Tvåprotonig bas + stark syra

$$Na_2CO_3 \xrightarrow{H_2O} 2\ Na^+ + {CO_3}^{2-}$$

$$[Na^+] = 2[Na_2CO_3]$$

$$[{CO_3}^{2-}]_1 = [Na_2CO_3]$$

$${CO_3}^{2-} + H_2O \rightleftarrows {HCO_3}^- + OH^-$$

$${HCO_3}^- + H_2O \rightleftarrows H_2CO_3 + OH^-$$

$$K_{b_2} = \frac{[{HCO_3}^-][OH^-]}{[{CO_3}^{2-}]_2} \Leftrightarrow [{HCO_3}^-] = \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]}$$

$$K_{b_1} = \frac{[H_2CO_3][OH^-]}{[{HCO_3}^-]} \Leftrightarrow [H_2CO_3] = \frac{K_{b_1}[{HCO_3}^-]}{[OH^-]}$$

$$K_w = [OH^-][H_3O^+] \Leftrightarrow [OH^-] = \frac{K_w}{[H_3O^+]}$$

$$[{CO_3}^{2-}]_1 = [{CO_3}^{2-}]_2 + [{HCO_3}^-] + [H_2CO_3]$$

$$[{CO_3}^{2-}]_1 = [{CO_3}^{2-}]_2 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + \frac{K_{b_1}[{HCO_3}^-]}{[OH^-]} = [{CO_3}^{2-}]_2 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + \frac{K_{b_1}\frac{K_{b_2}[{CO_3}^{2-}]}{[OH^-]}}{[OH^-]}$$

$$[{CO_3}^{2-}]_1 = [{CO_3}^{2-}]_2 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + \frac{K_{b_1}K_{b_2}[{CO_3}^{2-}]}{[OH^-]^2} = [{CO_3}^{2-}]_2\left(1 + \frac{K_{b_2}}{[OH^-]} + \frac{K_{b_1}K_{b_2}}{[OH^-]^2}\right) = [{CO_3}^{2-}]_2\left(\frac{[OH^-]^2 + K_{b_2}[OH^-] + K_{b_1}K_{b_2}}{[OH^-]^2}\right)$$

$$\left[{CO_3}^{2-}\right]_2 = \frac{[{CO_3}^{2-}]_1}{\left(\frac{[OH^-]^2 + K_{b_2}[OH^-] + K_{b_1}K_{b_2}}{[OH^-]^2}\right)} = \frac{[{CO_3}^{2-}]_1[OH^-]^2}{[OH^-]^2 + K_{b_2}[OH^-] + K_{b_1}K_{b_2}} = \frac{[{CO_3}^{2-}]_1\left(\frac{K_w}{[H_3O^+]}\right)^2}{\left(\frac{K_w}{[H_3O^+]}\right)^2 + K_{b_2}\frac{K_w}{[H_3O^+]} + K_{b_1}K_{b_2}}$$

$$\left[{CO_3}^{2-}\right]_2 = \frac{\frac{[{CO_3}^{2-}]_1{K_w}^2}{[H_3O^+]^2}}{\frac{{K_w}^2}{[H_3O^+]^2} + \frac{K_{b_2}K_w}{[H_3O^+]} + K_{b_1}K_{b_2}} = \frac{\frac{[{CO_3}^{2-}]_1{K_w}^2}{[H_3O^+]^2}}{\frac{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}{[H_3O^+]^2}} = \frac{[{CO_3}^{2-}]_1{K_w}^2}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$HA + H_2O \rightarrow A^- + H_3O^+$$

$$[HA]_1 = [A^-]$$

$$[Na^+] + [H_3O^+] = [OH^-] + [A^-] + [{HCO_3}^-] + 2[{CO_3}^{2-}]_2$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + [HA]_1 + \frac{K_{b_2}[{CO_3}^{2-}]_2}{[OH^-]} + 2\frac{[{CO_3}^{2-}]_1{K_w}^2}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + [HA]_1 + \frac{K_{b_2}\frac{[{CO_3}^{2-}]_1{K_w}^2}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}}{\frac{K_w}{[H_3O^+]}} + \frac{2\,[{CO_3}^{2-}]_1{K_w}^2}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + [HA]_1 + \frac{K_w\,K_{b_2}[Na_2CO_3][H_3O^+]}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2} + \frac{2{K_w}^2[Na_2CO_3]}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3] + [H_3O^+] = \frac{K_w}{[H_3O^+]} + [HA]_1 + \frac{K_w\,K_{b_2}[Na_2CO_3][H_3O^+] + 2{K_w}^2[Na_2CO_3]}{{K_w}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$2[Na_2CO_3][H_3O^+] + [H_3O^+]^2 = K_w\ + [HA]_1[H_3O^+] + \frac{K_w\,K_{b_2}[Na_2CO_3][H_3O^+]^2 + 2K_w{}^2[Na_2CO_3][H_3O^+]}{K_w{}^2 + K_{b_2}K_w\,[H_3O^+] + K_{b_1}K_{b_2}[H_3O^+]^2}$$

$$\begin{aligned}2K_w{}^2[Na_2CO_3][H_3O^+] + 2K_{b_2}K_w\,[Na_2CO_3][H_3O^+]^2 + 2K_{b_1}K_{b_2}[Na_2CO_3][H_3O^+]^3 + K_w{}^2[H_3O^+]^2 + K_{b_2}K_w\,[H_3O^+]^3 + K_{b_1}K_{b_2}[H_3O^+]^4 \\ = K_w{}^3 + K_{b_2}K_w{}^2[H_3O^+] + K_w\,K_{b_1}K_{b_2}[H_3O^+]^2 + K_w{}^2[HA]_1[H_3O^+] + K_{b_2}K_w\,[HA]_1[H_3O^+]^2 + K_{b_1}K_{b_2}[HA]_1[H_3O^+]^3 \\ + K_w\,K_{b_2}[Na_2CO_3][H_3O^+]^2 + 2K_w{}^2[Na_2CO_3][H_3O^+]\end{aligned}$$

$$\begin{aligned}2K_w{}^2[Na_2CO_3][H_3O^+] + 2K_{b_2}K_w\,[Na_2CO_3][H_3O^+]^2 + 2K_{b_1}K_{b_2}[Na_2CO_3][H_3O^+]^3 + K_w{}^2[H_3O^+]^2 + K_{b_2}K_w\,[H_3O^+]^3 + K_{b_1}K_{b_2}[H_3O^+]^4 \\ - K_w{}^3 - K_{b_2}K_w{}^2[H_3O^+] - K_w\,K_{b_1}K_{b_2}[H_3O^+]^2 - K_w{}^2[HA]_1[H_3O^+] - K_{b_2}K_w\,[HA]_1[H_3O^+]^2 - K_{b_1}K_{b_2}[HA]_1[H_3O^+]^3 \\ - K_w\,K_{b_2}[Na_2CO_3][H_3O^+]^2 - 2K_w{}^2[Na_2CO_3][H_3O^+] = 0\end{aligned}$$

$$\begin{aligned}K_{b_1}K_{b_2}[H_3O^+]^4 + [H_3O^+]^3\left(2K_{b_1}K_{b_2}[Na_2CO_3] + K_{b_2}K_w - K_{b_1}K_{b_2}[HA]_1\right) \\ + [H_3O^+]^2\left(2K_{b_2}K_w\,[Na_2CO_3] + K_w{}^2 - K_w\,K_{b_1}K_{b_2} - K_{b_2}K_w\,[HA]_1 - K_w\,K_{b_2}[Na_2CO_3]\right) \\ + [H_3O^+]\left(2K_w{}^2[Na_2CO_3] - K_{b_2}K_w{}^2 - K_w{}^2[HA]_1 - 2K_w{}^2[Na_2CO_3]\right) - K_w{}^3 = 0\end{aligned}$$

$$\begin{aligned}K_{b_1}K_{b_2}[H_3O^+]^4 + [H_3O^+]^3\left(K_{b_1}K_{b_2}(2[Na_2CO_3] - [HA]_1) + K_{b_2}K_w\right) + [H_3O^+]^2\left(K_{b_2}K_w\,[Na_2CO_3] + K_w{}^2 - K_w\,K_{b_1}K_{b_2} - K_{b_2}K_w\,[HA]_1\right) \\ + [H_3O^+]\left(-K_{b_2}K_w{}^2 - K_w{}^2[HA]_1\right) - K_w{}^3 = 0\end{aligned}$$

$$\begin{aligned}K_{b_1}K_{b_2}[H_3O^+]^4 + [H_3O^+]^3\left(K_{b_1}K_{b_2}(2[Na_2CO_3] - [HA]_1) + K_{b_2}K_w\right) + [H_3O^+]^2\left(K_{b_2}K_w\left([Na_2CO_3] - [HA]_1\right) + K_w\left(K_w - K_{b_1}K_{b_2}\right)\right) \\ - [H_3O^+]\left(K_w{}^2\left(K_{b_2} + [HA]_1\right)\right) - K_w{}^3 = 0\end{aligned}$$