ISYE 6644 Simulation (Summer 2023)

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Topic: 12

Project Group: 215

History

Yahtzee is a dice game that originates from a Canadian couple who creatively called it "The Yacht Game" because they would play it on their yacht with friends. In 1956, Polish toy maker and entrepreneur, Edwin S. Lowe, introduced this game into the market and it was later purchased by Milton Bradley in 1973. This game combines strategy, probability, and luck to achieve the highest score.

Rules

The round starts with the player rolling 5 dice. Then, the player decides which dice they wish to keep or re-roll for up to 3 rolls. Once the turn is over, a score is recorded on the scorecard for one of the categories based on the available points in the last roll. The scorecard is divided into an UPPER and LOWER section. The UPPER section consists of basic points focused on counting only multiples of a specific number on the dice. If the UPPER section of the scorecard exceeds 63 points, an additional 35 points are awarded. The LOWER section consists of more creative combinations where you aim to get 3 of a kind, 4 of a kind, a full house (3 of a kind + 2 of a kind), a small straight (4 sequential numbers), a large straight (5 sequential numbers), Yahtzee (5 of the same number), and chance (sum of all dice). The player can only score once per round, and all scores are final. The game ends after 13 rounds, and it is not necessary for all categories to have a non-zero value.

Table 1 below is an example scorecard that shows how the point system works for each category in the game of Yahtzee.

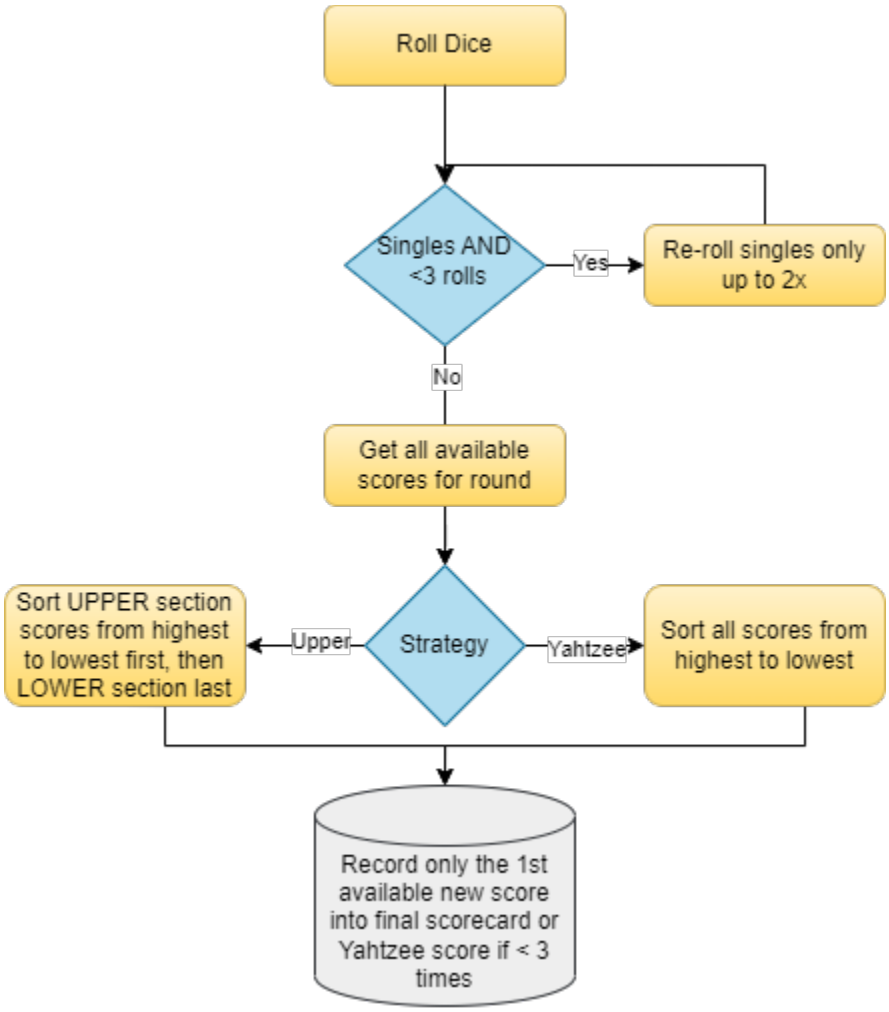
Table 1: Scorecard

Category	Points Available
UPPER	
Aces = 1	Count only 1s
Twos = 2	Count only 2s
Threes = 3	Count only 3s
Fours = 4	Count only 4s
Fives = 5	Count only 5s
Sixes = 6	Count only 6s
Bonus	*35; if UPPER score ≥ 63*
LOWER	
3 of a Kind	Add total of all dice
4 of a Kind	Add total of all dice
Full House	25
Small Straight	30
Large Straight	40
Yahtzee	50; up to 3 times
Chance	Add total of all dice

Strategy

As mentioned above, there are many different strategies involved in playing Yahtzee. One strategy is to complete the UPPER section first at the beginning of the game with the main goal of reaching 63 UPPER points to claim the additional 35 bonus points referenced as Upper in this report. Another is to continually prioritize the Yahtzee category because it gives the most points and can be achieved up to 3 times per game while the failed trails can still be counted towards the UPPER section referenced as Yahtzee in this report. Below is a generalized flowchart portraying the steps to simulate Yahtzee and the differentiation between the 2 strategies. Note that both strategies start off with the same final set of dice but differ in the order of how they are prioritized for recording.

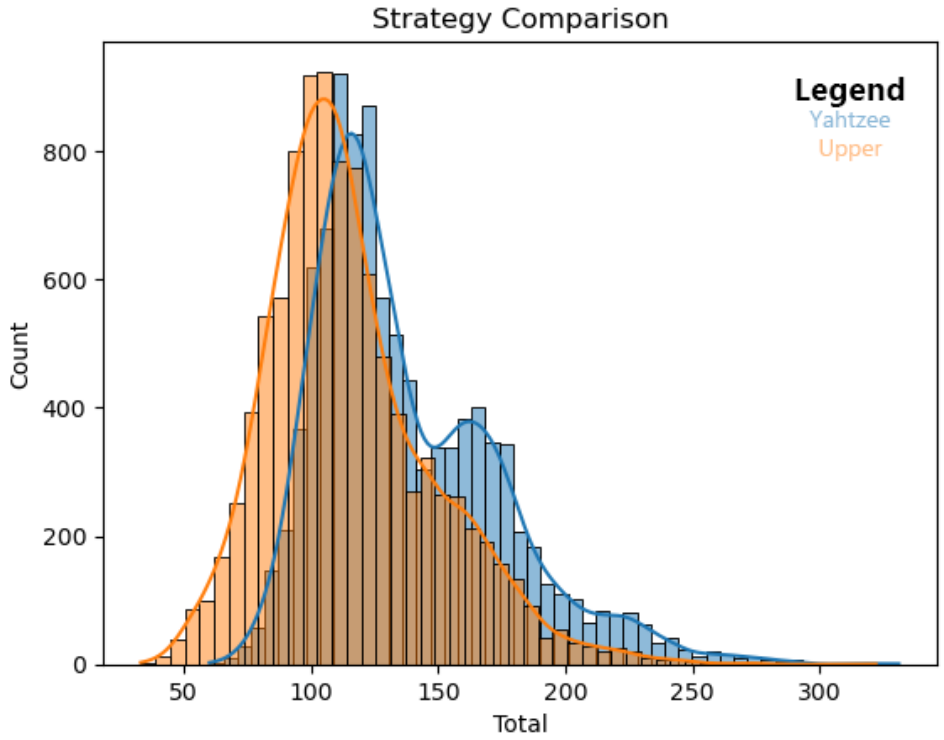
Figure 1: Yahtzee Flowchart



Results

The Upper strategy and the Yahtzee strategy were subjected to extensive testing and evaluation by conducting 10,000 simulations in Python. This simulation process used to obtain a comprehensive and accurate representation of how each strategy performed under various conditions and scenarios. In Figure 2, you will find a histogram visually illustrating the distributions of the Total scores for each strategy along with a trendline. Additionally, the summary statistics for each category have been compiled and are presented in Table 2, providing insights into the strategies' strengths and weaknesses across various scoring categories.

Figure 2: Strategy Histogram



The histogram of Total scores among the different strategies shows that the Upper strategy follows a normal distribution with a positive skew, while the Yahtzee strategy follows a multimodal distribution. Between the 2 strategies, the Upper strategy tends to have more trials with lower scores, and both strategies have a lot of overlap in the 100-130 range.

Table 2: Summary Statistics

	Yahtzee Strategy			Upper Strategy		
	Mean	**Min**	**Max**	**Mean**	**Min**	**Max**
UPPER						
Aces	1.3	0	4	1.4	0	5
Twos	3.7	0	8	3.9	0	10
Threes	5.8	0	15	6.1	0	15
Fours	8.0	0	16	8.4	0	20
Fives	9.8	0	25	10.4	0	25
Sixes	11.9	0	24	12.5	0	30
Bonus	*0.3*	*0*	*35*	*1.0*	*0*	*35*
LOWER						
Three of a Kind	18.0	0	28	13.3	0	28
Four of a Kind	14.3	0	29	8.6	0	29
Small Straight	1.5	0	30	0.7	0	30
Large Straight	0.1	0	40	0.0	0	40
Full House	24.5	0	25	19.8	0	25
Yahtzee	21.2	0	150	11.4	0	150
Chance	17.5	6	29	17.6	6	29
Total	137.9	60	331	115.2	29	273

In the UPPER section, the Upper strategy outperformed the Yahtzee strategy in every category, both in mean and max values. In the LOWER section, the Yahtzee strategy outperformed the Upper strategy in all categories except the Chance category. Overall, the Yahtzee strategy scored an average of 22.7 points more than the Upper strategy.

The confidence intervals can be calculated in python or by using the formula $CI = \mu \pm z(\frac{\sigma}{\sqrt{n}})$. Using a confidence level of 95%, the confidence interval for the Yahtzee strategy is [65.83 - 209.86], and the 95% confidence interval for the Upper strategy is [51.10 - 179.36].

For the Yahtzee strategy, $\frac{691}{10{,}000}$ (6.9%) simulations scored multiple Yahtzees per game, 3 of which exceeded the 3-score maximum and were recorded in the UPPER section, as noted in Table 2's Threes and Fives categories. For the Upper strategy, $\frac{288}{10{,}000}$ (2.9%) simulations successfully scored the 35 Bonus points.

Conclusion

The game of Yahtzee provides many opportunities for strategy, involving both probability and luck. While the number of potential strategies might be large, this report tests 2 common strategies. The strategy to focus on the UPPER section is justified by exceeding the 63-point threshold to gain a 35-point bonus. On the other hand, the strategy to focus on Yahtzee can be rewarding due to the high initial score and repeatability. The confidence interval for the Yahtzee strategy has both a higher lower and upper value than the Upper strategy; however, both strategies exhibit a large confidence interval, indicating high uncertainty in estimating the mean. When comparing the primary goals of each strategy, the Upper strategy was successful in 281 trials, while the Yahtzee strategy was successful in 691 trials. Additionally, the 50-point Yahtzee reward holds more value than the 35-point Bonus reward.

With the Yahtzee strategy having more successful trials, a higher-valued reward, and average higher Total score, it has proven to be the preferred strategy.