Trusses

Giacomo Boffi

http://intranet.dica.polimi.it/people/boffi-giacomo

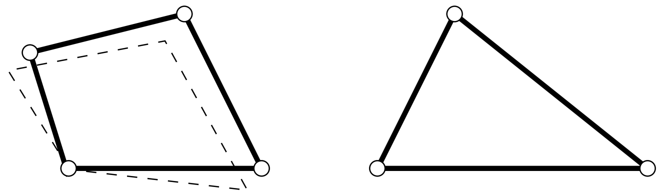
Dipartimento di Ingegneria Civile Ambientale e Territoriale
Politecnico di Milano

October 2, 2018

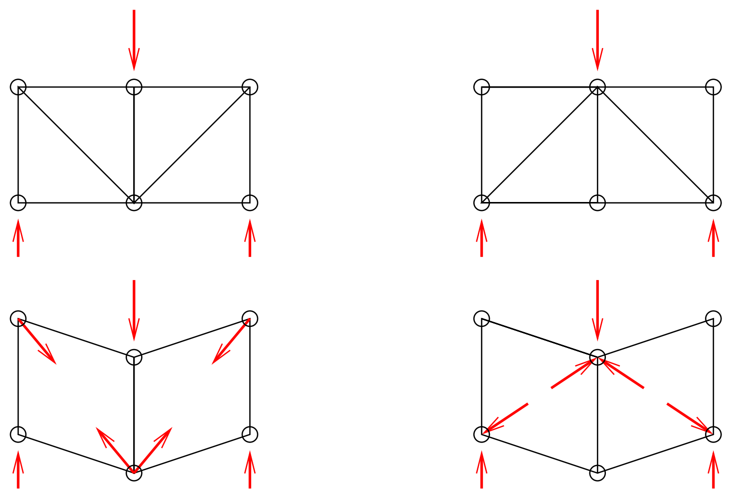
Introduction

Historical remarks

Triangles vs Quadrilaterals



Qualitative Analysis



Stability of Planar Trusses

$$2n = r + b$$

n number of nodes
r number of independent reaction components
b number of bars

Member Forces

- the truss must be in equilibrium
- *every* part of a truss must be in equilibrium
- we are free to choose the part for which we write the equations of equilibrium

Equilibrium of Joints

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Stability of Planar Trusses
Member Forces
Equilibrium of Joints
Equilibrium of Sections
Moment and Shear
Statically Indeterminate Trusses
Spatial Trusses
Joint Rigidity
Computer Aided Methods
Design of Trusses

Each joint must obey just two equations of equilibrium.

$$\sum F_x = 0, \qquad \sum F_y = 0.$$

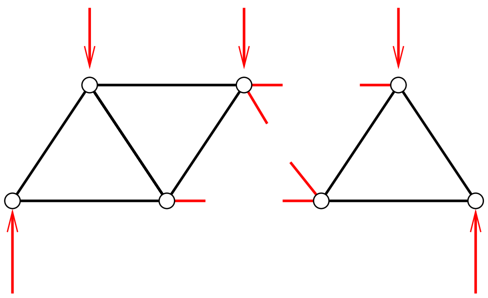
If we can single out a joint for which only two forces are unknown, we can determine the force values.

The equation of stability $2n = r + b$ means also that the number of unknowns $(r + b)$ must be equal to the number of independent equations that we can write $(2n)$.

Equilibrium of Sections

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Stability of Planar Trusses
Member Forces
Equilibrium of Joints
Equilibrium of Sections
Moment and Shear
Statically Indeterminate Trusses
Spatial Trusses
Joint Rigidity
Computer Aided Methods
Design of Trusses

If we cut a truss so that only three forces are unknown, we can write 3 equations of equilibrium to determine their values.



Moment and Shear

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Stability of Planar Trusses
Member Forces
Equilibrium of Joints
Equilibrium of Sections
Moment and Shear
Statically Indeterminate Trusses
Spatial Trusses
Joint Rigidity
Computer Aided Methods
Design of Trusses

Considering a section, you can see that the function of the forces in the chords is to equilibrate bending moment, and the function of diagonals is to equilibrate shear.

Drawing the diagrams of shear and moment for the truss considered as a beam gives you an insight on the forces that are requested in members and the required strength.

Indeterminate Trusses

What happens if

$$r + b > 2n \qquad ?$$

We have more unknowns (forces) than equations to determine them.
We can choose *arbitrarily* $r + b - 2n$ values for the forces and determine the other forces using the remaining equations.
Is this possible?
NO, because the additional restraints means that not every deformation is possible.
We have to choose the *hyper-static* forces so that the additional conditions on deformations are respected.

Spatial Trusses

The basic block is based on triangles and it is the *tetrahedron*.

For joints, we have to write 3 equations of equilibrium, hence a statically determined system must satisfy the condition

$$r + b = 3n.$$

For a section, we have at our disposal 6 equations of equilibrium.

Joint Rigidity

In many cases the chords can be made using a continuous piece of material (a steel beam, a wood member, etc) and it is inappropriate to section it so that we can connect the parts using a pinned connection.

This implies that the joint can develop a bending moment — these bending moments are usually small w/r to the bending moment resistance of members designed to resist the axial loads and can be disregarded in a preliminary design.

Computer Aided Methods

OpenSees (`opensees.berkeley.edu`)

Objectives

- Structural Efficiency
- Construction Effectiveness

Objectives

- Overall shape
- internal triangulation
- choice of material
- length of compressed members
- spacing of trusses
- position of nodes
- context

Configuration

Context often dictates the overall shape of the truss

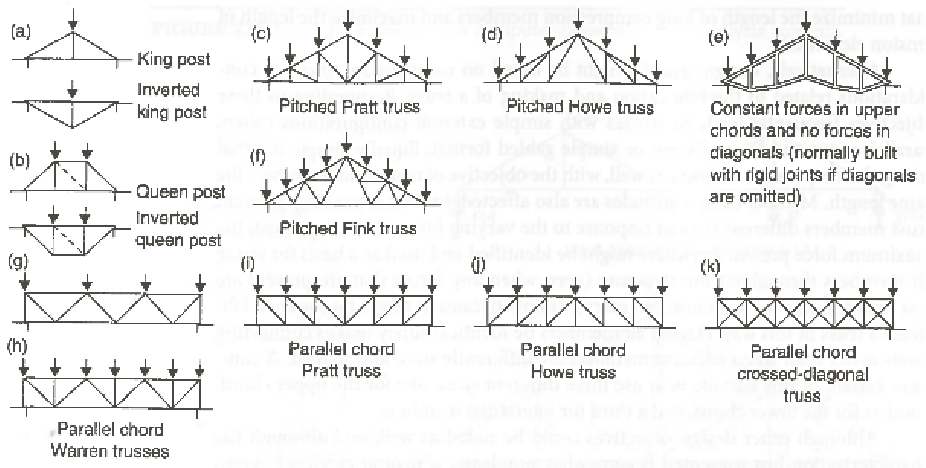
- roofs
- sheds
- availability of vertical space

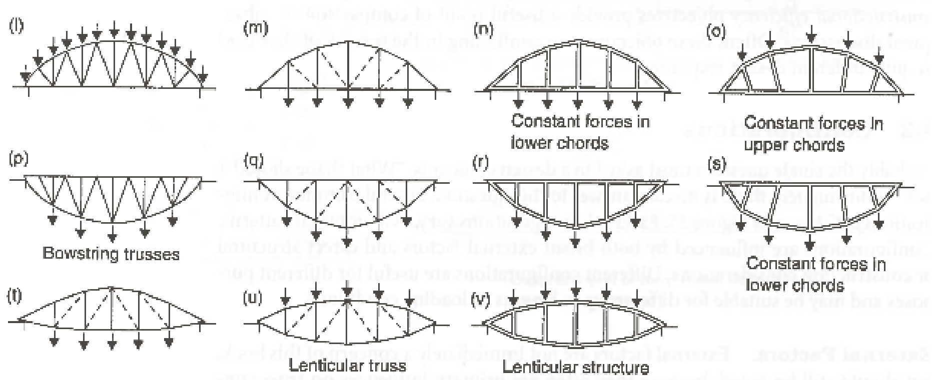
Configuration

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Design of Trusses
Objectives
Configuration
Depth of Trusses
Member Design

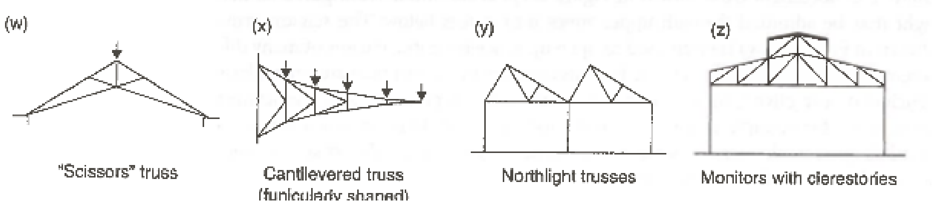
Basic form are to be found in e.g., "Structures" at page 150.

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Design of Trusses
Objectives
Configuration
Depth of Trusses
Member Design





Funicularly shaped trusses: depths vary with bending moment (horizontal components of chord forces are equal and diagonals are zero-force members under design loadings). Shaped trusses with no diagonals are built with rigid joints to handle varying loadings (see Chapter 9 on rigid-frame structures).



Configuration

Parallel Chord Trusses
are very common, they are typically designed as beams with continuous chords designed to resist the maximum moment.

Different diagonal configurations can be chosen, taking into consideration the need for reducing the length of compressed members, subjected to lateral instability.

Configuration

Funicular-Like Shaped Trusses
have s varying depth, chosen so that the loaded chord act as an arch and all the diagonal elements act only in terms of stiffening the compressed members and giving resistance to loads different from the loads that lead to design.

Depth of Trusses

As needed...

in general you can say that deep trusses, deep w/r to their span, are more efficient than relatively shallow ones.

- lightly loaded, closely spaced trusses $d/L \approx 1/20$
- trusses carrying secondary beams $d/L \approx 1/10$
- trusses carrying e.g., columns near the ground level $d/L \approx 1/4$ or the maximum permitted by the storey height.

Critical Loading

Different loading conditions (e.g., snow vs wind) determine *very different* actions in the members of the truss.
Our task is to identify, for each member, the critical loading.

Member Design

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Design of Trusses
Objectives
Configuration
Depth of Trusses
Member Design
Critical Loading
Member Design
Lateral Buckling

The individual member must be designed in compliance with a code that applies to the construction type and material used.
In general, you have to verify the *strength* of each member, the *connections* within members and (foremost?) the lateral stability of compressed members.

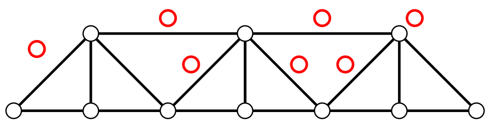
Lateral stability means that, for the same compressive axial load, a longer member requires a larger section to resist buckling.

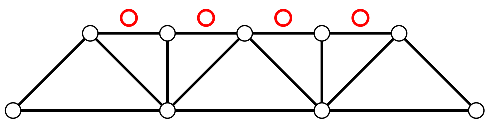
Another approach is to reduce the length of compressed members...

Lateral Buckling

Trusses
Giacomo Boffi
Introduction
Principles
Analysis of Trusses
Design of Trusses
Objectives
Configuration
Depth of Trusses
Member Design
Critical Loading
Member Design
Lateral Buckling

reducing the length of compressed members is a good idea: change the diagonal pattern.





Also the lateral bracing is important.

A 3D truss has the potential for better control of buckling.