vertical distance from y = 0 to bottom corner of window on lo
n_w2 = 4            # Number of windows on lower wall [-]

## Vault geometry
h_v = 1.25          # Height of the vault at mid-span with respect to its support c
w_v = 5.60          # Main span width of the vault [m]
t_v = 0.15          # Thickness of the vault [m]


## Slab information
max_delta = t2/2    # Maximum displacement, when the slab and wall lose connection
                    # restraining the wall [m]
```

2. Define material properties

This block is used to define the material properties of the wall and the connecting slab. In this case the friction coefficient is set to zero since the slab is directly supported by the vault, which is in return in a monolithical connection to the wall.

In [16]:
```
rho_masonry = 18000         # Density of the masonry [N/m^3]
mu_f = 0.0                  # Friction coefficient between slab and masonry wall
```

3. Define the loads and forces

In this paragraph, the loads and forces acting on the wall are defined. The following block defines the vertical load on top of the wall coming from a slab (S) or a superimposed wall (P). Note that the S-load will be used to directly calculate the restraining force due to friction between the wall and the slab. If you want to introduce a vertical load that does not produce any friction, please use the P-load.

In [7]:
```
P1 = 78288                  # vertical load on top of the upper wall [N]
S1 = 155075                 # slab load acting on top of the upper wall [N]
P2 = 248250                 # vertical load on top of the lower wall [N]
S2 = 0                      # slab load acting on top of the lower wall [N]
q_vault = 63000             # surface load acting on vault [N/m]
```

4. Calculation of overturning moment

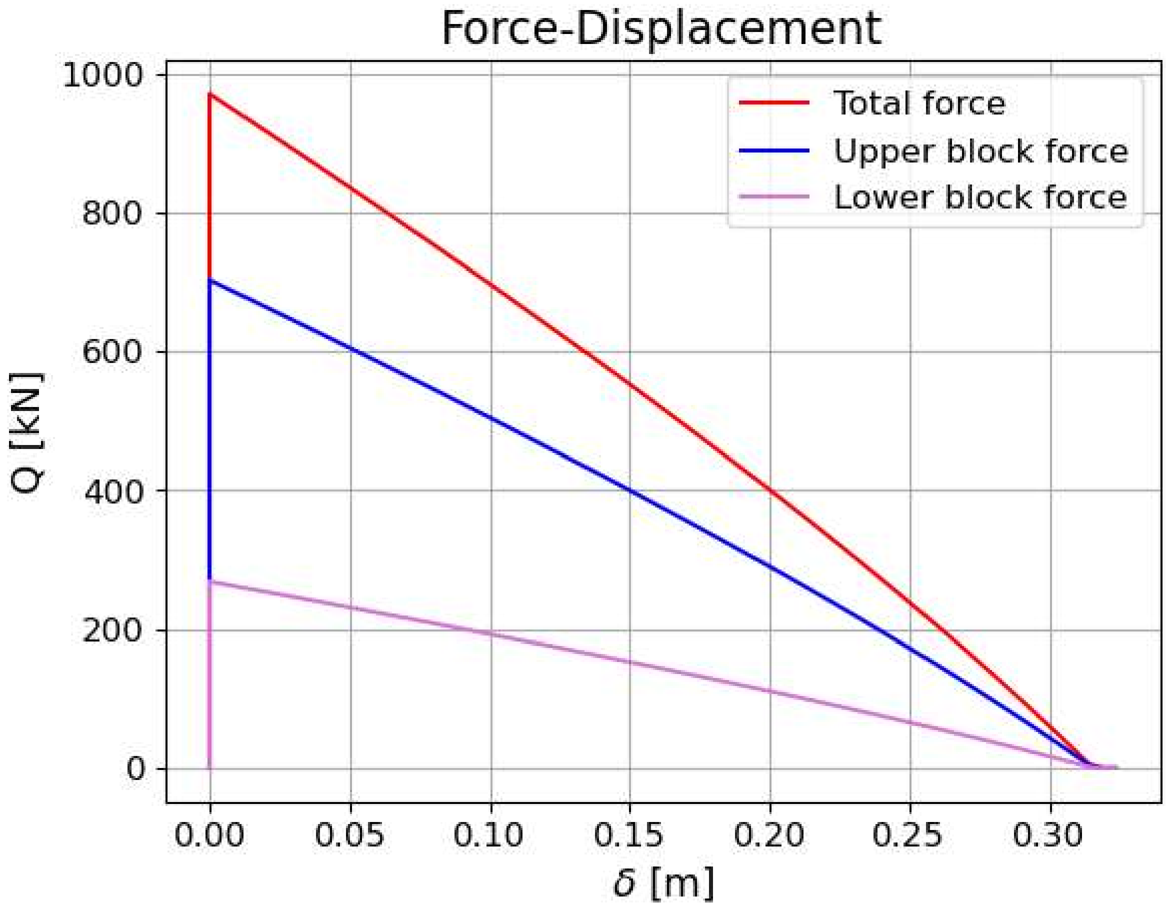
This paragraph calculates the pushover curve for the given case.

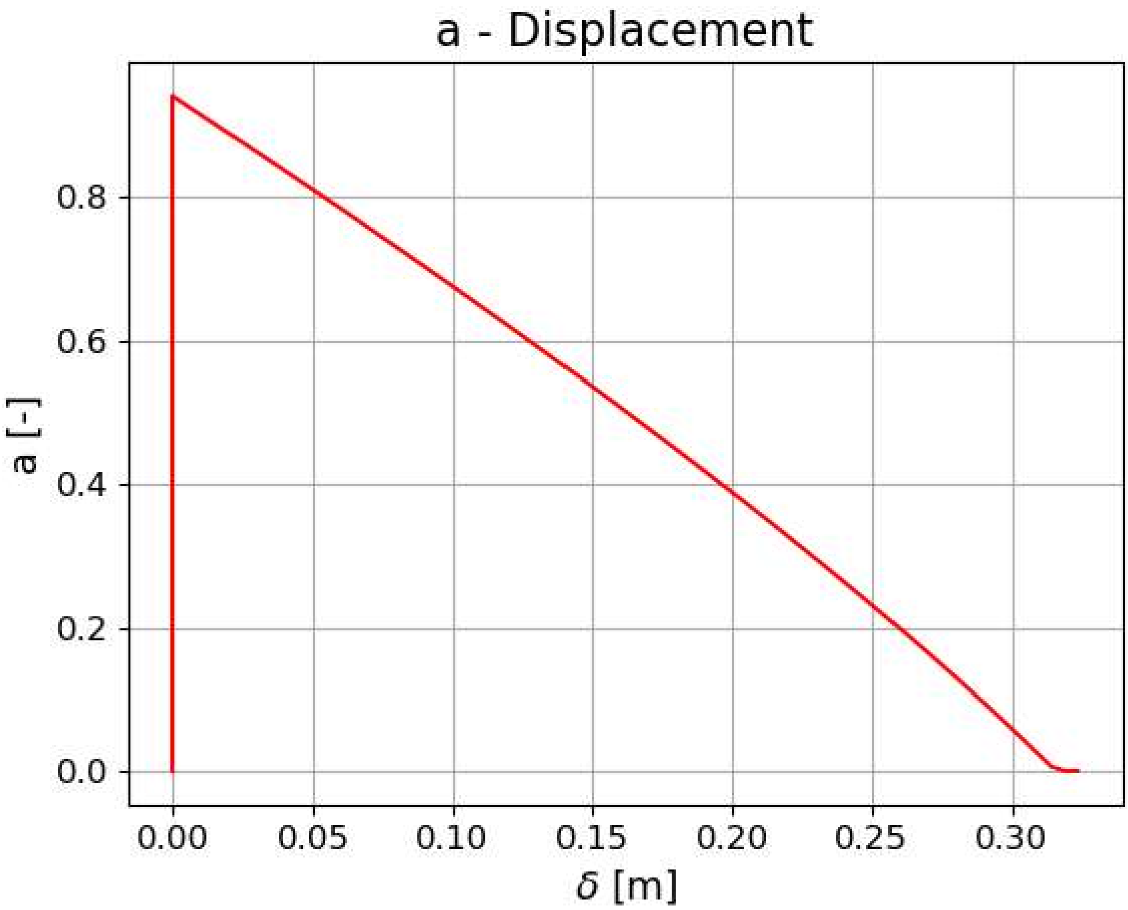
In [8]:
```python
# This function calculates the center of gravity of the wall, y-direction using the
def cog_fun(n_w,h_w,y_w,w_w,H,L,y_W):
    # n_w = number of windows in the wall block
    # h_w = height of the windows [m]
    # y_w = distance of ground to lower corner of the windows [m]
    # w_w = width of the windows [m]
    # H   = height of the wall block [m]
    # L   = length of the wall block [m]
    # y_W = height of the lower corner of the wall block above ground [m]
    A_w = n_w*h_w*w_w          # total area of windows [m^2]
    A = H*L                    # area of wall [m^2]
    ybar = H/2                 # centroid of wall with respect to its lower corner,
                               # rectangular profile with constant thickness [m]
    ybar_w = y_w+h_w/2-y_W     # centroid of wall with respect to lower corner of wa
                               # rectangular profile with constant thickness [m]

    COG_y = ((A*ybar)-(A_w*ybar_w))/(A-A_w)+y_W

    return COG_y

# This function calculates the horizontal thrust and the vertical load from the vau
def vault_fun(delta):
 # delta = displacement of the wall OOP [m]

    # defining displaced geometry
    w_v_new = w_v + delta                        # new width of the vault [m]
    l_v = math.sqrt((w_v/2)**2 + h_v**2)    # length of the diagonal connecting the
                                                 # mid-span of the vault [m]
    if w_v_new >= l_v*2:                         # check if the vault is still in place
        return np.array([0,0])
    else:
        h_v_new = math.sqrt(l_v**2-(w_v_new/2)**2) # new height of the vault [m]

     # calculate support loads
        R_h = (q_vault * w_v_new**2 /          # horizontal thrust [N]
              (8 * h_v_new))
        R_v = q_vault * w_v_new /2             # vertical support load [N]

    return np.array([R_v,R_h])

# In this function we find the alpha value corresponding to the part of the self we
# wall, representing the seismic load (Q1=alpha*G1 and Q2=alpha*G2), for an equilib
# at a given displacement delta
def alpha_fun(delta):

    delta_G1 = delta*(H1+H2-COG_1)/H1                          # Displacement at th
    delta_G2 = delta*COG_2/H2                                  # Displacement at th
    F2_it = F2
    if delta > H1:
        return [0,0]
    # Verify if the slab at cracking level is still in contact
    if delta > max_delta:
        F2_it=0
    # Calculate the thrust of the vault
    [R_v,R_h] = vault_fun(delta)
    if [R_v,R_h] == [0,0]:
        return [0,0]

    a = Symbol('a')
    F1 = Symbol('F1')
    # Equilibrium about the lower support point
    equation1 = Eq( -F1*Htot                                    # Moment from upper
                    -F2_it*H2                                   # Moment from middl
```

```
                    +a*G1*COG_1                                 # Moment from seism
                    +a*G2*COG_2                                 # Moment from seism
                    -(P1+S1)*t1/2                               # Moment from verti
                    +(P2+S2+R_v)*(delta-(t2/2))                 # Moment from verti
                    +G1*(delta_G1-(t1/2))                       # Moment from selfw
                    +G2*(delta_G2-(t2/2))                       # Moment from selfw
                    +R_h*H2                                     # Moment from the t
                , 0)

    # Equilibrium of the upper body about its rotation corner
    equation2 = Eq( -F1*H1                                      # Moment from upper
                    +a*G1*(COG_1-H2)                            # Moment from seism
                    +(P1+S1)*(t1/2-delta+d_t)                   # Moment from verti
                    +G1*(delta_G1+(t1/2)+d_t-delta)             # Moment from selfw
                    ,0)

    # Initial guesses to solve the equation (this is important to converge fast)
    a_guess = 1
    F1_guess = G1+G2

    # Solve the equation for alpha (a) and the upper reaction force (F1)
    [a,F1]=nsolve((equation1, equation2), (a, F1), (a_guess, F1_guess))

    return [a,F1]

## Derived wall properties
Htot=H1+H2          # total height of the wall [m]

# Calculating the net section of the wall (section without area of windows) [m^2]
A_net3 = H3*L-n_w1*h_w1*w_w1      #Net area of upper wall / Area of upper wall with
A_net4 = H4*L-n_w2*h_w2*w_w2      #Net area of lower wall / Area of lower wall with
A_net2 = H2*L-n_w2*(H2-y_w2)*w_w2 #Net area of lower moving block [m^2]

# Calculating the volume of the two wall blocks
V_net3 = A_net3*t1                #Net volume of upper wall / Volume of upper wall
V_net4 = A_net4*t2                #Net volume of lower wall / Volume of lower wall
V_net2 = A_net2*t2                #Net volume of lower block [m^3]
V_net1 = V_net3 + V_net4 - V_net2 #Net volume of upper block [m^3]
V_net_tot = V_net1 + V_net2       #Net volume of the whole wall [m^3]

#Calculating the centers of gravity with respect to the ground [m]
COG_3 = cog_fun(n_w1,h_w1,y_w1,w_w1,H3,L,H4)          # for upper wall
COG_4 = cog_fun(n_w2,h_w2,y_w2,w_w2,H4,L,0)           # for lower wall
COG_tot = (COG_3 * V_net3 + COG_4 * V_net4)/V_net_tot # for the whole wall
COG_2 = cog_fun(n_w2,(H2-y_w2),y_w2,w_w2,H2,L,0)      # for lower block/body
COG_1 = (COG_tot * V_net_tot - COG_2 * V_net2)/V_net1 # for upper block/body

# Difference between the thicknesses of the walls
d_t = t2-t1

## Derived loads and forces
G1 = rho_masonry*V_net1         # gravity load of the upepr wall (self-weight) [N]
G2 = rho_masonry*V_net2         # gravity load of the lower wall (self-weight) [N]
F2 = mu_f*S2                    # friction force from slab acting on top of the lowe

## Maximum earthquake force to start rocking
[alpha_max, F1_max] = alpha_fun(0)
Q_tot_max = alpha_max * (G1+G2)/1000  # Maximum total seismic load [kN]
Q_1_max = alpha_max * G1/1000         # Maximum seismic load on upper body[kN]
Q_2_max = alpha_max * G2/1000         # Maximum seismic load on lower body[kN]

## Plot force-displacement relationship of the overturning of the wall
delta_vals = np.linspace(0,H1,1000)
alpha_vals = np.zeros((np.size(delta_vals)))
```

```python
F1_vals = np.zeros((np.size(delta_vals)))

for i in range(np.size(delta_vals)): # Calculates the corresponding alpha and F1 fo
    [alpha_vals[i],F1_vals[i]]=alpha_fun(delta_vals[i])

# Transformation to only display values up to failure
alpha_vals = np.append(alpha_vals[alpha_vals>0],[0,0],axis=0)

delta_vals = delta_vals[: np.size(alpha_vals)]
F1_vals= F1_vals[:np.size(alpha_vals)]

q_tot_vals = alpha_vals * (G1+G2)/1000  # Maximum total seismic load [kN]
q_1_vals = alpha_vals * G1/1000         # Maximum seismic load on upper body[kN]
q_2_vals = alpha_vals * G2/1000         # Maximum seismic load on lower body[kN]

# Plot force-displacement relationship of the overturning of the wall
plt.figure()
plt.grid()

plt.plot([0,0],[0,Q_tot_max],color='red')
plt.plot(delta_vals,q_tot_vals,label='Total force',color='red')

plt.plot([0,0],[0,Q_1_max],color='blue')
plt.plot(delta_vals,q_1_vals,label='Upper block force',color='blue')

plt.plot([0,0],[0,Q_2_max],color='orchid')
plt.plot(delta_vals,q_2_vals,label='Lower block force',color='orchid')

plt.legend(fontsize=12)
plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('Q [kN]', fontsize=14)
plt.title('Force-Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)



# Plot acceleration-displacement relationship of the overturning of the wall
plt.figure()
plt.grid()
plt.plot([0,0],[0,alpha_max],color='red')
plt.plot(delta_vals,alpha_vals,color='red')
plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('a [-]', fontsize=14)
plt.title('a - Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
## Maximum earthquake force to start rocking
Q_tot_max = q_tot_vals[0]/1000           # Maximum total seismic load [kN]
Q_1_max = alpha_vals[0] * G1/1000        # Maximum seismic load on upper body[kN]
Q_2_max = alpha_vals[0] * G2/1000        # Maximum seismic load on lower body[kN]

a_max = alpha_vals[0]                    # Maximum alpha  [-]
acc_max = a_max*9.81                     # Maximum acceleration [-]

delta_max = delta_vals[-2]               # Maximum displacemet [m]

print(f"The maximum forces on each block are: Q1= {round(Q_1_max,2)}kN Q2={round(Q_
print('After reaching that limit, the wall will start rocking out of plane.')
print('This corresponds to acelerations of acc= '+str(round(acc_max,2))+' m/s2')
print(f"The a-factors is a={round(a_max,3)}")
print('The limit displacement corresponds to '+str(round(delta_max,3))+' m. After r
```

```
The maximum forces on each block are: Q1= 702.09kN Q2=268.07kN
After reaching that limit, the wall will start rocking out of plane.
This corresponds to acelerations of acc= 9.23 m/s2
The a-factors is a=0.941
The limit displacement corresponds to 0.319 m. After reaching this displacement th
e wall loses all resistance.
```





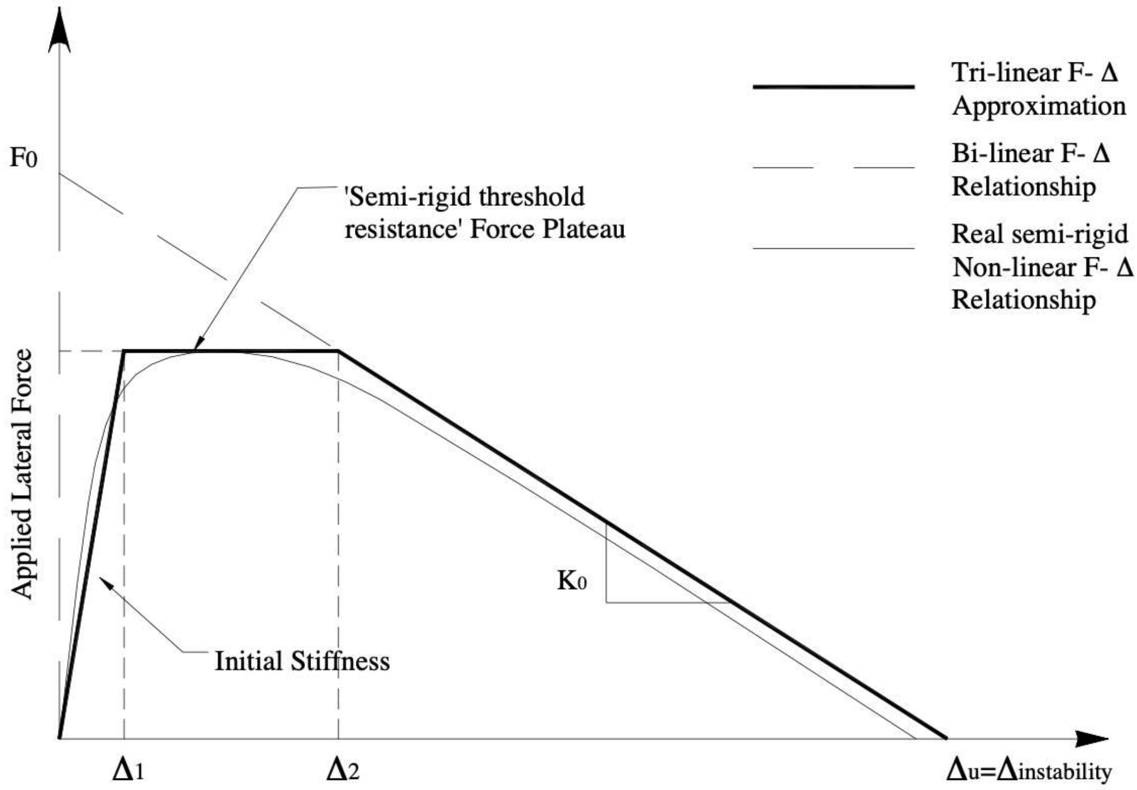
5. Computation of capacity curve

This section allows the analysis of the structural response in the context of the seismic spectrum. The goal is to compare the capacity curve to the ADRS response spectrum, which will allow the behaviour of the wall to be interpreted (e.g., indicating whether it would meet its design requirements).

The main input is the spectral acceleration (for a 475-year return period) for the wall in its given location, as well as its period. For users of the Swiss code, SIA 261 chapter 16.2.1.2 provides the horizontal soil acceleration for each seismic zone; otherwise it can be found in the relevant National Annex of the Eurocode.

As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, delta_1 and delta_2, and the maximum displacement As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, Δ_1 and Δ_2, and the maximum displacement capacity, Δ_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, Δ_u is approximated to be equivalent to Δ_o in this code., delta_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, delta_u is approximated to be equivalent to delta_o in this code.

In [9]:
```
display(Image(filename='Griffith_Trilinear.png'))
```



In [10]:
```
agr = 0.152           # Horizontal soil acceleration [g]

# Choose the coefficients for the trilinear model per Griffith et al. (2003)
d1 = 0.05             # Range from 0.05 to 0.20
d2 = 0.3              # Range from 0.25 to 0.50
```

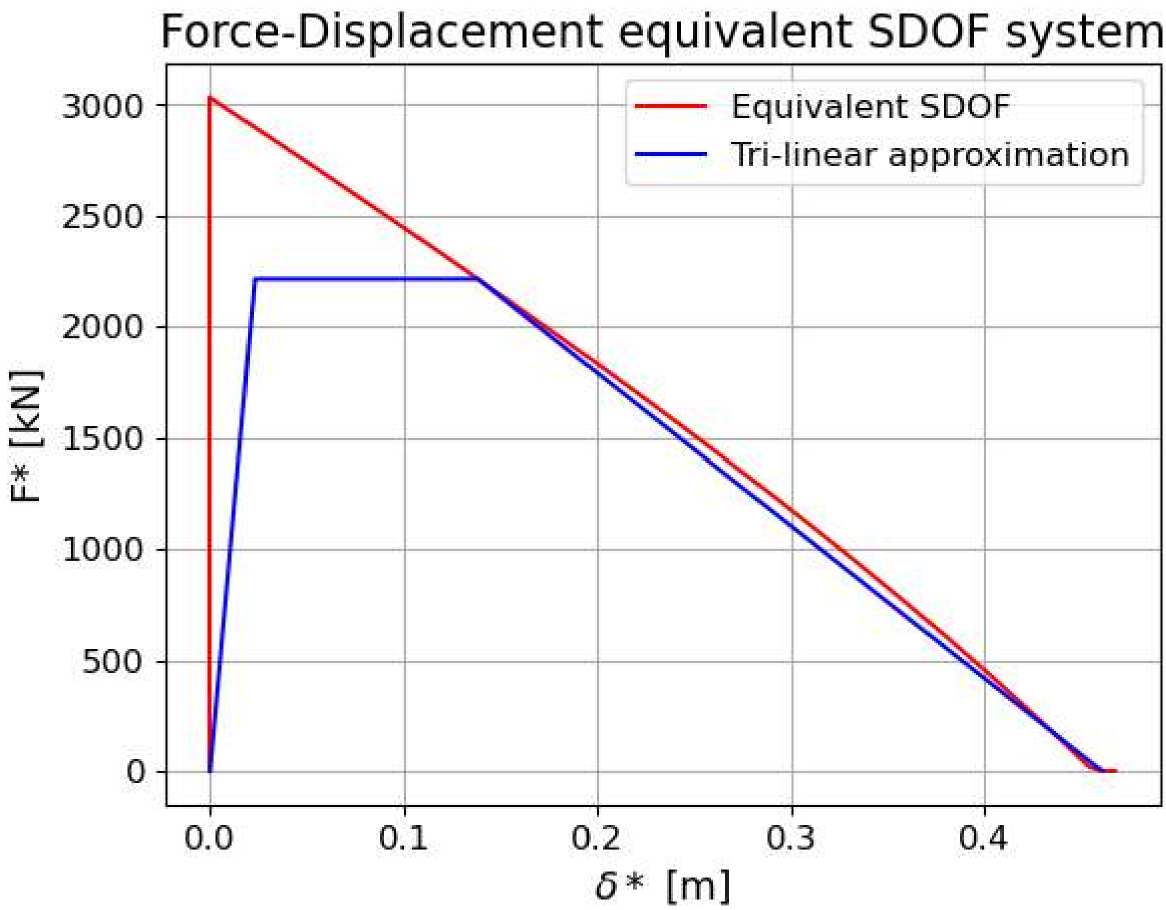
Response spectra are defined for single degree of freedom (DOF) systems. In this case, the mechanism involves more than one DOF. Thus, the capacity curve needs to be transformed into a capaciti curve for an equivalent SDOF system. Furthermore, after the transformation the pushover curve is aproximated as a tri-linear curve as proposed above (Griffith et al., 2003).

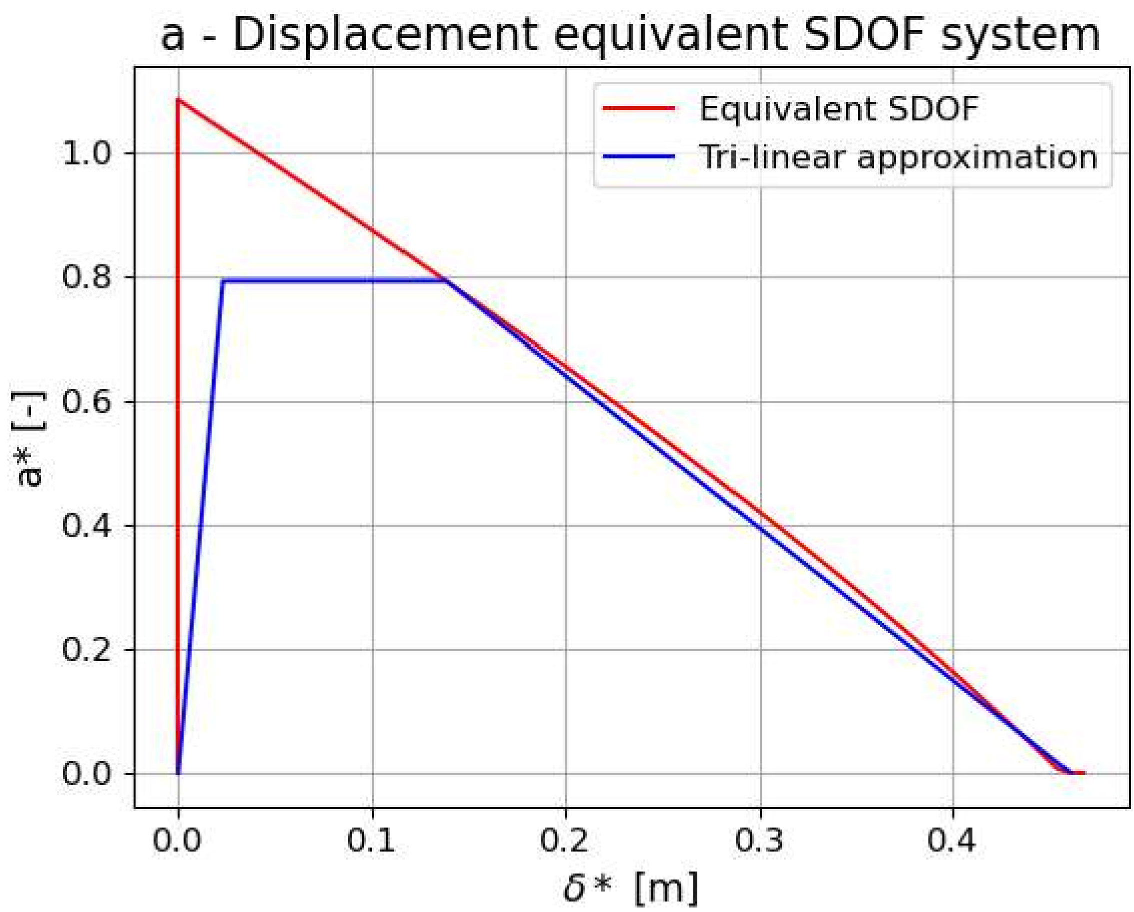
In [17]:
```
m = np.array([V_net1,V_net2])*rho_masonry                    # mass of wall [kg]
g = 9.81                                                      # gravitational constant
Qs = np.sum(m)*agr*g                                          # seismic force acting o
phi = np.divide([(Htot-COG_1)/(Htot-H1), COG_2/H2],(COG_2/H2))      # modal shape

# calculate effective properties
m_star = np.dot(m,phi)                                        # effective mass (SDOF)
Gamma = m_star / np.dot(m,phi**2)                             # participation factor [
Fb = q_tot_vals * 1000                                        # total shear force [N]
F_star = Fb/Gamma                                             # equivalent shear force
d_2 = delta_vals/H2*COG_2                                     # displacement at COG of
phi2 = phi[1]                                                 # modal value at COG of
d_star = d_2/(Gamma*phi2)                                     # effective displacement
a_star = F_star/m_star                                        # effective acceleration

# Tri-linear approximation of the pushover curve according to  Griffith et al. (200
[a_1,F1]=alpha_fun(d1*delta_max)
[a_2,F1]=alpha_fun(d2*delta_max)
F_star1 = a_1*(G1+G2)/Gamma
F_star2 = a_2*(G1+G2)/Gamma
a_star1 = F_star1/m_star
a_star2 = F_star2/m_star
d_star1 = (d1*delta_max)/H2*COG_2/(Gamma*phi2)
```

```
d_star2 = (d2*delta_max)/H2*COG_2/(Gamma*phi2)
d_star_max = d_star[-2]
F_star_max = F_star2
F_star_app = [0,F_star_max, F_star_max,0]
d_star_app = [0,d_star1,d_star2,d_star_max]
a_star_app = F_star_app/m_star
a_star_max = F_star_max/m_star

# Displacement capacities according to EN 1998-3
d_u1 = 0.4*d_star_max
d_u2 = 0.6*d_star_max

# Pushover curve of the equivalent SDOF Force-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,F_star[0]/1000],color='red')
plt.plot(d_star,np.divide(F_star,1000),label='Equivalent SDOF',color='red')
plt.plot(d_star_app,np.divide(F_star_app,1000),label='Tri-linear approximation',col

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('F* [kN]', fontsize=14)
plt.title('Force-Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

# Pushover curve of the equivalent SDOF Acceleration-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,a_star[0]],color='red')
plt.plot(d_star,a_star,label='Equivalent SDOF',color='red')
plt.plot(d_star_app,a_star_app,label='Tri-linear approximation',color='blue')

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('a* [-]', fontsize=14)
plt.title('a - Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
```

Out[17]:

```
(array([-0.2,  0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2]),
 [Text(0, -0.2, '−0.2'),
  Text(0, 0.0, '0.0'),
  Text(0, 0.2, '0.2'),
  Text(0, 0.4000000000000001, '0.4'),
  Text(0, 0.6000000000000001, '0.6'),
  Text(0, 0.8, '0.8'),
  Text(0, 1.0000000000000002, '1.0'),
  Text(0, 1.2000000000000002, '1.2')])
```





To compute the response spectrum, it is necessary to specify the soil type per SIA 261 Tableau 24 or the Eurocode National Annex for a particular country.

Alternatively, sometimes the response spectra are defined according to the limit state for analysis. The available limit states are: 'serviceability', 'damage limitation', 'significant damage' or 'near collapse. More information can be found in EC 8 chapter 4.2.3.

Finally, since this notebook works for a generalized case, the q-factor approach is not used. The use of the q-factor approach is dependent on the conditions described in 11.3.2.3, and hence to keep the calculation as useful as possible, the following spectra are elastic.

After the computation of the response spectrum, the capacity curve of the equivalent SDOF system can be plotted alongside the acceleration-displacement response spectrum. This provides a direct comparison between the specific behavior of the elements in question (in this case, the wall) and the spectrum of the desired limit state (either serviceability state, damage limitation, significant damage, or near collapse), and it can quickly give the spectral acceleration for a certain spectral displacement.

This displacement-based evaluation utilizes the equal displacement "rule" for periods smaller than T_c and the equal energy "rule" for periods larger than T_c. Thus, to interpret the results, extend the initial positive linear part of the capacity curve until it intersects the ADRS, and draw a vertical line at this intersection. If $T < T_c$, the actual displacement will be slightly larger than the displacement at the intersection of the capacity curve and the vertical line; if $T > T_c$, the actual displacement will be equivalent to the displacement at the intersection of the capacity curve and the vertical line. The displacement capacity is exceeded if the determined displacement is larger than the end of the capacity curve.

In [12]:
```
soil_type = 'E'        # specify the soil type (ranging from A to E)
```

In [13]:
```
### NOTHING TO BE CHANGED

## Determine the periods that define the spectrum according to the soil type, with
def spectrum_vals(soil_type):
    if soil_type == 'A':
        S = 1.00                                # [-]
        Tb  = 0.15                              # [s]
        Tc  = 0.40                              # [s]
        Td  = 2.00                              # [s]
    elif soil_type == 'B':
        S = 1.20                                # [-]
        Tb  = 0.15                              # [s]
        Tc  = 0.50                              # [s]
        Td  = 2.00                              # [s]
    elif soil_type == 'C':
        S = 1.15                                # [-]
        Tb  = 0.20                              # [s]
        Tc  = 0.60                              # [s]
        Td  = 2.00                              # [s]
    elif soil_type == 'D':
        S = 1.35                                # [-]
        Tb  = 0.20                              # [s]
        Tc  = 0.80                              # [s]
        Td  = 2.00                              # [s]
    elif soil_type == 'E':
        S = 1.40                                # [-]
        Tb  = 0.15                              # [s]
        Tc  = 0.50                              # [s]
        Td  = 2.00                              # [s]
```

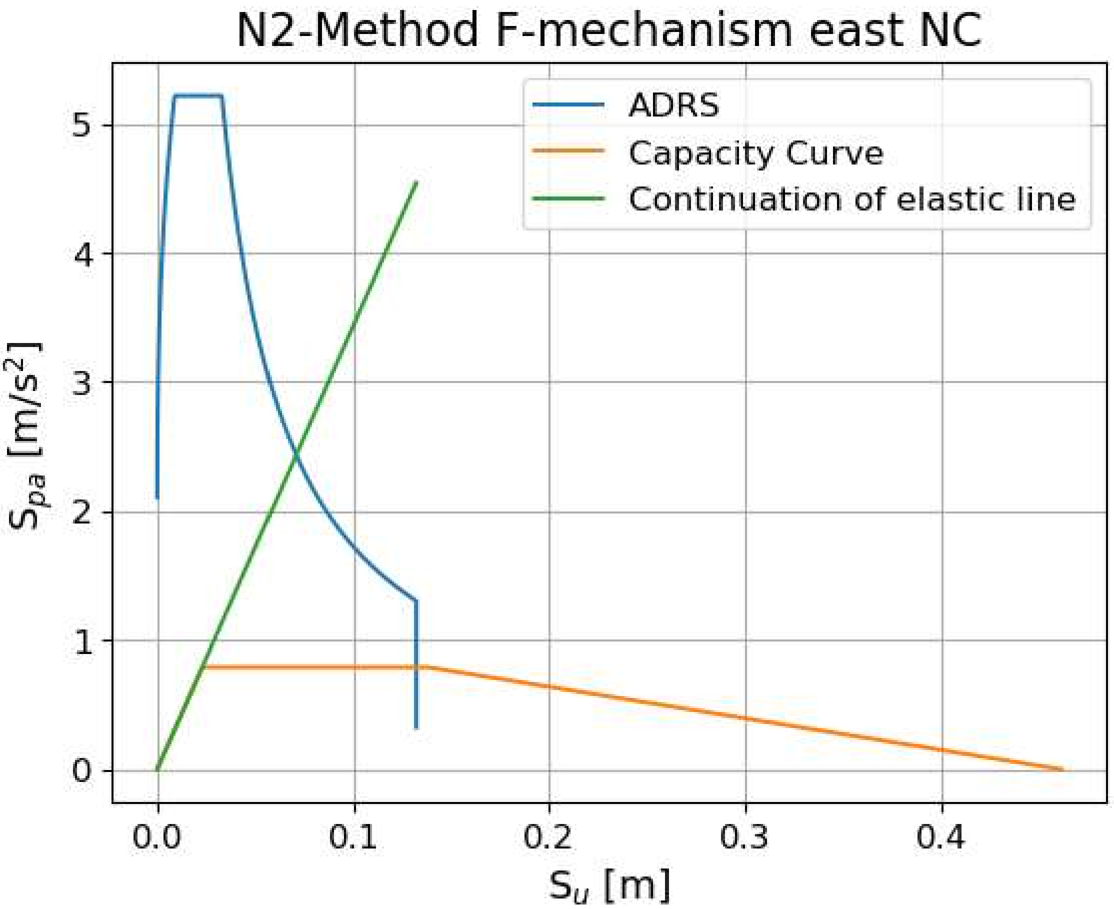
```
    return S,Tb,Tc,Td

## Determine the spectral acceleration according to the structure's fundamental per
def RS(T,agr,S,Tb,Tc,Td):

    # determine spectral ground acceleration for dimensionning
    g = 9.81                                            # [m/s^2]
    agd = agr*g                                         # [m/s^2]
    zeta = 0.05                                         # assume damping ratio to be 5% [-
    eta = max(np.sqrt(1/(0.5+10*zeta)),0.55)            # reduction coefficent [-]

    # compute spectrum according to period
    if T<Tb:
        Se = agd*S*(1+(T/Tb)*(eta*2.5-1))
    elif T>=Tb and T<Tc:
        Se = agd*S*eta*2.5
    elif T>=Tc and T<Td:
        Se = agd*S*eta*2.5*(Tc/T)
    else:
        Se = agd*S*eta*2.5*(Tc*Td/T**2)

    return Se
```

In [14]:

```
### NOTHING TO BE CHANGED

## Compute ADRS
# Initialize vectors
T_vals = np.linspace(0.001,4,1000)
Su_vals = np.zeros((len(T_vals),1))
Se_vals = np.zeros((len(T_vals),1))
[S,Tb,Tc,Td] = spectrum_vals(soil_type)

for i in range(0,1000):
    omega = 2*np.pi/T_vals[i]                                       # natural f
    Se_vals[i] = RS(T_vals[i],agr,S,Tb,Tc,Td)                       # spectral
    if T_vals[i] < Tc:
        Ry = max(Se_vals[i]*m_star/Qs,1)                            # R relatio
        wd_int = ((Ry-1)*Tc/T_vals[i] + 1)*Se_vals[i]/((omega**2)*Ry)  # displacem
        Su_vals[i] = min(wd_int,3*Se_vals[i]/omega**2)              # spectral
    else:
        Su_vals[i] = Se_vals[i]/(omega**2)                          # spectral

## Plot capacity curve NC
# plot results
plt.figure()
plt.grid()
plt.plot(Su_vals,Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals,Su_vals/d_star1*a_star_max,label="Continuation of elastic line")
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method F-mechanism east NC', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

def find_closest_pair(d1, f1, d2, f2):

    data_set1 = np.column_stack((d1, f1))
    data_set2 = np.column_stack((d2, f2))

    min_distance = float('inf')
    closest_pair = None
```

```python
    for point1 in data_set1:
        for point2 in data_set2:
            dist2d =point1 - point2
            distance = (dist2d[0]**2+dist2d[1]**2)**0.5
            if distance < min_distance:
                min_distance = distance
                closest_pair = (point1[0], point1[1], point2[0], point2[1])

    return closest_pair


closest = find_closest_pair(Su_vals, Se_vals, Su_vals[:round(len(Su_vals)/5)],Su_va
target_disp_NC = (closest[0]+closest[2])/2

print(f"The target displacement NC is of d={round(target_disp_NC,3)} m")
print(f"The NC limit displacement is d_u2={round(d_u2,3)} m")
if target_disp_NC<=d_u2:
    print('Therefore the NC limit state is verified')
else:
    print('Therefore the NC limit state is NOT verified!!!!')
```

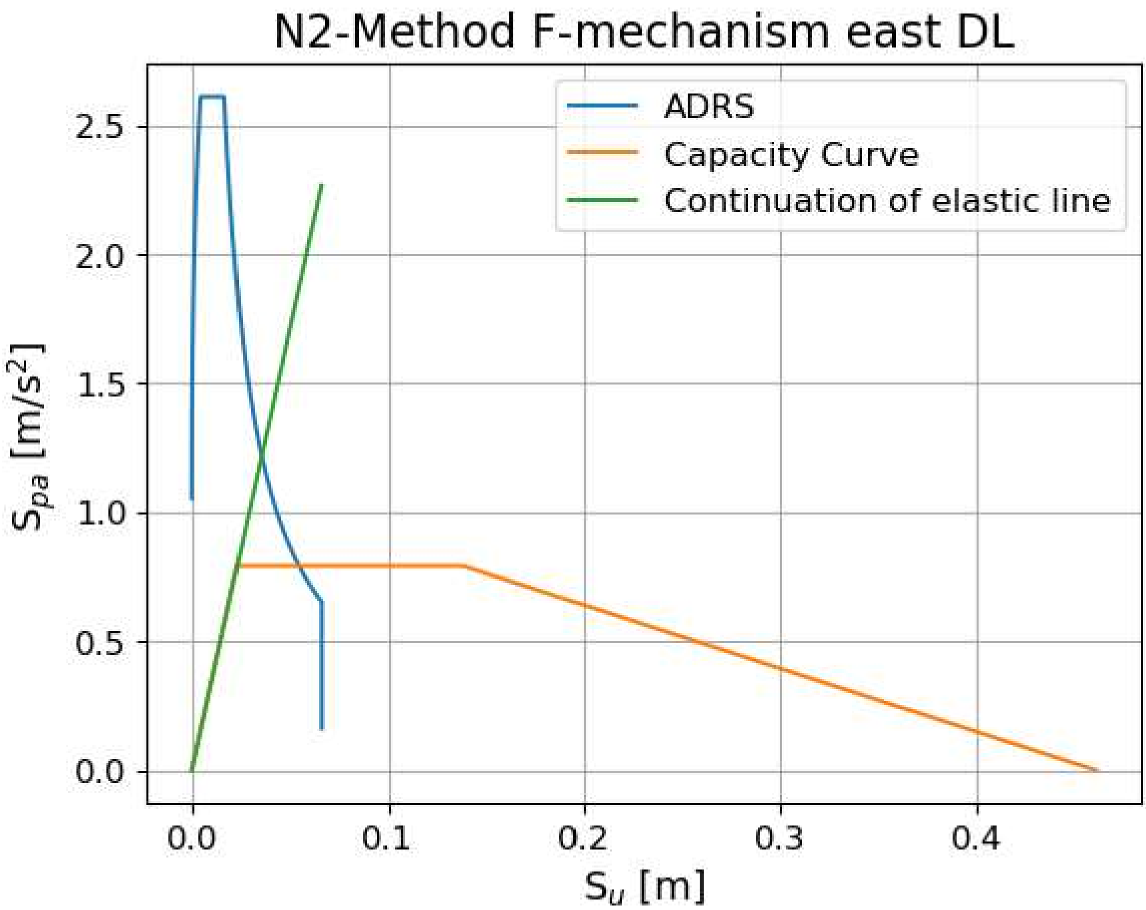
```
The target displacement NC is of d=0.074 m
The NC limit displacement is d_u2=0.277 m
Therefore the NC limit state is verified
```



In [15]:
```python
## Plot capacity curve DL
# plot results
plt.figure()
plt.grid()
plt.plot(0.5*Su_vals,0.5*Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/4)],Su_vals[:round(len(Su_vals)/4)]/d_star1*a_
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
```

```python
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method F-mechanism east DL', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

closest = find_closest_pair(0.5*Su_vals, 0.5*Se_vals, Su_vals[:round(len(Su_vals)/4
target_disp_DL = (closest[0]+closest[2])/2

print(f"The target displacement DL is of d={round(target_disp_DL,3)} m")
print(f"The DL limit displacement is d_u1={round(d_u1,3)} m")
if target_disp_DL<=d_u1:
    print('Therefore the DL limit state is verified')
else:
    print('Therefore the DL limit state is NOT verified!!!!')
```

```
The target displacement DL is of d=0.035 m
The DL limit displacement is d_u1=0.184 m
Therefore the DL limit state is verified
```



Created in Deepnote

Out-of-plane failure calculations

G-Mechanism Parish House West Facade

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Master Thesis 2023/2024

Supervisors: Prof. Dr. Katrin Beyer, Dr. Igor Tomic

Co-author: Caroline L. Heitmann

Caroline and I intensely collaborated on the elaboration of various notebooks for diffeent OOP mechanisms, generic and specifically applied to our respective buildings treated in our theses.

Introduction

This notebook can be used to calculate the push-over curve for an out-of-plane mechanisms of a masonry structure subject to seismic action. The mechanism in question is a G-mechanism (D'Ayala, 2002) and the calculations were performed for the west facade of the Parish House at St. Lawrence Church in Petrinja, Croatia.

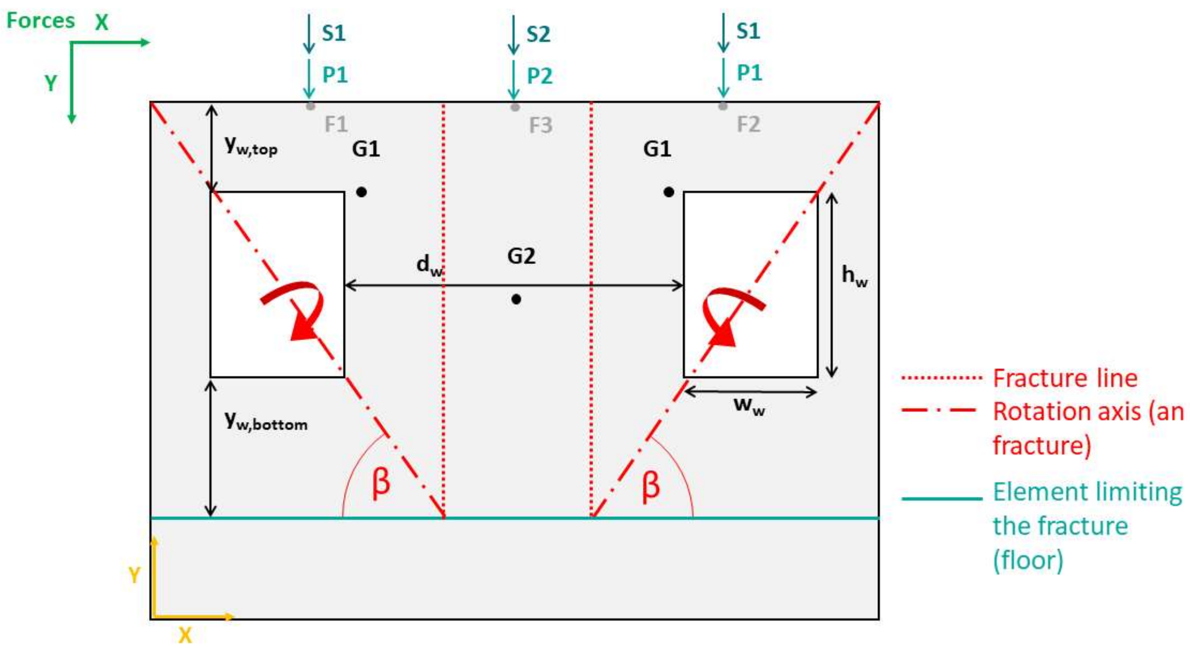
First, the geometric and material characteristics will be defined, then depending on these inputs, the displacement capacity for out-of-plane accelerations will be returned in form of a push-over graph.

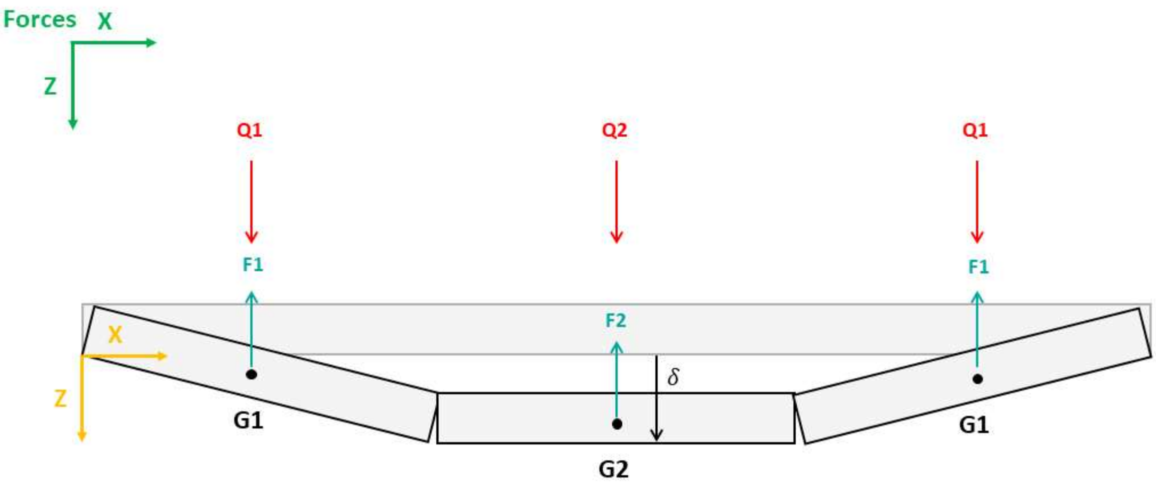
In [1]:
```
# Import the basic packages for the notebook
import numpy as np                    # Numerical library
import pandas as pd                   # Data analysis library
import matplotlib.pyplot as plt       # Plotting library
import matplotlib.ticker as maticker # Tick locating and formatting library
import matplotlib.colors as mcolors  # Colours
from sympy.solvers import solvers    # Library to solve equations
from sympy import Symbol, Eq, nsolve # For solving equations
import math
```

1. Define the wall geometry

The chosen example for this code is a plain wall moving out of plane around two diagonal axes. The diagonal cracks going through the window, as well as two vertical cracks separate the wall in three blocks. A sketch with the main dimensions is shown below. The displacement delta is measured at the center of the wall.

In [2]:
```
from IPython.display import Image, display
display(Image(filename='Mec_G_4.png'))
display(Image(filename='Mec_G_5.png'))
```





This block is used to define the geometry of the wall and the windows within.

In [4]:
```
## Wall geometry
y_w_top = 0.65        # Free space above window (until reaching next floor), y-direct
y_w_bottom = 0.9      # Free space below window (until reaching next floor), y-direct
d_w = 3.3             # Horizontal distance between the windows in the mechanism,x-di
h_w = 1.7             # Height of the window, y-direction [m]
w_w = 1.1             # Width of the window, x-direction
t = 0.55              # Thickness of wall,    z-direction [m]
# since it is a symmetric mechanism, it is considered that the cracks go from the u
# the two diagonal corners of a window until reaching the crack from the other side

## Slab information
max_delta = t/2       # Maximum drift (at the center crack), when the slab and wall l
                      # is no longer restraining the wall [m]
```

2. Define material properties

This block is used to define the material properties of the wall and the connecting slab.

In [3]:
```
rho_masonry = 18000          # Density of the masonry [N/m^3]
mu_f = 0.7                   # Friction coefficient between slab and masonry wall
```

3. Define the loads and forces

In this paragraph, the loads and forces acting on the wall are defined. The following block defines the vertical load on top of the wall coming from a slab (S) or a superimposed wall (P). Note that the S-load will be used to directly calculate the restraining force due to friction between the wall and the slab. If you want to introduce a vertical load that does not produce any friction, please use the P-load.

In [5]:

```
P1 = 11305              # vertical load on top of each triangular block [N]
S1 = 10700              # slab load acting on top of each triangular block [N]
P2 = 11479              # vertical load on top of the middle block [N]
S2 = 10865              # slab load acting on top of the middle block [N]
```

4. Calculation of overturning moment

This paragraph calculates the pushover curve for the given case.

In [6]:

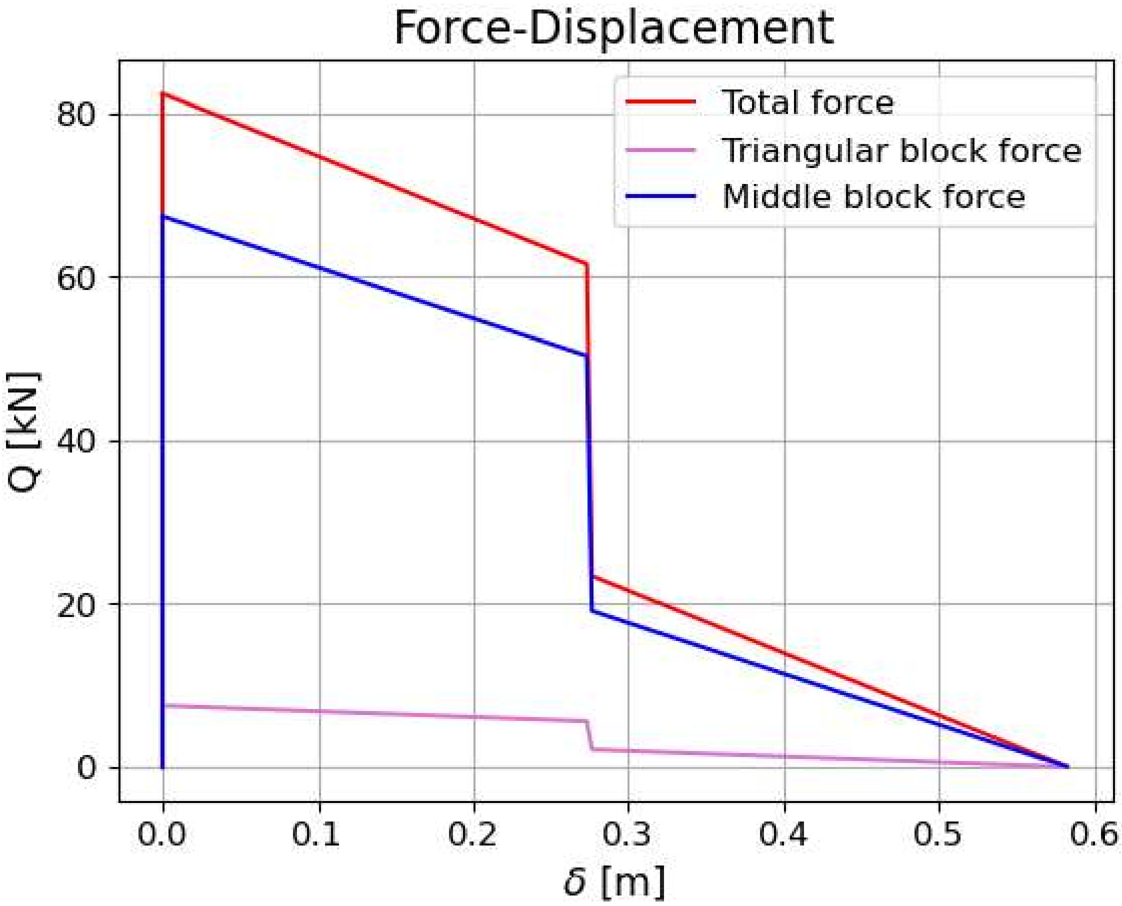
```
# This function calculates lever arm between the center of gravity of a triangular
def lever_arm(x,y):
    # H   = Total height of wall, y-direction [m]
    # L   = Total length of wall, x-direction [m]
    # t   = Thickness of wall,    z-direction [m]
    # x   = X-coordinate of the point for which the lever arm is searched [m]
    # y   = Y-coordinate of the point for which the lever arm is searched [m]
    y_top = H - y # Distance from top edge to center of gravity
    if x <= L/2:
        z_lever = (x * np.sin(beta)) - (y_top * np.cos(beta))
    else:
        x_bar = L-x
        z_lever = (x_bar * np.sin(beta)) - (y_top * np.cos(beta))

    return z_lever

# This function calculates the center of gravity of the moving triangular block, co
def COG_calc():
    # H          = Total height of wall, y-direction [m]
    # L          = Total length of wall, x-direction [m]
    # t          = Thickness of wall,    z-direction [m]
    # h_w        = Height of the window, y-direction [m]
    # w_w        = Width of the window,  x-direction [m]
    # x_w_out    = Distance between outer window edge and end of mechanism (horizon
    # y_w_top    = Distance between upper edge of window and floor (vertical) [m]
    # y_w_bottom = Distance between bottom edge of window and end of mechanism (ver
    # d_w        = Distance between the windows [m]
    COG_wall = np.array([2*L/3,2*H/3])
    COG_window = np.array([x_w_out + (2*w_w/3),y_w_bottom + (2*h_w/3)])

    V_brut  = H*L/2*t                          # volume of the moving wall block[m
    V_window =h_w*w_w/2*t                      # volume of opening in moving block
    V_net = V_brut - V_window                  # volume of the moving block withou

    COG_tot= ((COG_wall * V_brut) - (COG_window*V_window))/V_net
    return COG_tot


# This function calculates the maximum horizontal force that can be applied to the
# to stay in an equilibrium state
```

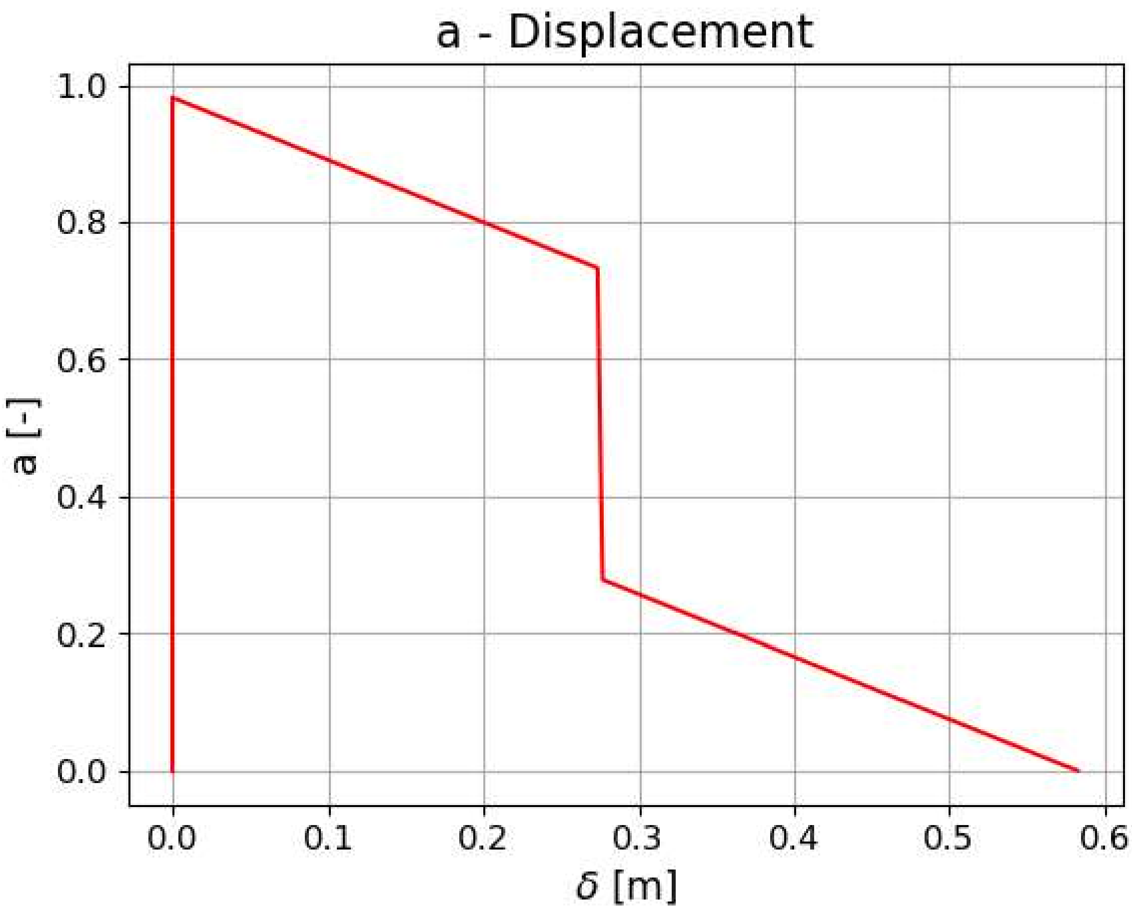
```python
def alpha_fun(delta):
    # delta        = Displacement at the top of the wall [m]
    delta_top_center = delta * z_top_center / z_delta
    delta_cog = delta * z_cog / z_delta
    delta_midblock2 = delta * z_midblock2/H


    if delta >= max_delta:
        F1_proj = 0
        F2_proj = 0
    else:
        F1_proj = F1
        F2_proj = F2

    # Define the symbol for the acceleration
    a = Symbol('a')


    # Define the equation
    equation = Eq(2*a*G1*z_cog/z_delta                                    # Seismic f
                - 2*F1_proj*z_top_center/z_delta                          # Friction
                + 2*G1_proj*(delta_cog-(t/2))/z_delta                     # Gravity f
                + 2*P1_proj*(delta_top_center-(t/2))/z_delta              # Vertical
                + 2*S1_proj*(delta_top_center-(t/2))/z_delta              # Slab load
                + a*G2/2                                                 # Seismic for
                - F2_proj                                                 # Friction
                + G2*(delta-t)/2/H                                        # Gravity f
                + P2*(delta-(t/2))/H                                      # Vertical
                + S2*(delta-(t/2))/H                                      # Slab load
                ,0)

    # Use nsolve to find the solution
    a_sol = nsolve(equation, a, 1.0)
    return a_sol

## Derived wall properties
beta = np.arctan(h_w/w_w)                      # angle of the crack
x_w_out = y_w_top / np.tan(beta)               # horizontal distance from corner of wi
x_w_in = y_w_bottom / np.tan(beta)             # vertical distance from bottom of wind
L_mid = d_w - (2*x_w_in)                       # length of the middle block [m]
L_t = w_w + x_w_in + x_w_out                   # length of the triangular blocks [m]
#print(L_t)
#print(L_mid)

H = h_w + y_w_top + y_w_bottom                 # total height of the mechanism [m]
L = w_w + x_w_out + x_w_in                     # total length of the triangular block

V_brut_t = H*L_t/4*t                           # volume of the moving wall block[m3]
V_window_t =h_w*w_w/2*t                        # volume of opening in moving block [m3
V_net_t = V_brut_t - V_window_t                # volume of the moving block without wi

V_net_mid = H * L_mid * t                      # volume of the middle block

COG = COG_calc()                               # center of gravity of the moving block

z_top_center = lever_arm(L/4,H)
z_cog = lever_arm(COG[0],COG[1])
z_delta = lever_arm(L/2,H)
z_midblock2 =H/2

## Derived loads and forces
G1 = rho_masonry*V_net_t                      # gravity load of the outer triangular w
G2 = rho_masonry*V_net_mid                    # gravity load of the middle wall (self-
F1 = mu_f*S1                                  # friction force from slab acting on top
```

```
F2 = mu_f*S2                                  # friction force from slab acting on top
# The projected parts of the actions that will create a moment around the turning c
P1_proj = P1 * np.cos(beta)
S1_proj = S1 * np.cos(beta)
G1_proj = G1 * np.cos(beta)

## Rocking
# Maximum earthquake force to start rocking
alpha_max  = alpha_fun(0)                     # Portion of gravity load [-]
Q1_max = G1*alpha_max/1000                    # Maximum seismic load on triangular blo
Q2_max = G2*alpha_max/1000                    # Maximum seismic load on middle block [
Q_tot_max=2*Q1_max+Q2_max

# Plot force-displacement relationship of the left overturning of the wall
delta_vals = np.linspace(0,H,1000)
alpha_vals = np.zeros((np.size(delta_vals)))

for i in range(np.size(delta_vals)):          # Calculates the corresponding alpha fo
                                              # at the COG of the triangular parts of
    alpha_vals[i]=alpha_fun(delta_vals[i])

# Transformation to only display values up to failure
alpha_vals = alpha_vals[alpha_vals>=0]
delta_vals = delta_vals[: np.size(alpha_vals)]

# Calculate the forces corresponding to the alpha factors on each block
q1_vals=alpha_vals*G1/1000                    # Seismic force on triangular block [kN]
q2_vals=alpha_vals*G2/1000                    # Seismic force on middle block [kN]
q_tot_vals=2*q1_vals+q2_vals

plt.figure()
plt.grid()
plt.plot([0,0],[0,Q_tot_max],label='Total force',color='red')
plt.plot(delta_vals,q_tot_vals,color='red')
plt.plot([0,0],[0,Q1_max],label='Triangular block force',color='orchid')
plt.plot(delta_vals,q1_vals,color='orchid')
plt.plot([0,0],[0,Q2_max],label='Middle block force',color='blue')
plt.plot(delta_vals,q2_vals, color='blue')

plt.legend(fontsize=12)
plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('Q [kN]', fontsize=14)
plt.title('Force-Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)



# Plot acceleration-displacement relationship of the overturning of the wall, where
# by gravitational force
plt.figure()
plt.grid()
plt.plot([0,0],[0,alpha_max],color='red')
plt.plot(delta_vals,alpha_vals,color='red')

plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('a [-]', fontsize=14)
plt.title('a - Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

## Maximum earthquake force to start rocking
a_max = alpha_vals[0]                       # Maximum alpha [-]
acc_max = a_max*9.81                        # Maximm acceleration [-]
```

```
delta_max = delta_vals[-2]                # Maximum displacemet [m]

print(f"The maximum forces on each block are: Q1= {round(Q1_max,2)}kN Q2={round(Q2_
print('After reaching that limit, the wall will start rocking out of plane.')
print('This corresponds to acelerations of acc= '+str(round(acc_max,2))+'m/s2')
print(f"The a-factor is a1={a_max} ")
print('The limit displacement corresponds to '+str(round(delta_max,3))+' m. After r
```

```
The maximum forces on each block are: Q1= 7.52kN Q2=67.42kN
After reaching that limit, the wall will start rocking out of plane.
This corresponds to acelerations of acc= 9.63m/s2
The a-factor is a1=0.9813453913122845
The limit displacement corresponds to 0.579 m. After reaching this displacement th
e wall loses all resistance.
```





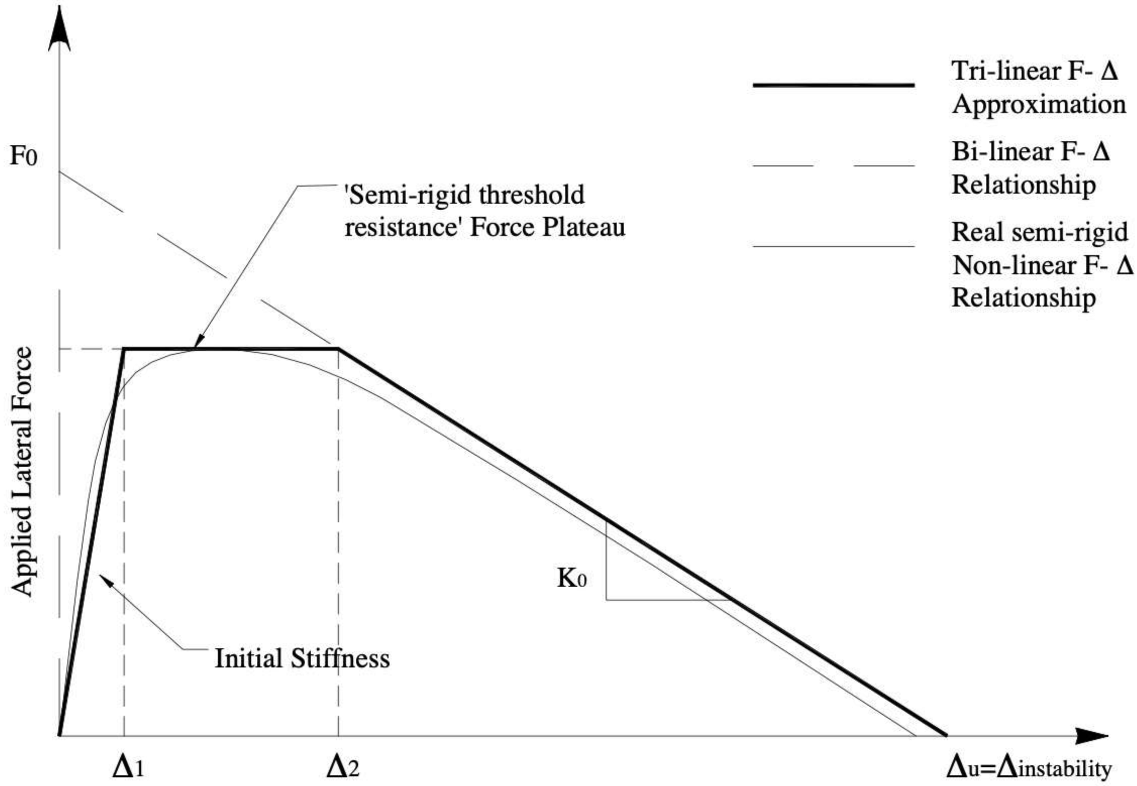
5. Computation of capacity curve

This section allows the analysis of the structural response in the context of the seismic spectrum. The goal is to compare the capacity curve to the ADRS response spectrum, which will allow the behaviour of the wall to be interpreted (e.g., indicating whether it would meet its design requirements).

The main input is the spectral acceleration (for a 475-year return period) for the wall in its given location, as well as its period. For users of the Swiss code, SIA 261 chapter 16.2.1.2 provides the horizontal soil acceleration for each seismic zone; otherwise it can be found in the relevant National Annex of the Eurocode.

As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, delta_1 and delta_2, and the maximum displacement As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, Δ_1 and Δ_2, and the maximum displacement capacity, Δ_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, Δ_u is approximated to be equivalent to Δ_o in this code., delta_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, delta_u is approximated to be equivalent to delta_o in this code.

In [7]:
```
display(Image(filename='Griffith_Trilinear.png'))
```



In [8]:
```
agr = 0.152        # Spectral acceleration for 475-year return period [g]

# Choose the coefficients for the tri-linear model per Griffith et al. (2003)
d1 = 0.075         # Range from 0.05 to 0.20
d2 = 0.45          # Range from 0.25 to 0.50
```

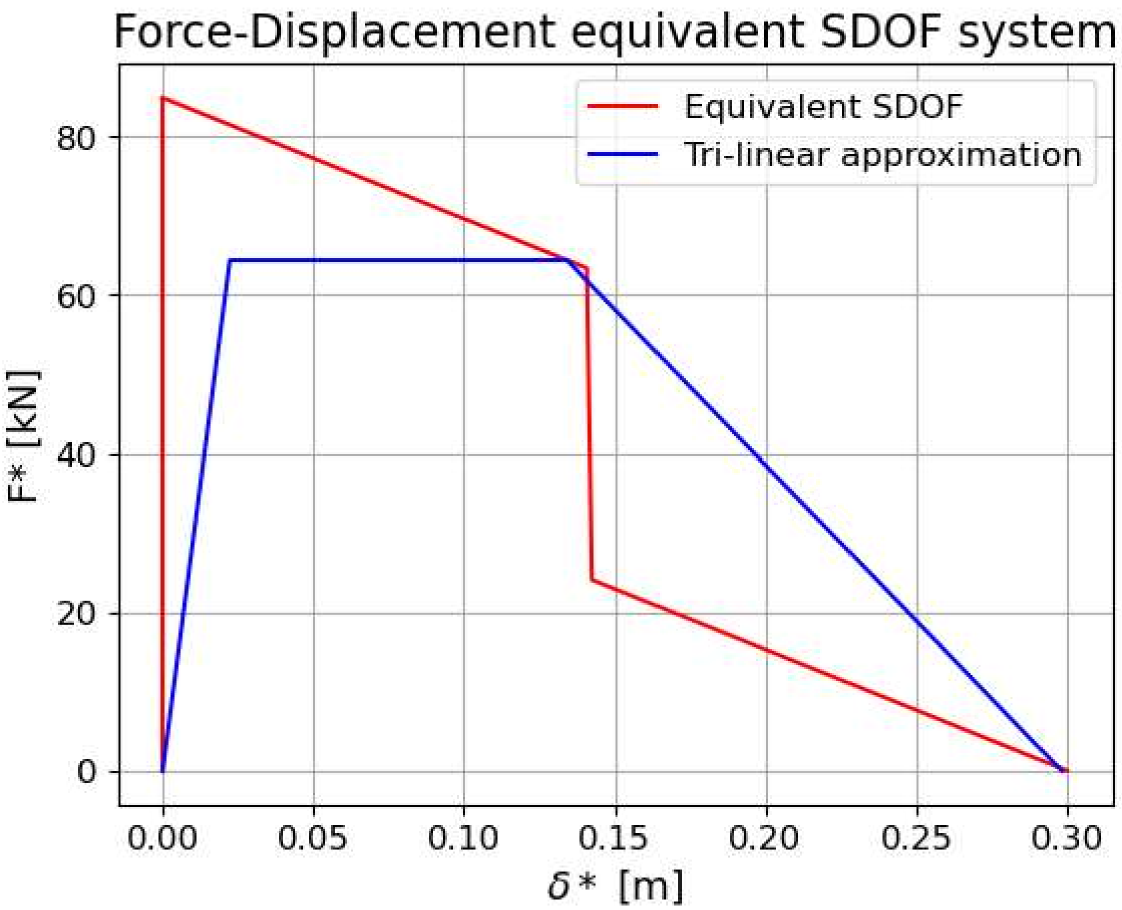
Response spectra are defined for single degree of freedom (DOF) systems. In this case, the mechanism involves more than one DOF. Thus, the capacity curve needs to be transformed into a capaciti curve for an equivalent SDOF system. Furthermore, after the transformation the pushover curve is aproximated as a tri-linear curve as proposed above (Griffith et al., 2003).

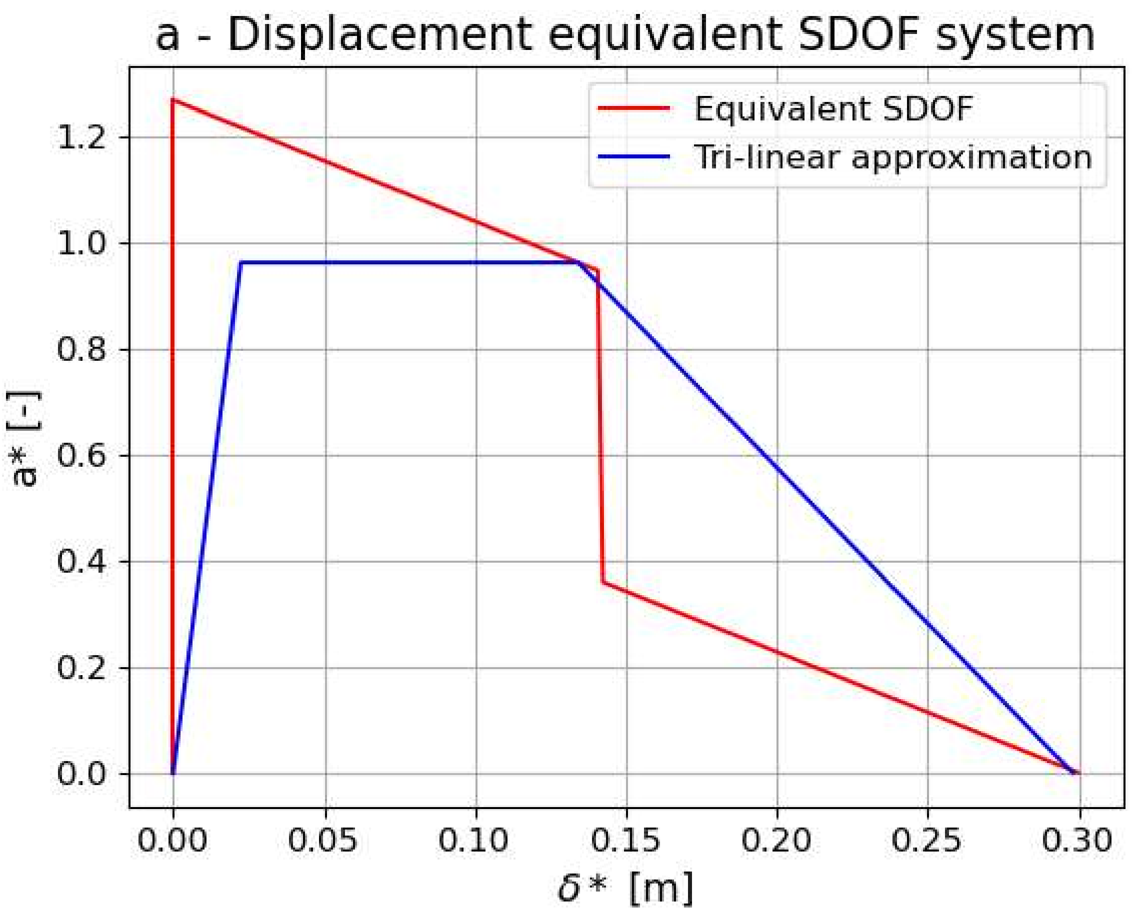
In [10]:
```
m = np.array([V_net_t,V_net_mid,V_net_t])*rho_masonry       # mass of wall [kg]
g = 9.81                                                   # gravitational constant
Qs = np.sum(m)*agr*g                                       # seismic force acting o
phi = np.divide([z_cog/z_delta, z_midblock2/H, z_cog/z_delta],z_midblock2/H)

# calculate effective properties
m_star = np.dot(m,phi)                                     # effective mass (SDOF)
Gamma = m_star / np.dot(m,phi**2)                          # participation factor [
Fb = q_tot_vals * 1000                                     # total shear force [N]
F_star = Fb/Gamma                                          # equivalent shear force
d_2 = delta_vals/H*z_midblock2                             # displacement at COG of
phi2 = phi[1]                                              # modal value at COG of
d_star = d_2/(Gamma*phi2)
a_star = F_star/m_star                                     # effective acceleration

# Tri-linear approximation of the pushover curve according to  Griffith et al. (200
a_1 = alpha_fun(d1*delta_max)
a_2 = alpha_fun(d2*delta_max)
```

```python
F_star1 = a_1*(2*G1+G2)/Gamma
F_star2 = a_2*(2*G1+G2)/Gamma
a_star1 = F_star1/m_star
a_star2 = F_star2/m_star
d_star1 = (d1*delta_max)/H*z_midblock2/(Gamma*phi2)
d_star2 = (d2*delta_max)/H*z_midblock2/(Gamma*phi2)
d_star_max = d_star[-2]
F_star_max = F_star2
F_star_app = [0,F_star_max, F_star_max,0]
d_star_app = [0,d_star1,d_star2,d_star_max]
a_star_app = F_star_app/m_star
a_star_max = F_star_max/m_star

# Displacement capacities according to EN 1998-3
d_u1 = 0.4*d_star_max
d_u2 = 0.6*d_star_max

# Pushover curve of the equivalent SDOF Force-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,F_star[0]/1000],color='red')
plt.plot(d_star,np.divide(F_star,1000),label='Equivalent SDOF',color='red')
plt.plot(d_star_app,np.divide(F_star_app,1000),label='Tri-linear approximation',col

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('F* [kN]', fontsize=14)
plt.title('Force-Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

# Pushover curve of the equivalent SDOF Acceleration-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,a_star[0]],color='red')
plt.plot(d_star,a_star,label='Equivalent SDOF',color='red')
plt.plot(d_star_app,a_star_app,label='Tri-linear approximation',color='blue')

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('a* [-]', fontsize=14)
plt.title('a - Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
```

Out[10]:

```
(array([-0.2,  0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4]),
 [Text(0, -0.2, '−0.2'),
  Text(0, 0.0, '0.0'),
  Text(0, 0.2, '0.2'),
  Text(0, 0.4000000000000001, '0.4'),
  Text(0, 0.6000000000000001, '0.6'),
  Text(0, 0.8, '0.8'),
  Text(0, 1.0000000000000002, '1.0'),
  Text(0, 1.2000000000000002, '1.2'),
  Text(0, 1.4000000000000001, '1.4')])
```





To compute the response spectrum, it is necessary to specify the soil type per SIA 261 Tableau 24 or the Eurocode National Annex for a particular country.

Alternatively, sometimes the response spectra are defined according to the limit state for analysis. The available limit states are: 'serviceability', 'damage limitation', 'significant damage' or 'near collapse. More information can be found in EC 8 chapter 4.2.3.

After the computation of the response spectrum, the capacity curve of the equivalent SDOF system can be plotted alongside the acceleration-displacement response spectrum. This provides a direct comparison between the specific behavior of the elements in question (in this case, the wall) and the spectrum of the desired limit state (either serviceability state, damage limitation, significant damage, or near collapse), and it can quickly give the spectral acceleration for a certain spectral displacement.

This displacement-based evaluation utilizes the equal displacement "rule" for periods smaller than T_c and the equal energy "rule" for periods larger than T_c. Thus, to interpret the results, extend the initial positive linear part of the capacity curve until it intersects the ADRS, and draw a vertical line at this intersection. If $T < T_c$, the actual displacement will be slightly larger than the displacement at the intersection of the capacity curve and the vertical line; if $T > T_c$, the actual displacement will be equivalent to the displacement at the intersection of the capacity curve and the vertical line. The displacement capacity is exceeded if the determined displacement is larger than the end of the capacity curve.

In [11]:
```
soil_type = 'E'        # specify the soil type (ranging from A to E)
```

In [12]:
```
## Determine the periods that define the spectrum according to the soil type, with
def spectrum_vals(soil_type):
    if soil_type == 'A':
        S = 1.00                                        # [-]
        Tb = 0.15                                       # [s]
        Tc = 0.40                                       # [s]
        Td = 2.00                                       # [s]
    elif soil_type == 'B':
        S = 1.20                                        # [-]
        Tb = 0.15                                       # [s]
        Tc = 0.50                                       # [s]
        Td = 2.00                                       # [s]
    elif soil_type == 'C':
        S = 1.15                                        # [-]
        Tb = 0.20                                       # [s]
        Tc = 0.60                                       # [s]
        Td = 2.00                                       # [s]
    elif soil_type == 'D':
        S = 1.35                                        # [-]
        Tb = 0.20                                       # [s]
        Tc = 0.80                                       # [s]
        Td = 2.00                                       # [s]
    elif soil_type == 'E':
        S = 1.40                                        # [-]
        Tb = 0.15                                       # [s]
        Tc = 0.50                                       # [s]
        Td = 2.00                                       # [s]

    return S,Tb,Tc,Td

## Determine the spectral acceleration according to the structure's fundamental per
def RS(T,agr,S,Tb,Tc,Td):

    # determine spectral ground acceleration for dimensionning
```

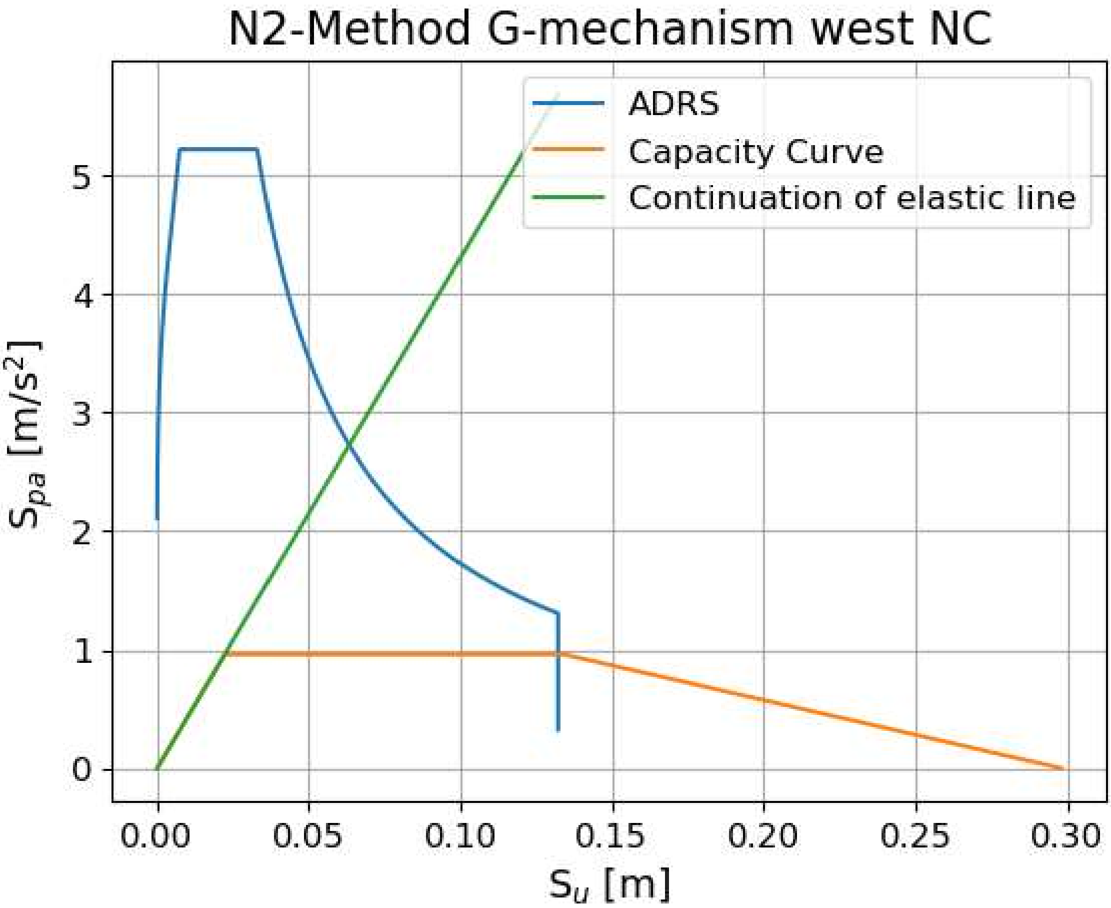
```
    g = 9.81                                         # [m/s^2]
    agd = agr*g                                      # [m/s^2]
    zeta = 0.05                                      # assume damping ratio to be 5% [-
    eta = max(np.sqrt(1/(0.5+10*zeta)),0.55)         # reduction coefficent [-]

    # compute spectrum according to period
    if T<Tb:
        Se = agd*S*(1+(T/Tb)*(eta*2.5-1))
    elif T>=Tb and T<Tc:
        Se = agd*S*eta*2.5
    elif T>=Tc and T<Td:
        Se = agd*S*eta*2.5*(Tc/T)
    else:
        Se = agd*S*eta*2.5*(Tc*Td/T**2)

    return Se
```

In [13]:

```
## Compute ADRS
# Initialize vectors
T_vals = np.linspace(0.001,4,1000)
Su_vals = np.zeros((len(T_vals),1))
Se_vals = np.zeros((len(T_vals),1))
[S,Tb,Tc,Td] = spectrum_vals(soil_type)

for i in range(0,1000):
    omega = 2*np.pi/T_vals[i]                                    # natural f
    Se_vals[i] = RS(T_vals[i],agr,S,Tb,Tc,Td)                    # spectral
    if T_vals[i] < Tc:
        Ry = max(Se_vals[i]*m_star/Qs,1)                         # R relatio
        wd_int = ((Ry-1)*Tc/T_vals[i] + 1)*Se_vals[i]/((omega**2)*Ry)   # displacem
        Su_vals[i] = min(wd_int,3*Se_vals[i]/omega**2)           # spectral
    else:
        Su_vals[i] = Se_vals[i]/(omega**2)                       # spectral

## Plot capacity curve NC
# plot results
plt.figure()
plt.grid()
plt.plot(Su_vals,Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/2)],Su_vals[:round(len(Su_vals)/2)]/d_star1*a_
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method G-mechanism west NC', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

def find_closest_pair(d1, f1, d2, f2):

    data_set1 = np.column_stack((d1, f1))
    data_set2 = np.column_stack((d2, f2))

    min_distance = float('inf')
    closest_pair = None

    for point1 in data_set1:
        for point2 in data_set2:
            dist2d =point1 - point2
            distance = (dist2d[0]**2+dist2d[1]**2)**0.5
            if distance < min_distance:
                min_distance = distance
                closest_pair = (point1[0], point1[1], point2[0], point2[1])
```

```
    return closest_pair


closest = find_closest_pair(Su_vals, Se_vals, Su_vals[:round(len(Su_vals)/2)],Su_va
target_disp_NC = (closest[0]+closest[2])/2

print(f"The target displacement NC is of d={round(target_disp_NC,3)} m")
print(f"The NC limit displacement is d_u2={round(d_u2,3)} m")
if target_disp_NC<=d_u2:
    print('Therefore the NC limit state is verified')
else:
    print('Therefore the NC limit state is NOT verified!!!!')
```

```
The target displacement NC is of d=0.063 m
The NC limit displacement is d_u2=0.179 m
Therefore the NC limit state is verified
```

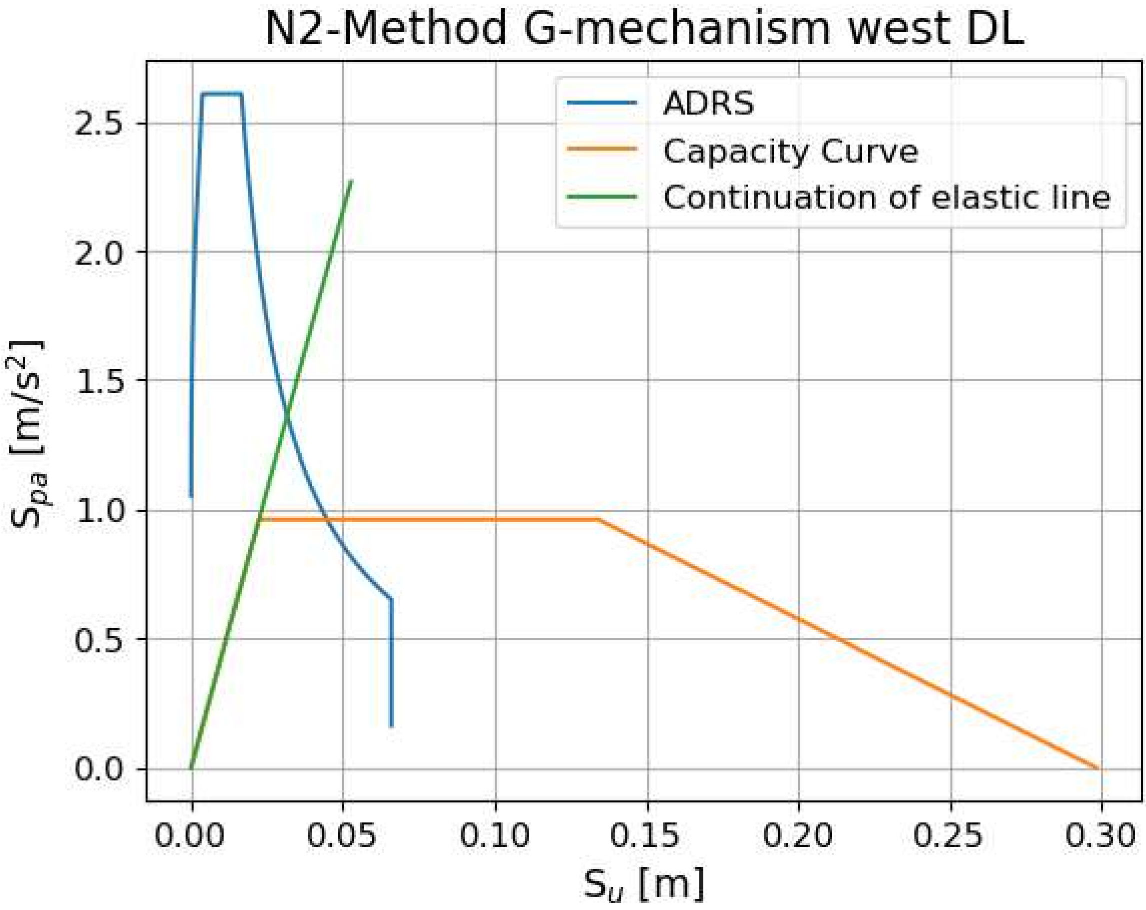


In [14]:

```
## Plot capacity curve DL
# plot results
plt.figure()
plt.grid()
plt.plot(0.5*Su_vals,0.5*Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/5)],Su_vals[:round(len(Su_vals)/5)]/d_star1*a_
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method G-mechanism west DL', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

closest = find_closest_pair(0.5*Su_vals, 0.5*Se_vals, Su_vals[:round(len(Su_vals)/5
target_disp_DL = (closest[0]+closest[2])/2

print(f"The target displacement DL is of d={round(target_disp_DL,3)} m")
```

```
print(f"The DL limit displacement is d_u1={round(d_u1,3)} m")
if target_disp_DL<=d_u1:
    print('Therefore the DL limit state is verified')
else:
    print('Therefore the DL limit state is NOT verified!!!!')
```

```
The target displacement DL is of d=0.032 m
The DL limit displacement is d_u1=0.119 m
Therefore the DL limit state is verified
```



Out-of-plane failure calculations

G-Mechanism Parish House East Facade

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Master Thesis 2023/2024

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Co-author: Caroline L. Heitmann

Caroline and I intensely collaborated on the elaboration of various notebooks for diffeent OOP mechanisms, generic and specifically applied to our respective buildings treated in our theses.

Introduction

This notebook can be used to calculate the push-over curve for an out-of-plane mechanisms of a masonry structure subject to seismic action. The mechanism in question is a G-mechanism (D'Ayala, 2002) and the calculations were performed for the east facade of the Parish House at St. Lawrence Church in Petrinja, Croatia.

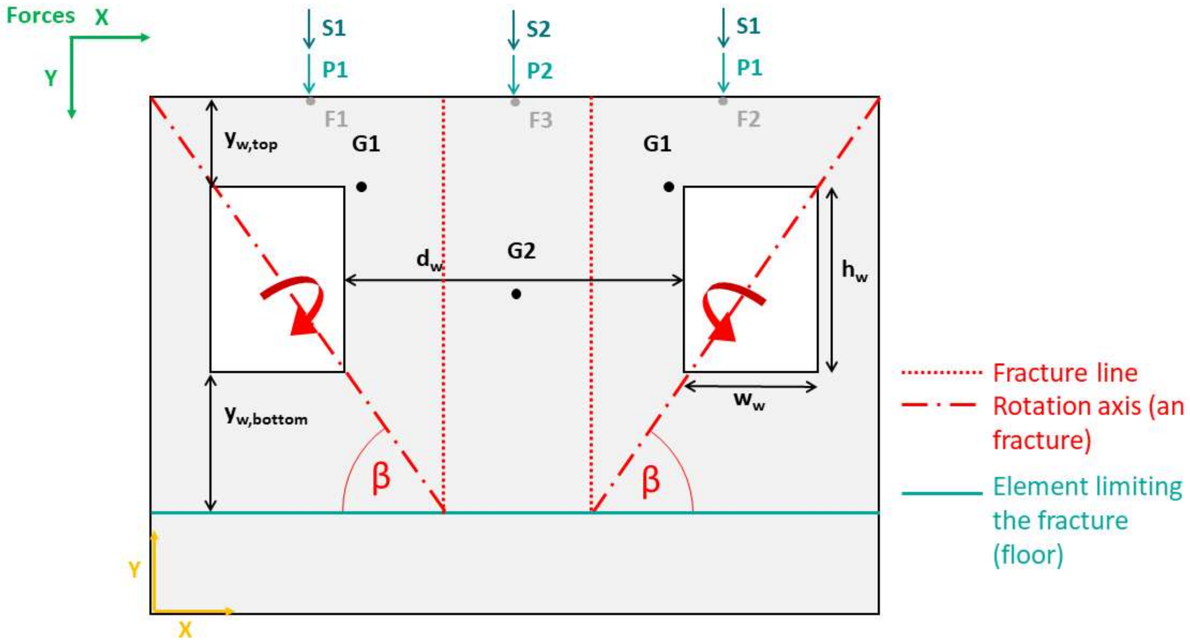
First, the geometric and material characteristics will be defined, then depending on these inputs, the displacement capacity for out-of-plane accelerations will be returned in form of a push-over graph.

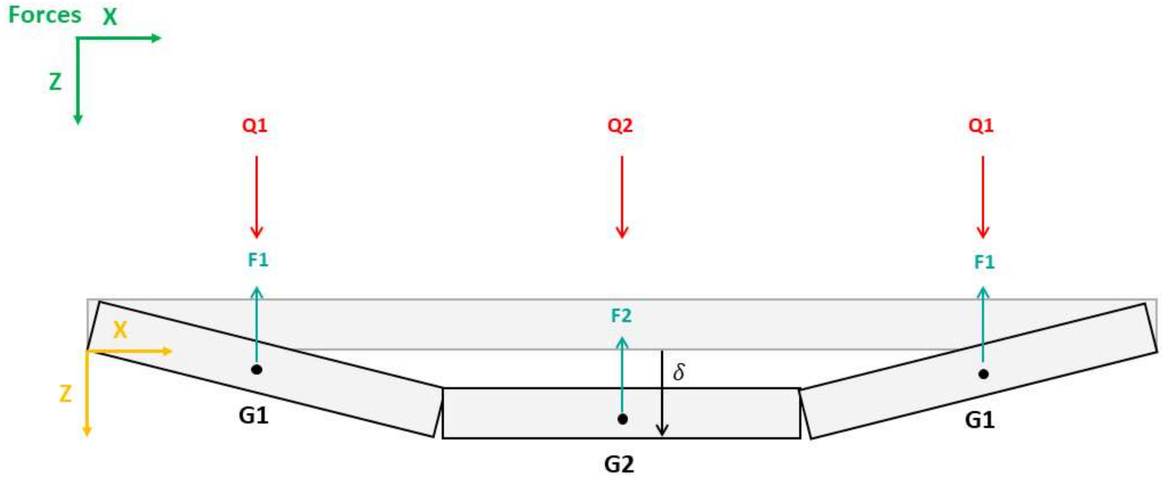
In [7]:
```
# Import the basic packages for the notebook
import numpy as np                    # Numerical library
import pandas as pd                   # Data analysis library
import matplotlib.pyplot as plt       # Plotting library
import matplotlib.ticker as maticker  # Tick locating and formatting library
import matplotlib.colors as mcolors   # Colours
from sympy.solvers import solvers     # Library to solve equations
from sympy import Symbol, Eq, nsolve  # For solving equations
import math
```

1. Define the wall geometry

The chosen example for this code is a plain wall moving out of plane around two diagonal axes. The diagonal cracks going through the window, as well as two vertical cracks separate the wall in three blocks. A sketch with the main dimensions is shown below. The displacement delta is measured at the center of the wall.

In [8]:
```
from IPython.display import Image, display
display(Image(filename='Mec_G_4.png'))
display(Image(filename='Mec_G_5.png'))
```





This block is used to define the geometry of the wall and the windows within.

In [9]:
```
## Wall geometry
y_w_top = 0.65          # Free space above window (until reaching next floor), y-direct
y_w_bottom = 0.9        # Free space below window (until reaching next floor), y-direct
d_w = 2.9               # Horizontal distance between the windows in the mechanism,x-di
h_w = 1.7               # Height of the window, y-direction [m]
w_w = 1.1               # Width of the window, x-direction
t = 0.55                # Thickness of wall,    z-direction [m]
# since it is a symmetric mechanism, it is considered that the cracks go from the u
# the two diagonal corners of a window until reaching the crack from the other side

## Slab information
max_delta = t/2         # Maximum drift (at the center crack), when the slab and wall l
                        # is no longer restraining the wall [m]
```

2. Define material properties

This block is used to define the material properties of the wall and the connecting slab. You can adapt the input to use the notebook for different cases.

In [10]:
```
rho_masonry = 18000             # Density of the masonry [N/m^3]
mu_f = 0.7                      # Friction coefficient between slab and masonry wall
```

3. Define the loads and forces

In this paragraph, the loads and forces acting on the wall are defined. The following block defines the vertical load on top of the wall coming from a slab (S) or a superimposed wall (P). Note that the S-load will be used to directly calculate the restraining force due to friction between the wall and the slab. If you want to introduce a vertical load that does not produce any friction, please use the P-load.

In [11]:

```
P1 = 11305              # vertical load on top of each triangular block [N]
S1 = 19000              # slab load acting on top of each triangular block [N]
P2 = 9329               # vertical load on top of the middle block [N]
S2 = 15678              # slab load acting on top of the middle block [N]
```

4. Calculation of overturning moment

This paragraph calculates the pushover curve for the given case.

In [12]:

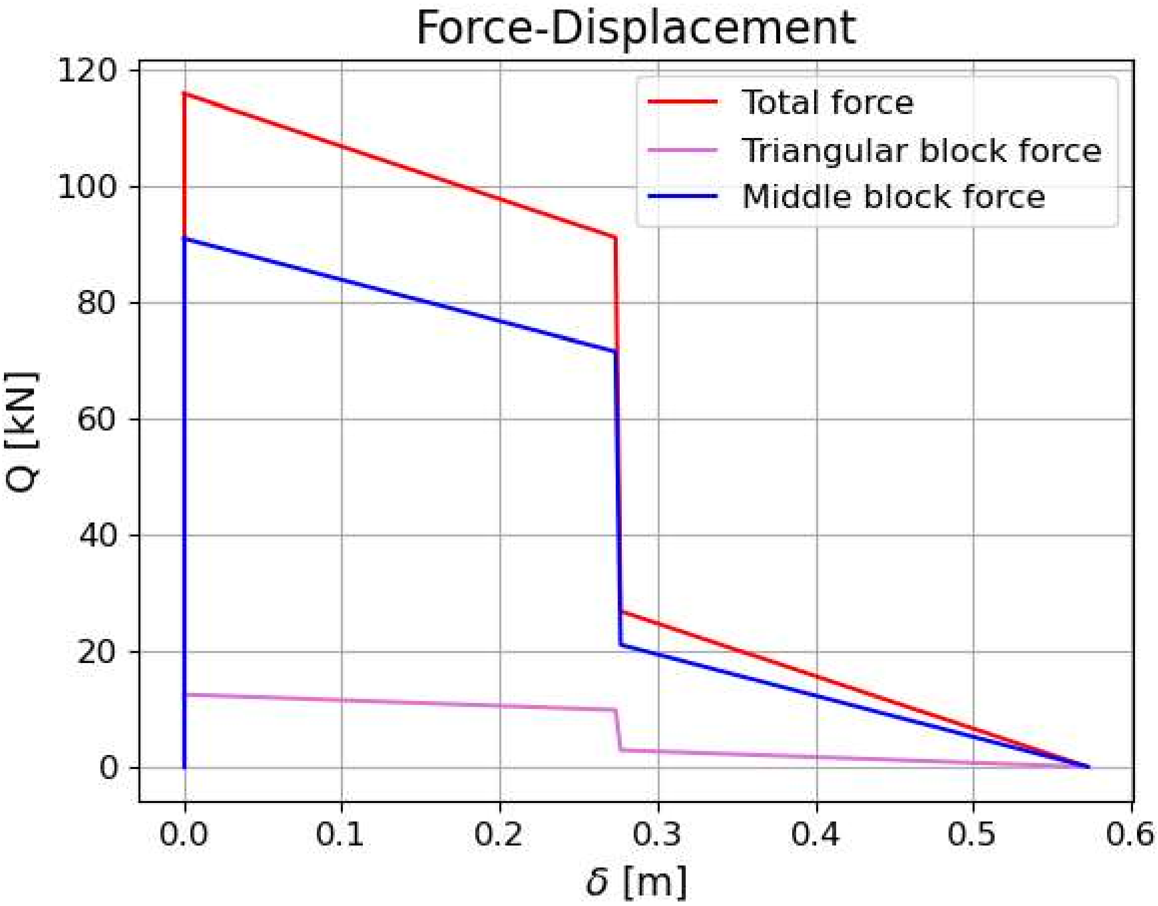
```
# This function calculates lever arm between the center of gravity of a triangular
def lever_arm(x,y):
    # H   = Total height of wall, y-direction [m]
    # L   = Total length of wall, x-direction [m]
    # t   = Thickness of wall,    z-direction [m]
    # x   = X-coordinate of the point for which the lever arm is searched [m]
    # y   = Y-coordinate of the point for which the lever arm is searched [m]
    y_top = H - y # Distance from top edge to center of gravity
    if x <= L/2:
        z_lever = (x * np.sin(beta)) - (y_top * np.cos(beta))
    else:
        x_bar = L-x
        z_lever = (x_bar * np.sin(beta)) - (y_top * np.cos(beta))

    return z_lever

# This function calculates the center of gravity of the moving triangular block, co
def COG_calc():
    # H          = Total height of wall, y-direction [m]
    # L          = Total length of wall, x-direction [m]
    # t          = Thickness of wall,    z-direction [m]
    # h_w        = Height of the window, y-direction [m]
    # w_w        = Width of the window,  x-direction [m]
    # x_w_out    = Distance between outer window edge and end of mechanism (horizon
    # y_w_top    = Distance between upper edge of window and floor (vertical) [m]
    # y_w_bottom = Distance between bottom edge of window and end of mechanism (ver
    # d_w        = Distance between the windows [m]
    COG_wall = np.array([2*L/3,2*H/3])
    COG_window = np.array([x_w_out + (2*w_w/3),y_w_bottom + (2*h_w/3)])

    V_brut  = H*L/2*t                               # volume of the moving wall block[m
    V_window =h_w*w_w/2*t                           # volume of opening in moving block
    V_net = V_brut - V_window                       # volume of the moving block withou

    COG_tot= ((COG_wall * V_brut) - (COG_window*V_window))/V_net
    return COG_tot


# This function calculates the maximum horizontal force that can be applied to the
# to stay in an equilibrium state
```

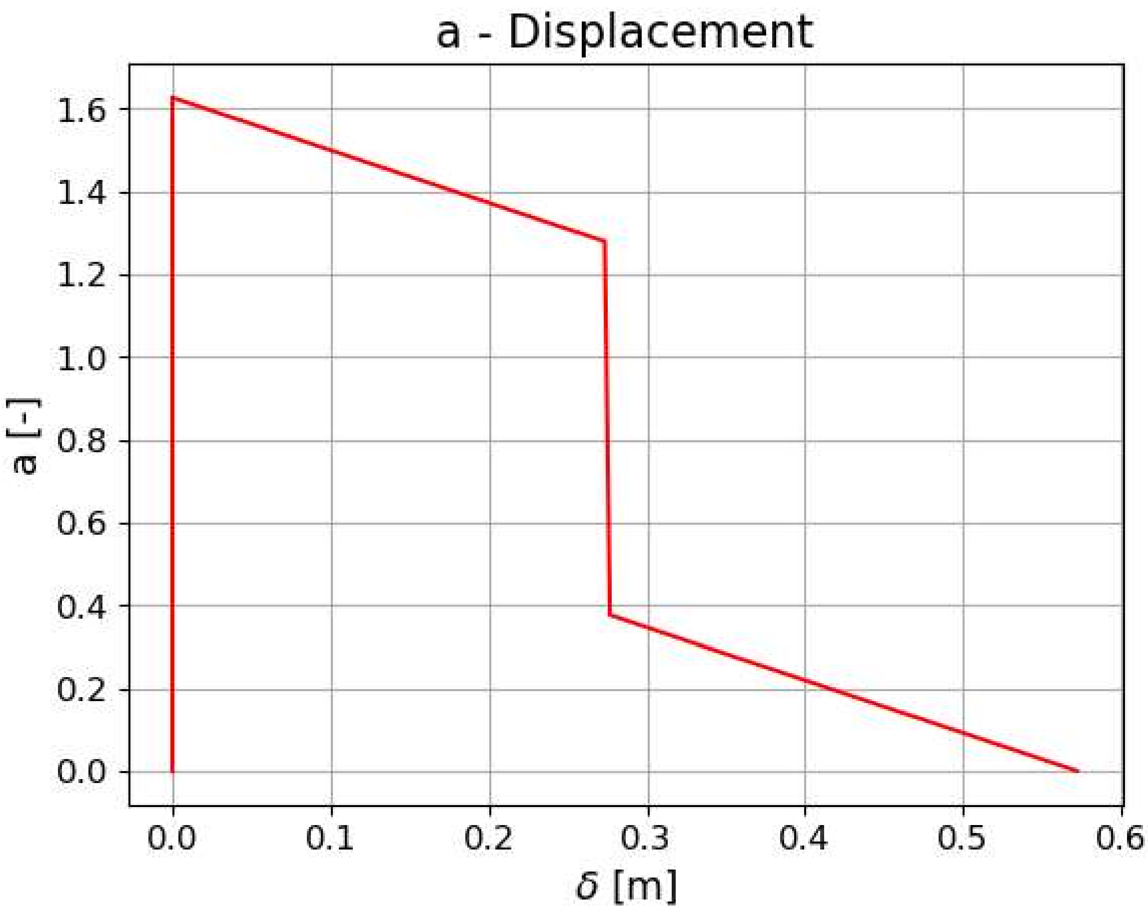
```python
def alpha_fun(delta):
    # delta         = Displacement at the top of the wall [m]
    delta_top_center = delta * z_top_center / z_delta
    delta_cog = delta * z_cog / z_delta
    delta_midblock2 = delta * z_midblock2/H


    if delta >= max_delta:
        F1_proj = 0
        F2_proj = 0
    else:
        F1_proj = F1
        F2_proj = F2

    # Define the symbol for the acceleration
    a = Symbol('a')


    # Define the equation
    equation = Eq(2*a*G1*z_cog/z_delta                                  # Seismic f
                  - 2*F1_proj*z_top_center/z_delta                      # Friction
                  + 2*G1_proj*(delta_cog-(t/2))/z_delta                 # Gravity f
                  + 2*P1_proj*(delta_top_center-(t/2))/z_delta          # Vertical
                  + 2*S1_proj*(delta_top_center-(t/2))/z_delta          # Slab load
                  + a*G2/2                                             # Seismic for
                  - F2_proj                                             # Friction
                  + G2*(delta-t)/2/H                                    # Gravity f
                  + P2*(delta-(t/2))/H                                  # Vertical
                  + S2*(delta-(t/2))/H                                  # Slab load
                  ,0)

    # Use nsolve to find the solution
    a_sol = nsolve(equation, a, 1.0)
    return a_sol

## Derived wall properties
beta = np.arctan(h_w/w_w)                     # angle of the crack
x_w_out = y_w_top / np.tan(beta)              # horizontal distance from corner of wi
x_w_in = y_w_bottom / np.tan(beta)            # vertical distance from bottom of wind
L_mid = d_w - (2*x_w_in)                      # length of the middle block [m]
L_t = w_w + x_w_in + x_w_out                  # length of the triangular blocks [m]
#print(L_t)
#print(L_mid)

H = h_w + y_w_top + y_w_bottom                # total height of the mechanism [m]
L = w_w + x_w_out + x_w_in                    # total length of the triangular block

V_brut_t = H*L_t/4*t                          # volume of the moving wall block[m3]
V_window_t =h_w*w_w/2*t                       # volume of opening in moving block [m3
V_net_t = V_brut_t - V_window_t               # volume of the moving block without wi

V_net_mid = H * L_mid * t                     # volume of the middle block

COG = COG_calc()                              # center of gravity of the moving block

z_top_center = lever_arm(L/4,H)
z_cog = lever_arm(COG[0],COG[1])
z_delta = lever_arm(L/2,H)
z_midblock2 =H/2

## Derived loads and forces
G1 = rho_masonry*V_net_t                     # gravity load of the outer triangular w
G2 = rho_masonry*V_net_mid                   # gravity load of the middle wall (self-
F1 = mu_f*S1                                 # friction force from slab acting on top
```

```
F2 = mu_f*S2                                  # friction force from slab acting on top
# The projected parts of the actions that will create a moment around the turning c
P1_proj = P1 * np.cos(beta)
S1_proj = S1 * np.cos(beta)
G1_proj = G1 * np.cos(beta)

## Rocking
# Maximum earthquake force to start rocking
alpha_max  = alpha_fun(0)                     # Portion of gravity load [-]
Q1_max = G1*alpha_max/1000                    # Maximum seismic load on triangular blo
Q2_max = G2*alpha_max/1000                    # Maximum seismic load on middle block [
Q_tot_max=2*Q1_max+Q2_max

# Plot force-displacement relationship of the left overturning of the wall
delta_vals = np.linspace(0,H,1000)
alpha_vals = np.zeros((np.size(delta_vals)))

for i in range(np.size(delta_vals)):          # Calculates the corresponding alpha fo
                                              # at the COG of the triangular parts of
    alpha_vals[i]=alpha_fun(delta_vals[i])

# Transformation to only display values up to failure
alpha_vals = alpha_vals[alpha_vals>=0]
delta_vals = delta_vals[: np.size(alpha_vals)]

# Calculate the forces corresponding to the alpha factors on each block
q1_vals=alpha_vals*G1/1000                    # Seismic force on triangular block [kN]
q2_vals=alpha_vals*G2/1000                    # Seismic force on middle block [kN]
q_tot_vals=2*q1_vals+q2_vals

plt.figure()
plt.grid()
plt.plot([0,0],[0,Q_tot_max],label='Total force',color='red')
plt.plot(delta_vals,q_tot_vals,color='red')
plt.plot([0,0],[0,Q1_max],label='Triangular block force',color='orchid')
plt.plot(delta_vals,q1_vals,color='orchid')
plt.plot([0,0],[0,Q2_max],label='Middle block force',color='blue')
plt.plot(delta_vals,q2_vals, color='blue')

plt.legend(fontsize=12)
plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('Q [kN]', fontsize=14)
plt.title('Force-Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)



# Plot acceleration-displacement relationship of the overturning of the wall, where
# by gravitational force
plt.figure()
plt.grid()
plt.plot([0,0],[0,alpha_max],color='red')
plt.plot(delta_vals,alpha_vals,color='red')

plt.xlabel('$\delta$ [m]', fontsize=14)
plt.ylabel('a [-]', fontsize=14)
plt.title('a - Displacement', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

## Maximum earthquake force to start rocking
a_max = alpha_vals[0]                        # Maximum alpha [-]
acc_max = a_max*9.81                         # Maximim acceleration [-]
```

```
delta_max = delta_vals[-2]                # Maximum displacemet [m]

print(f"The maximum forces on each block are: Q1= {round(Q1_max,2)}kN Q2={round(Q2_
print('After reaching that limit, the wall will start rocking out of plane.')
print('This corresponds to acelerations of acc= '+str(round(acc_max,2))+'m/s2')
print(f"The a-factor is a1={a_max} ")
print('The limit displacement corresponds to '+str(round(delta_max,3))+' m. After r
```

```
The maximum forces on each block are: Q1= 12.46kN Q2=90.82kN
After reaching that limit, the wall will start rocking out of plane.
This corresponds to acelerations of acc= 15.96m/s2
The a-factor is a1=1.6265672026107196
The limit displacement corresponds to 0.569 m. After reaching this displacement th
e wall loses all resistance.
```





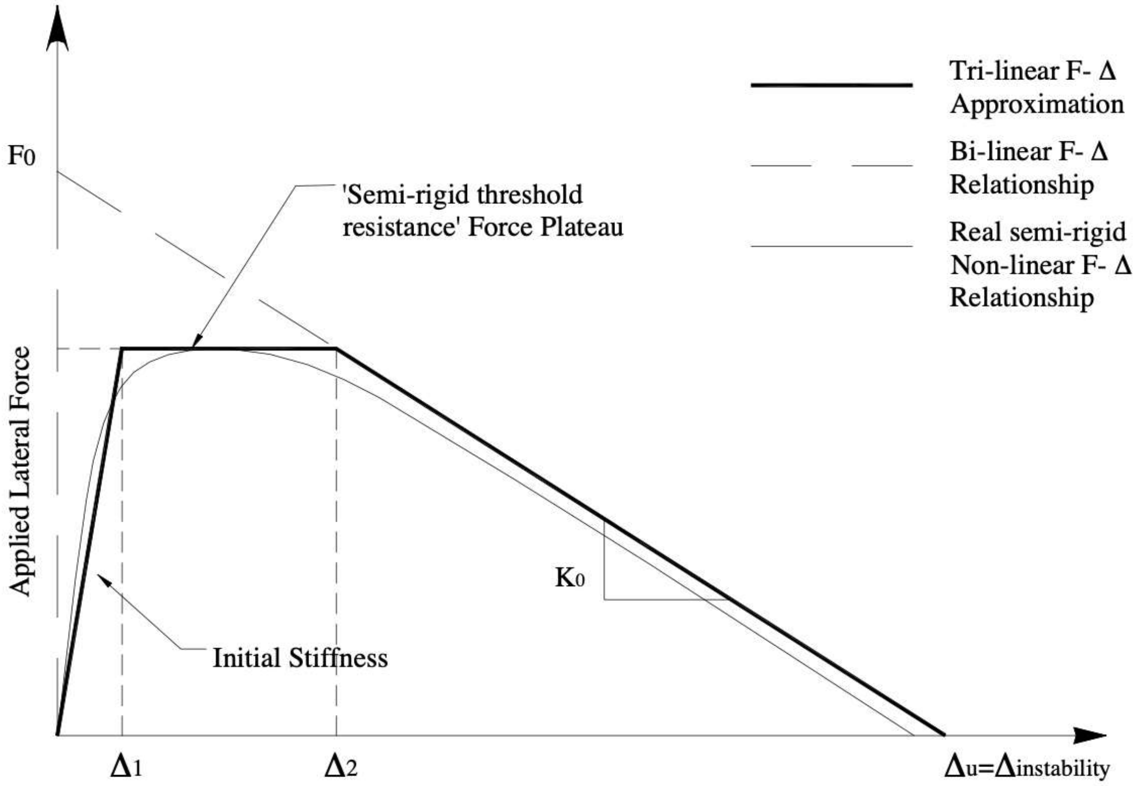
5. Computation of capacity curve

This section allows the analysis of the structural response in the context of the seismic spectrum. The goal is to compare the capacity curve to the ADRS response spectrum, which will allow the behaviour of the wall to be interpreted (e.g., indicating whether it would meet its design requirements).

The main input is the spectral acceleration (for a 475-year return period) for the wall in its given location, as well as its period. For users of the Swiss code, SIA 261 chapter 16.2.1.2 provides the horizontal soil acceleration for each seismic zone; otherwise it can be found in the relevant National Annex of the Eurocode.

As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, delta_1 and delta_2, and the maximum displacement As stated above, this section will provide an interpretation of the wall's behavior according to a trilinear force-displacement model. These model utilizes three points: two displacement parameters, Δ_1 and Δ_2, and the maximum displacement capacity, Δ_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, Δ_u is approximated to be equivalent to Δ_o in this code., delta_o. Below, the trilinear approximation as determined by Griffith et al. (2003) is plotted alongside the bilinear relationship and the real semi-rigid nonlinear relationship. According to Griffith et al. (2003), these coefficients depend on the material quality. Note that for this notebook, delta_u is approximated to be equivalent to delta_o in this code.

In [13]:
```
display(Image(filename='Griffith_Trilinear.png'))
```



In [14]:
```
agr = 0.152        # Spectral acceleration for 475-year return period [g]

# Choose the coefficients for the tri-linear model per Griffith et al. (2003)
d1 = 0.075         # Range from 0.05 to 0.20
d2 = 0.47          # Range from 0.25 to 0.50
```

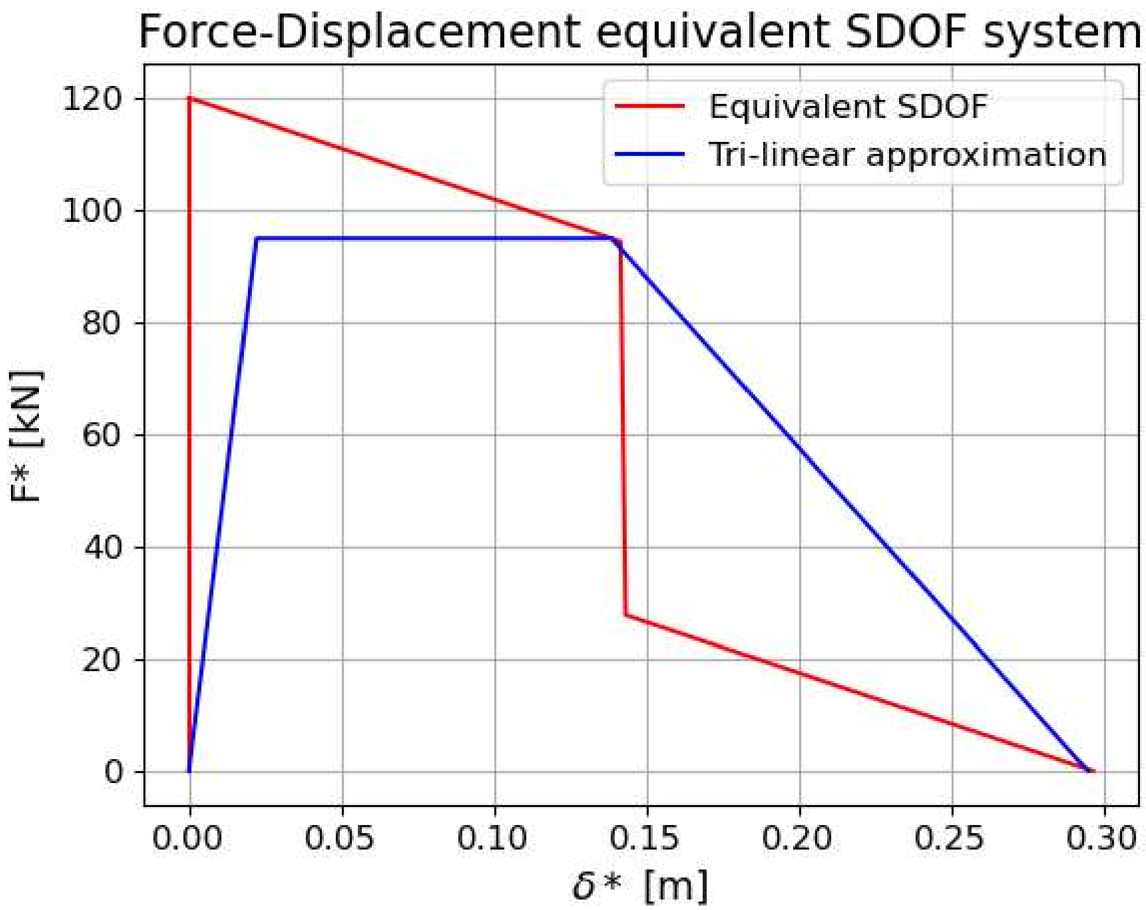
Response spectra are defined for single degree of freedom (DOF) systems. In this case, the mechanism involves more than one DOF. Thus, the capacity curve needs to be transformed into a capaciti curve for an equivalent SDOF system. Furthermore, after the transformation the pushover curve is aproximated as a tri-linear curve as proposed above (Griffith et al., 2003).

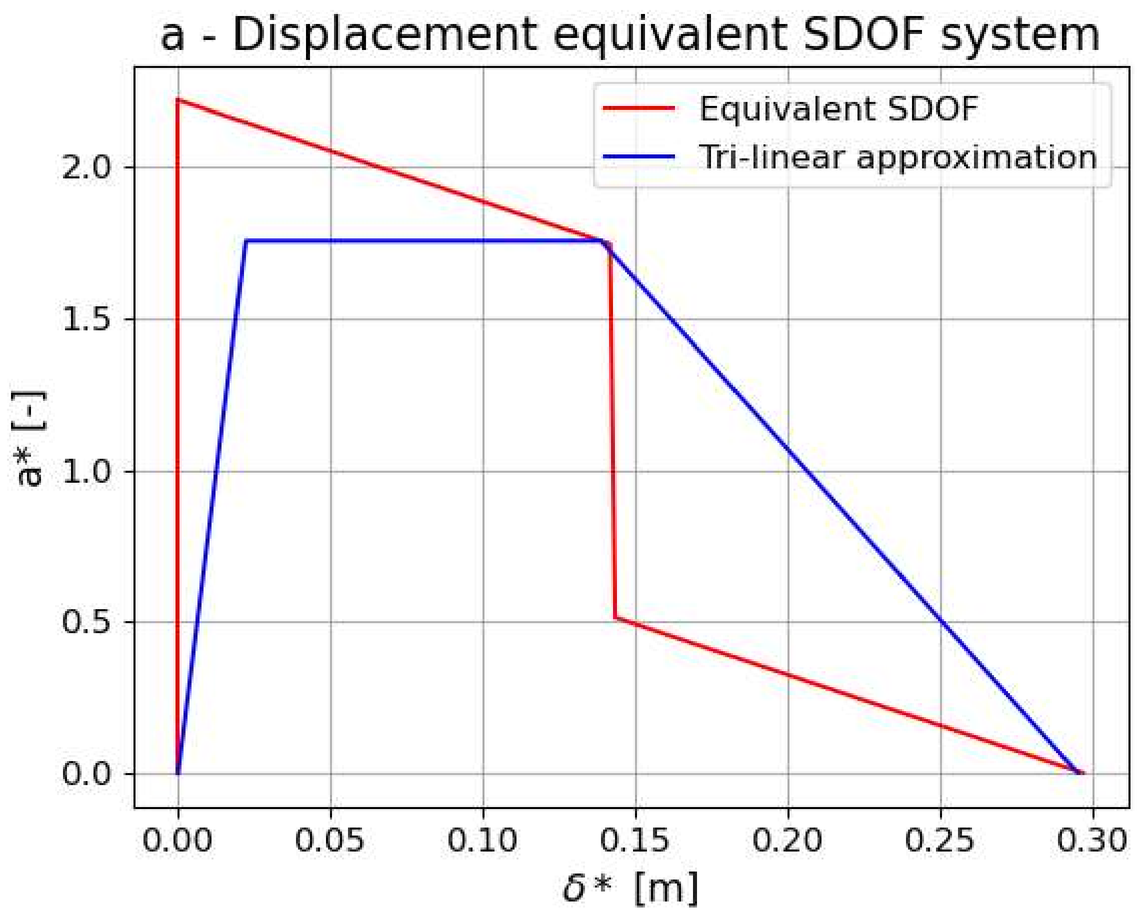
In [15]:
```
m = np.array([V_net_t,V_net_mid,V_net_t])*rho_masonry       # mass of wall [kg]
g = 9.81                                                   # gravitational constant
Qs = np.sum(m)*agr*g                                       # seismic force acting o
phi = np.divide([z_cog/z_delta, z_midblock2/H, z_cog/z_delta],z_midblock2/H)

# calculate effective properties
m_star = np.dot(m,phi)                                     # effective mass (SDOF)
Gamma = m_star / np.dot(m,phi**2)                          # participation factor [
Fb = q_tot_vals * 1000                                     # total shear force [N]
F_star = Fb/Gamma                                          # equivalent shear force
d_2 = delta_vals/H*z_midblock2                             # displacement at COG of
phi2 = phi[1]                                              # modal value at COG of
d_star = d_2/(Gamma*phi2)
a_star = F_star/m_star                                     # effective acceleration

# Tri-linear approximation of the pushover curve according to  Griffith et al. (200
a_1 = alpha_fun(d1*delta_max)
a_2 = alpha_fun(d2*delta_max)
```

```python
F_star1 = a_1*(2*G1+G2)/Gamma
F_star2 = a_2*(2*G1+G2)/Gamma
a_star1 = F_star1/m_star
a_star2 = F_star2/m_star
d_star1 = (d1*delta_max)/H*z_midblock2/(Gamma*phi2)
d_star2 = (d2*delta_max)/H*z_midblock2/(Gamma*phi2)
d_star_max = d_star[-2]
F_star_max = F_star2
F_star_app = [0,F_star_max, F_star_max,0]
d_star_app = [0,d_star1,d_star2,d_star_max]
a_star_app = F_star_app/m_star
a_star_max = F_star_max/m_star

# Displacement capacities according to EN 1998-3
d_u1 = 0.4*d_star_max
d_u2 = 0.6*d_star_max

# Pushover curve of the equivalent SDOF Force-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,F_star[0]/1000],color='red')
plt.plot(d_star,np.divide(F_star,1000),label='Equivalent SDOF',color='red')
plt.plot(d_star_app,np.divide(F_star_app,1000),label='Tri-linear approximation',col

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('F* [kN]', fontsize=14)
plt.title('Force-Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

# Pushover curve of the equivalent SDOF Acceleration-Displacement
plt.figure()
plt.grid()
plt.plot([0,0],[0,a_star[0]],color='red')
plt.plot(d_star,a_star,label='Equivalent SDOF',color='red')
plt.plot(d_star_app,a_star_app,label='Tri-linear approximation',color='blue')

plt.legend(fontsize=12)
plt.xlabel('$\delta*$ [m]', fontsize=14)
plt.ylabel('a* [-]', fontsize=14)
plt.title('a - Displacement equivalent SDOF system', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)
```

Out[15]:

```
(array([-0.5,  0. ,  0.5,  1. ,  1.5,  2. ,  2.5]),
 [Text(0, -0.5, '−0.5'),
  Text(0, 0.0, '0.0'),
  Text(0, 0.5, '0.5'),
  Text(0, 1.0, '1.0'),
  Text(0, 1.5, '1.5'),
  Text(0, 2.0, '2.0'),
  Text(0, 2.5, '2.5')])
```





To compute the response spectrum, it is necessary to specify the soil type per SIA 261 Tableau 24 or the Eurocode National Annex for a particular country.

Alternatively, sometimes the response spectra are defined according to the limit state for analysis. The available limit states are: 'serviceability', 'damage limitation', 'significant damage' or 'near collapse. More information can be found in EC 8 chapter 4.2.3.

After the computation of the response spectrum, the capacity curve of the equivalent SDOF system can be plotted alongside the acceleration-displacement response spectrum. This provides a direct comparison between the specific behavior of the elements in question (in this case, the wall) and the spectrum of the desired limit state (either serviceability state, damage limitation, significant damage, or near collapse), and it can quickly give the spectral acceleration for a certain spectral displacement.

This displacement-based evaluation utilizes the equal displacement "rule" for periods smaller than T_c and the equal energy "rule" for periods larger than T_c. Thus, to interpret the results, extend the initial positive linear part of the capacity curve until it intersects the ADRS, and draw a vertical line at this intersection. If $T < T_c$, the actual displacement will be slightly larger than the displacement at the intersection of the capacity curve and the vertical line; if $T > T_c$, the actual displacement will be equivalent to the displacement at the intersection of the capacity curve and the vertical line. The displacement capacity is exceeded if the determined displacement is larger than the end of the capacity curve.

In [16]:
```
soil_type = 'E'        # specify the soil type (ranging from A to E)
```

In [17]:
```
### NOTHING TO BE CHANGED

## Determine the periods that define the spectrum according to the soil type, with
def spectrum_vals(soil_type):
    if soil_type == 'A':
        S = 1.00                                # [-]
        Tb = 0.15                               # [s]
        Tc = 0.40                               # [s]
        Td = 2.00                               # [s]
    elif soil_type == 'B':
        S = 1.20                                # [-]
        Tb = 0.15                               # [s]
        Tc = 0.50                               # [s]
        Td = 2.00                               # [s]
    elif soil_type == 'C':
        S = 1.15                                # [-]
        Tb = 0.20                               # [s]
        Tc = 0.60                               # [s]
        Td = 2.00                               # [s]
    elif soil_type == 'D':
        S = 1.35                                # [-]
        Tb = 0.20                               # [s]
        Tc = 0.80                               # [s]
        Td = 2.00                               # [s]
    elif soil_type == 'E':
        S = 1.40                                # [-]
        Tb = 0.15                               # [s]
        Tc = 0.50                               # [s]
        Td = 2.00                               # [s]

    return S,Tb,Tc,Td

## Determine the spectral acceleration according to the structure's fundamental per
def RS(T,agr,S,Tb,Tc,Td):
```

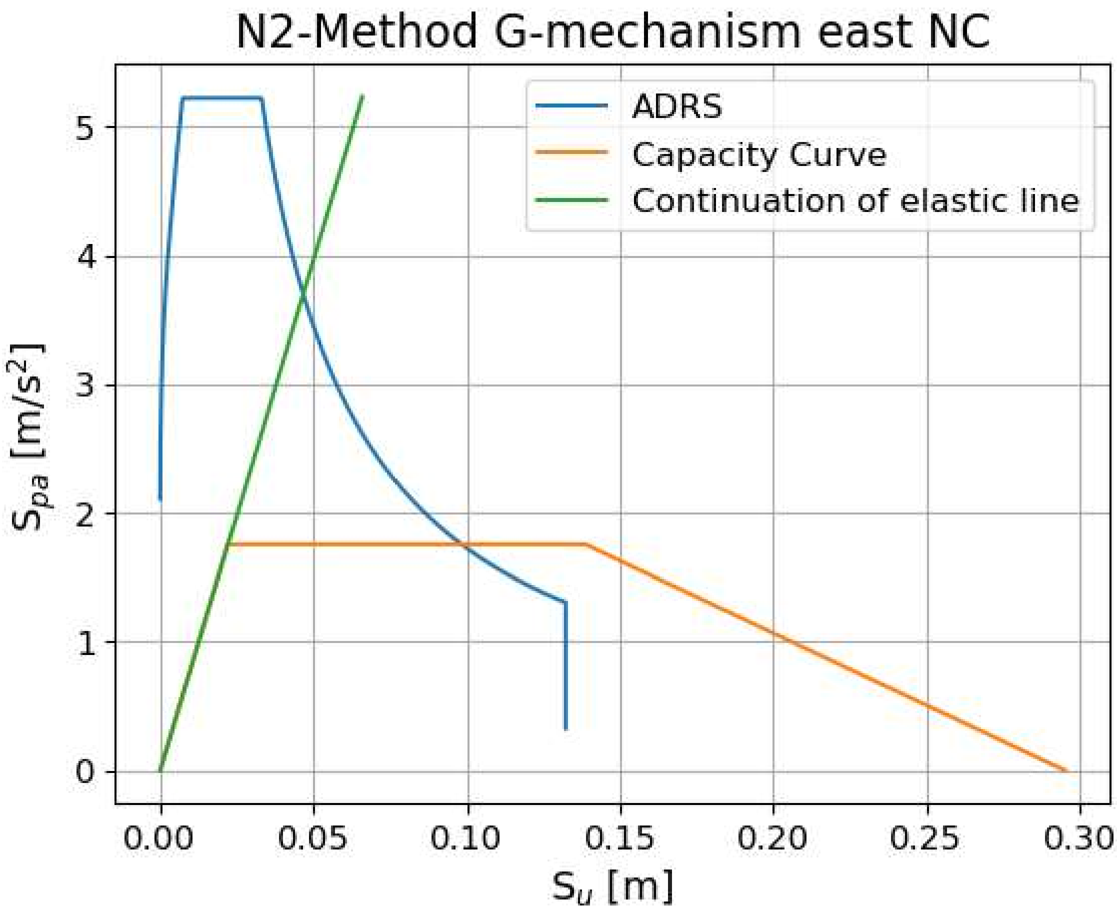
```python
    # determine spectral ground acceleration for dimensionning
    g = 9.81                                             # [m/s^2]
    agd = agr*g                                          # [m/s^2]
    zeta = 0.05                                          # assume damping ratio to be 5% [-
    eta = max(np.sqrt(1/(0.5+10*zeta)),0.55)             # reduction coefficent [-]

    # compute spectrum according to period
    if T<Tb:
        Se = agd*S*(1+(T/Tb)*(eta*2.5-1))
    elif T>=Tb and T<Tc:
        Se = agd*S*eta*2.5
    elif T>=Tc and T<Td:
        Se = agd*S*eta*2.5*(Tc/T)
    else:
        Se = agd*S*eta*2.5*(Tc*Td/T**2)

    return Se
```

In [18]:

```python
## Compute ADRS
# Initialize vectors
T_vals = np.linspace(0.001,4,1000)
Su_vals = np.zeros((len(T_vals),1))
Se_vals = np.zeros((len(T_vals),1))
[S,Tb,Tc,Td] = spectrum_vals(soil_type)

for i in range(0,1000):
    omega = 2*np.pi/T_vals[i]                                        # natural f
    Se_vals[i] = RS(T_vals[i],agr,S,Tb,Tc,Td)                        # spectral
    if T_vals[i] < Tc:
        Ry = max(Se_vals[i]*m_star/Qs,1)                             # R relatio
        wd_int = ((Ry-1)*Tc/T_vals[i] + 1)*Se_vals[i]/((omega**2)*Ry)   # displacem
        Su_vals[i] = min(wd_int,3*Se_vals[i]/omega**2)               # spectral
    else:
        Su_vals[i] = Se_vals[i]/(omega**2)                           # spectral

## Plot capacity curve NC
# plot results
plt.figure()
plt.grid()
plt.plot(Su_vals,Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/4)],Su_vals[:round(len(Su_vals)/4)]/d_star1*a_
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method G-mechanism east NC', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

def find_closest_pair(d1, f1, d2, f2):

    data_set1 = np.column_stack((d1, f1))
    data_set2 = np.column_stack((d2, f2))

    min_distance = float('inf')
    closest_pair = None

    for point1 in data_set1:
        for point2 in data_set2:
            dist2d =point1 - point2
            distance = (dist2d[0]**2+dist2d[1]**2)**0.5
            if distance < min_distance:
                min_distance = distance
```

```
                closest_pair = (point1[0], point1[1], point2[0], point2[1])

    return closest_pair


closest = find_closest_pair(Su_vals, Se_vals, Su_vals[:round(len(Su_vals)/2)],Su_va
target_disp_NC = (closest[0]+closest[2])/2

print(f"The target displacement NC is of d={round(target_disp_NC,3)} m")
print(f"The NC limit displacement is d_u2={round(d_u2,3)} m")
if target_disp_NC<=d_u2:
    print('Therefore the NC limit state is verified')
else:
    print('Therefore the NC limit state is NOT verified!!!!')
```

```
The target displacement NC is of d=0.047 m
The NC limit displacement is d_u2=0.177 m
Therefore the NC limit state is verified
```



In [19]:
```
## Plot capacity curve DL
# plot results
plt.figure()
plt.grid()
plt.plot(0.5*Su_vals,0.5*Se_vals,label='ADRS')
plt.plot(d_star_app,a_star_app,label='Capacity Curve')
plt.plot(Su_vals[:round(len(Su_vals)/10)],Su_vals[:round(len(Su_vals)/10)]/d_star1*
plt.legend(fontsize=12, loc='upper right')
plt.xlabel('S$_u$ [m]', fontsize=14)
plt.ylabel('S$_{pa}$ [m/s$^2$]', fontsize=14)
plt.title('N2-Method G-mechanism east DL', fontsize=16)
plt.xticks(fontsize=12)
plt.yticks(fontsize=12)

closest = find_closest_pair(0.5*Su_vals, 0.5*Se_vals, Su_vals[:round(len(Su_vals)/1
target_disp_DL = (closest[0]+closest[2])/2
```

```python
print(f"The target displacement DL is of d={round(target_disp_DL,3)} m")
print(f"The DL limit displacement is d_u1={round(d_u1,3)} m")
if target_disp_DL<=d_u1:
    print('Therefore the DL limit state is verified')
else:
    print('Therefore the DL limit state is NOT verified!!!!')
```

```
The target displacement DL is of d=0.023 m
The DL limit displacement is d_u1=0.118 m
Therefore the DL limit state is verified
```