Appendix A: Deep Reinforcement Learning

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In this appendix, we will go into further detail on how deep RL algorithms work. The aim here is not to give full treatment of all deep RL methods – Sutton and Barto (2018) is an excellent resource for theoretical underpinnings while the white papers behind the algorithms go into greater detail on modern approaches –; instead, this section is meant to serve as an abbreviated background view on model-free deep RL. We hope to introduce some of the important design choices that are made for model-free deep RL algorithms and will point to specific resources that give a more complete presentation. These sections presume some familiarity with gradient descent optimization – see Ruder (2017) for a thorough treatment. Throughout these sections, we'll consider the environment to be an MDP which makes for a cleaner presentation here and is also consistent with the referenced sources.

1 Gradient-based Model-free RL Algorithms

Policy gradients, value-based and actor-critic methods are the general classes of algorithms for gradient-based model-free RL. In this section, we will go over these algorithm classes to convey a broad sense of what they are attempting to achieve and the trade-offs between them.

1.1 Policy Gradients

The most straightforward way to optimize the RL objective is through a policy search, whereby the agent continually updates a policy parameterized by θ, π_θ – in deep RL, this parameterized policy will be a neural network. There are both gradient-free and gradient-based policy search methods that have been successful in practice (Salimans et al. 2017); but for the sake of brevity, we will focus on gradient-based methods, which are more commonly encountered. In policy gradient methods, algorithms are performing gradient ascent on the expected return, $J(\pi_\theta)$, to find the optimal policy parameters. At each gradient step, the following update is performed,

$$\theta_{t+1} = \theta_t + \alpha \nabla_\theta J(\pi_\theta).$$

Supposing that the agent has interacted with the environment and has collected some trajectories denoted by, τ_i, we can employ the policy gradient theorem – see Sutton and Barto (2018) for the proof of the policy gradient theorem – to estimate $\nabla J(\pi_\theta)$ as:

$$\nabla_\theta J(\pi_\theta) \approx \sum_{i,t} \nabla_\theta \log \pi_\theta(a_{i,t} \mid s_{i,t}) \left(\sum_{t'=t}^{H} \gamma^{t'-t} r(s_{i,t'}, a_{i,t'}) \right).$$

The intuition behind this approximation is exactly what we want: the agent will increase the probability of repeating the actions for the inputted states proportional to the size of the return. In practice, however, the empirical return tends to have high variance, resulting in very noisy gradients that impedes learning (Sutton and Barto 2018). To avoid this issue, many policy gradient algorithms replace the empirical return with a return estimator that has lower variance (Sutton and Barto 2018).

A sketch of a simple policy gradient algorithm is shown in Algorithm 1.

Algorithm 1 Vanilla On-policy Policy Gradient Algorithm

1: Input initial policy parameters θ_0
2: **for** $k = 0, 1, 2, \ldots$ **do**
3:     Generate a trajectory $\{s_i, a_i, r_i\}$ by following an exploration algorithm with the policy π_k (e.g. ϵ-greedy)
4:     **for** $t = 0, 1, \ldots, T-1$ **do**
5:         $G \leftarrow \sum_{j=t}^{T} \gamma^{j-t} r_j$
6:         $\theta_{k+1} \leftarrow \theta_k + \alpha \gamma^t G \nabla \log \pi_{\theta_k}(a_t|s_t)$

1.2 Value-based Methods

Another way to approach the RL problem is by estimating a value function, and then using the value function to retrieve a policy. Value functions attempt to quantify the goodness of being in a state or taking an action from a state. Given that the objective of RL is to maximize the cumulative expected reward, "goodness" refers to how high of a cumulative reward the agent can expect to receive. For example, the state-action-value or Q function finds the expected return from a state-action pair under a policy, π,

$$Q^\pi(s_t, a_t) = \mathbb{E}_{\tau \sim p_\pi(\tau|s_t, a_t)} \left[\sum_{t'=t}^{H} \gamma^{t'-t} r(s_t, a_t) \right].$$

There are more value functions that can used in practice (Schulman et al. 2018), but since Q-functions are commonly encountered, we will focus on Q-learning. From the above equation, we can define the Q function recursively,

$$Q^\pi(s_t, a_t) = r(s_t, a_t) + \gamma \mathbb{E}_{s_{t+1} \sim T(s_{t+1}|s_t, a_t),\, a_{t+1} \sim \pi(a_{t+1}|s_{t+1})} \left[Q^\pi(s_{t+1}, a_{t+1}) \right]$$

The motivation behind Q-learning is that if we know the Q function under certain conditions – e.g. that the state-action space is discrete and that all state-action pairs are visited ad infinitum (Sutton and Barto 2018, 131; Melo, n.d.) –, then we can easily find the optimal action at any state by selecting the action with the highest Q-value: $a_t^* = \operatorname*{argmax}_{a_t} Q(s_t, a_t)$. Using this optimal policy, we can write an equation for the optimal Q-function from the previous definitions,

$$Q^*(s_t, a_t) = r(s_t, a_t) + \gamma \mathbb{E}_{s_{t+1} \sim T(s_{t+1}|s_t, a_t)} \left[\max_{a_{t+1}} Q^*(s_{t+1}, a_{t+1}) \right]$$

The objective for deep Q-learning is to find an approximate Q-function that satisfies the above equation, where we represent our approximate Q-function, Q_ϕ, with a neural network that has the parameters of ϕ. Subtracting the left hand and right hand side of the recursive Q-function definition, we can define the temporal difference error, $\mathcal{E}$, for a state-action pair as

$$\mathcal{E} = r(s_t, a_t) + \gamma \max_a Q_\phi(s_{t+1}, a) - Q_\phi(s_t, a_t).$$

The goal is to find a Q_ϕ that sets the TD error to zero across all state-action pairs. The general process to achieve this objective is to sample trajectories from the environment, evaluate the TD Error across different state-action-reward transitions – with samples indexed by i –, and then perform the following gradient descent on the parameters of the Q-function neural network:

$$\phi_{t+1} = \phi_t - \alpha \nabla_\phi \sum_i \mathcal{E}_i^2$$

After the Q-function neural network or Q-network has been fitted, the agent will select actions by inputting the current state into the Q network and then identifying the state-action pair with the highest estimated Q value. The benefit of using neural networks to approximate Q-functions is that neural networks are able to generalize Q functions over unseen state-action pairs, whereas classic methods like TD-learning are not able to generalize (Mnih et al. 2015). But there a host of problems with getting neural-network-based value-learning agents to work in practice; see Mnih et al. (2015) and Hasselt, Guez, and Silver (2015) for more insight on these issues.

A notable concern with these methods is that they tend to suffer from bias which is introduced by fitting a value network to a target that incorporates an estimated value function (Hasselt, Guez, and Silver 2015). In review, a sketch of simple deep Q learning algorithm is shown in Algorithm 2.

Algorithm 2 Vanilla On-policy Q-learning Algorithm

1: Input initial Q-network parameters ϕ_0
2: **for** $k = 0, 1, 2, \ldots$ **do**
3:     **for** $t = 0, 1, \ldots, T - 1$ **do**
4:         Choose the action a_t using Q_{ϕ_k} according to an exploration algorithm (e.g. ϵ-greedy)
5:         Take action a_t and observe r_t, s_{t+1}
6:         $\mathcal{E} \leftarrow r_t + \gamma \max_a Q_{\phi_k}(s_{t+1}, a) - Q_{\phi_k}(s_t, a_t)$
7:         $\phi_{k+1} \leftarrow \phi_k - \alpha \nabla_{\phi_k} \mathcal{E}^2$

1.3 Actor-Critic

Actor-critic algorithms integrate the main components from both value-learning and policy gradient based methods. The "actor" attempts to learn the policy, and the "critic" attempts to learn a value function. While the general algorithm for the critic is exactly the same as mentioned in the value-learning section, the actor differs from policy gradient algorithms by incorporating the critic in its gradient ascent step. Instead of estimating the gradient of the return as in eq ..., the gradient of the return is estimated as

$$\nabla_\theta J(\pi_\theta) \approx \sum_{i,t} \nabla_\theta \log \pi_\theta(a_{i,t} \mid s_{i,t}) \, \hat{Q}(s_{i,t}, a_{i,t}).$$

where $\hat{Q}$ is a value function estimated by the critic. The benefit of actor-critic algorithms is that they can balance the issues with bias and variance that are common with value-learning and policy gradient methods respectively (Sutton and Barto 2018). Consequently, actor-critic algorithms have achieved many of the state of the art results in model-free deep RL (Haarnoja et al. 2018; Fujimoto, Hoof, and Meger 2018).

A simple sketch of an actor critic algorithm is shown in Algorithm 3.

Algorithm 3 Vanilla On-policy Actor-Critic Algorithm

1: Input initial policy and Q-network parameters θ_0, ϕ_0
2: **for** $k = 0, 1, 2, \ldots$ **do**
3:     Generate a trajectory $\{s_i, a_i, r_i\}$ by following an exploration algorithm with the policy π_k (e.g. ϵ-greedy)
4:     **for** $t = 0, 1, \ldots, T - 1$ **do**
5:         $\mathcal{E} \leftarrow r_t + \gamma \max\limits_a Q_{\phi_k}(s_{t+1}, a) - Q_{\phi_k}(s_t, a_t)$
6:         $\phi_{k+1} \leftarrow \phi_k - \alpha \nabla_{\phi_k} \mathcal{E}^2$
7:         $\theta_{k+1} \leftarrow \theta_k + \alpha \nabla \log \pi_{\theta_k}(a_t|s_t) Q_{\phi_k}(s_t, a_t)$

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2 Exploration

Exploration is a central component of any deep RL algorithm, and there are a variety of exploration algorithms that an agent can employ. In the sections above, we procedurally mentioned that the agent must select actions during training. Yet, how the agent selects these actions is very important because if an agent fails to explore the state-action space well, the agent will likely learn a bad policy. In this section, we will go over some common exploration algorithms and try to provide some practical insight.

2.1 Epsilon-Greedy Algorithm

The Epsilon-Greedy algorithm takes a very simple approach towards balancing exploration and exploitation. For some $\epsilon \in (0, 1)$ and typically close to 0, an Epsilon-Greedy agent exploits the best available action with probability $1 - \epsilon$ and explores a random action with probability ϵ. While simplistic, this algorithm has been effective on a range of problems notably Mnih et al. (2015).

2.2 Boltzmann Exploration

For algorithms that learn a value function, agents can select actions by sampling from a Boltzmann or Softmax distribution that is constructed over the value function. So if the agent is doing Q-learning, the probability for the agent to select the action, $a \in \mathcal{A}$, would be:

$$P(a) = \frac{e^{\frac{Q(s_t, a)}{\tau}}}{\sum\limits_{a'} e^{\frac{Q(s_t, a')}{\tau}}}$$

where $\tau \in (0, \infty)$ is called the temperature parameter and controls how much the agent will weight high Q scores over low Q scores. The advantage of using Boltzmann exploration is that since the probability of selecting an action is proportional to how good the agent thinks that action is, the agent can explore the space more efficiently than with Epsilon-Greedy exploration. This is because an Epsilon-Greedy agent will explore all actions equally including low reward yielding actions; in contrast, with Boltzmann exploration, an agent will spend less time on these unpromising actions.

2.3 Noise-Based Exploration

Injecting noise into an agent's action, observation or parameter space is another effective exploration method. With noise addition, there are a lot design choices that can be made. For instance, noise can be sampled from

different distributions (Plappert et al. 2018), scaled adaptively based on the policy(Plappert et al. 2018), or the noise can be sampled according to the state that the agent observes (Raffin and Stulp 2020). There are a variety of reasons why one would want to use these variations of noise injection, which are well discussed in Plappert et al. (2018) and Raffin and Stulp (2020).

2.4 Entropy-based Exploration

An increasingly common way to achieve exploration is by adding a function of the entropy term, $H(\pi(a|s))$, to the loss function of the policy network. While this introduces some bias as the agent no longer exclusively maximizes cumulative rewards, entropy-based exploration has been effective in practice at improving exploration, resulting in better agent performance. Ahmed et al. (2019) gives a very complete discussion on the benefits of using entropy.

There is a multitude of exploration algorithms that we have not touched on, so it is important to say that the algorithms mentioned above are generally suited for dense reward or "easy-exploration" problems. For "hard-exploration" problems – for instance, environments that have very few state-action pairs with significant rewards, like a maze, are seen as hard-exploration problems –, more intricate and bespoke exploration algorithms tend to outperform. We will not go over these more advanced exploration algorithms but a good example is seen in Ecoffet et al. (2021).

3 Replay Buffers vs. Parallel Workers

Another important algorithm design choice involves how an RL agent collects and samples past experiences. A significant issue that arises from sampling past experiences is correlation of samples Schaul et al. (2016). In the simple case that we have an RL agent that interacts with the environment for a few time steps and then uses this trajectory for its learning update, these samples, since they come from the same trajectory, will be strongly correlated. Learning from correlated samples is undesirable since this can result in agents getting stuck in local optima, which will impede learning, but there are other reasons why learning from a single trajectory causes problems – see Schaul et al. (2016) and Mnih et al. (2015) for more insight here. There are two commonly used solutions to this problem of correlated samples: Replay Buffers and Parallel Workers. If the agent is off-policy, the agent can employ a replay buffer – see Mnih et al. (2015) for a now classic example for a replay buffer-based agent. With a replay buffer, the agent will save state-action-reward transitions in the replay buffer, and during learning updates, the agent will sample transitions from the replay buffer. Since the replay buffer can be arbitrarily large and thus store transitions from numerous trajectories, sampling from the buffer will likely produce transitions that cover a broader region in state-action space than if the agent were to sample from one trajectory. On-policy agents, however, cannot use a replay buffer since these agents are not allowed to use samples from a policy different than the policy they are trying to improve – replay buffers are generally large enough to hold transitions from old policies. The solution to de-correlate samples for on-policy agents is to use parallel workers, whereby multiple workers explore the environment according to the current policy. Due to stochasticity that can come from exploration and the environment, the workers will go on different trajectories in state-action space. The on-policy agent will collect samples across these workers in a learning step, and then repeat the process of collecting and sampling transitions with an updated policy. The idea behind using parallel workers is that sampling over multiple trajectories will de-correlate samples – see Babaeizadeh et al. (2017) on more details here. In practice, using a large replay buffer, on the order of millions of transitions, or a modest number of parallel, around 4-16 for simple RL problems, is very important to achieve good agent performance for off-policy and on-policy algorithms respectively.

4 Next Steps

Reinforcement learning is a huge, constantly evolving field, and we've only touched on a small sliver of RL in our study. In this appendix, we briefly introduced some of the major design decisions that are made with model-free deep RL algorithms, but there are many more decisions that are made with current state of the art algorithms. In table . . . , we present some commonly used deep RL algorithms. Readers that are interested in learning more about the current state of deep RL should read the papers referenced in table . . . ; Mnih et al. (2015), Haarnoja et al. (2018) and (**PPO?**) are good places to start. We have also not given any space to discuss model-based or gradient-free techniques which also warrant attention – see Janner et al. (2019) and Salimans et al. (2017) for more here. A benefit of model-free deep RL over these other approaches is that there has been more of a general focus on model-free deep RL, so there are widely available packages

like Stable Baselines and TensorFlow Agents which implement model-free deep RL agents. It is important to keep in mind though that deep RL has only become established in the last decade, and with the large amount of open research questions, deep RL may evolve significantly in the near future.

Algorithm	Description	Model	Policy	Method
MBPO	Model-based Policy Optimization [1]	Model-based	On-policy	Policy Gradient
MCTS	Monte Carlo Tree Search [2]	Model-based	Either	Any
A2C	Advantage Actor Critic [3]	Model-free	On-policy	Actor-critic
A3C	Asynchronous A2C [4]	Model-free	On-policy	Actor-critic
TRPO	Trust Region Policy Optimization [5]	Model-free	On-policy	Policy Gradient
PPO	Proximal Policy Optimization [6]	Model-free	On-policy	Actor-critic
DQN	Deep Q Networks [7]	Model-free	Off-policy	Value-Based
DDPG	Deep Deterministic Policy Gradient [8]	Model-free	Off-policy	Actor-critic
TD3	Twin Delayed DDPG [9]	Model-free	Off-policy	Actor-critic
SAC	Soft Actor Critic [10]	Model-free	Off-policy	Actor-critic
IMPALA	Importance Weighted Actor Learner [11]	Model-free	Off-policy	Actor-critic

References

Ahmed, Zafarali, Nicolas Le Roux, Mohammad Norouzi, and Dale Schuurmans. 2019. "Understanding the Impact of Entropy on Policy Optimization." *arXiv:1811.11214 [Cs, Stat]*, June. http://arxiv.org/abs/1811.11214.

Babaeizadeh, Mohammad, Iuri Frosio, Stephen Tyree, Jason Clemons, and Jan Kautz. 2017. "Reinforcement Learning Through Asynchronous Advantage Actor-Critic on a GPU." *arXiv:1611.06256 [Cs]*, March. http://arxiv.org/abs/1611.06256.

Ecoffet, Adrien, Joost Huizinga, Joel Lehman, Kenneth O. Stanley, and Jeff Clune. 2021. "Go-Explore: A New Approach for Hard-Exploration Problems." *arXiv:1901.10995 [Cs, Stat]*, February. http://arxiv.org/abs/1901.10995.

Fujimoto, Scott, Herke van Hoof, and David Meger. 2018. "Addressing Function Approximation Error in Actor-Critic Methods." *arXiv:1802.09477 [Cs, Stat]*, October. http://arxiv.org/abs/1802.09477.

Haarnoja, Tuomas, Aurick Zhou, Pieter Abbeel, and Sergey Levine. 2018. "Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor." *arXiv:1801.01290 [Cs, Stat]*, August. http://arxiv.org/abs/1801.01290.

Hasselt, Hado van, Arthur Guez, and David Silver. 2015. "Deep Reinforcement Learning with Double Q-Learning." *arXiv:1509.06461 [Cs]*, December. http://arxiv.org/abs/1509.06461.

Janner, Michael, Justin Fu, Marvin Zhang, and Sergey Levine. 2019. "When to Trust Your Model: Model-Based Policy Optimization." *arXiv:1906.08253 [Cs, Stat]*, November. http://arxiv.org/abs/1906.08253.

Melo, Francisco S. n.d. "Convergence of Q-Learning: A Simple Proof," 4.

Mnih, Volodymyr, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, et al. 2015. "Human-Level Control Through Deep Reinforcement Learning." *Nature* 518 (7540): 529–33. https://doi.org/10.1038/nature14236.

Plappert, Matthias, Rein Houthooft, Prafulla Dhariwal, Szymon Sidor, Richard Y. Chen, Xi Chen, Tamim Asfour, Pieter Abbeel, and Marcin Andrychowicz. 2018. "Parameter Space Noise for Exploration." *arXiv:1706.01905 [Cs, Stat]*, January. http://arxiv.org/abs/1706.01905.

Raffin, Antonin, and Freek Stulp. 2020. "Generalized State-Dependent Exploration for Deep Reinforcement Learning in Robotics." *arXiv:2005.05719 [Cs, Stat]*, May. http://arxiv.org/abs/2005.05719.

Ruder, Sebastian. 2017. "An Overview of Gradient Descent Optimization Algorithms." *arXiv:1609.04747 [Cs]*, June. http://arxiv.org/abs/1609.04747.

Salimans, Tim, Jonathan Ho, Xi Chen, Szymon Sidor, and Ilya Sutskever. 2017. "Evolution Strategies as a Scalable Alternative to Reinforcement Learning." *arXiv:1703.03864 [Cs, Stat]*, September. http://arxiv.org/abs/1703.03864.

Schaul, Tom, John Quan, Ioannis Antonoglou, and David Silver. 2016. "Prioritized Experience Replay." *arXiv:1511.05952 [Cs]*, February. `http://arxiv.org/abs/1511.05952`.

Schulman, John, Philipp Moritz, Sergey Levine, Michael Jordan, and Pieter Abbeel. 2018. "High-Dimensional Continuous Control Using Generalized Advantage Estimation." *arXiv:1506.02438 [Cs]*, October. `http://arxiv.org/abs/1506.02438`.

Sutton, Richard S, and Andrew G Barto. 2018. *Reinforcement Learning: An Introduction.* MIT press.

References

[1] Michael Janner, Justin Fu, Marvin Zhang, and Sergey Levine. When to Trust Your Model: Model-Based Policy Optimization. *arXiv:1906.08253 [cs, stat]*, November 2019. arXiv: 1906.08253.

[2] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction.* MIT press, 2018.

[3] Volodymyr Mnih, Adrià Puigdomènech Badia, Mehdi Mirza, Alex Graves, Timothy P. Lillicrap, Tim Harley, David Silver, and Koray Kavukcuoglu. Asynchronous Methods for Deep Reinforcement Learning. *arXiv:1602.01783 [cs]*, June 2016. arXiv: 1602.01783.

[4] Mohammad Babaeizadeh, Iuri Frosio, Stephen Tyree, Jason Clemons, and Jan Kautz. Reinforcement Learning through Asynchronous Advantage Actor-Critic on a GPU. *arXiv:1611.06256 [cs]*, March 2017. arXiv: 1611.06256.

[5] John Schulman, Sergey Levine, Philipp Moritz, Michael I. Jordan, and Pieter Abbeel. Trust Region Policy Optimization. *arXiv:1502.05477 [cs]*, April 2017. arXiv: 1502.05477.

[6] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal Policy Optimization Algorithms. *arXiv:1707.06347 [cs]*, August 2017. arXiv: 1707.06347.

[7] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin Riedmiller, Andreas K. Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg, and Demis Hassabis. Human-level control through deep reinforcement learning. *Nature*, 518(7540):529–533, February 2015.

[8] Timothy P. Lillicrap, Jonathan J. Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. *arXiv:1509.02971 [cs, stat]*, July 2019. arXiv: 1509.02971.

[9] Scott Fujimoto, Herke van Hoof, and David Meger. Addressing Function Approximation Error in Actor-Critic Methods. *arXiv:1802.09477 [cs, stat]*, October 2018. arXiv: 1802.09477.

[10] Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, and Sergey Levine. Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement Learning with a Stochastic Actor. *arXiv:1801.01290 [cs, stat]*, August 2018. arXiv: 1801.01290.

[11] Lasse Espeholt, Hubert Soyer, Remi Munos, Karen Simonyan, Volodymir Mnih, Tom Ward, Yotam Doron, Vlad Firoiu, Tim Harley, Iain Dunning, Shane Legg, and Koray Kavukcuoglu. IMPALA: Scalable Distributed Deep-RL with Importance Weighted Actor-Learner Architectures. *arXiv:1802.01561 [cs]*, June 2018. arXiv: 1802.01561.