Svag treprotonig syra + stark bas

$$H_3A + H_2O \rightleftarrows H_2A^- + H_3O^+$$

$$H_2A^- + H_2O \rightleftarrows HA^{2-} + H_3O^+$$

$$HA^{2-} + H_2O \rightleftarrows A^{3-} + H_3O^+$$

$$K_{a_1} = \frac{[H_2A^-][H_3O^+]}{[H_3A]_2} \Leftrightarrow [H_2A^-] = \frac{K_{a_1}[H_3A]_2}{[H_3O^+]}$$

$$K_{a_2} = \frac{[HA^{2-}][H_3O^+]}{[H_2A^-]} \Leftrightarrow [HA^{2-}] = \frac{K_{a_2}[H_2A^-]}{[H_3O^+]}$$

$$K_{a_3} = \frac{[A^{3-}][H_3O^+]}{[HA^{2-}]} \Leftrightarrow [A^{3-}] = \frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$K_w = [OH^-][H_3O^+] \Leftrightarrow [OH^-] = \frac{K_w}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 + [H_2A^-] + [HA^{2-}] + [A^{3-}]$$

$$[H_3A]_1 = [H_3A]_2 + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{K_{a_2}[H_2A^-]}{[H_3O^+]} + \frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]} + \frac{K_{a_3}\frac{K_{a_2}\frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2 + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]} + \frac{K_{a_3}\frac{K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$[H_3A]_1 = [H_3A]_2\left(1 + \frac{K_{a_1}}{[H_3O^+]} + \frac{K_{a_2}K_{a_1}}{[H_3O^+]^2} + \frac{K_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]^3}\right) \Leftrightarrow$$

$$[H_3A]_2 = \frac{[H_3A]_1}{1 + \frac{K_{a_1}}{[H_3O^+]} + \frac{K_{a_2}K_{a_1}}{[H_3O^+]^2} + \frac{K_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]^3}} = \frac{[H_3A]_1}{\frac{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]^3}}$$

$$= \frac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$BOH \xrightarrow{H_2O} B^+ + OH^-$$

$$[BOH]_1 = [B^+]$$

$$[B^+] + [H_3O^+] = [OH^-] + [H_2A^-]_2 + 2[HA^{2-}] + 3[A^{3-}]$$

$$[BOH]_1 + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]}{[H_3O^+]} + 2\frac{K_{a_2}[H_2A^-]}{[H_3O^+]} + 3\frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$[BOH]_1 + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + 2\frac{K_{a_2}[H_2A^-]}{[H_3O^+]} + 3\frac{K_{a_3}[HA^{2-}]}{[H_3O^+]}$$

$$[BOH]_1 + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{2K_{a_2}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]} + \frac{3K_{a_3}\frac{K_{a_2}[H_2A^-]}{[H_3O^+]}}{[H_3O^+]}$$

$$[BOH]_1 + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{2K_{a_2}K_{a_1}[H_3A]_2}{[H_3O^+]^2} + \frac{3K_{a_3}\frac{K_{a_1}[H_3A]_2}{[H_3O^+]}}{[H_3O^+]}$$

$$[BOH]_1 + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_3A]_2}{[H_3O^+]} + \frac{2K_{a_2}K_{a_1}[H_3A]_2}{[H_3O^+]^2} + \frac{3K_{a_3}K_{a_2}K_{a_1}[H_3A]_2}{[H_3O^+]^3}$$

$$[BOH]_1 + [H_3O^+]$$
$$= \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}\dfrac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}}{[H_3O^+]}$$
$$+ 2\,\frac{K_{a_2}\dfrac{K_{a_1}\dfrac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}}{[H_3O^+]}}{[H_3O^+]}$$
$$+ 3\,\frac{K_{a_3}\dfrac{K_{a_2}\dfrac{K_{a_1}\dfrac{[H_3A]_1[H_3O^+]^3}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}}{[H_3O^+]}}{[H_3O^+]}}{[H_3O^+]}$$

$$[BOH]_1 + [H_3O^+] - \frac{K_w}{[H_3O^+]}$$
$$= \frac{K_{a_1}[H_3A]_1[H_3O^+]^2}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}} + \frac{2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+]}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$
$$+ \frac{3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$[BOH]_1 + [H_3O^+] - \frac{K_w}{[H_3O^+]} = \frac{K_{a_1}[H_3A]_1[H_3O^+]^2 + 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] + 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1}{[H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}}$$

$$\left([BOH]_1 + [H_3O^+] - \frac{K_w}{[H_3O^+]}\right)\left([H_3O^+]^3 + K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}\right)$$
$$= K_{a_1}[H_3A]_1[H_3O^+]^2 + 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] + 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1$$

$$[BOH]_1[H_3O^+]^3 + [BOH]_1K_{a_1}[H_3O^+]^2 + [BOH]_1K_{a_2}K_{a_1}[H_3O^+] + [BOH]_1K_{a_3}K_{a_2}K_{a_1} + [H_3O^+]^4 + K_{a_1}[H_3O^+]^3 + K_{a_2}K_{a_1}[H_3O^+]^2$$
$$+ K_{a_3}K_{a_2}K_{a_1}[H_3O^+] - K_w[H_3O^+]^2 - K_wK_{a_1}[H_3O^+] - K_wK_{a_2}K_{a_1} - \frac{K_wK_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]}$$
$$= K_{a_1}[H_3A]_1[H_3O^+]^2 + 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] + 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1$$

$$[H_3O^+]^4 + [BOH]_1[H_3O^+]^3 + K_{a_1}[H_3O^+]^3 + [BOH]_1K_{a_1}[H_3O^+]^2 + K_{a_2}K_{a_1}[H_3O^+]^2 - K_w[H_3O^+]^2 - K_{a_1}[H_3A]_1[H_3O^+]^2$$
$$+ [BOH]_1K_{a_2}K_{a_1}[H_3O^+] + K_{a_3}K_{a_2}K_{a_1}[H_3O^+] - K_wK_{a_1}[H_3O^+] - 2K_{a_2}K_{a_1}[H_3A]_1[H_3O^+] + [BOH]_1K_{a_3}K_{a_2}K_{a_1} - K_wK_{a_2}K_{a_1}$$
$$- 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1 - \frac{K_wK_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]} = 0$$

$$[H_3O^+]^4 + [H_3O^+]^3([BOH]_1 + K_{a_1}) + [H_3O^+]^2([BOH]_1K_{a_1} + K_{a_2}K_{a_1} - K_w - K_{a_1}[H_3A]_1) + [H_3O^+]([BOH]_1K_{a_2}K_{a_1} + K_{a_3}K_{a_2}K_{a_1} - K_wK_{a_1}$$
$$- 2K_{a_2}K_{a_1}[H_3A]_1) + [BOH]_1K_{a_3}K_{a_2}K_{a_1} - K_wK_{a_2}K_{a_1} - 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1 - \frac{K_wK_{a_3}K_{a_2}K_{a_1}}{[H_3O^+]} = 0$$

$$[H_3O^+]^5 + [H_3O^+]^4([BOH]_1 + K_{a_1}) + [H_3O^+]^3([BOH]_1K_{a_1} + K_{a_2}K_{a_1} - K_w - K_{a_1}[H_3A]_1) + [H_3O^+]^2([BOH]_1K_{a_2}K_{a_1} + K_{a_3}K_{a_2}K_{a_1} - K_wK_{a_1}$$
$$- 2K_{a_2}K_{a_1}[H_3A]_1) + [H_3O^+]([BOH]_1K_{a_3}K_{a_2}K_{a_1} - K_wK_{a_2}K_{a_1} - 3K_{a_3}K_{a_2}K_{a_1}[H_3A]_1) - K_wK_{a_3}K_{a_2}K_{a_1} = 0$$

$$[H_3O^+]^5 + [H_3O^+]^4([BOH]_1 + K_{a_1}) + [H_3O^+]^3(K_{a_1}([BOH]_1 - [H_3A]_1 + K_{a_2}) - K_w) + [H_3O^+]^2(K_{a_1}([BOH]_1K_{a_2} + K_{a_3}K_{a_2} - K_w$$
$$- 2K_{a_2}[H_3A]_1)) + [H_3O^+](K_{a_2}K_{a_1}([BOH]_1K_{a_3} - K_w - 3K_{a_3}[H_3A]_1)) - K_wK_{a_3}K_{a_2}K_{a_1} = 0$$