Exercise 1*

Draw a graphical representation of the complete lattice $(2^{\{a,b,c\}}, \subseteq)$ and compute supremum and infimum of the following sets:

1. $\sqcap\{\{a\}, \{b\}\} = ?$

2. $\sqcup\{\{a\}, \{b\}\} = ?$

3. $\sqcap\{\{a\}, \{a,b\}, \{a,c\}\} = ?$

4. $\sqcup\{\{a\}, \{a,b\}, \{a,c\}\} = ?$

5. $\sqcap\{\{a\}, \{b\}, \{c\}\} = ?$

6. $\sqcup\{\{a\}, \{b\}, \{c\}\} = ?$

7. $\sqcap\{\{a\}, \{a,b\}, \{b\}, \emptyset\} = ?$

8. $\sqcup\{\{a\}, \{a,b\}, \{b\}, \emptyset\} = ?$

Exercise 2

Prove that for any partially ordered set $(D, \sqsubseteq)$ and any $X \subseteq D$, if supremum of X ($\sqcup X$) and infimum of X ($\sqcap X$) exist then they are uniquely defined. (Hint: use the definition of supremum and infimum and antisymmetry of $\sqsubseteq$.)
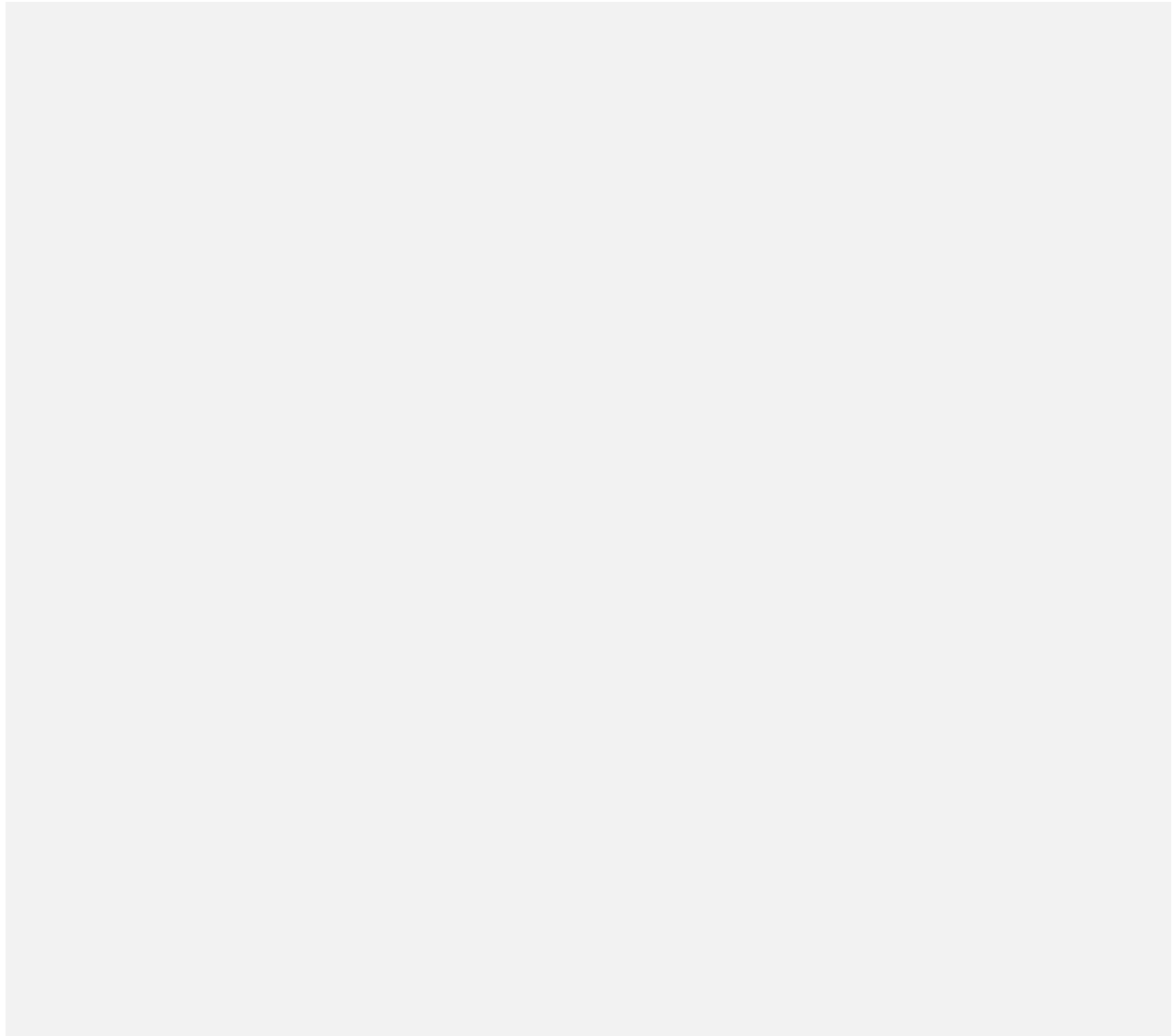
Exercise 3

Let $(D, \sqsubseteq)$ be a complete lattice. What are $\sqcup \emptyset$ and $\sqcap \emptyset$ equal to?

Exercise 4*

Consider the complete lattice $(2^{\{a,b,c\}}, \subseteq)$. Define a function $f : 2^{\{a,b,c\}} \rightarrow 2^{\{a,b,c\}}$ such that f is monotonic.
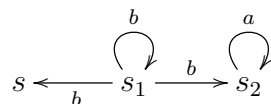
- Compute the greatest fixed point by using directly the Tarski's fixed point theorem.

- Compute the least fixed point by using the Tarski's fixed point theorem for finite lattices (i.e. by starting from $\bot$ and by applying repeatedly the function f until the fixed point is reached).
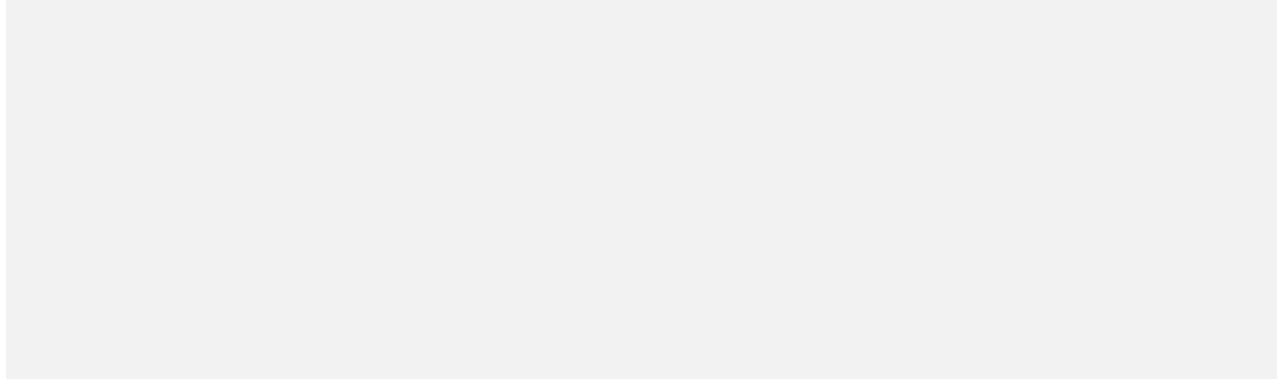
Exercise 5

Consider the following labelled transition system.

$$s \xleftarrow{\quad b \quad} s_1 \xrightarrow{\quad b \quad} s_2$$

with $s_1 \overset{b}{\circlearrowright}$ and $s_2 \overset{a}{\circlearrowright}$

Compute for which sets of states $[\![X]\!] \subseteq \{s, s_1, s_2\}$ the following formulae are true.

- $X = \langle a \rangle t\!\!t \vee [b]X$

- $X = \langle a \rangle t\!\!t \vee ([b]X \wedge \langle b \rangle t\!\!t)$

Exercise 6 (optional)

Solve Exercise 4.10 from *Reactive Systems*, page 83.