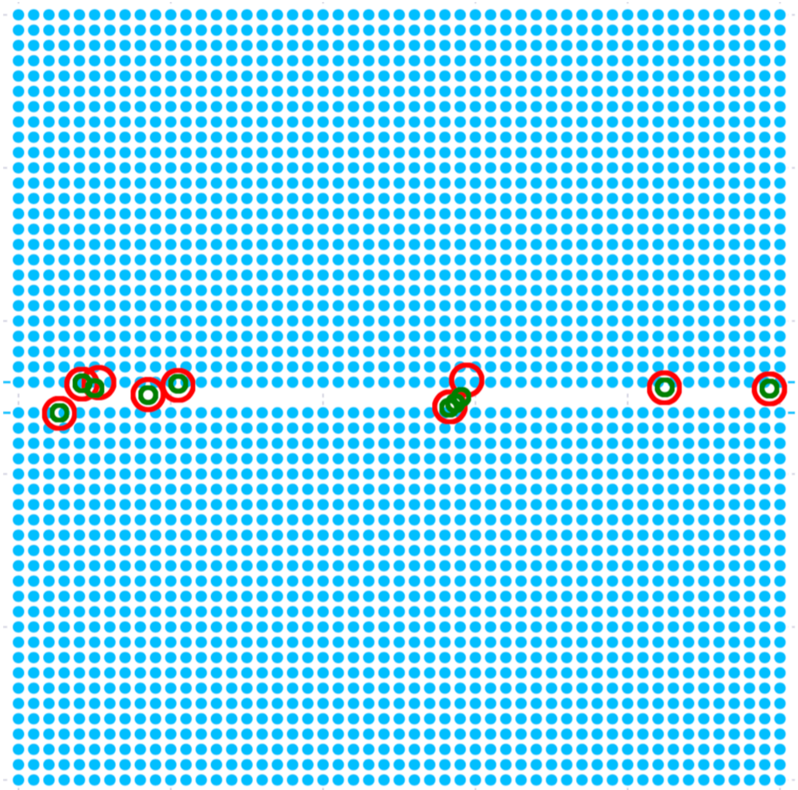
EPSRC Centre for Doctoral Training in Industrially Focused Mathematical Modelling



Improved Source Reconstruction from Hydrophone Data

Bogdan Toader

Contents

1. Introduction

We want to estimate the positions of ships in a shipping lane, given a small number of observations from hydrophones.

Background

This project in collaboration with the National Physical Laboratory (NPL) is motivated by the EU Marine Strategy Framework Directive which requires EU member states to assess the level of marine noise in their seas. To achieve this, we are interested in using mathematical modelling and numerical techniques to estimate the positions and the sound levels of the ships in a particular shipping lane, given only a small number of observations from hydrophones (underwater microphones). Solving this problem may have other potential applications like monitoring the ship activity in a harbour.

The problem of reconstructing the locations and estimating the source levels can be studied at different levels of complexity, and so we consider two approaches. Our first approach considers where, given a snapshot in time with a number of unknown ships present in the shipping lane, we want to estimate their locations and source levels based on measurements recorded by the hydrophones at that specific point in time. We extend the static problem by considering a dynamic version of the problem, where the ships are moving in the shipping lane, measurements are taken periodically, and we estimate their locations and source levels.

Our project forms part of an ongoing NPL programme. In [1] they consider three approaches: one based on modelling of the sound propagation, one based on measurements taken by the hydrophones, and one which combines these using a compressive sensing framework; the last approach gave the best results. The compressive sensing solution to the static problem was improved upon in a subsequent InFoMM miniproject [2], which also introduced an approach for solving the dynamic problem. It was concluded that the solution to the dynamic problem gives better results than the solution to the static problem. The three static approaches, together with a detailed background of the modelling of the sound sources and propagation, are presented in [3].

2. Problem

We first consider a discretised version of the problem, where the region of interest consists of a 501 x 501 grid.

We assume that the region of interest is a 50 km × 50 km square, which we discretise using a 501 × 501 grid, so each square represents a 100 m × 100 m area. We assume that the ships (or sources) can only be located at grid points in an area of 21 × 501 points defining a shipping lane in the middle of the region of interest, while the hydrophones (or receivers) are situated outside the shipping lane. We have 10 receivers that record the sound level at 8 frequencies. Therefore, we are taking 10 × 8 measurements. An example of a configuration is shown in Figure 1.

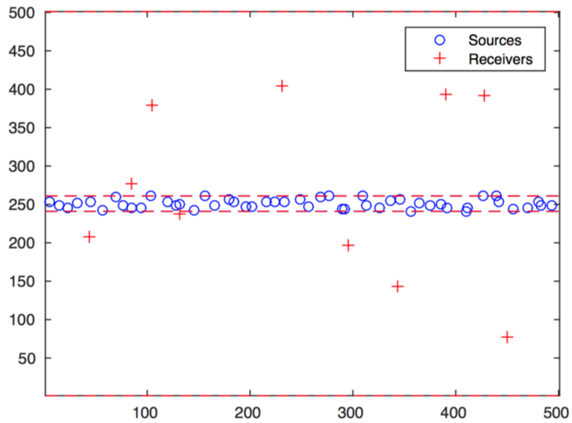


Figure 1: An example of a receiver and source configuration in the region of interest. The region in the centre is the shipping lane. Image taken with permission from [2].

We represent the magnitude of the emitted sound at each location in the region of interest as an entry in the vector x. Therefore, x is a vector with n = 10521 entries. However, since there is no sound emitted at most of the locations (only ships emit sound, and there is only a very small number of ships in the shipping lane), most entries of x are zero – we say that x is **sparse**. The way the sound propagates from each source to each receiver is captured by a measurement matrix A, which has one column corresponding to each location in the shipping lane and one row for each measurement, so it is an $m \times n$ matrix with $m = 80$ and $n = 10521$. Therefore, the product Ax gives a vector y which has $m = 80$ entries consisting of the measured sound level at each receiver on each frequency. Our problem is formulated as

$$Ax = y, \tag{1}$$

where x is unknown. When the measurement matrix (or the sensing matrix) A has more columns than rows, we say that the system is **underdetermined**, in which case there is no unique solution. In this situation, the compressed sensing approach is to find the *sparsest solution*, that is, the solution with the least number of ships. This is found with high probability if we solve an *optimisation* problem where we search for the solution to (1) while minimising

$$|x|_1 = \sum_{i=1}^{n} |x_i|. \tag{2}$$

Similarly, we can extend this approach to the case when the ships are moving. By assuming that the ships travel in straight paths and with the same speed, we obtain a measurement matrix M where each column represents a potential path.

Prior Progress

In the previous work done by NPL and continued during the InFoMM miniproject [2], the solution to the static problem has been refined, and the dynamic formulation has been setup. The overall sound level estimation in the region of interest was estimated accurately by the static approach. However, trying to take a deeper look at the sound emitted by each ship was impossible, due to the fact that the reconstruction of each individual source did not prove to be very exact. Using the dynamic formulation on the other hand, we are able to estimate the ship locations more accurately. A key aim of our project was to better understand why this is the case.

Glossary of Terms

- Region of interest: The 50km × 50km area where the ships and receivers are located, that we discretise using the 501 × 501 grid.
- Shipping lane: The 21 × 501 area in the middle of the region of interest where the ships are located.
- Static problem: The problem with the ships not moving in the shipping lane.
- Dynamic problem: The problem with the ships traveling along straight paths.
- Correlation: A number between 0 and 1 which characterises how similar two different ship locations (or paths) are.
- Dimensions of the problem: The number of measurements, the number of possible locations (or paths) and the number of ships.
- Undersampling ratio: Ratio of number of measurements to number of possible locations (or paths).
- Sparsity ratio: Ratio of number of ships to the number of measurements.
- Grid free approach: An approach to modelling the static problem where we do not discretise the region of interest using a grid.

> We analyse the main factors that potentially contribute to the improved performance of the dynamic problem over the static problem: correlation, undersampling ratio and sparsity ratio.

Issues to Analyse

In order to understand better why the dynamic approach gives better results than the static approach, we analyse the following factors:

- Correlations
- Undersampling ratio
- Sparsity ratio

Finally, we will also explore a grid-free approach to the static problem in order to avoid the conflicting requirements of (1) having a fine grid to accurately pinpoint the ships and (2) wanting low correlation between entries for ease of reconstructing the positions.

3. Dynamic Problem

The question we seek to answer in this section is why the formulation of the dynamic problem gives better results than the static problem. In the compressed sensing literature, there are two main factors that influence the performance on a specific problem: the correlation between the columns of the sensing matrix A and the relationship between the dimensions of the problem. We compare three problems: the Static problem, the dynamic problem where all the ships are moving with the same speed (Dynamic1), and the dynamic problem where ships move with four different speeds (Dynamic4).

Correlation

For a given problem represented as a linear system, the correlation between two different columns of the sensing matrix A (which in our case represent either ship locations or paths) is a number between 0 and 1 which indicates how similar the two columns are. The higher the number, the harder it is for the recovery algorithm to distinguish between the two.

In Figure 2, we show the correlation between the top-centre location in the shipping lane and all the other locations in the Static problem (left) and the correlation between the horizontal path that starts at the top-centre location in the shipping lane and the horizontal paths that start at all the other locations in the lane in the Dynamic1 problem (right). Each line is a contour of the correlation function. We see that, in the dynamic case, the correlation decays more rapidly than in the static case, implying that the recovery algorithm can distinguish more easily between the different paths.

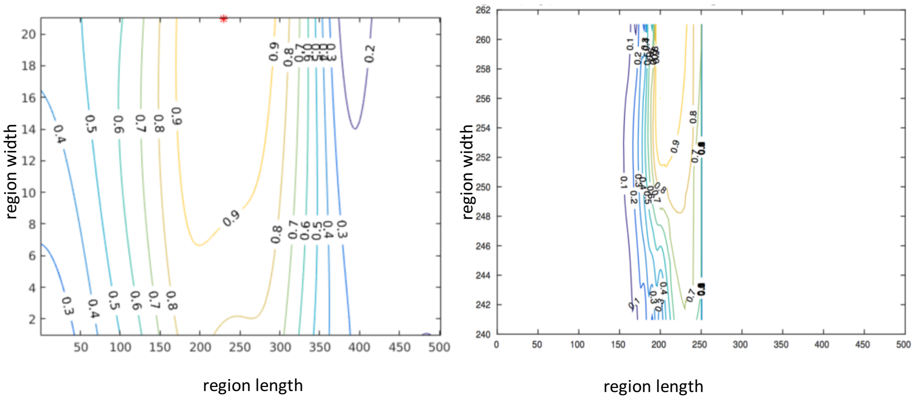


Figure 2: Correlation between ship locations (left [2]) and correlation between ship paths (right). Each line is a contour of the correlation function. The red dot is the reference ship location with which we calculate correlation.

In Figure 3, we show how the fraction of correlation values that are higher than the given correlation value varies with the given correlation value. This shows clearly that correlation in the dynamic problems is improved. The fraction decreases for higher given correlation values.

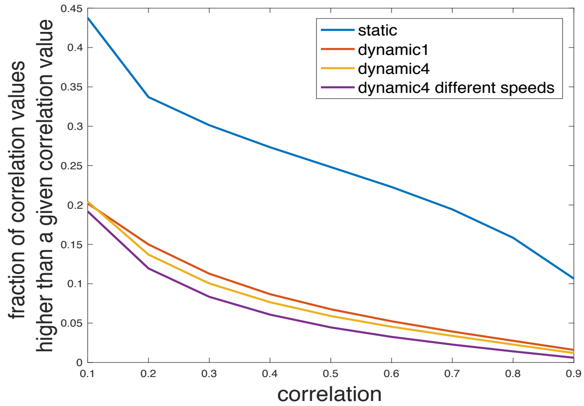


Figure 3: The fraction of correlation values between different columns that are higher than correlation values between 0.1 and 0.9.

Dimensions

Lastly, we look at how the dimensions of the problem vary between the Static, Dynamic1 and Dynamic4 problems. By allowing different speeds for the ships, the number of potential paths and therefore the number of columns in the sensing matrix A changes. Since the number of measurements we are taking is the same (10 receivers × 8 frequencies × 50 time steps), the undersampling ratio of the matrix changes.

We use four ship speeds; V, $0.1V$, $0.3V$ and $0.6V$ and the number of paths grows from 140620 (the number of paths when all ships have the same speed) to 523851 (it is less than 4×140620 because of the discretisation of the paths on the grid. For the static problem, the dimensions of the matrix are 80 : 10521 (with an undersampling ratio of 0.007), while in the dynamic problem with one speed, the dimensions are 80×50 : 140620, so the undersampling ratio is improved (4000/140620 ≈ 0.028). However, when we have four different speeds, the dimensions of the matrix are 80×50 : 523851, and because 4000/523851 ≈ 0.007, the undersampling ratio is now the similar to the static problem.

Lastly, the sparsity ratio becomes smaller as we increase the number of rows. For a specific number of ships whose locations we want to estimate, the sparsity ratio is smaller in the dynamic approach than in the static approach since we take we take 80 measurements for each time t = 1,…,T. The sparsity ratio does not change however when we add more speeds. The three cases are shown more clearly in Table 1.

	Static	Dynamic1	Dynamic4
Rows	80	80 × 50	80 × 50
Columns	10521	140620	523851
Undersampling ratio	0.007	0.028	0.007
Sparsity ratio for 10 ships	0.125	0.0025	0.0025

Table 1: The three problems and their dimensions

Numerical Experiments

For each problem, we ran the implementation of our solution using randomly generated source configurations for different numbers of sources (for 2, 5, 10, 15, 20 and 25 sources) and we compared the distances between true locations/paths and estimated location/paths.

The results of the experiments showed that the performance is similar in Dynamic1 and Dynamic4: Dynamic4 is slightly worse, but there is a considerable difference between the dynamic cases and the static case. The main question we are trying to answer is what contributes to the better outcome in finding the sources of the dynamic problem compared to the static problem.

The better results of the dynamic problem compared with the static problem are explained by improved correlation and increased number of measurements and signal dimensions.

While the Dynamic1 problem has an improved undersampling ratio over the Static problem, the Dynamic4 problem has the same undersampling ratio as the Static case; see Table 1. However, the fact that Dynamic1 gives better results than Dynamic4 implies that the improved undersampling in Dynamic1 outweighs the improved correlation in Dynamic4. Furthermore, both Dynamic cases have improved correlation and improved sparsity ratio compared with the Static case.

To sum up, the improved correlation for the dynamic problem means that signals with a higher number of non-zeros can be accurately reconstructed. We believe that the more favourable correlation, combined with the fact that the number of nonzeros to reconstruct does not increase (though the measurement and signal dimensions do), is the reason why improved results are observed for the dynamic problem compared to the static problem.

A grid-free model avoids the conflicting need for a finer mesh with the requirement of low correlation of the recovery algorithm.

4. Grid-free Approach

When discretising the shipping lane, we need to have a fine mesh in order to capture the ship locations with enough accuracy that the results are useful. However, having a finer grid conflicts with the low correlation between ship locations that is required by recovery algorithms in traditional compressed sensing. Because the measurement depends continuously on the data, refining the mesh leads to high correlation.

In order to avoid this issue, in this section we describe a grid-free approach to compressed sensing (a technique which is often called super-resolution) [4]. While previously we assumed that the region of interest is a grid and ships are located at points on the grid, we will now model each ship as a point in a continuous 2D domain and the sound will be modelled using a point-spread function. Since the models for source location and sound propagation are continuous, such a treatment is a natural fit to our problem.

Preliminary Results

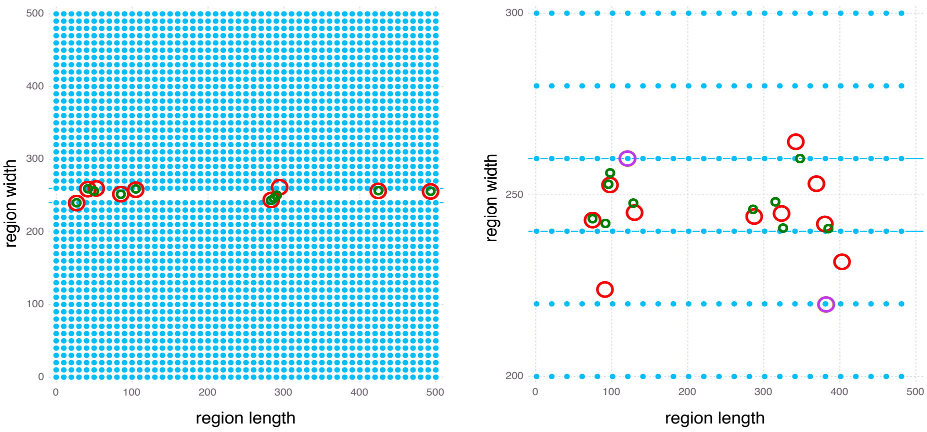


Figure 4: Grid-free numerical experiments. The blue dots are the receivers, the green circles are the true ship locations, while the red and the purple circles are the estimated ship locations. The ships at the red circles are estimated to emit a larger sound than those at the purple circles.

We ran a few numerical experiments using an existing approach to grid-free compressed sensing [5] with the same dimensions as our gridded solution. We have seen that, if the number of receivers is high enough as seen in the left panel in Figure 4, the recovery works very well, except when the sources are very close together. However, when we reduce the number of receivers, as shown in the right panel in Figure 4, the recovery performance degrades.

In order for our results to be realistic, we have to make a trade-off between accuracy of locations of the ships and number of receivers. By placing the receivers not only close to the shipping lane, but also inside it, we can obtain a receiver configuration which performs reasonably well. We use this configuration to run a set of more rigorous experiments to compare how good the grid-free approach is in comparison with the gridded approach. We used the same receiver configuration in both implementations and ran each test (2, 5, 10, 15, 20 and 25 ships) and compared the distances between the true and the estimated ship locations.

We found that the performance of the grid-free approach is poorer when the number of ships is low. However, in the more realistic scenario when there are between 15 and 25 ships in the shipping lane the results of the two approaches are comparable. This is a good starting point for a more in-depth analysis of the receiver configuration and the factors that affect the performance of the grid-free approach to our problem.

5. Conclusions

In this miniproject we have carried out a more in-depth analysis of why the formulation of the dynamic problem gives better results than the static problem, and we also proposed an alternative, grid-free model for our problem.

We conclude that correlation plays a large role in driving performance of the dynamic case, but also that the sparsity of the solution is important, since we keep the number of ships constant but we make more measurements. Lastly, the undersampling ratio allows the Dynamic1 problem to give slightly better results than the Dynamic4 problem.

On the other hand, in order to avoid the conflict between the requirement for a fine mesh and the need of the recovery algorithm for low coherence, we looked at a grid-free model. We ran a few experiments on a basic model (the Cauchy kernel) and showed that, for a different model of sound propagation, we achieve similar results to the gridded static formulation.

6. Potential Impact

The results of the project are informing NPL's work on using sensor networks for environmental monitoring both in a general context as well as specifically for measuring marine noise, which is mandated by the EU Marine Strategy Framework Directive and requires EU member states to assess the environmental status of their seas and oceans.

Peter Harris, Principal Research Scientist at NPL, said "*The mini-project has led to a substantial increase in our understanding of the problem, and has suggested some new and promising avenues for future possible study*".

References

1. P Harris et al (2015) *Investigation into the design of a marine noise monitoring network using a compressive sensing approach*. UACE2015 - 3rd Underwater Acoustics Conference and Exhibition, **1**, 425-432

2. R Philip (2015) *Source reconstruction from hydrophone data*. InFoMM CDT Miniproject Report

3. P Harris et al (2016) *Monitoring anthropogenic ocean sound from shipping using an acoustic sensor network and a compressive sensing approach*. Sensors **16**, 415

4. B Recht et al (2015) *Superresolution without separation.* 6th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, CAMSAP 2015, Cancun, Mexico, 45-48

5. B Recht et al (2015) *The alternating descent conditional gradient method for sparse inverse problems.* 6th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, CAMSAP 2015, Cancun, Mexico, 57-60