Chase maneuvers using Lambert's problem

This document describes the implementation of the chase maneuver software and how to interact with the main script.

Chase maneuvers or intercept trajectories are those that answer the question: "How do you get from point A to point B in space in a given amount of time?" [*Curtis, 2010]*. The most common solution to these kind of maneuvers is found using Lambert's problem. For this purpose two spacecraft initially in two non-coplanar geocentric elliptical orbits are to rendezvous after a given time. One of the spacecraft engages on a chase maneuver in order to intercept the second spacecraft. The intercept trajectory is determined as a bi-impulsive orbital transfer from one of the orbits to the other. Although effects from the oblateness of the Earth, atmospheric drag, solar radiation pressure and third-body gravitation would perturb the actual trajectory of the spacecraft, they will be neglected for this analysis. The only force acting on the spacecraft is the gravitational pull of earth.

The software allows the user to define the orbits and the initial positions of the spacecraft as well as the desired time after which the rendezvous should take place, it then displays in the console the orbital elements of the transfer and initial orbits and the delta-v requirement for the mission to take place.

Interacting with the script

The following example shows a typical user interaction with the script. The user's input is marked with a bold font.

```
< Chase maneuvers >

 < Inputs >


***************

Initial orbit

***************


Input the semimajor axis (kilometers)

(semimajor axis > 0)

?14000


Input the orbital eccentricity (non-dimensional)

(0 <= eccentricity < 1)

?0.3
```

```
Input the orbital inclination (degrees)
(0 <= inclination <= 180)
?45

Input the right ascension of the ascending node (degrees)
(0 <= right ascension of the ascending node <= 360)
?20

Input the argument of perigee (degrees)
(0 <= argument of perigee <= 360)
?190

Input the true anomaly (degrees)
(0 <= true anomaly <= 360)
?0

***********
Final orbit
***********
Input the semimajor axis (kilometers)
(semimajor axis > 0)
?17000

Input the orbital eccentricity (non-dimensional)
(0 <= eccentricity < 1)
?0.2

Input the orbital inclination (degrees)
(0 <= inclination <= 180)
?60

Input the right ascension of the ascending node (degrees)
(0 <= right ascension of the ascending node <= 360)
?30
```

```
Input the argument of perigee (degrees)
(0 <= argument of perigee <= 360)
?100

Input the true anomaly (degrees)
(0 <= true anomaly <= 360)
?120



Choose the orbital direction

  1 - posigrade

  2 - retrograde

 Select 1 or 2
?1



Input the transfer time in seconds
(transfer time > 0)
?7000

< Results >

Orbital elements of the initial orbit
-------------------------------------
        sma (km)              eccentricity          inclination (deg)         argper (deg)
 +1.40000000000000e+04  +3.00000000000000e-01  +4.50000000000000e+01  +1.90000000000000e+02

       raan (deg)          true anomaly (deg)       period (min)
 +2.00000000000000e+01  +0.00000000000000e+00  +1.64855436911759e+04

Orbital elements of the transfer orbit after the first impulse
---------------------------------------------------------------
        sma (km)              eccentricity          inclination (deg)         argper (deg)
```

```
 +1.41999882300583e+04  +4.10108242303906e-01  +6.38268869619928e+01  +1.27899535068550e+02


      raan (deg)          true anomaly (deg)       period (min)

 +2.36209026120571e+01  +5.99641194144845e+01  +1.68400430651964e+04


Orbital elements of the transfer orbit prior to the final impulse
-----------------------------------------------------------------
        sma (km)              eccentricity          inclination (deg)         argper (deg)

 +1.41999882300583e+04  +4.10108242303906e-01  +6.38268869619928e+01  +1.27899535068550e+02


      raan (deg)          true anomaly (deg)       period (min)

 +2.36209026120571e+01  +1.77786895286189e+02  +1.68400430651964e+04


Orbital elements of the final orbit
-----------------------------------
        sma (km)              eccentricity          inclination (deg)         argper (deg)

 +1.70000000000000e+04  +2.00000000000000e-01  +6.00000000000000e+01  +1.00000000000000e+02


      raan (deg)          true anomaly (deg)       period (min)

 +3.00000000000000e+01  +1.20000000000000e+02  +2.20589381625801e+04


Initial delta-v vector and magnitude


x-component of delta-v      -2636.952520  m/s

y-component of delta-v        974.628422  m/s

z-component of delta-v      -1414.222268  m/s


delta-v magnitude            3146.973749  m/s


Final delta-v vector and magnitude


x-component of delta-v       -125.021599  m/s

y-component of delta-v        758.936085  m/s

z-component of delta-v        434.341742  m/s


delta-v magnitude             883.327306  m/s
```

```
Total delta-v                4030.301055  m/s
```

```
Transfer time                7000.000000  seconds
```

The following provides a brief description of the orbital elements displayed by the script.

sma (km) – semimajor axis of the orbit given in kilometers

eccentricity – orbital eccentricity as a non-dimensional parameter

inclination (deg) – orbital inclination given in degrees

argper (deg) – argument of perigee given in degrees

raan (deg) – right ascension of the ascending node given in degrees

true anomaly (deg) – true anomalies given in degrees

period (min) – orbital period given in miunutes

The script also displays a graphical representation of the problem as it can be seen in Figure1.The initial, final and transfer orbits as well as the initial position the spacecraft and the rendezvous point are presented for each solution. Through the interactive features of MATALB graphics the user can change the view point or check the basic three-dimensional geometry of the orbits. A legend is also provided in the upper right corner.

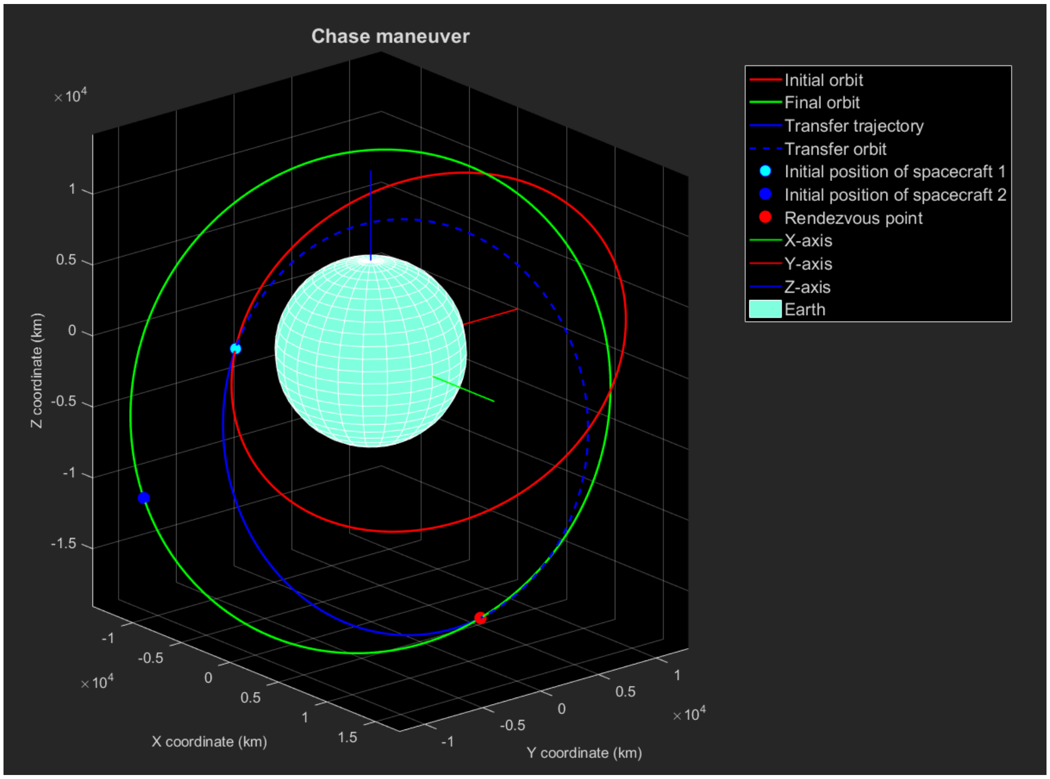

Figure 1 – Chase maneuver

Mathematical Analysis

The following analysis is summarized from [*Curtis (2010)*].

The subscripts “1” for the chaser spacecraft and its orbit, subscript “2” for the target spacecraft and its orbit, and subscript “3” for the transfer trajectory, will be used next.

Suppose that spacecraft 1 and spacecraft 2 are in two non-coplanar geocentric elliptical orbits. The orbital elements and the position of the spacecraft in these orbits are known. At a time instant, spacecraft 1 engages on a chase maneuver in order to rendezvous with spacecraft 2, after a time interval Δt.

First we calculate the angular momentum *h*, with the eccentricity *e* and the semimajor axis *a*, that are given as inputs to the problem.

$$h_1 = \sqrt{\frac{a_1 \mu}{(1-e_1^2)}}$$

$$h_2 = \sqrt{\frac{a_2 \mu}{(1-e_2^2)}}$$

Where μ is the gravitational parameter of Earth.

Having found the angular momentum of the two orbits the algorithm implemented in *sv_from_oe.m* can be used to calculate the state vectors of the two spacecraft at the beginning of the chase maneuver. They will be denoted as $\vec{r}_1$ and $\vec{v}_1$ for spacecraft 1 and $\vec{r}_2$ and $\vec{v}_2$ for spacecraft 2.

Next we have to move spacecraft 2 along its orbit to the position 2’ that it will occupy after the time interval Δt, when it will be intercepted by spacecraft 2. To do that, we must first calculate the time passed since perigee at point 2. Since we know the position of spacecraft 2 in its orbit, i.e. the true anomaly, the eccentric anomaly is found from as,

$$E_2 = 2\tan^{-1}\left(\sqrt{\frac{1-e_2}{1+e_2}}\tan\frac{\theta_2}{2}\right)$$

Substituting this value into Kepler’s equation (*kepler_equation.m*) yields the time passed since perigee.

$$t_2 = \frac{T_2}{2\pi}\left(E_2 - e_2 \sin E_2\right)$$

After the time interval Δt, spacecraft 2 will be at the rendezvous position 2’.

$$t_{2'} = t_2 + \Delta t$$

The corresponding mean anomy is,

$$M_{e2'} = 2\pi \frac{t_{2'}}{T_2}$$

By replacing this value for the mean anomaly into Kepler's equation yields,

$$E_{2'} - e_2 \sin E_{2'} = M_{e2'}$$

The true anomaly at position 2' is then found to be,

$$\theta_{2'} = 2\tan^{-1}\left(\sqrt{\frac{1+e_2}{1-e_2}} \tan\frac{E_{2'}}{2}\right)$$

The state vector of spacecraft 2 at position 2' can now be calculated using again the algorithm implemented in *sv_from_oe.m*.

The intercept trajectory is determined using Lambert's problem. Substituting $\vec{r}_1$ and $\vec{r}_{2'}$ along with the value of the time interval Δt into the algorithm implemented in *Lambert.m* which yields the values of the velocity vectors at the beginning of the chase maneuver, $\vec{v}_1)_2$, and at the end of the chase maneuver , $\vec{v}_{2'})_2$. From these we can calculate the initial $\Delta\vec{v}_1$ required to transfer spacecraft 1 on the intercept trajectory.

$$\Delta\vec{v}_1 = \vec{v}_1)_2 - \vec{v}_1$$

and the final $\Delta\vec{v}_2$ required to transfer spacecraft 1 on orbit 2, where it will intercept with spacecraft 2,

$$\Delta\vec{v}_2 = \vec{v}_2 - \vec{v}_{2'})_2$$

The Δv requirement for this intercept maneuver will be the sum of the magnitudes of these two vectors,

$$\Delta v = \left\|\Delta\vec{v}_1\right\| + \left\|\Delta\vec{v}_2\right\|$$

The last thing left to do is to calculate the orbital elements of the transfer trajectory. This can be achieved by substituting the vectors $\vec{r}_1$ and $\vec{v}_1)_2$ into to the algorithm implemented in *oe_from_sv.m.*

References

[1] Bate, R.R., Mueller, D.D., White, J.E. (1971); *Fundamentals of Astrodynamics*; Dover Publications, New York.

[2] Curtis, H. D. (2010). *Orbital mechanics for engineering students*, Elsevier ButterworthEarth Fact Sheet