Home Assignment

20-05-2010
due 03-06-2010

For every problem, copy the text of the problem, give a BRIEF sumary of the procedure(s) you'll use to solve the problem, detail relevant steps in your procedure, giving also SOME intermediate numerical result and clearly state the required answers.

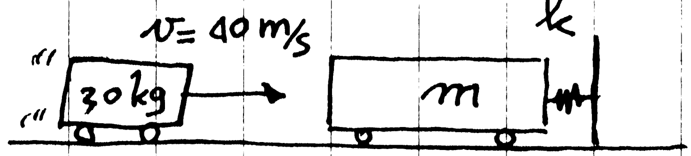
Your manuscripts will be collected after the class of 03-06-2010, ~ 18:00.

You may choose the format of your submissions: electronic (PDF only), printed or handwritten.(*)

(*) Nicely handwritten, that is... also, pretty please <u>do not</u> email a scan or a photo(**) of your manuscript.

(**) Already happened!

IMPACT



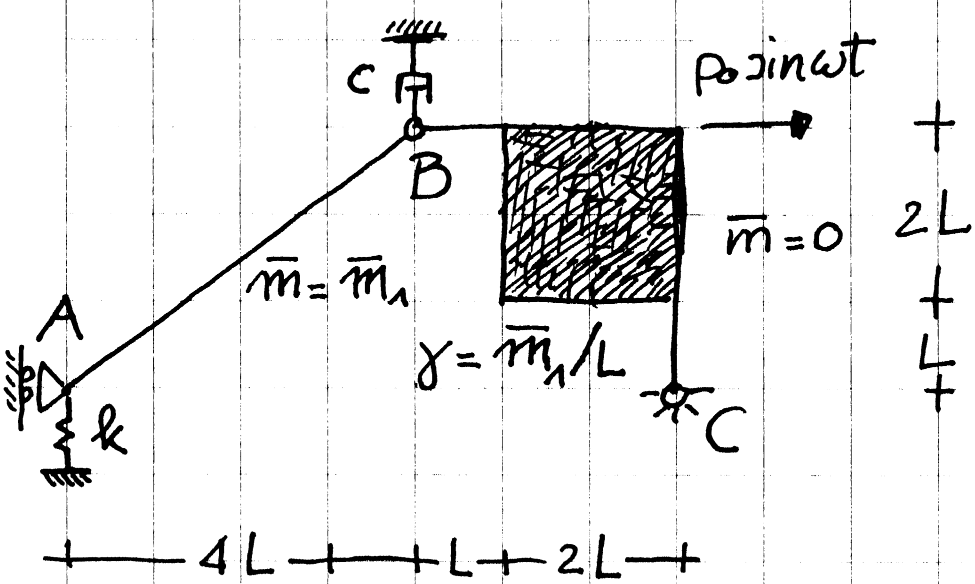
A mass $m_0 = 40$ kg hits a SDOF system with unknown mass m and unknown stiffness k.

After collision, the two masses are "glued" together, and the maximum displacement is measured as $x_{max} = 48$ mm, while the maximum velocity is measured as $\dot{x}_{max} = 240$ mm/s.

Compute the natural frequency of the «ORIGINAL» SDOF system.

GENERALIZED COORDINATES

RIGID SDOF SYSTEM



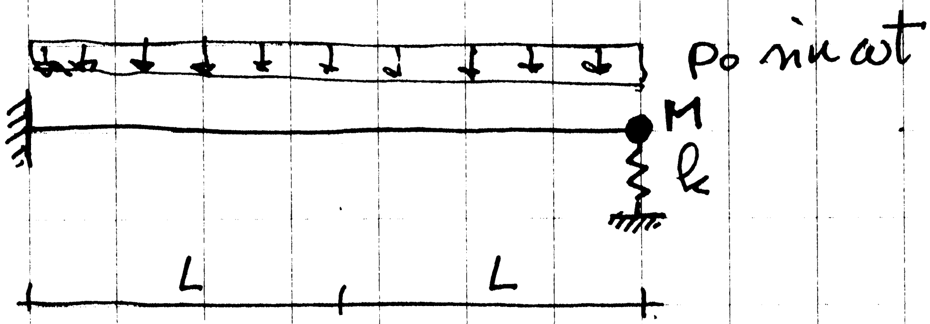
The system above is composed of two rigid bodies, hinged to each other in B, $\overline{AB}$ has unit mass $\overline{m} = \overline{m}_1$, const, while $\overline{BC}$ is massless, but is rigidly connected to a rigid square body, with unit mass $\gamma = \overline{m}_1/L$, const.

The motion of the SDOF is constrained by a spring (stiffness k) in A and a damper (damping coefficient c).

WRITE THE EQ. OF MOTION USING GENERALIZED COORDINATE.

GENERALIZED COORDINATES

FLEXIBLE SYSTEMS



The beam in figure is clamped at the left and is supported by a spring

$$k = EJ/40L^3$$

at the right.

The elastic stiffness and unit mass, EJ and $\bar{m}$, are constant.

At the right end, the beam supports a point mass

$$M = 12\,\bar{m}L$$

Using an adequate shape function, write the equation of motion.

- Numerical Integration -

A SDOF is characterized by

$k = 32000$ N/m,

$m = 1800$ kg,

$\xi = 7\%$,

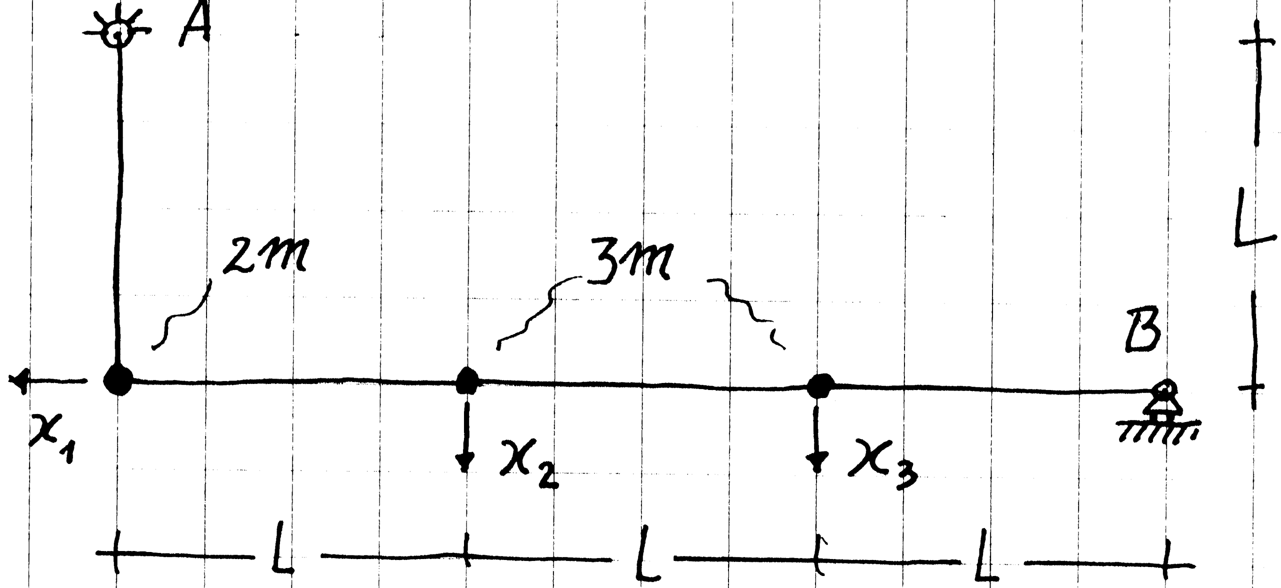
$f_y = 2.5$ kN,

and is subjected to a load

$$p(t) = 3\,\text{kN} \cdot \begin{cases} t + 12t^2 - 64t^3 & 0 \le t \le 0.25\,s \\ 0 & \text{otherwise} \end{cases}$$

Disregarding non-linear behaviour, give the equation of motion $x(t)$ for rest initial conditions, and compute the response with central differences, constant acceleration and linear acceleration with $h = 0.005$ s. Plot and compare the results with exact solution. OPTIONAL Repeat considering non-linear behaviour

MDOF SYSTEM



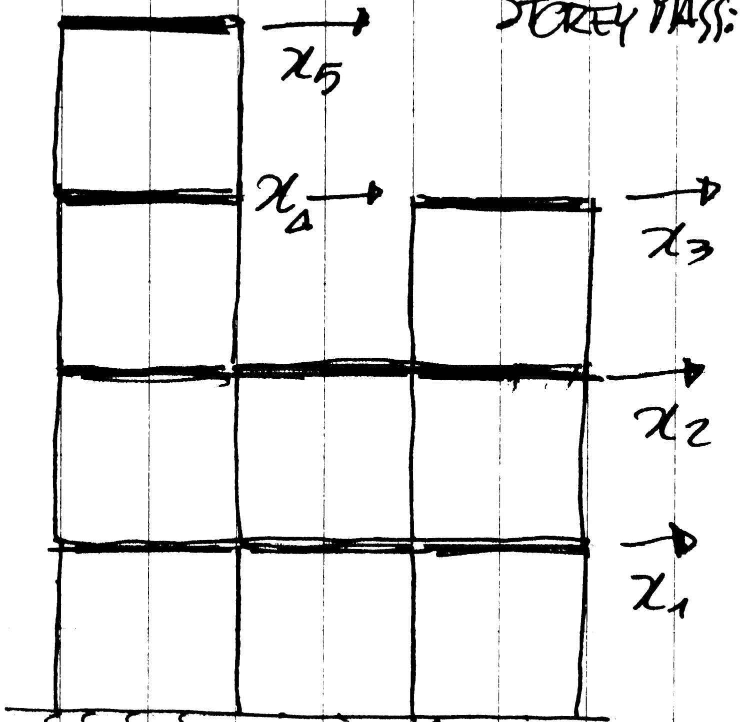
$EJ = \text{const}$ $\qquad \bar{m} = 0$

$$EJ/mL^3 = 2500 \left(\frac{rad}{s}\right)^2$$

The beam mass is negligible with respect to the supported masses.

1. Using the 3 DOF evidenced in the figure, compute $\mathbf{F}$ (disregarding axial deformability).
2. Compute all the eigenvalues and eigenvectors of the 3 DOF system
3. Compute the rotations in A during free vibrations due to $\dot{x}_0 = \{1, 0, 0\}^T \frac{L}{2500\,\text{seconds}}$

RAYLEIGH–RITZ ∘ SUBSPACE ITERATION



STOREY MASS: $m_1 = m_2 = 3m$

$m_3 = m_4 = m_5 = m$

STOREY STIFFNESS:

1) $4k$
2) $4k$
3) $2k$
4) $2k$
5) $2k$

NB: $\underline{K}$ is not diagonal.

$$\underline{\underline{\Phi}}^T = \begin{bmatrix} 1 & 2 & 3 & 3 & 4 \\ 1 & 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}$$

Find the first 3 eigen- using the Ritz base on the left. collect Ritz eigenv. in $\underline{\underline{Z}}_0$

Do one subspace iteration

$$\underline{\underline{\Phi}}_1 = \underline{K}^{-1} \underline{M} \, \underline{\underline{\Phi}} \, \underline{\underline{Z}}_0$$

and compute $\underline{\underline{Z}}_1$ for the new Ritz base $\underline{\underline{\Phi}}_1$.

Compare the results in terms of eigenvalues.