Interplanetary Mission Design

This document describes the implementation of the interplanetary mission design software and how to interact with the main script.

The program can be used to define interplanetary trajectories between any two planets of the solar system, using the method of patched conics. It allows the user to specify the desired planet and date for departure and arrival and displays the results in the console together with a 3D graphical representation of the heliocentric trajectory. In the case when the departure and arrival dates are chosen in such a manner that would make the mission completely unrealistic or virtually impossible, the user will be shown an error message and will be asked to redefine the mission parameters.

Interacting with the script

The following example shows a typical user interaction with the script. The user's input is marked with a bold font.

```
< Interplanetary mission design program >


  < Inputs >


Departure parameters


  Choose the planet of deprarture


  1 - Mercury
  2 - Venus
  3 - Earth
  4 - Mars
  5 - Jupiter
  6 - Saturn
  7 - Uranus
  8 - Neptune
  9 - Pluto
? 3


Input the calendar date for departure
(1 <= day <= 31/ 1 <= month <= 12/ year = four digits)
```

```
(Example: 1/12/2005)
? 15/8/2005

Input the universal time for departure
(0 <= hours <= 24: 0 <= minutes <= 60: 0 <= seconds <= 60)
(Example: 12:00:00)
? 0:0:0

Input the altitude of the circular departure parking orbit
? 190

Departure parameters

  Choose the planet of arrival

  1 - Mercury
  2 - Venus
  3 - Earth
  4 - Mars
  5 - Jupiter
  6 - Saturn
  7 - Uranus
  8 - Neptune
  9 - Pluto
? 4

Input the calendar date for arrival
(1 <= day <= 31/ 1 <= month <= 12/ year = four digits)
(Example: 1/12/2005)
? 15/3/2006

Input the universal time for arrival
(0 <= hours <= 24: 0 <= minutes <= 60: 0 <= seconds <= 60)
(Example: 12:00:00)
? 8:30:15
```

Input the altitude of the circular capture orbit

? **300**

< Results >

Departure planet Earth

Departure calendar date 15/8/2005

Departure universal time 0:0:0

Departure julian date 2453597.500000

Arrival planet Mars

Arrival calendar date 15/3/2006

Arrival universal time 8:30:15

Arrival julian date 2453809.854340

Transfer time 212.354340 days

Heliocentric ecliptic orbital elements of the departure planet

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sma (AU)	eccentricity	inclination (deg)	argper (deg)
+1.00000010719028e+00	+1.67080823655031e-02	-6.82712373564529e-04	+1.14511069431326e+02

raan (deg)	true anomaly (deg)	longper(deg)	period (days)
+3.48454825182714e+02	+2.19238224181515e+02	+1.02965894614039e+02	+3.65257013255628e+02

Heliocentric ecliptic orbital elements of the transfer orbit

prior to reaching the sphere of influence of the arrival planet

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sma (AU)	eccentricity	inclination (deg)	argper (deg)
+1.34405158402362e+00	+2.46864076366525e-01	+3.05825990754311e+00	+3.55668675971044e+02

raan (deg) true anomaly (deg) period (min)

```
 +3.22198466955492e+02  +4.33698393145309e+00  +5.69144311168632e+02


Heliocentric ecliptic orbital elements of the transfer orbit
prior to reaching the sphere of influence of the arrival planet
---------------------------------------------------------------
        sma (AU)              eccentricity          inclination (deg)         argper (deg)
 +1.34405158402362e+00  +2.46864076366525e-01  +3.05825990754311e+00  +3.55668675971044e+02

       raan (deg)          true anomaly (deg)       period (min)
 +3.22198466955492e+02  +1.52443889044627e+02  +5.69144311168632e+02


Heliocentric ecliptic orbital elements of the arrival planet
--------------------------------------------------------------
        sma (AU)              eccentricity          inclination (deg)         argper (deg)
 +1.52365783237832e+00  +9.34197102317202e-02  +1.85017129104839e+00  +2.86506756012675e+02

       raan (deg)          true anomaly (deg)       longper(deg)             period (days)
 +4.95609676998296e+01  +1.34292648994542e+02  +3.36067723712505e+02  +6.86957178986249e+02



Departure velocity vector and magnitude

x-component of departure velocity                              20.609502  km/s
y-component of departure velocity                              25.765976  km/s
z-component of departure velocity                               1.762616  km/s
departure velocity magnitude                                    1.762616  km/s

Arrival velocity vector and magnitude

x-component of arrival velocity                               -20.458782  km/s
y-component of arrival velocity                                -4.331110  km/s
z-component of arrival velocity                                -0.852806  km/s
departure velocity magnitude                                   20.929585  km/s

Hyperbolic excess velocity vector and magnitude at departure
```

```
x-component of hyperbolic excess velocity at departure        2.838426  km/s

y-component of hyperbolic excess velocity at departure        2.338532  km/s

z-component of hyperbolic excess velocity at departure        1.762932  km/s

hyperbolic excess velocity at departure magnitude             4.078398  km/s


Hyperbolic excess velocity vector and magnitude at arrival


x-component of hyperbolic excess velocity at arrival          1.339208  km/s

y-component of hyperbolic excess velocity at arrival          2.035801  km/s

z-component of hyperbolic excess velocity at arrival         -1.255317  km/s

hyperbolic excess velocity at arrival magnitude               2.741128  km/s


<Planetary departure parameters>


Altitude of the parking orbit                                              190.000000  km

Period of the parking orbit                                                 88.289628  min

Speed of the space vehicle in its circular orbit                             7.790262  km/s

Radius to periapsis of the departure hyperbola                            6568.000000  km

Eccentricity of the departure hyperbola                                      1.274079

Speed of the space vehicle at the periapsis of the departure hyperbola      11.747753  km/s

Delta_v for departure                                                        3.957491  km/s


<Planetary rendezvous parameters>


Altitude of the capture orbit                                              300.000000  km

Period of the capture orbit                                                112.963060  min

Speed of the space vehicle in its circular orbit                             3.411458  km/s

Radius to periapsis of the arrival hyperbola                              3680.000000  km

Eccentricity of the arrival   hyperbola                                      1.645623

Speed of the space vehicle at the periapsis of the arrival hyperbola         5.548862  km/s

Delta_v for arrival                                                          2.137405  km/s


Total delta_v for the mission                                              6.094895  km/s
```

The following provides a brief description of the orbital elements displayed by the script.

sma (AU) – semimajor axis of the orbit given in kilometers

eccentricity – orbital eccentricity as a non-dimensional parameter

inclination (deg) – orbital inclination given in degrees

argper (deg) – argument of perigee given in degrees

raan (deg) – right ascension of the ascending node given in degrees

true anomaly (deg) – true anomalies given in degrees

longper (deg) – longitude of perigee given in degrees

period (min) – orbital period given in miunutes

The semimajor axis unit of measure is given in AU (astronomical unit). 1 AU represents roughly the distance between the Sun and Earth, and it is a common unit of measure used in the Solar System. Its exact value is 149,597,871,000 km.

The script also outputs a graphical representation of the problem as it can be seen in Figure 1.The planetary and transfer orbits as well as the initial and final positions of the planets are presented for each solution. Through the interactive features of MATALB graphics the user can change the view point or check the basic three-dimensional geometry of the orbits. A legend is also provided in the upper right corner.

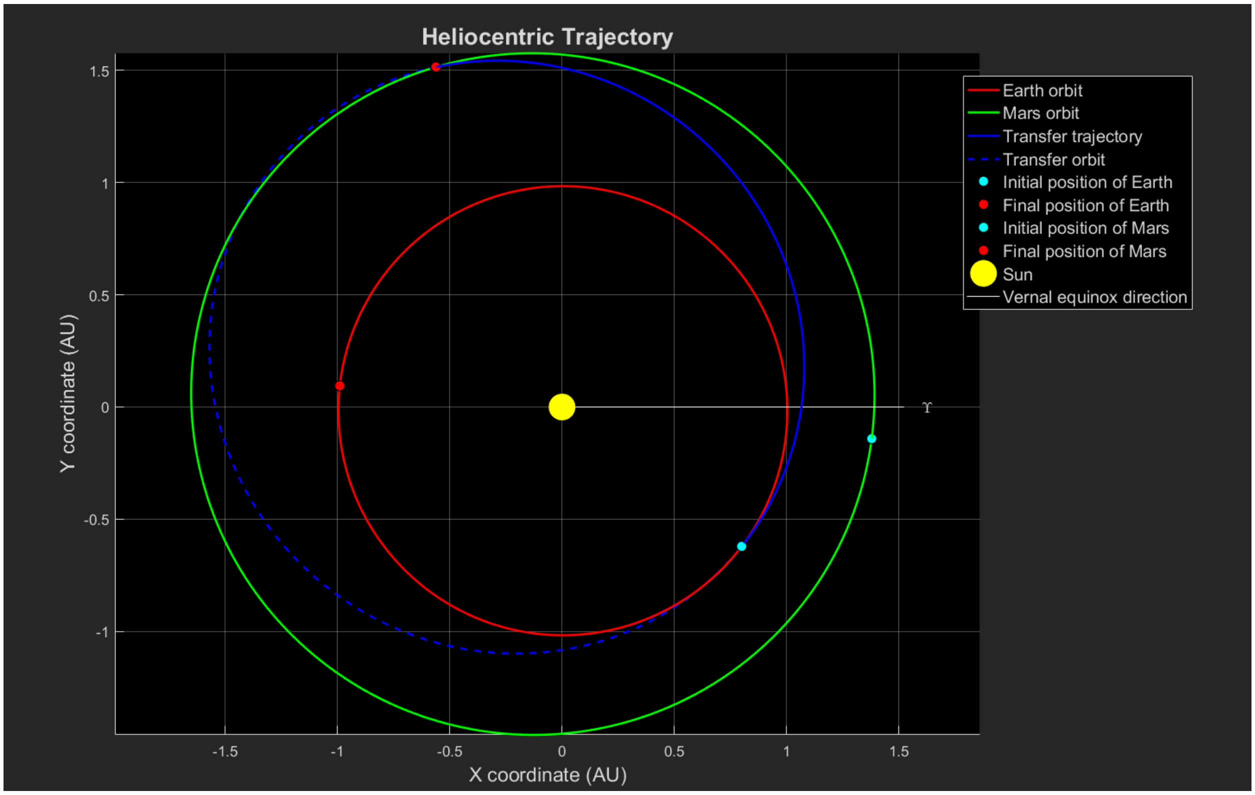

Figure 1 – Interplanetary trajectory between Earth and Mars

Analysis

To find the interplanetary trajectory using the method of patched conics the following approach, summarized from [*Curtis (2010)*], is presented below.

Method of patched conics

In the method of patched conics the mission is divided in three phases. The first phase is the heliocentric trajectory between the two planets. For this, the solution to Lambert's problem is employed. This approach allows for the computation of the minimum energy trajectory, using the time of flight and the positions of the planets. The second one is the hyperbolic departure phase which takes the space vehicle from a circular parking orbit around the planet of departure up to the sphere of influence of that planet. The last phase, the arrival phase, is concerned with the trajectory from the sphere of influence of the arrival planet up to a circular capture orbit around the respective planet. In this way three conics are patched together.

It is assumed that inside the spheres of influence the only force acting on the space vehicle is the gravitational pull of the planets, and outside of them the only force is the gravitational pull of the sun. Any other forces or perturbations are neglected.

1. Heliocentric trajectory

To analyze the trajectory between the departure and the arrival planet we must first start with the heliocentric trajectory. The spheres of influence may be considered as points coinciding with the centers of the planets due to the vastness of the distances involved in interplanetary trajectories. Thus, the state vector of the space vehicle crossing the sphere of influence coincides with that of the planet, at that moment in time.

The first step is determine the state vector of planet 1 at departure and the state vector of planet 2 at arrival. This is implemented in the file *planet_oe_and_sv.m* . In this way we find the state vector of planet 1, $\vec{R}_1$ and $\vec{V}_1$ at moment *t*, and the state vector of planet 2, $\vec{R}_2$ and $\vec{V}_2$ at moment $t+\Delta t$, where Δt represents the time difference between departure and arrival.

The next step involves finding the space vehicle's transfer trajectory between the two planets. The heliocentric position vector of the space vehicle at moment *t* coincides with the position vector of planet 1, $\vec{R}_1$, and at moment $t+\Delta t$ with the position vector of planet 2, $\vec{R}_2$.

Because the departure and arrival times are given in a day, month, year and universal time format we must transform them in Julian days, and subtract the results in order to find the time of flight. This is implemented in the file *Julian0.m*.

Having the two vectors $\vec{R}_1$ and $\vec{R}_2$, and knowing the time of flight between them we can apply the solution to Lambert's problem (*Lambert.m*) to obtain the departure velocity $\vec{V}_D$ and the arrival velocity $\vec{V}_A$ relative to the sun. Then, either using the vectors $\vec{R}_1$ and $\vec{V}_D$, or vectors $\vec{R}_2$ and $\vec{V}_A$ the orbital elements of the heliocentric transfer orbit will be obtained (*oe_from_sv.m*).

The space vehicle's hyperbolic excess velocity upon departing from the sphere of influence of planet 1 is:

$$\vec{v}_{\infty})_{Departure} = \vec{V}_D - \vec{V}_1$$

In a similar fashion we obtain the hyperbolic excess speed of the space vehicle upon arriving at the sphere of influence of planet 2,

$$\vec{v}_{\infty})_{Arrival} = \vec{V}_A - \vec{V}_2$$

The hyperbolic excess speeds are:

$$v_{\infty})_{Departure} = \left\| \vec{V}_D - \vec{V}_1 \right\|$$

$$v_{\infty})_{Arrival} = \left\| \vec{V}_A - \vec{V}_2 \right\|$$

2. The departure phase

The departure phase starts from a circular parking orbit around planet 1. The altitude of the parking orbit will be denoted as a_p. To find the radius to periapsis of the departure hyperbola we must add to the altitude of the parking orbit the radius of planet 1, r_1. Thus we have,

$$r_p = a_p + r_1$$

This yields the speed of the space vehicle at the periapsis of the departure hyperbola

$$v_p = \sqrt{\left[v_{\infty})_{Departure} \right]^2 + 2\frac{\mu_1}{r_p}}$$

where μ_1 represents the gravitational parameter of planet 1 (*astronomical_data.m*).

The speed of the space vehicle in the circular parking orbit is,

$$v_{c1} = \sqrt{\frac{\mu_1}{r_p}}$$

Thus, the delta-v requirement at departure can be found to be,

$$\Delta v_{Departure} = v_p - v_{c1}$$

The eccentricity of the departure hyperbola is,

$$e_{Departure} = 1 + \frac{r_p \left[v_{\infty})_{Departure} \right]^2}{\mu_1}$$

The period of the circular parking orbit can be computed as,

$$T_{parking} = \frac{2\pi}{\sqrt{\mu_1}} r_p^{\ 3/2}$$

3. The arrival phase

The assumption made is that upon arrival at planet 2, the space vehicle will embark on a circular capture orbit around the planet. To do this a delta-v must be applied at the periapsis of the arrival hyperbola to put the space vehicle on the circular orbit.

The radius at periapsis of the arrival hyperbola around planet 2 can be computed by adding to the altitude at periapsis of the arrival hyperbola the radius of planet 2,

$$r_p)_{hyp} = a_p)_{hyp} + r_2$$

The speed of the space vehicle at the periapsis of the arrival hyperbola is,

$$v_p)_{hyp} = \sqrt{\left[v_\infty)_{Arrival}\right]^2 + \frac{2\mu_2}{r_p)_{ell}}}$$

where μ_2 is the gravitational parameter of planet 2.

The speed of the space vehicle in the circular orbit is,

$$v_{c2} = \sqrt{\frac{\mu_2}{r_p)_{hyp}}}$$

We can now find the delta-v requirement at arrival

$$\Delta v_{Arrival} = v_p)_{hyp} - v_{c2}$$

The eccentricity of the departure hyperbola is,

$$e_{Arrival} = 1 + \frac{r_p)_{hyp}\left[v_\infty)_{Arrival}\right]^2}{\mu_2}$$

The period of the circular parking orbit can be computed as,

$$T_{parking} = \frac{2\pi}{\sqrt{\mu_2}} r_p)_{hyp}^{\ 3/2}$$

References

[1] Bate, R.R., Mueller, D.D., White, J.E. (1971); *Fundamentals of Astrodynamics*; Dover Publications, New York.

[2] Curtis, H. D. (2010). *Orbital mechanics for engineering students*, Elsevier ButterworthEarth Fact Sheet