Beams

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October 26, 2018

Introduction

Definition:

- a prevalent dimension (the span),
- loads act perpendicular to the prevalent dimension, so no axial forces but shear and bending moment,
- the cross section dimensions are the depth (parallel to loading) and the width.

We restrict our attention to rectilinear beams but curvilinear beams are possible as well.

Historical remarks:

- Galileo Galilei,
- Coulomb,
- De Saint Venant.

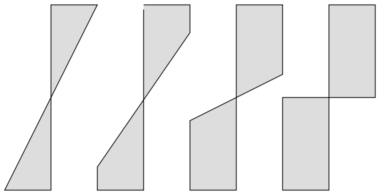
Steel Beams

The design of a steel beam in most cases coincides with choosing a section in a catalog of commonly available steel members.
The section you are going to choose has to satisfy two essential requirements in terms of strength and deformability.

- Strength: the section must be strong enough to resist the maximum value of the bending moment,
$$f_y W \geq M_{\text{max}} \quad \rightarrow \quad W \geq M_{\text{max}}/f_y.$$
- Deformability: the maximum deflection must be less than a fraction of the beam span, $\Delta \leq L/k$.
  The value of k depends on the load cases used to compute the deflection (live vs dead vs combination) and the function that the beam has to perform (support a roof, a floor, a wall, etc).
  E.g., for a simply supported beam subjected to a distributed load, supporting a wall
$$\Delta = \frac{5wL^4}{384EJ} \leq \frac{L}{200} \quad \rightarrow \quad J \geq \frac{1000wL^3}{384EJ}.$$

Steel beams, plastic behavior

Steel is an elastic-plastic material, if the strain exceeds a certain amount the normal stress remains constant, $\sigma = f_y$.
What happens when $M = Wf_y$ and the load is increasing?



When the external bending moment increases, the outer fibers are further pulled-pushed and in a larger part of the section the stress is equal to f_y — the part of the section that is still elastic is, OTOH, smaller and smaller and the elastic stiffness of the section is diminishing.
At some point, when in all the section $\sigma = \pm f_y$ the elastic stiffness is equal to 0. In terms of strength we have reached the *plastic moment*, in terms of stiffness we have formed a *plastic hinge*.

Steel beams, plastic behavior

For a rectangular beam $W = bd^2/6$ and $M_y = f_y bd^2/6$ is the maximum elastic moment.
When $\sigma = \pm f_y$ we have $T = C = f_y bd/2$ and their arm is $a = d/2$, hence $M_p = f_y bd^2/4$ is the moment of plasticization, that turns the section into a plastic hinge.
For a rectangular section $M_p/M_y = 1.5$, for different shapes we have different ratios.
For common I shaped sections, the ratio has an average value around 1.12, for a circular bar it's about 1.7 and it's 2.0 for a diamond shaped bar.

Shear

In most cases, when dealing with wide flange beams, T beams, channels it is possible to estimate the design value of the shear strength taking into account only the web area A_w,

$$V_{Rd} = 0.6 f_y A_w.$$

RC Principles

Concrete resists very well compression, resists very poorly tension.
A plain concrete beam when loaded will tend to crack vertically in zones of (relatively) large bending moments and diagonally in zones of high shear and eventually break down for small loadings.
To avoid the crack growth and the eventual failure we dispose some tension resistant material perpendicular to the cracks' directions, usually steel bars, longitudinal steel bars to equilibrate bending moments and vertical stirrups (U-shaped bars) or diagonally bent bars to equilibrate shear forces.

Principles of reinforcement design

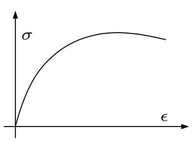
Design of reinforcement, especially the longitudinal reinforcement, is a balancing act.

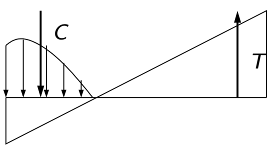
If you provide not enough steel, the section will collapse by breakage of the reinforcement, but also providing too much steel leads to a bad design in which the failure of the beam happens when the compressed concrete is crushed.
Concrete has a degree of ductility when compressed, but when its deformation is excessive it breaks suddenly.

A RC beam correctly designed has the right amount of reinforcement, so that the first yielding happens in the steel and we reach a limit state in which most of the concrete is safely plasticized and the steel deformations are still under control.

In this case a possible failure of a beam is preceded by a visual increase of deflections. Further, the possibility of a plastic behavior leads to an energy dissipation that is useful to control the response to seismic excitation.

RC design at the limit state





$$T = A_s f_y, \qquad C \approx 0.8 x b f_c, \qquad x = \frac{A_s f_y}{0.8 b f_c}$$

To have a correct ductility, a good design choice is to assign appropriate values to the steel and concrete deformations.

RC Member Sizes

Min Overall Depth (U.S. practice)

Member	Simply Supported	H+F	F+F	Cantilever
Slabs	L/20	L/24	L/28	L/10
Beams	L/16	L/18	L/21	L/8

Overall depth h is the reinforcement depth d plus the bottom concrete cover.