Svag treprotonig syra + svag enprotonig bas

$$H_3A + H_2O \rightleftarrows H_2A^- + H_3O^+$$

$$H_2A^- + H_2O \rightleftarrows HA^{2-} + H_3O^+$$

$$HA^{2-} + H_2O \rightleftarrows A^{3-} + H_3O^+$$

$$K_{a_1} = \frac{[H_2A^-][H_3O^+]}{[H_3A]_2} \Leftrightarrow [H_2A^-] = \frac{K_{a_1}[H_3A]_2}{[H_3O^+]}$$

$$K_{a_2} = \frac{[HA^{2-}][H_3O^+]}{[H_2A^-]} \Leftrightarrow [HA^{2-}] = \frac{K_{a_2}[H_2A^-]}{[H_3O^+]}$$

$$K_{a_3} = \frac{[A^{3-}][H_3O^+]}{[HA^{2-}]} \Leftrightarrow [A^{3-}] = \frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 + [H_2A^-] + [HA^{2-}] + [A^{3-}]$$

$$[H_3A]_1 = [H_3A]_2 + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{K_{a_2}[H_2A^-]}{[H_3O^+]} + \frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]} + \frac{K_{a_3}\frac{K_{a_2}\frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]} + \frac{K_{a_3}\frac{K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 \left(1 + \frac{K_{a_1}}{[H_3O^+]} + \frac{K_{a_2}K_{a_1}}{[H_3O^+]^2} + \frac{K_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]^3}\right) \Leftrightarrow$$

$$[H_3A]_2 = \frac{[H_3A]_1}{1 + \frac{K_{a_1}}{[H_3O^+]} + \frac{K_{a_2}K_{a_1}}{[H_3O^+]^2} + \frac{K_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]^3}} = \frac{[H_3A]_1}{\frac{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]^3}}$$

$$= \frac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$K_w = [OH^-][H_3O^+] \Leftrightarrow [OH^-] = \frac{K_w}{[H_3O^+]}$$

$$B + H_2O \rightleftarrows HB^+ + OH^-$$

$$K_b = \frac{[HB^+][OH^-]}{[B]_2} \Leftrightarrow [HB^+] = \frac{K_b[B]_2}{[OH^-]} = \frac{K_b[B]_2}{\frac{K_w}{[H_3O^+]}} = \frac{K_b[B]_2[H_3O^+]}{K_w}$$

$$[B]_1 = [B]_2 + [HB^+] = [B]_2 + \frac{K_b[B]_2}{[OH^-]} = [B]_2\left(1 + \frac{K_b}{[OH^-]}\right) = [B]_2\left(\frac{[OH^-] + K_b}{[OH^-]}\right) \Leftrightarrow$$

$$[B]_2 = \frac{[B]_1}{\left(\frac{[OH^-] + K_b}{[OH^-]}\right)} = \frac{[B]_1[OH^-]}{[OH^-] + K_b}$$

$$[B]_2 = \frac{[B]_1\frac{K_w}{[H_3O^+]}}{\frac{K_w}{[H_3O^+]} + K_b} = \frac{\frac{[B]_1K_w}{[H_3O^+]}}{\frac{K_w + K_b[H_3O^+]}{[H_3O^+]}} = \frac{[B]_1K_w}{K_w + K_b[H_3O^+]}$$

$$[HB^+] + [H_3O^+] = [OH^-] + [H_2A^-]_2 + 2[HA^{2-}] + 3[A^{3-}]$$

$$\frac{K_b[B]_2[H_3O^+]}{K_w} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]}{[H_3O^+]} + 2\frac{K_{a_2}[H_2A^-]}{[H_3O^+]} + 3\frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+]\dfrac{[B]_1K_w}{K_w + K_b[H_3O^+]}}{K_w} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + 2\frac{K_{a_2}[H_2A^-]}{[H_3O^+]} + 3\frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+][B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{2K_{a_2}\dfrac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]} + \frac{3K_{a_3}\dfrac{K_{a_2}[H_2A^-]}{[H_3O^+]}}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+][B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{2K_{a_2}K_{a_1}[H_3A]_2}{[H_3O^+]^2} + \frac{3K_{a_3}\dfrac{K_{a_2}\dfrac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+][B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{2K_{a_2}K_{a_1}[H_3A]_2}{[H_3O^+]^2} + \frac{3K_{a_3}K_{a_2}K_{a_1}[H_3A]_2}{[H_3O^+]^3}$$

$$\frac{K_b[H_3O^+][B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+]$$

$$= \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}\dfrac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}}{[H_3O^+]}$$

$$+ 2\frac{K_{a_2}\dfrac{K_{a_1}\dfrac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}}{[H_3O^+]}}{[H_3O^+]}$$

$$+ 3\frac{K_{a_3}\dfrac{K_{a_2}\dfrac{K_{a_1}\dfrac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+][B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+]$$

$$= \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_1[H_3O^+]^2}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}} + \frac{2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$+ \frac{3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$\frac{K_b[H_3O^+][B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_1[H_3O^+]^2 + 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] + 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$\frac{K_b[H_3O^+]^2[B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+]^2 = K_w + \frac{K_{a_1}[H_3A]_1[H_3O^+]^3 + 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^2 + 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$\frac{K_b[H_3O^+]^5[B]_1 + K_bK_{a_1}[H_3O^+]^4[B]_1 + K_bK_{a_2}K_{a_1}[H_3O^+]^3[B]_1 + K_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+]^2[B]_1}{K_w\ + K_b[H_3O^+]} + [H_3O^+]^5 + K_{a_1}[H_3O^+]^4 + K_{a_2}K_{a_1}[H_3O^+]^3$$
$$+ K_{a_3}K_{a_2}K_{a_1}[H_3O^+]^2$$
$$= K_w[H_3O^+]^3 + K_wK_{a_1}[H_3O^+]^2 + K_wK_{a_2}K_{a_1}[H_3O^+] + K_wK_{a_3}K_{a_2}K_{a_1} + K_{a_1}[H_3A]_1[H_3O^+]^3 + 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^2$$
$$+ 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]$$

$$K_b[H_3O^+]^5[B]_1 + K_bK_{a_1}[H_3O^+]^4[B]_1 + K_bK_{a_2}K_{a_1}[H_3O^+]^3[B]_1 + K_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+]^2[B]_1 + K_w\,[H_3O^+]^5 + K_w\,K_{a_1}[H_3O^+]^4$$
$$+ K_w\,K_{a_2}K_{a_1}[H_3O^+]^3 + K_w\,K_{a_3}K_{a_2}K_{a_1}[H_3O^+]^2 + K_b[H_3O^+]^6 + K_bK_{a_1}[H_3O^+]^5 + K_bK_{a_2}K_{a_1}[H_3O^+]^4 + K_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+]^3$$
$$= {K_w}^2[H_3O^+]^3 + {K_w}^2K_{a_1}[H_3O^+]^2 + {K_w}^2K_{a_2}K_{a_1}[H_3O^+] + {K_w}^2K_{a_3}K_{a_2}K_{a_1} + K_wK_{a_1}[H_3A]_1[H_3O^+]^3$$
$$+ 2K_wK_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^2 + 3K_wK_{a_3}K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] + K_wK_b[H_3O^+]^4 + K_wK_{a_1}K_b[H_3O^+]^3 + K_wK_bK_{a_2}K_{a_1}[H_3O^+]^2$$
$$+ K_wK_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+] + K_bK_{a_1}[H_3A]_1[H_3O^+]^4 + 2K_bK_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^3 + 3K_bK_{a_3}K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^2$$

$$K_b[H_3O^+]^5[B]_1 + K_bK_{a_1}[H_3O^+]^4[B]_1 + K_bK_{a_2}K_{a_1}[H_3O^+]^3[B]_1 + K_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+]^2[B]_1 + K_w\,[H_3O^+]^5 + K_w\,K_{a_1}[H_3O^+]^4$$
$$+ K_w\,K_{a_2}K_{a_1}[H_3O^+]^3 + K_w\,K_{a_3}K_{a_2}K_{a_1}[H_3O^+]^2 + K_b[H_3O^+]^6 + K_bK_{a_1}[H_3O^+]^5 + K_bK_{a_2}K_{a_1}[H_3O^+]^4 + K_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+]^3$$
$$- {K_w}^2[H_3O^+]^3 - {K_w}^2K_{a_1}[H_3O^+]^2 - {K_w}^2K_{a_2}K_{a_1}[H_3O^+] - {K_w}^2K_{a_3}K_{a_2}K_{a_1} - K_wK_{a_1}[H_3A]_1[H_3O^+]^3$$
$$- 2K_wK_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^2 - 3K_wK_{a_3}K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] - K_wK_b[H_3O^+]^4 - K_wK_{a_1}K_b[H_3O^+]^3 - K_wK_bK_{a_2}K_{a_1}[H_3O^+]^2$$
$$- K_wK_bK_{a_3}K_{a_2}K_{a_1}[H_3O^+] - K_bK_{a_1}[H_3A]_1[H_3O^+]^4 - 2K_bK_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^3 - 3K_bK_{a_3}K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]^2 = 0$$

$$K_b[H_3O^+]^6 + [H_3O^+]^5\left(K_b[B]_1 + K_w\ + K_bK_{a_1}\right) + [H_3O^+]^4\left(K_bK_{a_1}[B]_1 + K_w\,K_{a_1} + K_bK_{a_2}K_{a_1} - K_wK_b - K_bK_{a_1}[H_3A]_1\right)$$
$$+ [H_3O^+]^3\left(K_bK_{a_2}K_{a_1}[B]_1 + K_w\,K_{a_2}K_{a_1} + K_bK_{a_3}K_{a_2}K_{a_1} - {K_w}^2 - K_wK_{a_1}[H_3A]_1 - K_wK_{a_1}K_b - 2K_bK_{a_2}K_{a_1}[H_3A]_1\right)$$
$$+ [H_3O^+]^2\left(K_bK_{a_3}K_{a_2}K_{a_1}[B]_1 + K_w\,K_{a_3}K_{a_2}K_{a_1} - {K_w}^2K_{a_1} - 2K_wK_{a_2}K_{a_1}[H_3A]_1 - K_wK_bK_{a_2}K_{a_1} - 3K_bK_{a_3}K_{a_2}K_{a_1}[H_3A]_1\right)$$
$$- [H_3O^+]\left({K_w}^2K_{a_2}K_{a_1} + 3K_wK_{a_3}K_{a_2}K_{a_1}[H_3A]_1 + K_wK_bK_{a_3}K_{a_2}K_{a_1}\right) - {K_w}^2K_{a_3}K_{a_2}K_{a_1} = 0$$

$$K_b[H_3O^+]^6 + [H_3O^+]^5\left(K_b\left([B]_1 + K_{a_1}\right) + K_w\right) + [H_3O^+]^4\left(K_bK_{a_1}\left([B]_1 - [H_3A]_1 + K_{a_2}\right) + K_w\left(K_{a_1} - K_b\right)\right)$$
$$+ [H_3O^+]^3\left(K_{a_2}K_{a_1}\left(K_b[B]_1 + K_w\ + K_bK_{a_3} - 2K_b[H_3A]_1\right) - K_w\left(K_w + K_{a_1}[H_3A]_1 + K_{a_1}K_b\right)\right)$$
$$+ [H_3O^+]^2\left(K_{a_2}K_{a_1}\left(K_bK_{a_3}[B]_1 + K_w\,K_{a_3} - 2K_w[H_3A]_1 - K_wK_b - 3K_bK_{a_3}[H_3A]_1\right) - {K_w}^2K_{a_1}\right)$$
$$- [H_3O^+]\left(K_wK_{a_2}K_{a_1}\left(K_w + 3K_{a_3}[H_3A]_1 + K_bK_{a_3}\right)\right) - {K_w}^2K_{a_3}K_{a_2}K_{a_1} = 0$$