

## Chapter 2

# Practical Work

### 2.1 Technical details

Our implementation of the algorithms has taken into account the need for an efficient and fast testing system. Testing has been done on only one byte of plaintext, in ECB mode, since this is generally the fastest mode of operation. We have used an OOP approach in order to provide a flexible, modular and easy to extend system. Our initial attempts to use other modes of operation seemed impractical at the time in terms of the duration of the tests.

**Table creation** For table creation, in order to avoid duplicate chains we always test for the uniqueness of the starting points. In order to do this effectively, we have used a binary search tree that before generating any chain verifies if the starting point has not already been used. We also establish an explicit bijection between the start and end points. After the chain generation is complete, before writing the table on the disk we efficiently sort the endpoints. Writing to the table now is only possible because we have a bijection between the start and end points.

**Comparison with the endpoints** Since our tables are now sorted according to the endpoints, we make the comparison between our intermediate key and the endpoints using binary and exponential search, to perform a maximum of  $O(n \log(n))$  operations. Other small improvements are that instead of performing string comparison we compare the memory buffers. Also, we make extensive use of the heap memory to store our data since it removes the need for copying the same values repeatedly.

#### 2.1.1 DES Exhaustive Key Search

This implementation of the DES exhaustive key search starts with a counter set to 0. The counter consists of 32 bits that are distributed amongst the last 5 bytes of the key currently tested. Therefore the running time of the algorithm depends on the position of the most significant bit that has the value 1. It takes about 200 minutes for the counter to discover the key with the first bit 1 and the others set to 0 (11941s). The fifth bit is reached within 774s - 12 min, the

sixth in 382s( 6min), the seventh in 192s ( 3min), therefore the time required to discover the key increases exponentially. Given that, by the time the first bit is reached, half of the search space has been verified, this gives us an average of 200 minutes to discover the key.

Example - for the key:

11111110111111011111101110001110001100100101110101101111100101

the running time was 2309s - 38.5 minutes

## 2.2 Hellman TMTO

The implementation of the Hellman method will be provided as a C++ header file, attached to this report.

### Table values

*Success rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	6	11	9	17	23	32	47	57
1024	-	7	5	5	13	13	20	30	47	-
2048	3	1	8	9	12	15	24	34	-	-

The success rate is relative to 1000 tests. it has remained consistently lower than 6 % .

We have also tested the hypothesis whether it might be easier or more difficult to find a key if it has already been processed a certain number of times. We have applied the step function on a randomly generated key many times in order to test the convergence of the chains in our tables. Contrary to our expectations, of a bigger success rate, we got less hits than for entirely random keys. This means, that at least when used to recover the key of the DES, the Hellman method is really deployable.

The other tables are provided at the end of this document.

**Reduction function** Our choice of a reduction function has taken into account its prospective speed as the first argument for its usage. For a given ciphertext, we take the 7 most significant bits out of its last 4 bytes. From the fifth to last byte we take its four less significant bits.

The function takes these bits from the five less significant bytes

✓	✓	✓	✓	✓	✓	✓	✓	
---	---	---	---	---	---	---	---	--

And for the fifth to last byte:

			✓	✓	✓	✓		
--	--	--	---	---	---	---	--	--

We have chosen this function because it corresponds to a memory buffer copy, an operation modulo  $2^5$  and an assignment. It is thus very fast to compute, much faster than the 32 less significant bits. Unfortunately, it behaved worse on almost every test we performed.

**Time to compute on Copacabana** We will consider the table  $t = 1024$  and  $m = 4194304$ . Using the same line of reasoning as in exercise 6, and assuming that the key could be found using the input table, we obtain that the average cryptanalysis time is  $\frac{nr. encryptions}{15 \cdot 48 \cdot 10^8} = \frac{185250}{15 \cdot 48 \cdot 10^8} = 0.00000257s$

## 2.3 Rivest TMTO

For our tables, we do not store the chain length along with the start and end points. Since the efficiency of our buffer comparison is so high, once we have an alarm, we perform a comparison of the ciphertexts at every step of the recreation of the chain. The choice of the parameter  $d$  should be according to the average chain length that we desire. We have tried to accommodate the values of  $t$  and  $d$  such that  $t$  is not much higher than  $2^d$  and the other way around.

## 2.4 Comparison between methods

In our case, Rivest has had a slightly lower average rate, but it still performs quite well. The case where  $d = 7$  has performed better. Perhaps the reason for this algorithm's not so stellar performance is the low convergence of the chains in the Hellman tables for our setting.

After performing our tests, it seems that we have erroneously measured the memory access time. More precisely, we have omitted to take into account the time needed to match a candidate to the endpoints. The time required to do so using the Rivest method is much lower than the time required by our implementation of Hellman, since we break our search immediately after we get the first set of alarms. We have also measured the success rate of our algorithms if we continue iterating even after we reach a distinguished point during the online phase, in the hope that even if we did not merge with an existing chain we might be able to do so in the future. Maybe surprisingly enough only negligible variations are to be observed in the recovery rate.

# Hellman TMTO results

Success rate

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	6	11	9	17	23	32	47	57
1024	-	7	5	5	13	13	20	30	47	-
2048	3	1	8	9	12	15	24	34	-	-

Alarm rate

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	1.88	4.398	8.198	16.625	31.227	66.231	124.314	253.625
1024	-	3.88	8.259	15.58	30.471	62.953	124.195	254.876	527.055	-
2048	7.64	15.158	30.552	63.326	125.499	257.383	524.175	1032.41	-	-

Average number of encryptions

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	826.57	1254.72	1955.39	3391.18	5917.86	11948.6	21434.4	43889.7
1024	-	2279.45	3898.95	6309.28	11357.8	22619	42880.8	86870.2	185250	-
2048	7169.69	12181.5	22796.2	45897.4	87367.4	178308	367626	702149	-	-

Read table time

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.027287	0.037347	0.077952	0.145621	0.284914	0.552151	1.14965	2.23479
1024	-	0.013632	0.026276	0.035899	0.07003	0.14807	0.29567	0.568602	1.12763	-
2048	0.005136	0.01075	0.035985	0.046705	0.079979	0.13963	0.283689	0.548732	-	-

Encrypt time

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000416524	0.000608301	0.000934446	0.00159589	0.00274543	0.0057239	0.0100153	0.0204408
1024	-	0.00110904	0.00186312	0.002971	0.00533433	0.010543	0.0198321	0.0402688	0.0847293	-
2048	0.00343169	0.0058463	0.0109038	0.0218218	0.0412658	0.08078	0.167071	0.364431	-	-

Test time

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.00158009	0.00232139	0.00357258	0.00580993	0.0100439	0.0200469	0.0438221	0.0852149
1024	-	0.0028273	0.00448556	0.00683111	0.0116088	0.0213902	0.038945	0.0775758	0.166565	-
2048	0.00711442	0.0115494	0.0205248	0.0393977	0.0728286	0.13808	0.280968	0.60854	-	-

# Hellman with custom function

The alarm rates are similar but the success rate is lower.

*Success rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	4	10	6	18	24	36	41	-
1024	-	4	13	9	12	17	24	30	-	-
2048	1	9	6	10	11	19	21	-	-	-

*Alarm rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	1.942	3.96	8.219	16.034	31.731	65.416	129.448	-
1024	-	4.255	7.863	16.371	30.767	62.695	126.863	260.875	-	-
2048	8.415	16.058	32.561	63.553	133.385	254.319	519.574	-	-	-

*Average number of encryptions*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	834.548	1171.64	1929.33	3260.41	6016.88	11558.9	22870.6	-
1024	-	2533.63	3762.86	6585.53	11632.3	22677.4	43651	93134.1	-	-
2048	7810.69	12746.3	24240	45706	94669.1	174358	354674	-	-	-

*Read table time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.032423	0.066682	0.132445	0.26367	0.522328	1.03105	2.09723	-
1024	-	0.015936	0.033856	0.064677	0.144506	0.247852	0.500674	0.994911	-	-
2048	0.004536	0.016726	0.018952	0.063087	0.12429	0.143207	0.50257	-	-	-

*Encrypt time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000654366	0.000916259	0.00156048	0.00257075	0.00483137	0.0090826	0.0180006	-
1024	-	0.0019674	0.00295258	0.00531078	0.0068251	0.0121478	0.0222111	0.0502736	-	-
2048	0.00484498	0.00648876	0.012598	0.0237749	0.052015	0.090551	0.183524	-	-	-

*Test time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.00214478	0.00311558	0.00490363	0.00795825	0.0140707	0.0256465	0.0783554	-
1024	-	0.00462325	0.00666009	0.0112641	0.0139652	0.0232679	0.0410689	0.0895754	-	-
2048	0.00945544	0.0121429	0.0223817	0.0413127	0.0863203	0.146799	0.293194	-	-	-

## Table size

The table size is the same, regarding of the algorithm used.

*Table size*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	1,0 MB	2,1 MB	4,2 MB	8,4 MB	16,8 MB	33,6 MB	67,1 MB	134,2 MB
1024	-	524,3 kB	1,0 MB	2,1 MB	4,2 MB	8,4 MB	16,8 MB	33,6 MB	67,1 MB	-
2048	262,1 kB	524,3 kB	1,0 MB	2,1 MB	4,2 MB	8,4 MB	16,8 MB	33,6 MB	-	-

## Using two tables

The comparative statistics for two tables of size 1024 x 2097152 with two functions versus one table of 1024 x 40194394 with one function. As it can be seen, the combination of two different tables with two different functions has reduced the number of collisions and lead to a higher number of successes.

Measurement	1024-2097152-LSB	1024-2097152-CUSTOM	both tables	1024-4194304-LSB
Number of successes:	30	24	55	47
Average number of alarms:	254.876	267.932	500.005	527.055
Average number of encryptions:	86870.2	95054.1	173307	185250
Table read time(seconds):	0.568602	0.0516563	1.07059	1.12763
Average encrypting time(seconds):	0.0402688	0.0516563	0.0814892	0.0847293
Average test time(seconds):	0.0775758	0.0905885	0.153262	0.166565

# Rivest-LSB, d=7

Statistical data:

*Success rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	3	3	4	6	14	27	28	39
1024	-	1	0	2	1	10	12	27	46	-
2048	1	0	0	1	8	10	11	25	-	-

*Alarm rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.751	1.52	3.009	6.031	11.77	23.633	47.316	91.736
1024	-	0.87	1.77	3.607	7.153	13.56	28.684	54.824	110.432	-
2048	0.967	1.921	3.755	7.545	15.075	30.109	60.082	117.89	-	-

*Average number of encryptions*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	607.356	697.026	890.538	1272.31	1981.72	3503.32	6305.36	11780.3
1024	-	1181.36	1354.43	1679.69	2373.78	3474.59	6350.99	11124.2	21161.8	-
2048	2268.67	2482.82	2874.19	3699.01	5380.43	8625.19	15193.8	27793.1	-	-

*Read table time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.021385	0.046203	0.081063	0.170726	0.37845	0.643534	2.55669	4.89161
1024	-	0.019859	0.042712	0.076657	0.159459	0.310815	0.416429	1.31612	2.62924	-
2048	0.009586	0.02289	0.045014	0.080103	0.156802	0.306007	0.593901	1.2024	-	-

*Encrypt time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000358332	0.000400107	0.000516005	0.000795209	0.00114063	0.00208564	0.00520924	0.00836875
1024	-	0.000859513	0.00119848	0.00129112	0.00171893	0.00263655	0.00552649	0.0095004	0.0162402	-
2048	0.00188977	0.00208579	0.00231552	0.00294245	0.00439546	0.00737082	0.0129624	0.0234513	-	-

*Test time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000869975	0.00128629	0.00207692	0.00375936	0.00637659	0.0128481	0.0615064	0.120275
1024	-	0.0018223	0.00238418	0.00360741	0.00555714	0.00931853	0.0198355	0.0371093	0.0995058	-
2048	0.00350362	0.0040866	0.00491755	0.00673086	0.0105012	0.0193791	0.0345446	0.0658356	-	-

# Rivest-LSB, d=11

Statistical data for Rivest d=11 using LSB

*Success rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	5	7	5	10	11	12	-	-
1024	-	1	2	1	8	8	12	-	-	-
2048	0	0	3	6	9	8	-	-	-	-

*Alarm rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	1.149	2.846	4.566	9.17	22.048	43.163	-	-
1024	-	2.644	5.672	9.696	18.298	44.922	75.446	-	-	-
2048	4.622	8.932	18.266	37.908	71.634	162.158	-	-	-	-

*Average number of encryptions*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	689.707	889.302	1212.47	1933.81	3899.52	7031.62	-	-
1024	-	1823.01	2652.32	4039.83	6703.07	15464.8	24756.1	-	-	-
2048	4946.34	7382.6	13592.5	24558.3	46610.1	100205	-	-	-	-

*Read table time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.039504	0.079287	0.103238	0.221085	0.62203	1.21466	-	-
1024	-	0.019721	0.039589	0.086928	0.166311	0.334427	0.52224	-	-	-
2048	0.011264	0.019364	0.037705	0.069938	0.1286	0.283627	-	-	-	-

*Encrypt time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000635409	0.000775561	0.000981688	0.00142652	0.00273554	0.00619449	-	-
1024	-	0.00164321	0.00242858	0.00377778	0.0062616	0.0133059	0.01878	-	-	-
2048	0.00452778	0.00682616	0.0109959	0.0197597	0.0349132	0.0762174	-	-	-	-

*Test time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.00139452	0.00198964	0.00295242	0.00479048	0.00919519	0.0211038	-	-
1024	-	0.00300194	0.00458419	0.00732528	0.0124014	0.0262952	0.0378502	-	-	-
2048	0.00779797	0.0118388	0.0192515	0.0338108	0.0601655	0.129387	-	-	-	-



# Rivest-custom function, d=11

Statistical data for Rivest d=11 using custom function R.

*Success rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	4	3	5	8	9	12	-	-
1024	-	3	3	3	4	10	15	20	-	-
2048	1	3	5	6	4	9	17	27	-	-

*Alarm rate*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	1.29	2.274	5.015	9.564	20.341	42.999	-	-
1024	-	2.363	4.891	9.05	20.651	43.926	75.373	170.549	-	-
2048	4.355	8.833	18.082	38.658	71.621	151.576	305.977	615.334	-	-

*Average number of encryptions*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	720.755	885.371	1345.27	1993.48	3746.82	6459.9	-	-
1024	-	1780.7	2557.44	3834.57	7152.32	14437.5	24400.3	53862	-	-
2048	4702.44	7437.61	12820.2	26258.8	46215.7	90223.1	190830	371544	-	-

*Read table time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.020288	0.046788	0.072139	0.143997	0.30013	0.541175	-	-
1024	-	0.017461	0.025791	0.04855	0.1293	0.178755	0.550827	1.13346	-	-
2048	0.008522	0.020675	0.043082	0.08063	0.154513	0.258994	0.289505	0.536621	-	-

*Encrypt time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000348295	0.000444406	0.000658977	0.000959509	0.00177931	0.00301587	-	-
1024	-	0.00110853	0.00153586	0.00230296	0.0039628	0.00767056	0.0163986	0.0400612	-	-
2048	0.00338252	0.00550068	0.00910765	0.01816	0.0319376	0.048589	0.085483	0.165066	-	-

*Test time*

t\m	16384	32768	65536	131072	262144	524288	1048576	2097152	4194304	8388608
512	-	-	0.000805207	0.00131677	0.00221073	0.00379083	0.00733685	0.0134469	-	-
1024	-	0.00213786	0.00314245	0.00504023	0.00882712	0.0166349	0.0352526	0.0831724	-	-
2048	0.00568151	0.00924226	0.0153296	0.0307549	0.0544799	0.0800006	0.13777	0.265578	-	-