Spatially Variable Deconvolution for Lightsheet Microscopy

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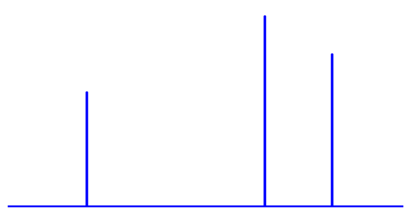
Lightsheet microscopy

Consider the problem of localising k non-negative point sources on the interval $[0,1]$, namely finding their locations $t_1,\dots,t_k$ and magnitudes $a_1,\dots,a_k \geq 0$ from m noisy samples $y_1,\dots,y_m$ which consist of the convolution of the input signal with a known kernel ϕ (e.g. Gaussian $\phi(t)=e^{-t^2/\sigma^2}$) and additive noise $\mathbf{w}$ bounded by δ.

Input signal

x is the discrete measure we want to reconstruct.

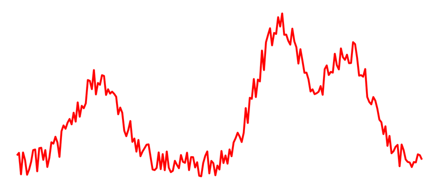


$$x(t)=\sum_{i=1}^{k} a_i \cdot \delta_{t_i},$$

t_i and a_i $(i=1,\dots,k)$ are the locations and magnitudes of the point sources.

Measured signal

y_i are the samples we use to reconstruct x.



$$y_j=\int_{[0,1]} \phi(t-s_j)x(\mathrm{d}t)+w_j,$$

where s_j $(j=1,\dots,m)$ are sampling locations and $\mathbf{w}=[w_1,\dots,w_m]^T$ with $\|\mathbf{w}\|_2\leq\delta$ is the noise.

Model

The measure x can be recovered exactly by solving the TV norm minimisation problem over non-negative measures z on $[0,1]$:

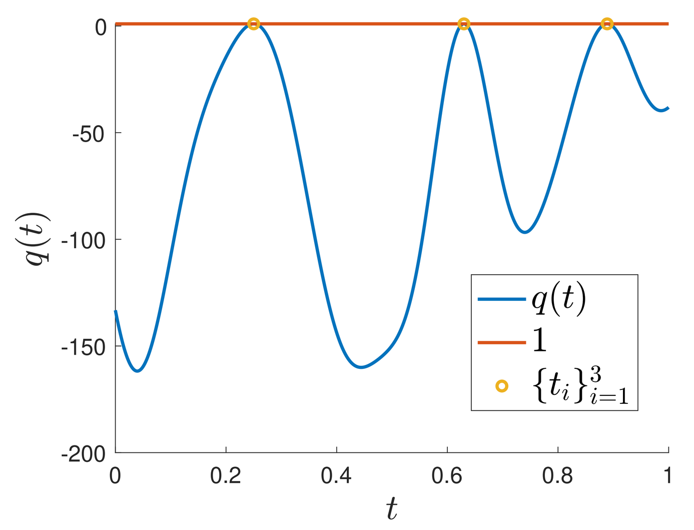
$$\min_{z\geq 0}\|z\|_{TV} \quad \text{subject to} \quad y_j=\int_{[0,1]}\phi(t-s_j)x(\mathrm{d}t), \quad \forall j=1,\dots,m, \tag{1}$$

or its dual:

$$\max_{\lambda\in\mathbb{R}^m} y^T\lambda \quad \text{subject to} \quad \sum_{j=1}^{m}\lambda_j\phi(t-s_j)\leq 1, \quad \forall t\in[0,1]. \tag{2}$$

Having the solution λ to the dual problem (2), the locations $\{t_i\}_{i=1}^k$ in the input signal are given by the global maximisers of the dual certificate:

$$q(t)=\sum_{j=1}^{m}\lambda_j\phi(t-s_j).$$



Fitting the PSF

Algorithm

We solve $\min_{\lambda\in\mathbb{R}^m} f(\lambda)$ for the exact penalty f:

$$f(\lambda)=-y^T\lambda+\Pi\cdot\max\{\sup_s\sum_{j=1}^{m}\lambda_j\phi(s-s_j)-1,0\}$$

in the set $Q=\{\lambda\in\mathbb{R}^m \mid \|\lambda\|_\infty\leq B\}$. As described in [1], at every iteration p:

1. Compute the piece-wise linear model:

$$\hat{f}_p(\lambda)=\max_{0\leq l\leq p}\left[f(\lambda_l)+\langle g(\lambda_l),\lambda-\lambda_l\rangle\right],$$

   where $g(\lambda_l)$ are some subgradients of f at λ_l.

2. Find the optimal value $\hat{f}_p^*$ of the current model $\hat{f}_p$ and the smallest value f_p^* of the objective f so far.

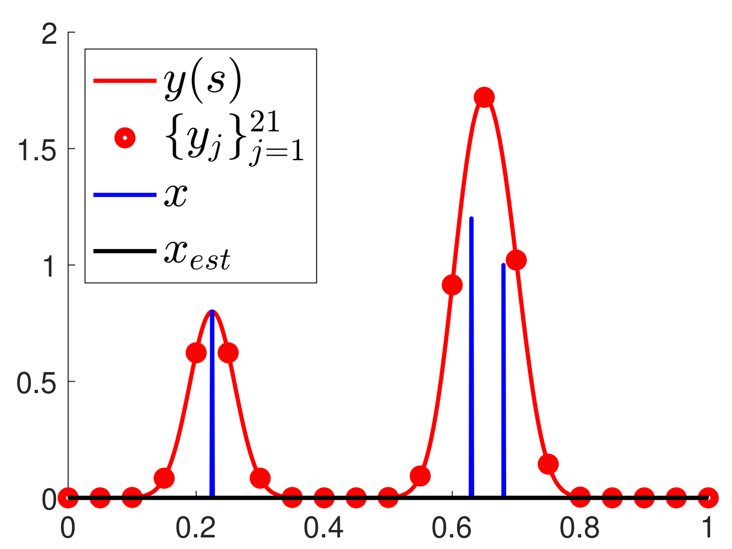
3. Calculate the next iterate λ_{p+1} as the projection of λ_p on the level set $\mathcal{L}_p(\alpha)=\{\lambda\in Q \mid \hat{f}_p(\lambda)\leq\ell_p(\alpha)\}$ where $\ell_p(\alpha)=(1-\alpha)\hat{f}_p^*+\alpha f_p^*$ for some $\alpha\in(0,1)$.

Results – simulated data

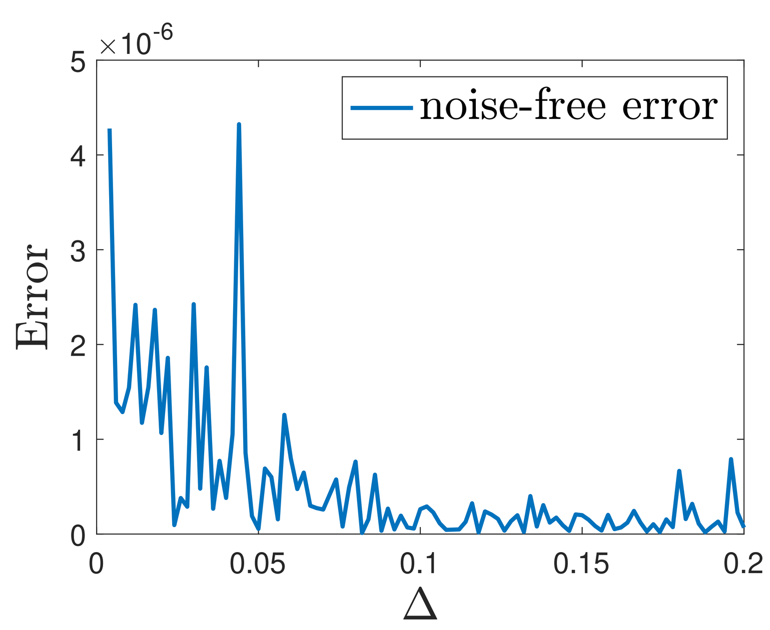
Results – real data

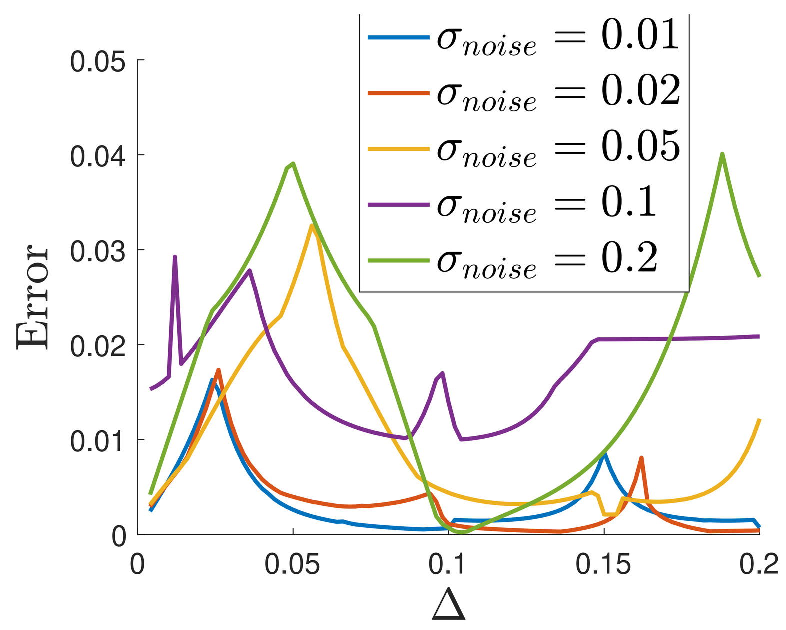
Beads

- **Exact measurements:** the level method finds the correct locations of the point sources with high accuracy – see the localisation error for two sources as the distance between them goes to zero in the bottom left figure.
- **Noisy measurements:** As the distance between two sources decreases, they are replaced with one source with higher intensity in the reconstructed signal.



Example of setup with noise-free measurements, solved to accuracy 10^{-6}.

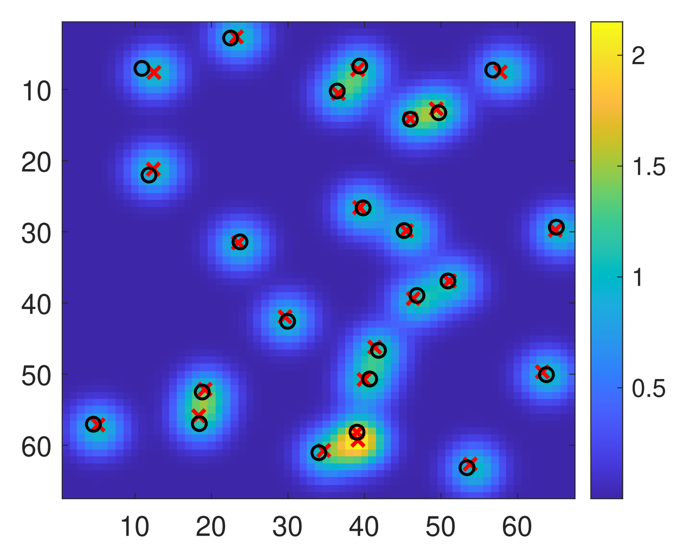




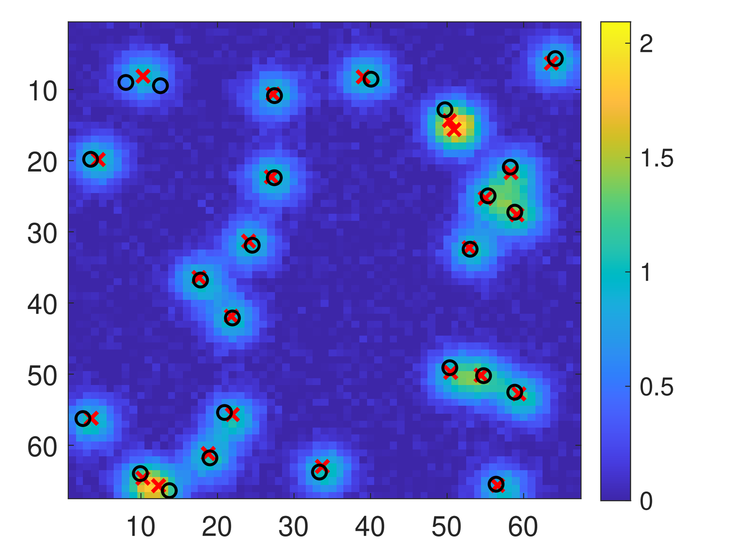
Localisation error for two point sources as the distance Δ between them increases from 0.004 to 0.2 in the noise-free case (left) and noisy case for Gaussian noise with standard deviation 0.01, 0.02, 0.05, 0.1, 0.2 (right). The convolution kernel is Gaussian with $\sigma=0.05$, intensities are 1 and $m=21$ equispaced samples.

Marchantia

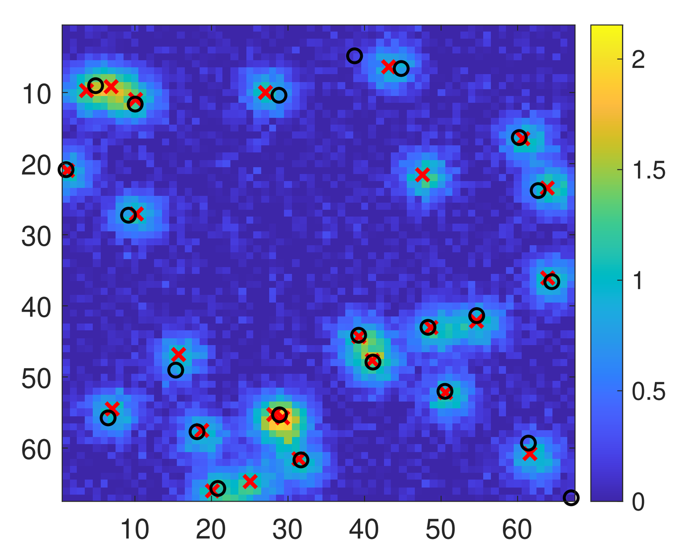
Examples of $k=25$ sources in an image with 67×67 pixels (giving $m=4489$ measurements), Gaussian kernel with $\sigma=0.05$ and level method parameters $\Pi=100$ and $B=100$. Red crosses × are true sources and black circles ○ are estimated sources.



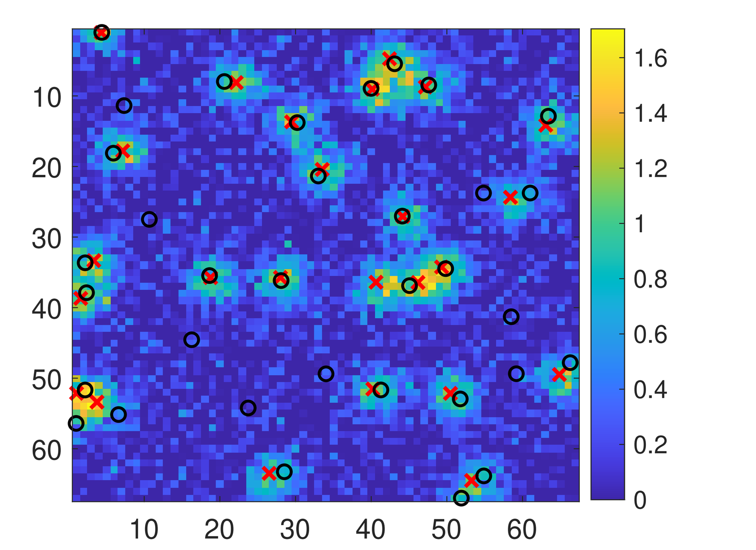
Noise-free measurements.



Gaussian noise with standard deviation 0.05.



Gaussian noise with standard deviation 0.1.



Gaussian noise with standard deviation 0.2.

References

[1] Y. Nesterov. *Introductory Lectures on Convex Optimization: A Basic Course*. Springer Publishing Company, 2014.

Acknowledgments

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