# Dynamics of Structures 2012-2013

1st home assignment due on Tuesday 2013-05-28

## Contents



## Instructions

For each of the *six problems* copy the text of the problem, summarize the procedure you'll be using, detail all relevant steps including part of intermediate numerical results as you see fit, *clearly state* the required answers.

The maximum scores for each exercise are respectively (and approximately) 8, 11, 20, 16, 20 and 25, for a total of 100 points for a *perfect* submission.

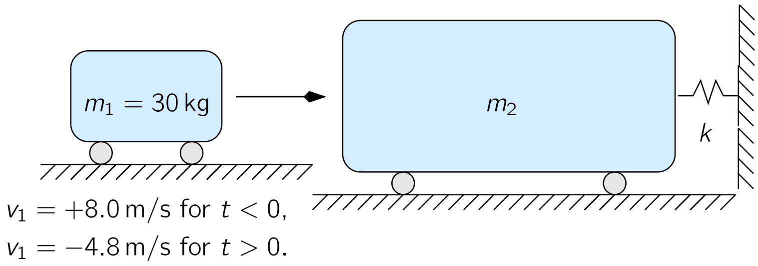
Please submit your homework by e-mail before 14:30, as a PDF[1] attachment. The spreadsheets and/or the sources of the programs that you used for computing your solutions should be sent as separate attachments in the same e-mail.

As an alternative to e-mail submission, you can give me a printed (or *nicely* handwritten) copy of your homework before the class of May 28th.

While you can (and should) discuss the problems with your colleagues, your paper **must** be strictly the result of your individual effort or it will be completely disregarded.

[1]Please check that all the needed fonts are included in the PDF file before sending. If you use Word, http://www.bc.edu/content/dam/files/libraries/pdf/embed-fonts.pdf is a good reference.

# 1 Impact (elastic rebound)




A free body, mass $m_1 = 30\,\mathrm{kg}$ and velocity $v = 8\,\mathrm{m\,s^{-1}}$, impacts an undamped SDOF system, mass $m_2$ and stiffness $k$.

The impact is, hypothetically, a perfect elastic impact, meaning that *also* the energy is conserved during the impact and that the duration of the contact is infinitesimal.

After the impact the free body has a negative velocity, $v = -4.8\,\mathrm{m}$ and the amplitude of the harmonic motion of the SDOF is $x_{2,\mathrm{max}} = 32\,\mathrm{mm}$.

Determine the mass and the stiffness of the SDOF.

# 2 Dynamical Testing

You want to determine the mass $m$, the stiffness $k$ and the damping ratio $\zeta$ of a small, one storey building that can be modeled as a single degree of freedom system.

A series of 4 dynamical test is performed, loading the building with a vibrodyne and measuring the amplitude $\rho$ and the phase difference $\theta$ of the steady state motion (note that the number of measurements taken is greater than what is strictly requested, as it is recognized that there are sources of uncertainties in the experimental setup).

In each test the load amplitude is $p_0 = 1600\,\mathrm{N}$, while the excitation frequencies $\omega_n$ (with $n = 1, \ldots, 4$) are different.

The measurements are summarized in the following table

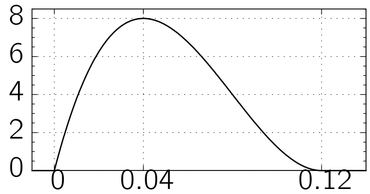
| $n$ | $f_n/1\mathrm{Hz}$ | $\rho_n/1\mu\mathrm{m}$ | $\theta_n/1^\circ$ |
|---|---|---|---|
| 1 | 6.0 | 44.29 | 12.0 |
| 2 | 7.0 | 93.44 | 30.7 |
| 3 | 8.0 | 119.25 | 131.6 |
| 4 | 9.0 | 42.54 | 162.4 |

Give your best estimate of $m$, $\zeta$ and $k$.

## 3 Numerical Integration

A single degree of freedom system, with a mass $m = 3\,\mathrm{kg}$, a stiffness $k = 1200\,\mathrm{N\,m^{-1}}$ and a damping ratio $\zeta = 0.10$ is at rest when it is subjected to an external force $p(t)$:

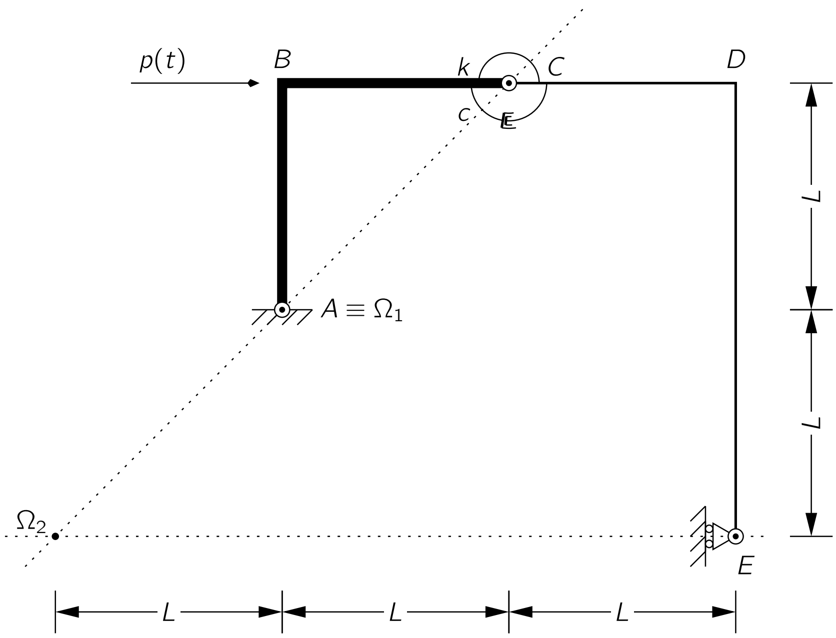
$$p(t) = \begin{cases} p_0 \dfrac{(t_0 - t)^2 t}{t_0^3} & \text{for } 0.0 \le t \le t_0, \\ 0.0 & \text{otherwise.} \end{cases}$$




where $t_0 = 0.12\,\mathrm{s}$ and $p_0 = 54\,\mathrm{N}$.

(1) Give the analytical solution of the equation of motion in the interval $0 \le t \le 10t_0$. (2) Integrate the equation of motion numerically, using the algorithm of constant acceleration with a time step $h = t_0/30$ in the same interval. (3) Plot your results (both the exact response and the numerical solution) in a meaningful manner. (4) [Optional] Repeat the numerical integration assuming an elasto-plastic spring with a yield strength $f_y = 6\,\mathrm{N}$ and plot your results.

## 4 Generalized Coordinates (rigid bodies)




The articulated system in figure, composed by

- two rigid bars, (1) ABC and (2) CDE,

- three kinematical constraints, (1) a hinge in A, (2) an internal hinge in C and (3) a vertical roller in E,
- two deformable constraints, (1) a rotational spring[2] in C, its stiffness $k$, (2) a rotational dashpot[3] in E, its damping coefficient $c$,

is excited by a time varying horizontal force applied in D, $p(t)$

Considering that the bar ABC has a constant unit mass $\overline{m}$, with $\overline{m}\,L = m$ while all the other parts of the system are massless, and using $v_{\mathrm{E}}$ (the vertical displacement of the roller initially in E) as the generalized coordinate

1. compute the generalized parameters $m^*$, $c^*$ and $k^*$,
2. compute the generalized loading $p^*(t)$ and
3. write the equation of dynamic equilibrium.

# 5 Rayleigh quotient

A straight, uniform beam of length $L$ is clamped at $x = 0$ and is simply supported at $x = L$. It is possible to approximate its free vibrations using a shape function $\psi(x)$,

$$v(x, t) = Z_0\,\psi(x)\,\sin(\omega t), \qquad 0 \le x \le L.$$

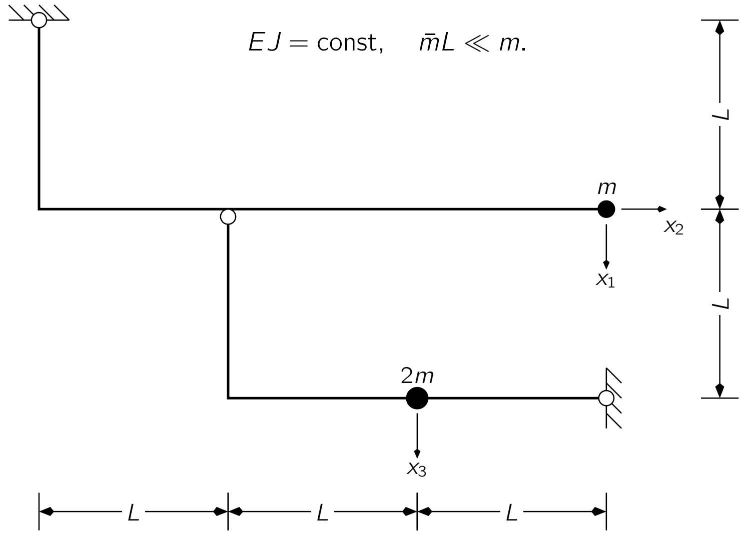
Using a polynomial shape function $\psi(x) = \sum_{i=0}^{n} a_i (x/L)^i$ and imposing the kinematical conditions at $x = 0$ ($\psi(0) = 0$, $\psi'(0) = 0$), you'll find that the constant term and the linear term in the polynomial must be equal to zero.

1. Using a shape function $\psi(x) = \xi^2 + a\xi^3$, $\xi = {}^{x}/_{L}$, such that $\psi(L) = 0$ estimate, by the means of Rayleigh quotient, $\omega^2$.
2. Using a shape function $\psi(x) = \xi^2 + a\xi^3 + b\xi^4$ such that $\psi(L) = 0$ and $M(L) = 0$, compute a new estimate of $\omega^2$.
3. [Optional] Using a shape function $\psi(x) = \xi^2 + a\xi^3 + b\xi^4 + c\xi^5$ you can obey all the constraints and still have at your disposal a free parameter. Find the minimum value of the Rayleigh quotient computed as a function of the free parameter.

[2] A rotational spring gives, on each connected beam, a couple proportional and opposite to the *relative* rotation between the beams. Note that indicating with $\phi_{AB}$ the rotation of beam $A$ relative to beam $B$, it is $\phi_{AB} = -\phi_{BA}$.

[3] Same as a rotational spring, but the couple is proportional to the time derivative of the relative rotation.

# 6 3 DOF System




A three hinged arch supports two different bodies of negligible dimensions, whose total mass is much greater than the mass of the structure. Axial and shear deformations can be neglected.

1. Discuss the choice of the dynamical degrees of freedom given in figure.

With the positions $\omega_0^2 = {}^k\!/\!_m$ and $k = {}^{EJ}\!/\!_{L^3}$,

2. compute the three eigenvalues of the system and the corresponding eigenvectors, normalizing the eigenvectors with respect to the mass matrix $\boldsymbol{M}$ ($\boldsymbol{\Psi}$ being the eigenvectors' matrix, it must be $\boldsymbol{\Psi}^T\boldsymbol{M}\boldsymbol{\Psi} = m\boldsymbol{I}$).

Considering that the system is at rest when $t = 0$ and is then loaded by

$$\boldsymbol{p}(t) = \frac{kL}{2000}\begin{Bmatrix}0\\1\\0\end{Bmatrix}\sin(\frac{\omega_0}{6}t),$$

(3) write the three *modal* equations of motion, (4) integrate the modal equations of motion and write the three equations of modal displacement, $q_i = q_i(t)$ (you should be able to write your results in terms of the unit length $L$), (5) find the analytical expression of $u_3 = u_3(t)$, showing your intermediate results and (6) plot $u_3$ in the interval $0 \le \omega_0\, t \le 10$.

**Hint**

$$\boldsymbol{F} = \boldsymbol{K}^{-1} = \frac{L^3}{EJ}\frac{1}{6}\begin{bmatrix}408 & -98 & 53\\ -98 & \cdots & -13\\ 53 & -13 & \cdots\end{bmatrix}, \qquad \boldsymbol{K} = \frac{EJ}{L^3}\frac{3}{200}\begin{bmatrix}11 & 19 & -42\\ 19 & \cdots & 22\\ -42 & 22 & \cdots\end{bmatrix}.$$