Dynamics of Structures 2009-2010

last home assignment

Instructions

This assignment is due no later than Sunday 2011/02/06. You must check in your assignment by email, as a PDF attachment.[1] No Word documents, thank you.

For each of the *six problems* summarize the procedure you'll be using, write down all relevant steps (including some intermediate numerical results as you see fit), *clearly state* the required answers.

Points for each problem are $\approx$ 10 10 15 40 40 35 for exact results and clean developments.

1 Impact

A body, its mass $m_1 = 120\,\text{kg}$, is dropped from an height $h = 2.5\,\text{m}$ above an undamped *SDOF* system. The collision is inelastic (i.e., the two bodies are perfectly bound after the collision).

The amplitude of the ensuing harmonic motion is $x_{\text{max}} = 0.0135\,\text{m}$, while the period of vibration is $T = 0.83\,\text{sec}$

What are the mass m_0 and the stiffness k_0 of the *SDOF* system?

2 Vibration Isolation

A rotating machine is characterized by

— its mass, $m = 92\,000\,\text{kg}$;

— its working frequency, $f_\text{w} = 120\,\text{Hz}$,

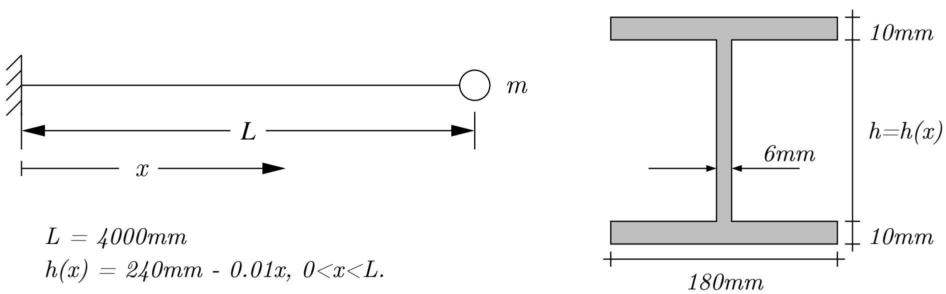
[1] Please check that all needed fonts are included in the file before sending. In doubt, http://en.allexperts.com/q/Microsoft-Word-1058/2009/8/embedding-pdf-file.htm.

— the value of the unbalanced load it exerts on its supports, $f_{\mathrm{w}} = 4200\,\mathrm{N}$.

Design a suspension system for the machine knowing that it is necessary to reduce the transmitted force to 1200 N and that, to reduce the vibration amplitude during transients, the suspension must have a viscous damping ratio of 5%.

3 Generalized Coordinates (flexible systems)

Using Rayleigh's ratio, estimate the natural frequency of vibration of a tapered beam that supports a point mass $m = 10\,000\,\mathrm{kg}$ in each of the following two hypothesis: *a*) the beam mass is negligible with respect to m and *b*) the beam mass is not negligible.



The system and the beam section are described in the figure above. The beam material (low carbon steel) has a Young modulus $E = 206\,\mathrm{GPa}$ and a specific mass $\rho = 7.85\,\mathrm{mg/mm^3}$.

4 Numerical Integration

A single degree of freedom system, with a mass $m = 200\,\mathrm{kg}$, a stiffness $k = 24\,\frac{\mathrm{MN}}{\mathrm{m}}$ and a viscous damping ratio $\zeta = 0.06$ is at rest when it is subjected to an external force $p(t)$:

$$p(t) = \begin{cases} (6.00 - 5.00\,\mathrm{t/s})\,\mathrm{MN} & \text{for } 0.00 \le t \le 0.20\,\mathrm{s}, \\ 0.00 & \text{otherwise.} \end{cases}$$

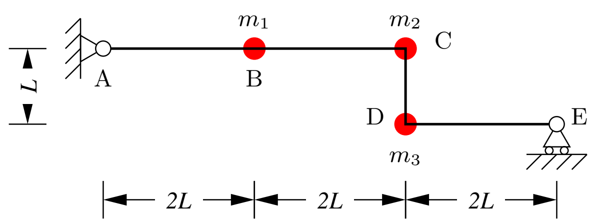
1. Find the exact response in the time interval $0.00 \le t \le 0.30\,\mathrm{s}$ and give the analytical expressions of the equation of motion in the forced and in the free response phases.

2. Integrate the equation of motion numerically using the algorithm of *constant acceleration*, with two different integration steps $h_1 = 10.0\,\mathrm{ms}$ and $h_2 = 0.25\,\mathrm{ms}$, in the time interval $0.0\,\mathrm{s} \le t \le 0.3\,\mathrm{s}$. Report the details of (only) the first 10 steps of the computations.

3. As above, using the *linear acceleration* algorithm.

4. Plot your results in a meaningful manner and comment your results.

5 Multiple *DOF* System



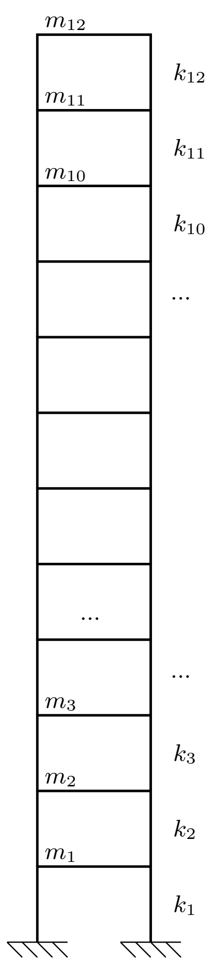
$$m_1 = 4m, \quad m_2 = 3m, \quad m_3 = 3m; \qquad EJ = \text{const.}$$

An uniform, simply supported beam sustains three point masses, the beam mass being negligible with respect to the sustained mass. Consider negligible also the axial and shear deformations of the beam.

1. Write the structural matrices, find the eigenvalues solving the determinantal equation, find the eigenvectors of the system.

2. Examine the behaviour of the system when the support in A is subjected to an assigned vertical motion, $v_A = v_A(t)$.

6 Rayleigh-Ritz, Subspace Iteration, Derived Ritz Vectors



The structure on the left can be analyzed as a shear type building. Individual storey masses are given by

$$m_n = 480\,\mathrm{t} - n \times 5\,\mathrm{t}$$

(in tonnes), while individual storey stiffnesses are

$$k_n = 1200.0\,\frac{\mathrm{MN}}{\mathrm{m}} - (n-1)^2 \times 8.5\,\frac{\mathrm{MN}}{\mathrm{m}},$$

(please remember that individual storey stiffnesses differ from the corresponding diagonal elements of $\boldsymbol{K}$)[a]

1. Estimate the lowest three eigenvalues and eigenvectors of the structure[b] using the Rayleigh-Ritz procedure, denoting the Ritz base with $\hat{\boldsymbol{\Phi}}_0$ and the Ritz coordinates eigenvector matrix with $\boldsymbol{Z}$.

2. Do one subspace iteration, deriving a new set of Ritz base vectors, $\hat{\boldsymbol{\Phi}}_1 = \boldsymbol{K}^{-1}\boldsymbol{M}\boldsymbol{\Phi}\boldsymbol{Z}_0$.

3. Estimate the lowest three eigenpairs using the upgraded Ritz base $\hat{\boldsymbol{\Phi}}_1$ and give a graphical representation of the eigenvectors.

4. Find the first three Derived Ritz Vectors from a loading proportional to the mass distribution and compare them with the eigenvectors that you have just computed.

[a] For example, the stiffness matrix element $k_{9,9}$ is given by

$$k_9 + k_{10} = 656.0\,\frac{\mathrm{MN}}{\mathrm{m}} + 511.5\,\frac{\mathrm{MN}}{\mathrm{m}} = 1167.5\,\frac{\mathrm{MN}}{\mathrm{m}}.$$

[b] The eigenvectors $\boldsymbol{\psi}_i$ of the structure are different from the eigenvectors in Ritz coordinates $\boldsymbol{z}_i$.