Svag tvåprotonig syra + svag enprotonig bas

$$H_2A + H_2O \rightleftarrows HA^- + H_3O^+$$

$$HA^- + H_2O \rightleftarrows A^{2-} + H_3O^+$$
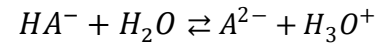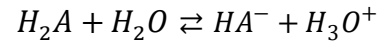
$$K_{a_1} = \frac{[HA^-][H_3O^+]}{[H_2A]_2} \Leftrightarrow [HA^-] = \frac{K_{a_1}[H_2A]_2}{[H_3O^+]}$$

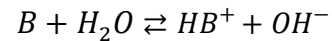$$K_{a_2} = \frac{[A^{2-}][H_3O^+]}{[HA^-]} \Leftrightarrow [A^{2-}] = \frac{K_{a_2}[HA^-]}{[H_3O^+]}$$

$$[H_2A]_1 = [H_2A]_2 + [HA^-] + [A^{2-}]$$

$$[H_2A]_1 = [H_2A]_2 + \frac{K_{a_1}[H_2A]_2}{[H_3O^+]} + \frac{K_{a_2}[HA^-]}{[H_3O^+]} = [H_2A]_2 + \frac{K_{a_1}[H_2A]_2}{[H_3O^+]} + \frac{K_{a_2}\frac{K_{a_1}[H_2A]_2}{[H_3O^+]}}{[H_3O^+]} = [H_2A]_2 + \frac{K_{a_1}[H_2A]_2}{[H_3O^+]} + \frac{K_{a_2}K_{a_1}[H_2A]_2}{[H_3O^+]^2}$$

$$[H_2A]_1 = [H_2A]_2\left(1 + \frac{K_{a_1}}{[H_3O^+]} + \frac{K_{a_2}K_{a_1}}{[H_3O^+]^2}\right) = [H_2A]_2\left(\frac{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}{[H_3O^+]^2}\right) \Leftrightarrow$$

$$[H_2A]_2 = \frac{[H_2A]_1}{\frac{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}{[H_3O^+]^2}} = \frac{[H_2A]_1[H_3O^+]^2}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}$$

$$K_w = [OH^-][H_3O^+] \Leftrightarrow [OH^-] = \frac{K_w}{[H_3O^+]}$$

$$B + H_2O \rightleftarrows HB^+ + OH^-$$

$$K_b = \frac{[HB^+][OH^-]}{[B]_2} \Leftrightarrow [HB^+] = \frac{K_b[B]_2}{[OH^-]} = \frac{K_b[B]_2}{\frac{K_w}{[H_3O^+]}} = \frac{K_b[B]_2[H_3O^+]}{K_w}$$

$$[B]_1 = [B]_2 + [HB^+] = [B]_2 + \frac{K_b[B]_2}{[OH^-]} = [B]_2\left(1 + \frac{K_b}{[OH^-]}\right) = [B]_2\left(\frac{[OH^-] + K_b}{[OH^-]}\right) \Leftrightarrow$$

$$[B]_2 = \frac{[B]_1}{\left(\dfrac{[OH^-] + K_b}{[OH^-]}\right)} = \frac{[B]_1[OH^-]}{[OH^-] + K_b}$$

$$[B]_2 = \frac{[B]_1\dfrac{K_w}{[H_3O^+]}}{\dfrac{K_w}{[H_3O^+]} + K_b} = \frac{\dfrac{[B]_1 K_w}{[H_3O^+]}}{\dfrac{K_w + K_b[H_3O^+]}{[H_3O^+]}} = \frac{[B]_1 K_w}{K_w + K_b[H_3O^+]}$$

$$[HB^+] + [H_3O^+] = [OH^-] + [HA^-] + 2[A^{2-}]$$

$$\frac{K_b[B]_2[H_3O^+]}{K_w} + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_{a_1}[H_2A]_2}{[H_3O^+]} + 2\frac{K_{a_2}[HA^-]}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+][B]_2}{K_w} + [H_3O^+] = \frac{K_w + K_{a_1}[H_2A]_2}{[H_3O^+]} + \frac{2K_{a_2}\dfrac{K_{a_1}[H_2A]_2}{[H_3O^+]}}{[H_3O^+]}$$

$$\frac{K_b[H_3O^+][B]_2}{K_w} + [H_3O^+] = \frac{K_w + K_{a_1}[H_2A]_2}{[H_3O^+]} + \frac{2K_{a_2}K_{a_1}[H_2A]_2}{[H_3O^+]^2}$$

$$\frac{K_b[H_3O^+]^3[B]_2}{K_w} + [H_3O^+]^3 = K_w[H_3O^+] + K_{a_1}[H_3O^+][H_2A]_2 + 2K_{a_2}K_{a_1}[H_2A]_2$$

$$\frac{K_b[H_3O^+]^3\dfrac{[B]_1K_w}{K_w + K_b[H_3O^+]}}{K_w} + [H_3O^+]^3$$

$$= K_w\,[H_3O^+] + K_{a_1}[H_3O^+]\frac{[H_2A]_1[H_3O^+]^2}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}} + 2K_{a_2}K_{a_1}\frac{[H_2A]_1[H_3O^+]^2}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}$$

$$\frac{K_b[H_3O^+]^3[B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+]^3 = K_w\,[H_3O^+] + \frac{K_{a_1}[H_2A]_1[H_3O^+]^3}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}} + \frac{2K_{a_2}K_{a_1}[H_2A]_1[H_3O^+]^2}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}$$

$$\frac{K_b[H_3O^+]^3[B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+]^3 = K_w\,[H_3O^+] + \frac{K_{a_1}[H_2A]_1[H_3O^+]^3 + 2K_{a_2}K_{a_1}[H_2A]_1[H_3O^+]^2}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}$$

$$\frac{K_b[H_3O^+]^2[B]_1}{K_w + K_b[H_3O^+]} + [H_3O^+]^2 = K_w + \frac{K_{a_1}[H_2A]_1[H_3O^+]^2 + 2K_{a_2}K_{a_1}[H_2A]_1[H_3O^+]}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}$$

$$K_b[H_3O^+]^2[B]_1 + K_w\,[H_3O^+]^2 + K_b[H_3O^+]^3$$
$$= K_w{}^2 + K_w\,K_b[H_3O^+]$$
$$+ \frac{K_w\,K_{a_1}[H_2A]_1[H_3O^+]^2 + 2K_w\,K_{a_2}K_{a_1}[H_2A]_1[H_3O^+] + K_b K_{a_1}[H_2A]_1[H_3O^+]^3 + 2K_b K_{a_2}K_{a_1}[H_2A]_1[H_3O^+]^2}{[H_3O^+]^2 + K_{a_1}[H_3O^+] + K_{a_2}K_{a_1}}$$

$$K_b[H_3O^+]^4[B]_1 + K_b K_{a_1}[H_3O^+]^3[B]_1 + K_b K_{a_2}K_{a_1}[H_3O^+]^2[B]_1 + K_w\,[H_3O^+]^4 + K_w\,K_{a_1}[H_3O^+]^3 + K_w\,K_{a_2}K_{a_1}[H_3O^+]^2 + K_b[H_3O^+]^5$$
$$+ K_b K_{a_1}[H_3O^+]^4 + K_b K_{a_2}K_{a_1}[H_3O^+]^3$$
$$= K_w{}^2[H_3O^+]^2 + K_w{}^2 K_{a_1}[H_3O^+] + K_w{}^2 K_{a_2}K_{a_1} + K_w\,K_b[H_3O^+]^3 + K_w\,K_b K_{a_1}[H_3O^+]^2 + K_w\,K_b K_{a_2}K_{a_1}[H_3O^+]$$
$$+ K_w\,K_{a_1}[H_2A]_1[H_3O^+]^2 + 2K_w\,K_{a_2}K_{a_1}[H_2A]_1[H_3O^+] + K_b K_{a_1}[H_2A]_1[H_3O^+]^3 + 2K_b K_{a_2}K_{a_1}[H_2A]_1[H_3O^+]^2$$

$$K_b[H_3O^+]^4[B]_1 + K_b K_{a_1}[H_3O^+]^3[B]_1 + K_b K_{a_2}K_{a_1}[H_3O^+]^2[B]_1 + K_w\,[H_3O^+]^4 + K_w\,K_{a_1}[H_3O^+]^3 + K_w\,K_{a_2}K_{a_1}[H_3O^+]^2 + K_b[H_3O^+]^5$$
$$+ K_b K_{a_1}[H_3O^+]^4 + K_b K_{a_2}K_{a_1}[H_3O^+]^3 - K_w{}^2[H_3O^+]^2 - K_w{}^2 K_{a_1}[H_3O^+] - K_w{}^2 K_{a_2}K_{a_1} - K_w\,K_b[H_3O^+]^3 - K_w\,K_b K_{a_1}[H_3O^+]^2$$
$$- K_w\,K_b K_{a_2}K_{a_1}[H_3O^+] - K_w\,K_{a_1}[H_2A]_1[H_3O^+]^2 - 2K_w\,K_{a_2}K_{a_1}[H_2A]_1[H_3O^+] - K_b K_{a_1}[H_2A]_1[H_3O^+]^3$$
$$- 2K_b K_{a_2}K_{a_1}[H_2A]_1[H_3O^+]^2 = 0$$

$$K_b[H_3O^+]^5 + [H_3O^+]^4\left(K_b[B]_1 + K_w + K_bK_{a_1}\right) + [H_3O^+]^3\left(K_bK_{a_1}[B]_1 + K_wK_{a_1} + K_bK_{a_2}K_{a_1} - K_wK_b - K_bK_{a_1}[H_2A]_1\right)$$
$$+ [H_3O^+]^2\left(K_bK_{a_2}K_{a_1}[B]_1 + K_wK_{a_2}K_{a_1} - K_w^2 - K_wK_bK_{a_1} - K_wK_{a_1}[H_2A]_1 - 2K_bK_{a_2}K_{a_1}[H_2A]_1\right)$$
$$- [H_3O^+]\left(K_w^2K_{a_1} + K_wK_bK_{a_2}K_{a_1} + 2K_wK_{a_2}K_{a_1}[H_2A]_1\right) - K_w^2K_{a_2}K_{a_1} = 0$$

$$K_b[H_3O^+]^5 + [H_3O^+]^4\left(K_b([B]_1 + K_b) + K_w\right) + [H_3O^+]^3\left(K_bK_{a_1}\left([B]_1 - [H_2A]_1 + K_{a_2}\right) + K_w\left(K_{a_1} - K_b\right)\right)$$
$$+ [H_3O^+]^2\left(K_bK_{a_1}\left(K_{a_2}[B]_1 - 2K_{a_2}[H_2A]_1 - K_w\right) + K_w\left(K_{a_2}K_{a_1} - K_{a_1}[H_2A]_1 - K_w\right)\right)$$
$$- [H_3O^+]\left(K_wK_{a_1}\left(K_w + K_bK_{a_2} + 2K_{a_2}[H_2A]_1\right)\right) - K_w^2K_{a_2}K_{a_1} = 0$$