An Approach for Incremental Updating Approximations in Variable Precision Rough Sets while Attribute Generalized

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***Abstract*—Rough set theory (RST) for knowledge updating have been successfully applied in data mining and it's correlative domains. As a special type of probabilistic rough set model, Variable precision rough sets (VPRS) model is an extension of RST. For an information system, the VPRS model allows a flexible approximation boundary region by using a precision variable and has a better tolerance ability for inconsistent data. However, the approximations of a concept may change when an information system varies. The approach for incremental updating of approximations while attribute generalizing in VPRS should be considered. In this paper, an incremental model and its algorithm for updating approximations of a concept based on VPRS are proposed when attribute generalized. Examples are employed to validate the feasibility of this approach.**

***Keywords*—Granular Computing, Variable Precision Rough Sets, Probabilistic rough sets, Incremental Updating, Approximations.**

I. Introduction

Granular computing (GrC), introduced by Zadeh and Lin [11], [12], has been widely and successfully used in machine learning, pattern recognition and data mining. The three most essential theories of GrC are Fuzzy Logic Theory (Word Computing), Rough Set Theory (RST) and Quotient Space Theory. The GrC realized by RST has been widely and successfully developed.

RST, introduced by Z. Pawlak in 1982, is a powerful mathematical tool for dealing with inexact, uncertain or vague information in decision situations [13], [14], [15], [16], [17]. The idea of RST is based on the indiscernibility relation and any set of all indiscernible objects is called an elementary set, which forms basic granule of knowledge about the universe. Variable Precision Rough Sets (VPRS) model was proposed by Ziarko due to errors existing in the information system [18].

Today, data are growing at an unprecedented rate. The information system evolves over time. Approaches for updating knowledge incrementally are getting more and more popular. An information system is composed by the attributes, the objects, and the domain of attributes' values. The previous work for updating knowledge from information systems are mainly focused on these three aspects: the attribute (features) generalized [1], [2], [3], [4], the objects (instances) changed [5], [6], [7], [8] and the attributes' values changed [9], [10]. All these work may help people to obtain the dynamic knowledge with different viewpoints.

In this paper, we focus on the first case when the attributes generalized. The variation of the attributes may affect the granularity of the knowledge space. Decision rules induced by lower and upper approximations may also alter. In the earlier studies, a method for updating approximations of a concept incrementally was discussed by Chan [1] when adding or deleting an attribute in a complete system. A method for updating approximations in an incomplete information system based on the characteristic relation was presented by Li et al [2] when an attribute set varies over time. However, the errors in the system are not considered in their work. Our study here is to investigate the method for incremental learning knowledge based on VPRS which can deal with errors in the complete information system.

The remaining of the paper is organized as follows. Section 2 includes the elementary background introduction to VPRS. The approach and algorithm for updating approximations of a concept in the complete information system are presented in Section 3. Section 4 shows an example to validate the proposed models. The paper ends with conclusions and further research topics in Section 5.

II. Preliminaries

In this section, we first briefly introduce some basic concepts, notations and results of rough sets as well as their extensions [13], [14], [18], [19], [20], [21], [22], [23], [24], [25].

Definition 1: A complete information system is defined as a tuple $S = (\mathbb{U}, \mathbb{A}, V, f)$, where $\mathbb{U}$ is a non-empty finite set of objects, called the universe. $\mathbb{A}$ is a non-empty finite set of attributes, $\mathbb{A} = C \bigcup D$, C denotes the set of condition attributes and D denotes the set of decision attributes,

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$C \bigcap D = \emptyset$. $V = \bigcup_{a \in \mathbb{A}} V_a$, where V_a is a domain of the attribute a. $f : \mathbb{U} \times \mathbb{A} \to V$ is an information function such that $f(x, a) \in V_a$ for every $x \in \mathbb{U}$, $a \in \mathbb{A}$. $f(x_i, a_j)$ denotes the value of object x_i on attribute a_j.

Definition 2: Let $B \subseteq \mathbb{A}$ be a subset of attributes. The information set for any object $x \in \mathbb{U}$ can be denoted by

$$\overrightarrow{x}_B = \{f(x, a) : a \in B\} \tag{1}$$

An equivalence relation called the indiscernibility relation, denoted by $IND(B)$, is defined as

$$IND(B) = \left\{(x, y)|\ (x, y) \in \mathbb{U} \times \mathbb{U},\ \overrightarrow{x}_B = \overrightarrow{y}_B\right\} \tag{2}$$

Two objects x, y satisfying the relation $IND(B)$ are indiscernible by attributes from B.

Definition 3: Let $B \subseteq \mathbb{A}$ be a subset of attributes. The information set for any $E \in \mathbb{U}/B$ could be denoted by

$$\overrightarrow{E}_B = \overrightarrow{x}_B,\ x \in E \tag{3}$$

where, $\forall x, y \in E$, $\overrightarrow{x}_B = \overrightarrow{y}_B$.

Definition 4: Let X and Y be non-empty subsets of the universe $\mathbb{U}$. Let

$$c(X, Y) = \begin{cases} 1 - |X \bigcap Y| / |X|, & |X| > 0 \\ 0, & |X| = 0 \end{cases} \tag{4}$$

where $|\bullet|$ denotes the cardinality of a set, $c(X, Y)$ is called the relative degree of misclassification.

Definition 5: Let $0 \leq \beta < 0.5$. The majority inclusion relation is defined as

$$X \overset{\beta}{\subseteq} Y \Leftrightarrow c(X, Y) \leq \beta \tag{5}$$

"Majority" requires that the number of public elements between X and Y be larger than half of the number of elements in X.

Definition 6: Let $X \subseteq \mathbb{U}$, and R be an equivalence relation. $\mathbb{U}/R = \{E_1, E_2, \cdots, E_t\}$. The β-lower approximation of X is defined as:

$$\underline{R}_\beta(X) = \bigcup \left\{E \in \mathbb{U}/R|\ E \overset{\beta}{\subseteq} X\right\} \tag{6}$$

or, equivalently,

$$\underline{R}_\beta(X) = \bigcup \{E \in \mathbb{U}/R|\ c(E, X) \leq \beta\} \tag{7}$$

The β-upper approximation of X is defined as

$$\overline{R}_\beta(X) = \bigcup \{E \in \mathbb{U}/R|\ c(E, X) < 1 - \beta\} \tag{8}$$

The β-boundary region of X is defined as

$$BND_\beta(X) = \bigcup \{E \in \mathbb{U}/R|\ \beta < c(E, X) < 1 - \beta\} \tag{9}$$

Proposition 1: Let $B, P \subseteq \mathbb{A}$, $B \bigcap P = \emptyset$, and $\mathbb{U}/IND(B) = \{E_1, E_2, \cdots, E_t\}$. Then, we have:

$$\mathbb{U}/IND(B \bigcup P) = \{E_{ij}|i = 1, 2, \cdots, t; j = 1, 2, \cdots, l_i\}$$

where $\forall i \in \{1, 2, \cdots, t\}, E_i/P = \{E_{i1}, E_{i2}, \cdots, E_{il_i}\}$.

Proposition 2: Let $P \subset B \subseteq \mathbb{A}$, and $\mathbb{U}/IND(B) = \{E_1, E_2, \cdots, E_t\}$. Suppose $E, F \in \mathbb{U}/IND(B)$, and E, F are different equivalence classes, it means $\overrightarrow{E}_B \neq \overrightarrow{F}_B$. Then, E and F can be combined into a equivalence class, denoted by $CB(E, F)$.

$$CB(E, F) = \begin{cases} 0, & \overrightarrow{E}_P = \overrightarrow{F}_P \ or\ \overrightarrow{E}_{(B-P)} \neq \overrightarrow{F}_{(B-P)} \\ 1, & \overrightarrow{E}_P \neq \overrightarrow{F}_P \ and\ \overrightarrow{E}_{(B-P)} = \overrightarrow{F}_{(B-P)} \end{cases}$$

where, "0" denotes E, F can't be combined into one equivalence class, and "1" denotes E, F can be combined into one equivalence class.

III. Updating β-Upper and β-Lower Approximations Incrementally

As mentioned above, the approach shows that by using the equivalence classes we can indeed realize updating approximations incrementally in the complete information system. This feature can contribute to handle dynamic attribute generalization effectively, because the upper and lower approximations of a concept are defined according to a specific attribute set and they will be changed if the attribute set evolves with time. In other words, knowledge needs updating. Based on this method, we present the process of updating the approximations of a concept incrementally in the complete information system when the attribute set is changing dynamically in VPRS.

A. The incremental algorithm

A complete information system $S = (\mathbb{U}, B \bigcup D, V, f)$, where U is a non-empty set with n objects, $B \bigcap D = \emptyset$, B is a set of condition attributes and D is a set of decision attributes. $\mathbb{U}/D = \{D_1, \cdots, D_j, \cdots, D_v\}$, $\mathbb{U}/B = \{E_1, \cdots, E_i, \cdots, E_u\}$.

At first, we update the equivalence classes incrementally. Then we update the β-upper and β-lower approximations incrementally. The simple flowchart of the incremental algorithm for updating approximations is shown in Fig.1.

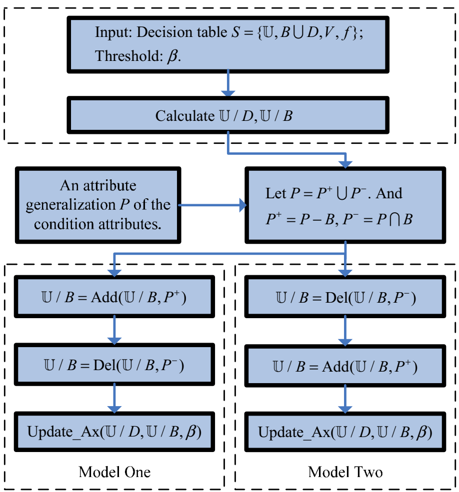

Fig. 1. The flowchart of the incremental algorithm

Here, we give two models. For an attribute generalization P of the condition attributes, let $P = P^{+} \bigcup P^{-}$, where $P^{+} = P - B$ and $P^{-} = P \bigcap B$. It means the attribute set P^{+} is added into B, and the attribute set P^{-} is deleted from B.

Model One: First, add P^{+} into B; Second, delete P^{-} from B.

Model Two: First, delete P^{-} from B; Second, add P^{+} into B.

B. A pseudo-incremental algorithm

Here, we present the pseudo-incremental algorithms. Algorithm 1 is the algorithm of adding an attribute set; Algorithm 2 is the algorithm of deleting an attribute set; Algorithm 3 is the algorithm of updating the approximations.

Algorithm 1: Add the attribute set P into B

Function: Add($\mathbb{U}/B$ **,** P**)**
begin
  for *each* $E_i \in \mathbb{U}/B$ **do**
    compute $E_i/P = \{E_{i1,} E_{i1,} \cdots, E_{il_i}\}$;
  end
  let $B^{+} = B \bigcup P$ **then**
  $\mathbb{U}/B^{+} = \{E_{ij} | i = 1, 2, \cdots, u; j = 1, 2, \cdots, l_i\}$;
  return $\mathbb{U}/B^{+}$
end

Algorithm 2: Del the attribute set P from B

Function: Del($\mathbb{U}/B$ **,** P**)**
begin
  initialize: s=0 and $B_s^{-} = \emptyset$;
  for *each* $E_i \in U/B$ **do**
    let $IsFind =$ **false**;
    for $k = 1$ *to* s **do**
      if $\overrightarrow{E_{iP}} \neq \overrightarrow{B_{k\ P}^{-}}$ *and* $\overrightarrow{E_{i(B-P)}} == \overrightarrow{B_{k\ (B-P)}^{-}}$ **then**
        $IsFind =$ **true**;
        break;
      end
    end
    if $IsFind ==$ ***true*** **then**
      $B_k^{-} \leftarrow B_k^{-} \bigcup E_i$;
    else
      $s = s{+}1$ and $B_s^{-} \leftarrow E_i$;
    end
  end
  let $B^{-} = B - P$ **then**
  $\mathbb{U}/B^{-} = \{B_k^{-} | k = 1, 2, \cdots, s\}$;
  return $\mathbb{U}/B^{-}$;
end

Algorithm 3: Update Approximations

Function: Update_Ax($\mathbb{U}/B$ **,** $\mathbb{U}/D$**,** β**)**
begin
  for *each* $D_j \in \mathbb{U}/D$ **do**
    initialize: $\underline{R}_{\beta}(D_j) = \emptyset$ and $\overline{R}_{\beta}(D_j) = \emptyset$
    for *each* $E_i \in \mathbb{U}/B$ **do**
      if $1 - |E_i \bigcap D_j|/|E_i| \leq \beta$ **then**
        $\underline{R}_{\beta}(D_j) \leftarrow \underline{R}_{\beta}(D_j) \bigcup E_i$
      end
      if $1 - |E_i \bigcap D_j|/|E_i| < 1 - \beta$ **then**
        $\overline{R}_{\beta}(D_j) \leftarrow \overline{R}_{\beta}(D_j) \bigcup E_i$
      end
    end
  end
  return $\underline{R}_{\beta}(D_j)$ and $\overline{R}_{\beta}(D_j)$ $(\forall D_j \in \mathbb{U}/D)$
end

TABLE I
A COMPLETE INFORMATION SYSTEM

U	a_1	a_2	a_3	a_4	a_5	d
x_1	1	1	1	1	3	Y
x_2	1	1	1	2	2	Y
x_3	1	1	2	1	3	Y
x_4	1	1	2	2	1	Y
x_5	1	2	1	1	3	Y
x_6	1	2	1	2	2	Y
x_7	1	2	2	1	3	N
x_8	1	2	2	2	1	N
x_9	2	1	1	1	3	N
x_{10}	2	1	1	2	2	N
x_{11}	2	2	1	1	3	N
x_{12}	2	2	1	2	2	N

IV. AN ILLUSTRATION

In this section, we provide an example to show how to use the above approach and algorithm to update the interesting knowledge dynamically. The information system is given in Table 1, let $\mathbb{U} = \{x_1, x_2, \cdots, x_{12}\}$, $B = \{a_1, a_3, a_5\}$, $D = \{d\}$. Then, $\mathbb{U}/B = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, $\mathbb{U}/D = \{D_1, D_2\} = \{Y, N\}$, where $E_1 = \{x_1, x_5\}$, $E_2 = \{x_2, x_6\}$, $E_3 = \{x_3, x_7\}$, $E_4 = \{x_4, x_8\}$, $E_5 = \{x_9, x_{11}\}$, $E_6 = \{x_{10}, x_{12}\}$, $D_1 = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $D_2 = \{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$. Now, an attribute generalization $P = \{a_1, a_2, a_3, a_4\} = P^{-} \bigcup P^{+}$, where $P^{-} = \{a_1, a_3\}$ and $P^{+} = \{a_2, a_4\}$. Here, it means the attribute set P^{-} is deleted from B, and the attribute set P^{+} is added into B.

A. Model One

(1). Add P^{+} into B.

Let $B^{+} = B \bigcup P^{+} = \{a_1, a_2, a_3, a_4, a_5\}$.

Since $\mathbb{U}/B = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, according to Proposition 1, we can calculate $\mathbb{U}/B^{+}$ as followis.

$$\begin{aligned} E_1/P^{+} &= \{\{x_1\}, \{x_5\}\} = \{E_{11}, E_{12}\} \\ E_2/P^{+} &= \{\{x_2\}, \{x_6\}\} = \{E_{21}, E_{22}\} \\ E_3/P^{+} &= \{\{x_3\}, \{x_7\}\} = \{E_{31}, E_{32}\} \end{aligned}$$

$$\begin{aligned}E_4/P^+ &= \{\{x_4\},\{x_8\}\} = \{E_{41},E_{42}\}\\ E_5/P^+ &= \{\{x_9\},\{x_{11}\} = \{E_{51},E_{52}\}\}\\ E_6/P^+ &= \{\{x_{10}\},\{x_{12}\}\} = \{E_{61},E_{62}\}\end{aligned}$$

Above all,

$$\begin{aligned}\mathbb{U}/B^+ &= \{E_{11},E_{12},E_{21},E_{22},E_{31},E_{32},\\ &\quad E_{41},E_{42},E_{51},E_{52},E_{61},E_{62}\}\\ &= \{F_1,F_2,\cdots,F_{12}\}.\end{aligned}$$

where, $F_1 = \{x_1\}$, $F_2 = \{x_2\}$, $F_3 = \{x_3\}$, $F_4 = \{x_4\}$, $F_5 = \{x_5\}$, $F_6 = \{x_6\}$, $F_7 = \{x_7\}$, $F_8 = \{x_8\}$, $F_9 = \{x_9\}$, $F_{10} = \{x_{10}\}$, $F_{11} = \{x_{11}\}$, $F_{12} = \{x_{12}\}$.

(2). Delete P^- from B^+.

Let $B^* = B^+ - P^- = \{a_2,a_4,a_5\}$.

Since $\mathbb{U}/B^+ = \{F_1,F_2,\cdots,F_{12}\}$, according to Proposition 2, we calculate $\mathbb{U}/B^*$. Due to

$$\begin{aligned}\overrightharpoon{F}_{1B^*} &= \overrightharpoon{F}_{3B^*} = \overrightharpoon{F}_{9B^*}\\ \overrightharpoon{F}_{2B^*} &= \overrightharpoon{F}_{10B^*}\\ \overrightharpoon{F}_{5B^*} &= \overrightharpoon{F}_{7B^*} = \overrightharpoon{F}_{11B^*}\\ \overrightharpoon{F}_{6B^*} &= \overrightharpoon{F}_{12B^*}\end{aligned}$$

So, F_1, F_3 and F_9 can be combined into one equivalence class; F_2 and F_{10} can be combined into one equivalence class; F_5, F_7 and F_{11} can be combined into one equivalence class; F_6 and F_{12} can be combined into one equivalence class.

Above all,

$$\begin{aligned}\mathbb{U}/B^* &= \{F_1\bigcup F_3\bigcup F_9, F_2\bigcup F_{10}, F_4,\\ &\quad F_5\bigcup F_7\bigcup F_{11}, F_6\bigcup F_{12}, F_8\}\\ &= \{G_1,G_2,G_3,G_4,G_5,G_6\}.\end{aligned}$$

where, $G_1 = F_1\bigcup F_3\bigcup F_9 = \{x_1,x_3,x_9\}$, $G_2 = F_2\bigcup F_{10} = \{x_2,x_{10}\}$, $G_3 = F_4 = \{x_4\}$, $G_4 = F_5\bigcup F_7\bigcup F_{11} = \{x_5,x_7,x_{11}\}$, $G_5 = F_6\bigcup F_{12} = \{x_6,x_{12}\}$, $G_6 = F_8 = \{x_8\}$.

(3) Update Approximations. Let $\beta = 0.4$.

According to Algorithm 3, we update β-lower approximation and β-upper approximation.

Since $\mathbb{U}/D = \{D_1,D_2\}$ and $\mathbb{U}/B^* = \{G_1,G_2,G_3,G_4,G_5,$
$G_6\}$, then we have:

$$\begin{array}{ll} c(G_1,D_1) = 1/3 & c(G_1,D_2) = 2/3\\ c(G_2,D_1) = 1/2 & c(G_2,D_2) = 1/2\\ c(G_3,D_1) = 0 & c(G_3,D_2) = 1\\ c(G_4,D_1) = 2/3 & c(G_4,D_2) = 1/3\\ c(G_5,D_1) = 1/2 & c(G_5,D_2) = 1/2\\ c(G_6,D_1) = 1 & c(G_6,D_2) = 0\end{array}$$

So, the β-lower approximation and β-upper approximation of D_1, D_2 are calculated as follows.

$$\begin{aligned}\underline{R}^*_{0.4}(D_1) &= \bigcup\{G\in\mathbb{U}/B^*|\ c(G,D_1)\le 0.4\}\\ &= G_1\bigcup G_3 = \{x_1,x_3,x_9,x_4\}\\ \overline{R}^*_{0.4}(D_1) &= \bigcup\{G\in\mathbb{U}/B^*|\ c(G,D_1)< 0.6\}\\ &= G_1\bigcup G_2\bigcup G_3\bigcup G_5\\ &= \{x_1,x_3,x_9,x_2,x_{10},x_4,x_6,x_{12}\}\\ \underline{R}^*_{0.4}(D_2) &= \bigcup\{G\in\mathbb{U}/B^*|\ c(G,D_2)\le 0.4\}\\ &= G_4\bigcup G_6 = \{x_5,x_7,x_{11},x_8\}\\ \overline{R}^*_{0.4}(D_2) &= \bigcup\{G\in\mathbb{U}/B^*|\ c(G,D_2)< 0.6\}\\ &= G_2\bigcup G_4\bigcup G_5\bigcup G_6\\ &= \{x_2,x_{10},x_4,x_5,x_7,x_{11},x_6,x_{12},x_8\}\end{aligned}$$

B. Model Two

(1). Delete P^- from B.

Let $B^- = B - P^- = \{a_5\}$.

Since $\mathbb{U}/B = \{E_1,E_2,E_3,E_4,E_5,E_6\}$, according to Proposition 2, we calculate $\mathbb{U}/B^-$. Due to

$$\begin{aligned}\overrightharpoon{E}_{1B^-} &= \overrightharpoon{E}_{3B^-} = \overrightharpoon{E}_{5B^-}\\ \overrightharpoon{E}_{2B^-} &= \overrightharpoon{E}_{6B^-}\end{aligned}$$

So, E_1, E_3 and E_5 can be combined into one equivalence class; E_2 and E_6 can be combined into one equivalence class.

Above all,

$$\begin{aligned}\mathbb{U}/B^- &= \{E_1\bigcup E_3\bigcup E_5, E_2\bigcup E_6, E_4\}\\ &= \{F_1,F_2,F_3\}.\end{aligned}$$

where, $F_1 = E_1\bigcup E_3\bigcup E_5 = \{x_1,x_3,x_5,x_7,x_9,x_{11}\}$, $F_2 = E_2\bigcup E_6 = \{x_2,x_6,x_{10},x_{12}\}$, $F_3 = E_4 = \{x_4,x_8\}$.

(2). Add P^+ into B^-.

Let $B^* = B^-\bigcup P^+ = \{a_2,a_4,a_5\}$.

Since $\mathbb{U}/B^- = \{F_1,F_2,F_3\}$, according to Proposition 1, we calculate $\mathbb{U}/B^*$.

$$\begin{aligned}F_1/P^+ &= \{\{x_1,x_3,x_9\},\{x_5,x_7,x_{11}\}\} = \{F_{11},F_{12}\}\\ F_2/P^+ &= \{\{x_2,x_{10}\},\{x_6,x_{12}\}\} = \{F_{21},F_{22}\}\\ F_3/P^+ &= \{\{x_4\},\{x_8\}\} = \{F_{31},F_{32}\}\end{aligned}$$

Above all,

$$\begin{aligned}\mathbb{U}/B^* &= \{E_{11},E_{21},E_{31},E_{12},E_{22},E_{32}\}\\ &= \{G_1,G_2,G_3,G_4,G_5,G_6\}.\end{aligned}$$

where, $G_1 = E_{11} = \{x_1,x_3,x_9\}$, $G_2 = E_{21} = \{x_2,x_{10}\}$, $G_3 = E_{31} = \{x_4\}$, $G_4 = E_{12} = \{x_5,x_7,x_{11}\}$, $G_5 = E_{22} = \{x_6,x_{12}\}$, $G_6 = E_{32} = \{x_8\}$.

(3). Update Approximations. Let $\beta = 0.3$.

According to Algorithm 3, we update β-lower approximation and β-upper approximation.

Since $\mathbb{U}/D = \{D_1, D_2\}$ and $\mathbb{U}/B^* = \{G_1, G_2, G_3, G_4, G_5, G_6\}$, then

$$\begin{array}{ll} c(G_1,D_1)=1/3 & c(G_1,D_2)=2/3 \\ c(G_2,D_1)=1/2 & c(G_2,D_2)=1/2 \\ c(G_3,D_1)=0 & c(G_3,D_2)=1 \\ c(G_4,D_1)=2/3 & c(G_4,D_2)=1/3 \\ c(G_5,D_1)=1/2 & c(G_5,D_2)=1/2 \\ c(G_6,D_1)=1 & c(G_6,D_2)=0 \end{array}$$

So, the β-lower approximation and β-upper approximation of D_1, D_2 are calculated as follows.

$$\begin{aligned} \underline{R}^*_{0.3}(D_1) &= \bigcup\{G \in \mathbb{U}/B^* |\ c(G,D_1) \le 0.3\} \\ &= G_3 = \{x_4\} \\ \overline{R}^*_{0.3}(D_1) &= \bigcup\{G \in \mathbb{U}/B^* |\ c(G,D_1) < 0.7\} \\ &= G_1 \bigcup G_2 \bigcup G_3 \bigcup G_4 \bigcup G_5 \\ &= \{x_1,x_3,x_9,x_2,x_{10},x_4,x_5,x_7,x_{11},x_6,x_{12}\} \\ \underline{R}^*_{0.3}(D_2) &= \bigcup\{G \in \mathbb{U}/B^* |\ c(G,D_2) \le 0.3\} \\ &= G_6 = \{x_8\} \\ \overline{R}^*_{0.3}(D_2) &= \bigcup\{G \in \mathbb{U}/B^* |\ c(G,D_2) < 0.7\} \\ &= G_1 \bigcup G_2 \bigcup G_4 \bigcup G_5 \bigcup G_6 \\ &= \{x_1,x_3,x_9,x_2,x_{10},x_4,x_5,x_7,x_{11},x_6,x_{12},x_8\} \end{aligned}$$

To sum up, the two incremental models give an intuitive understanding for attributes updating, which help the people to make a fast and reliable decision making and extraction of relevant knowledge from dynamically changing massive databases.

V. Conclusion

This paper presents a pseudo-incremental algorithm for incremental updating approximations of a concept in the information system based on VPRS. It is obvious that we can obtain the updated approximations by the previous results without beginning from scratch. Examples are employed to validate this approach. In the future work, we will simulate the proposed algorithm by using computer programs, the semantic analysis as well as the applications of the proposed approach in real data mining systems will be also carried out.

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