Tvåprotonig bas + svag syra

$$Na_2CO_3 \xrightarrow{H_2O} 2\ Na^+ + {CO_3}^{2-}$$

$$[Na^+] = 2[Na_2CO_3]$$

$$[{CO_3}^{2-}]_1 = [Na_2CO_3]$$

$${CO_3}^{2-} + H_2O \rightleftarrows {HCO_3}^- + OH^-$$

$${HCO_3}^- + H_2O \rightleftarrows H_2CO_3 + OH^-$$

$$K_{b_2} = \frac{[{HCO_3}^-][OH^-]}{[{CO_3}^{2-}]_2} \Leftrightarrow [{HCO_3}^-] = \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]}$$

$$K_{b_1} = \frac{[H_2CO_3][OH^-]}{[{HCO_3}^-]} \Leftrightarrow [H_2CO_3] = \frac{K_{b_1}[{HCO_3}^-]}{[OH^-]}$$

$$K_w = [OH^-][H_3O^+] \Leftrightarrow [OH^-] = \frac{K_w}{[H_3O^+]}$$

$$[{CO_3}^{2-}]_1 = [{CO_3}^{2-}]_2 + [{HCO_3}^-] + [H_2CO_3]$$

$$[{CO_3}^{2-}]_1 = [{CO_3}^{2-}]_2 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + \frac{K_{b_1}[{HCO_3}^-]}{[OH^-]} = [{CO_3}^{2-}]_2 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + \frac{K_{b_1}\frac{K_{b_2}[{CO_3}^{2-}]}{[OH^-]}}{[OH^-]}$$

$$[{CO_3}^{2-}]_1 = [{CO_3}^{2-}]_2 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + \frac{K_{b_1}K_{b_2}[{CO_3}^{2-}]}{[OH^-]^2} = [{CO_3}^{2-}]_2\left(1 + \frac{K_{b_2}}{[OH^-]} + \frac{K_{b_1}K_{b_2}}{[OH^-]^2}\right) = [{CO_3}^{2-}]_2\left(\frac{[OH^-]^2 + K_{b_2}[OH^-] + K_{b_1}K_{b_2}}{[OH^-]^2}\right)$$

$$\left[{CO_3}^{2-}\right]_2 = \frac{[{CO_3}^{2-}]_1}{\left(\frac{[OH^-]^2 + K_{b_2}[OH^-] + K_{b_1}K_{b_2}}{[OH^-]^2}\right)} = \frac{[{CO_3}^{2-}]_1[OH^-]^2}{[OH^-]^2 + K_{b_2}[OH^-] + K_{b_1}K_{b_2}} = \frac{[{CO_3}^{2-}]_1\left(\frac{K_w}{[H_3O^+]}\right)^2}{\left(\frac{K_w}{[H_3O^+]}\right)^2 + K_{b_2}\frac{K_w}{[H_3O^+]} + K_{b_1}K_{b_2}}$$

$$\left[{CO_3}^{2-}\right]_2 = \frac{\frac{[{CO_3}^{2-}]_1{K_w}^2}{[H_3O^+]^2}}{\frac{{K_w}^2}{[H_3O^+]^2} + \frac{K_{b_2}K_w}{[H_3O^+]} + K_{b_1}K_{b_2}} = \frac{\frac{[{CO_3}^{2-}]_1{K_w}^2}{[H_3O^+]^2}}{\frac{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}{[H_3O^+]^2}} = \frac{[{CO_3}^{2-}]_1{K_w}^2}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$HA + H_2O \rightleftarrows A^- + H_3O^+$$

$$K_a = \frac{[A^-][H_3O^+]}{[HA]_2} \Leftrightarrow [A^-] = \frac{K_a[HA]_2}{[H_3O^+]}$$

$$[HA]_1 = [HA]_2 + [A^-] = [HA]_2 + \frac{K_a[HA]_2}{[H_3O^+]} = [HA]_2\left(1 + \frac{K_a}{[H_3O^+]}\right) = [HA]_2\left(\frac{[H_3O^+] + K_a}{[H_3O^+]}\right) \Leftrightarrow$$

$$[HA]_2 = \frac{[HA]_1}{\left(\frac{[H_3O^+] + K_a}{[H_3O^+]}\right)} = \frac{[HA]_1[H_3O^+]}{[H_3O^+] + K_a}$$

$$[Na^+] + [H_3O^+] = [OH^-] + [A^-] + [HCO_3{}^-] + 2[{CO_3}^{2-}]_2$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_a[HA]_2}{[H_3O^+]} + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + 2\frac{[{CO_3}^{2-}]_1{K_w}^2}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_a \frac{[HA]_1[H_3O^+]}{[H_3O^+] + K_a}}{[H_3O^+]} + \frac{K_{b_2}[{CO_3}^{2-}]_2}{\frac{K_w}{[H_3O^+]}} + \frac{2[{CO_3}^{2-}]_1 {K_w}^2}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_a[HA]_1}{[H_3O^+] + K_a} + \frac{K_{b_2}[H_3O^+][{CO_3}^{2-}]_2}{K_w} + \frac{2{K_w}^2[{CO_3}^{2-}]_1}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_a[HA]_1}{[H_3O^+] + K_a} + \frac{K_{b_2}[H_3O^+]\frac{[{CO_3}^{2-}]_1 {K_w}^2}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}}{K_w} + \frac{2{K_w}^2[{CO_3}^{2-}]_1}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + \frac{K_a[HA]_1}{[H_3O^+] + K_a} + \frac{K_w K_{b_2}[H_3O^+][Na_2CO_3]}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2} + \frac{2{K_w}^2[Na_2CO_3]}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3][H_3O^+] + [H_3O^+]^2 = K_w + \frac{K_a[HA]_1[H_3O^+]}{[H_3O^+] + K_a} + \frac{K_w K_{b_2}[H_3O^+]^2[Na_2CO_3] + 2{K_w}^2[H_3O^+][Na_2CO_3]}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$\begin{aligned}
&2[Na_2CO_3][H_3O^+]^2 + [H_3O^+]^3 + 2K_a[Na_2CO_3][H_3O^+] + K_a[H_3O^+]^2 \\
&\quad = K_w[H_3O^+] + K_w K_a + K_a[HA]_1[H_3O^+] \\
&\quad + \frac{K_w K_{b_2}[H_3O^+]^3[Na_2CO_3] + 2{K_w}^2[H_3O^+]^2[Na_2CO_3] + K_w K_a K_{b_2}[H_3O^+]^2[Na_2CO_3] + 2{K_w}^2 K_a[H_3O^+][Na_2CO_3]}{{K_w}^2 + K_{b_2}K_w[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}
\end{aligned}$$

$$\begin{aligned}
&2{K_w}^2[Na_2CO_3][H_3O^+]^2 + {K_w}^2[H_3O^+]^3 + 2{K_w}^2 K_a[Na_2CO_3][H_3O^+] + {K_w}^2 K_a[H_3O^+]^2 + 2K_{b_2}K_w[Na_2CO_3][H_3O^+]^3 + K_{b_2}K_w[H_3O^+]^4 \\
&\quad + 2K_a K_{b_2}K_w[Na_2CO_3][H_3O^+]^2 + K_a K_{b_2}K_w[H_3O^+]^3 + 2K_{b_1}K_{b_2}[Na_2CO_3][H_3O^+]^4 + K_{b_1}K_{b_2}[H_3O^+]^5 \\
&\quad + 2K_a K_{b_1}K_{b_2}[Na_2CO_3][H_3O^+]^3 + K_a K_{b_1}K_{b_2}[H_3O^+]^4 \\
&\quad = {K_w}^3[H_3O^+] + {K_w}^3 K_a + {K_w}^2 K_a[HA]_1[H_3O^+] + K_{b_2}{K_w}^2[H_3O^+]^2 + {K_w}^2 K_{b_2}K_a[H_3O^+] + K_a K_{b_2}K_w[HA]_1[H_3O^+]^2 \\
&\quad + K_w K_{b_1}K_{b_2}[H_3O^+]^3 + K_w K_a K_{b_1}K_{b_2}[H_3O^+]^2 + K_a K_{b_1}K_{b_2}[HA]_1[H_3O^+]^3 + K_w K_{b_2}[H_3O^+]^3[Na_2CO_3] \\
&\quad + 2{K_w}^2[H_3O^+]^2[Na_2CO_3] + K_w K_a K_{b_2}[H_3O^+]^2[Na_2CO_3] + 2{K_w}^2 K_a[H_3O^+][Na_2CO_3]
\end{aligned}$$

$$2K_w{}^2[Na_2CO_3][H_3O^+]^2 + K_w{}^2[H_3O^+]^3 + 2K_w{}^2K_a[Na_2CO_3][H_3O^+] + K_w{}^2K_a[H_3O^+]^2 + 2K_{b_2}K_w\,[Na_2CO_3][H_3O^+]^3 + K_{b_2}K_w\,[H_3O^+]^4$$
$$+ 2K_aK_{b_2}K_w\,[Na_2CO_3][H_3O^+]^2 + K_aK_{b_2}K_w\,[H_3O^+]^3 + 2K_{b_1}K_{b_2}[Na_2CO_3][H_3O^+]^4 + K_{b_1}K_{b_2}[H_3O^+]^5$$
$$+ 2K_aK_{b_1}K_{b_2}[Na_2CO_3][H_3O^+]^3 + K_aK_{b_1}K_{b_2}[H_3O^+]^4 - K_w{}^3[H_3O^+] - K_w{}^3K_a - K_w{}^2K_a[HA]_1[H_3O^+] - K_{b_2}K_w{}^2[H_3O^+]^2$$
$$- K_w{}^2K_{b_2}K_a[H_3O^+] - K_aK_{b_2}K_w\,[HA]_1[H_3O^+]^2 - K_w\,K_{b_1}K_{b_2}[H_3O^+]^3 - K_w\,K_aK_{b_1}K_{b_2}[H_3O^+]^2 - K_aK_{b_1}K_{b_2}[HA]_1[H_3O^+]^3$$
$$- K_w\,K_{b_2}[H_3O^+]^3[Na_2CO_3] - 2K_w{}^2[H_3O^+]^2[Na_2CO_3] - K_w\,K_aK_{b_2}[H_3O^+]^2[Na_2CO_3] - 2K_w{}^2K_a[H_3O^+][Na_2CO_3] = 0$$

$$K_{b_1}K_{b_2}[H_3O^+]^5 + [H_3O^+]^4\left(K_{b_2}K_w\, + 2K_{b_1}K_{b_2}[Na_2CO_3] + K_aK_{b_1}K_{b_2}\right)$$
$$+ [H_3O^+]^3\left(K_w{}^2 + 2K_{b_2}K_w\,[Na_2CO_3] + K_aK_{b_2}K_w\, + 2K_aK_{b_1}K_{b_2}[Na_2CO_3] - K_w\,K_{b_1}K_{b_2} - K_aK_{b_1}K_{b_2}[HA]_1\right.$$
$$\left. - K_w\,K_{b_2}[Na_2CO_3]\right)$$
$$+ [H_3O^+]^2\left(2K_w{}^2[Na_2CO_3] + K_w{}^2K_a\, + 2K_aK_{b_2}K_w\,[Na_2CO_3] - K_{b_2}K_w{}^2 - K_aK_{b_2}K_w\,[HA]_1 - K_w\,K_aK_{b_1}K_{b_2} - 2K_w{}^2[Na_2CO_3]\right.$$
$$\left. - K_w\,K_aK_{b_2}[Na_2CO_3]\right) + [H_3O^+]\left(2K_w{}^2K_a[Na_2CO_3] - K_w{}^3 - K_w{}^2K_a[HA]_1 - K_w{}^2K_{b_2}K_a - 2K_w{}^2K_a[Na_2CO_3]\right) - K_w{}^3K_a$$
$$= 0$$

$$K_{b_1}K_{b_2}[H_3O^+]^5 + [H_3O^+]^4\left(K_{b_1}K_{b_2}(2[Na_2CO_3] + K_a) + K_{b_2}K_w\right)$$
$$+ [H_3O^+]^3\left(K_w{}^2 + K_{b_2}K_w\,[Na_2CO_3] + K_aK_{b_2}K_w\, + 2K_aK_{b_1}K_{b_2}[Na_2CO_3] - K_w\,K_{b_1}K_{b_2} - K_aK_{b_1}K_{b_2}[HA]_1\right)$$
$$+ [H_3O^+]^2\left(K_w{}^2K_a\, + K_aK_{b_2}K_w\,[Na_2CO_3] - K_{b_2}K_w{}^2 - K_aK_{b_2}K_w\,[HA]_1 - K_w\,K_aK_{b_1}K_{b_2}\right)$$
$$+ [H_3O^+]\left(-K_w{}^3 - K_w{}^2K_a[HA]_1 - K_w{}^2K_{b_2}K_a\right) - K_w{}^3K_a = 0$$

$$K_{b_1}K_{b_2}[H_3O^+]^5 + [H_3O^+]^4\left(K_{b_1}K_{b_2}(2[Na_2CO_3] + K_a) + K_{b_2}K_w\right)$$
$$+ [H_3O^+]^3\left(K_{b_1}K_{b_2}(2K_a[Na_2CO_3] - K_a[HA]_1 - K_w) + K_w\left(K_w\, + K_{b_2}[Na_2CO_3] + K_aK_{b_2}\right)\right)$$
$$+ [H_3O^+]^2\left(K_aK_{b_2}K_w\left(Na_2CO_3 - [HA]_1 - K_{b_1}\right) + K_w{}^2\left(K_a - K_{b_2}\right)\right) - [H_3O^+]\left(K_w{}^2\left(K_w\, + K_a[HA]_1 + K_{b_2}K_a\right)\right) - K_w{}^3K_a = 0$$