Dynamics of Structures 2012-2013

Summer Assignment, due by Friday, 30 August 2013.

Contents

1 SDOF model of a continuous system 1

2 Vibration Isolation 2

3 Generalized Coordinates 3

4 Rayleigh Quotient 3

5 Derived Ritz Vectors 4

6 2 DOF System 5

Instructions

The maximum scores for each one of the six problems are respectively (and approximately) 15, 25, 15, 20, 20 and 25 (120 points total).

For each one of the six problems **a:** copy the text of the problem, **b:** summarize the procedure you'll be using, **c:** write down all the relevant steps (showing parts of the intermediate numerical results as you see fit), **d:** clearly state the required answers.

Please submit your homework by email (giacomo.boffi@polimi.it) by Friday 30 August as a PDF[1] attachment. If necessary you can submit spreadsheets, program sources etc in *separate* attachments in the same email.

You can (and should) discuss the problems with your colleagues, but your paper **must** be strictly the result of your individual effort.

1 SDOF model of a continuous system

A reinforced concrete ($E = 30\,\text{GPa}$, $\rho = 2500\,\text{kg}\,\text{m}^{-3}$) chimney is a hollow, truncated cone with the following geometrical characteristics: (a) total height $H = 48\,\text{m}$, (b) constant thickness $t = 0.250\,\text{m}$, (c) base external radius $R_0 = 2.25\,\text{m}$, (d) top external radius $R_\text{H} = 1.75\,\text{m}$.

[1] Please check that all the needed fonts are included in the PDF file before sending. If you use Word, http://www.bc.edu/content/dam/files/libraries/pdf/embed-fonts.pdf is a good reference.

1. Using an appropriate shape function and computing the required integrals using the Simpson rule with 8 intervals, find the Rayleigh quotient's approximation to the fundamental frequency of vibration of the tower.

An annular platform, its weight being 30% of the weight of the chimney, is installed at height $H_\mathrm{p} = 42\,\mathrm{m}$.

2. Proceding as above, compute the new fundamental frequency.

3. Repeat, taking into account the geometrical stiffness.

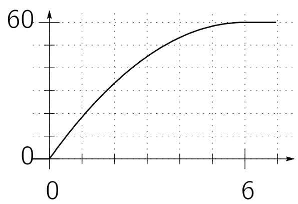
2 Vibration Isolation

An industrial machine has a mass $m_0 = 120\,000\,\mathrm{kg}$ and transmits to its rigid supports a vertical harmonic force of amplitude $1.2\,\mathrm{kN}$ at $60\,\mathrm{Hz}$ when it reaches the steady state regime.

1. Determine the stiffness k_susp of an undamped, elastic suspension system for the machine, so that the amplitude of the s-s transmitted force is no greater than $300\,\mathrm{N}$.

The transient has a duration t_0, with $t_0 = 6\,\mathrm{s}$, and the rotational velocity of the unbalanced mass, expressed in Hz, varies according to the following equation:

$$f(t) = 60\,\mathrm{Hz} \times \begin{cases} \left(2 - \frac{t}{t_0}\right)\frac{t}{t_0} & 0 \le t \le t_0, \\ 1 & t_0 \le t. \end{cases}$$



2. determine the maximum transmitted force during the transient response of the undamped suspension,

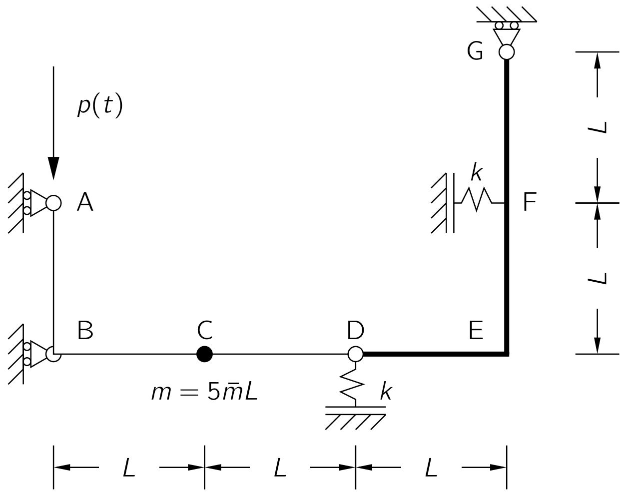
3. design two alternative suspension systems, with assigned damping ratio equal to 2% and 10% and the same TR, and find the maximum transmitted force during the transient for each one of the damped suspension systems,

4. comment your results (max 5 lines).

Tip Do you remember that the unbalanced forces in a rotating system are proportional to the square of the angular velocity?

You may write, with sufficient accuracy, $p(t) = \text{factor} \times \omega^2(t)\ \sin(\omega(t)t)$.

3 Generalized Coordinates (rigid bodies)



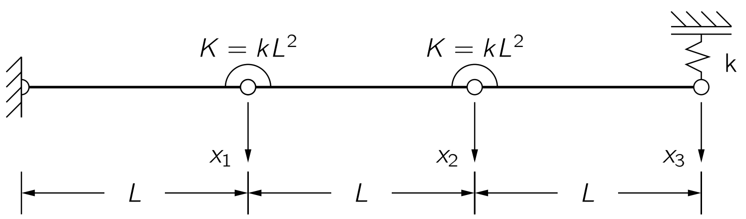
The structure in figure is an articulated SDOF system composed by

- two rigid bars, (1) ABCD and (2) DEFG,
- three fixed external constraints, (1) a vertical roller in A, (2) a vertical roller in B and (3) a horizontal roller in G,
- one internal constraint, a hinge in D
- two deformable external constraints, (1) a vertical spring in D, its stiffness k, (2) a horizontal spring in E, its stiffness k.

The bar DEFG has a unit mass $\bar{m}$, the bar ABCD is massless and supports, in C, a body of negligible dimensions, whose mass is $m = 5\bar{m}L$.
Using v_A (the vertical displacement of A) as the generalized coordinate

1. compute the generalized parameters m^* and k^*,
2. compute the generalized loading $p^\star(t)$ considering a vertical force applied in A, $p(t) = p_0 \sin \omega t$. and
3. write the equation of dynamic equilibrium.

4 Rayleigh Quotient



The system in figure is composed by 3 rigid bars of constant unit mass, $\bar{m}$, and of equal lenght L.

Starting with a trial shape $\boldsymbol{\phi} = \begin{Bmatrix} 2 & 2 & 1 \end{Bmatrix}^T$ (i.e., $\boldsymbol{x} = Z_0 \boldsymbol{\phi} \sin \omega t$) give the successive Rayleigh estimates R_{00}, R_{01} and R_{11} of ω^2, the natural frequency of vibration of the system.

This problem is more conveniently approached if you determine the structural matrices of the system.

The deformation energy V can be expressed in terms of the *relative* rotations across the springs and also in terms of the stiffness matrix $\boldsymbol{K}$ coefficients k_{ij} and the nodal displacements:

$$V = \frac{1}{2}\sum_{n=1}^{2} K_n\theta_n^2 + \frac{1}{2}kx_3^2, \qquad V = \frac{1}{2}\sum_i\sum_j k_{ij}x_ix_j.$$

where each θ_n is the *relative* rotation across the spring number n, $n = 1, \ldots, 5$. The relative rotations depend on the nodal displacements, $\theta_n = \theta_n(\boldsymbol{x})$ and, for example, it is

$$\theta_2 = \frac{x_3 - x_2}{L} - \frac{x_2 - x_1}{L} = \frac{x_3 - 2x_2 + x_1}{L}.$$

In the same manner, equating an explicit expression of the kinetic energy, in terms of the velocities of the centres of mass of the rods and their angular velocities, to its matrix formulation ($T = 1/2\ \sum_i\sum_j m_{ij}\dot{x}_i\dot{x}_j$) you can determine the coefficients of the mass matrix.

5 Derived Ritz Vectors

A 48 DOF structural model is described by the mass matrix M and the stiffness matrix, $\boldsymbol{K}$ that you can download following the above links.

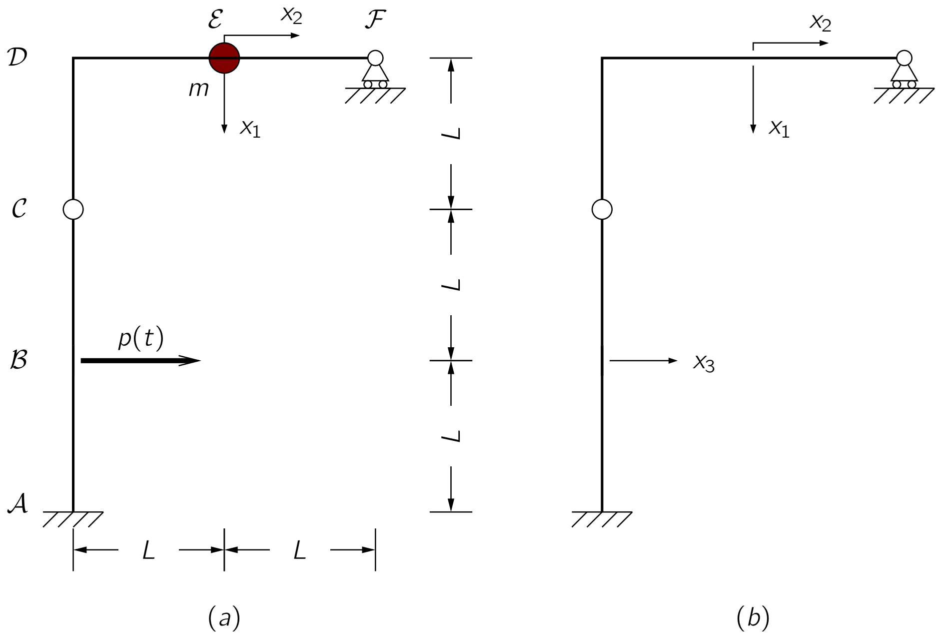
1. Compute the first 3 Derived Ritz Vectors using an initial load vector proportional to the mass distribution,
2. compute the first 3 eigenvalues and eigenvectors of the model, using only one iteration of the Rayleigh-Ritz procedure and using the DRVs as the initial Ritz base,
3. compute the same eigenvalues and eigenvectors using a library procedure and compare with your previous results.

Tips

1. The matrices at `ftp://math.nist.gov/pub/MatrixMarket2/Harwell-Boeing/bcsstruc1/bcsstm01.mtx.gz` and `ftp://math.nist.gov/pub/MatrixMarket2/Harwell-Boeing/bcsstruc1/bcsstk01.mtx.gz` are stored in the so called Matrix Market format (see `http://math.nist.gov/MatrixMarket/formats.html`).

   Note that the files are, after decompression, ASCII text files that you can open with any text editor (notepad, vim, emacs, whatever).
2. The same page provides links to implementations in different programming languages of functions to read this format. In particular it provides a Matlab function, `http://math.nist.gov/MatrixMarket/mmio/matlab/mmread.m` that you can use to get the structural matrices into your script.
3. Had you any problem, remember that you can open the files in an editor and that the format itself is, in our simple case, simple as well: after a header, there are lines of three numbers, (a) the row number i, (b) the column number j, (c) the value, $a_{ij} = a_{ji}$ (note that off diagonal terms are given only once).
4. The mass matrix is not definite positive, it is possible that you have to perform a static condensation to get some of the requested results.

6 2 DOF System



In figure (a) it is shown a 2 DOF model of a structure composed of two uniform beams (flexural stiffness EJ, unit mass $\bar{m}$) that supports a body of mass m.

> The suspended mass being much greater than the total mass of the beams, $m \gg 5\bar{m}L$, you can consider $\bar{m} \approx 0$ and use the 2 DOF model of figure (a).

In figure (b) it is shown a different model of the same structure, where an additional, non dynamical degree of freedom has been introduced.

Using the 2 DOF model of figure (a) **1:** compute the structural matrices $\boldsymbol{M}$ and $\boldsymbol{K}$, **2:** compute the eigenvalues and the normalized eigenvectors of the system.

The system, initially at rest, is then subjected to a horizontal force applied in $\mathcal{B}$

$$p(t) = p_o \cos(3\omega_0) \qquad \text{with } \omega_0^2 = \frac{EJ}{mL^3}.$$

3: write the modal equations of motion and find the modal responses, **4:** plot the displacement component $x_2(t)$ in the interval $0 \le \omega_0 t \le 10$, **5:** with x_3 as defined in figure (b), plot $x_3(t)$ in the same interval.

Tip Do you remember *static condensation*? The stiffness matrix of the structural model in figure (b), where a non-dynamical degree of freedom has been introduced, is

$$\boldsymbol{K}_{3\times 3} = \frac{3}{29}\frac{EJ}{L^3}\begin{bmatrix} +76 & -12 & +30 \\ -12 & +08 & -20 \\ +30 & -20 & +79 \end{bmatrix}.$$