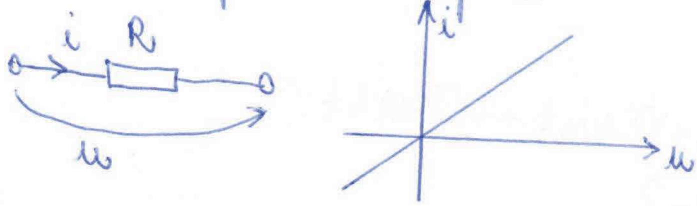
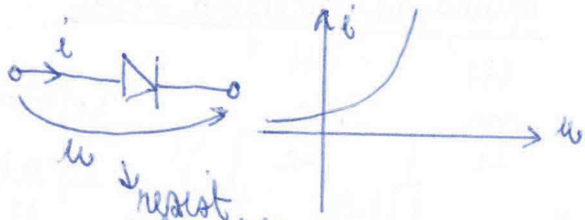
Analiza circuitelor în regim permanent nesinusoidal

Circuite liniare

- în circuitul liniar toate elementele de circuit au o caracteristică liniară (u și i variază după o linie dreaptă).



dioda = R neliniar



rezist. neliniară

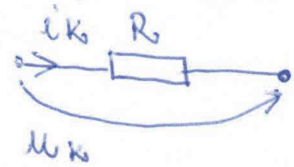
- fie un circuit liniar la care tensiunile aplicate sunt nesinusoidale de tipul

$$u(t)=U_0+\sum_{1}^{\infty}U_K\sqrt{2}\sin(K\omega t+\alpha_K)=U_0+U^{(1)}\sqrt{2}\sin(\omega t+\alpha_1)+U^{(2)}\sqrt{2}\sin(2\omega t+\alpha_2)+\dots$$

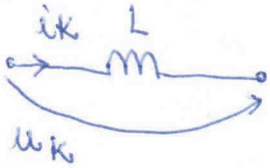
$$\underline{U}=U_0+\underline{U}^{(1)}+\underline{U}^{(2)}+\dots \qquad \underline{U}^{(1)}=U^{(1)}e^{j\alpha_1}$$

Analiza în regim permanent a acestui circuit se face pe fiecare armonică în parte cu ajutorul calculului în complex. Armonica de ordinul „n” a tensiunii determină apariția armonicii de ordinul n a curentului.

Impedanța complexă a elementelor ideale de circuit corespunzătoare armonicii K este

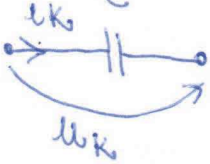
- 

$u_K=R\cdot i_K$ $\boxed{\underline{Z}_R^{(K)}=R}$ $\underline{I}^{(K)}=\dfrac{\underline{U}^{(K)}}{R}$ $kdi=kdu$

- 

$u_K=L\dfrac{di_K}{dt}$ $\boxed{\underline{Z}_L^{(K)}=j\omega KL}$ $\Rightarrow \underline{I}^{(K)}=\dfrac{\underline{U}^{(K)}}{j\omega KL}$, $kdi<kdu$

kdi — coef de distorsiune al crt; kdu — coef de distorsiune al tensiunii

- 

$i_K=C\dfrac{du_K}{dt}$ $\boxed{\underline{Z}_C^{K}=\dfrac{-j}{\omega KC}}$ $\Rightarrow \underline{I}^{(K)}=\dfrac{\underline{U}^{(K)}}{\dfrac{-j}{K\omega C}}$, $kdi>kdu$

$$i(t)=I_0+\sum_K I_K\sqrt{2}\sin(K\omega t+\beta_K)$$

Regimul componentelor de curent continuu pt crt și tens se det pe o rețea separată a cărei structură diferă de cea pe care se studiază regimul armonicelor de ordinul 1, 2, ...

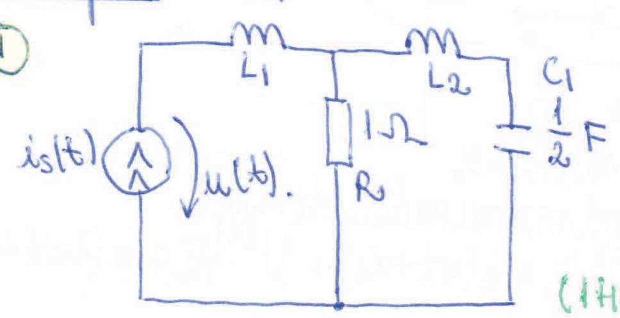
Deoarece în c.c $u_C = ct \Rightarrow i_C = C\frac{du_C}{dt} = 0$ și condensatorul se înlocuiește cu un rezistor $R \to \infty$ (întrerupere)

Similar, deoarece $i_L = ct \Rightarrow u_L = L\frac{di_L}{dt} = 0$ și bobina se înlocuiește cu un rezistor $R \to 0$ (scurtcircuit)

Aplicații cu o singură sursă

exemplu.

1

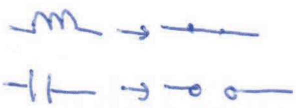


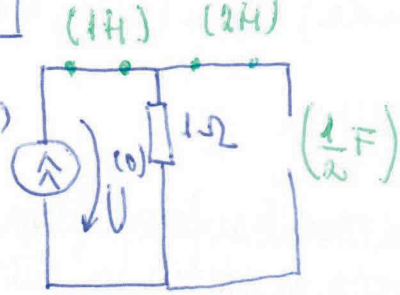
$i_s(t) = 2+\sqrt{2}\sin t + 2\sqrt{2}\cos 2t$

a) $u(t) = ?$

$U_{ef} = ?$

cc





$\boxed{U^{(0)}} = I_s^{(0)} \cdot 1 = 2 \cdot 1 = \boxed{2V}$

unde $I_s^{(0)} = 2$

k=1 (armonica fundamentală) $i_s^{(1)}(t) = \sqrt{2}\sin t$

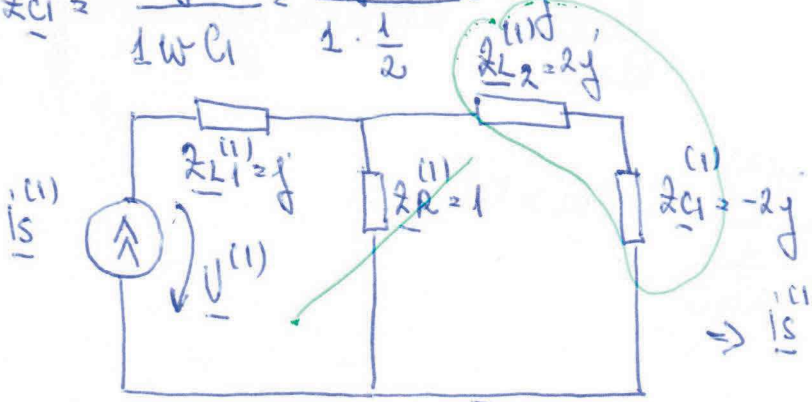
$\omega = 1$ rad/s

$i_s^{(1)}(t) = \sqrt{2}\sin t \to \underline{I}_s^{(1)} = 1e^{j0} = 1$

$\underline{Z}_{L1}^{(1)} = j(1)\omega L_1 = 1j$

$\underline{Z}_{L2}^{(1)} = j(1)\omega L_2 = 2j$

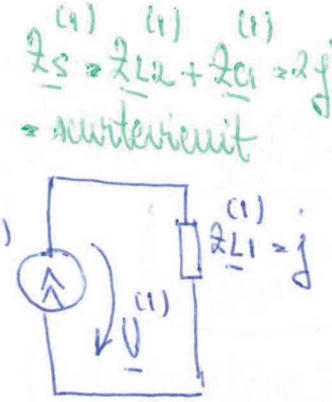
$\underline{Z}_{C1}^{(1)} = \frac{-j}{1\omega C_1} = \frac{-j}{1 \cdot \frac{1}{2}} = -2j$



$\underline{Z}_S^{(1)} = \underline{Z}_{L2}^{(1)} + \underline{Z}_{C1}^{(1)} = 2j - 2j = 0$

= scurtcircuit

$\Rightarrow$



$\underline{U}^{(1)} = \underline{Z}_{L1}^{(1)} \cdot \underline{I}_s^{(1)} = 1 \cdot j = 1e^{j\frac{\pi}{2}}$

$\rightarrow \boxed{u^{(1)}(t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right)}$

k = 2 (armonica de ordinul 2)

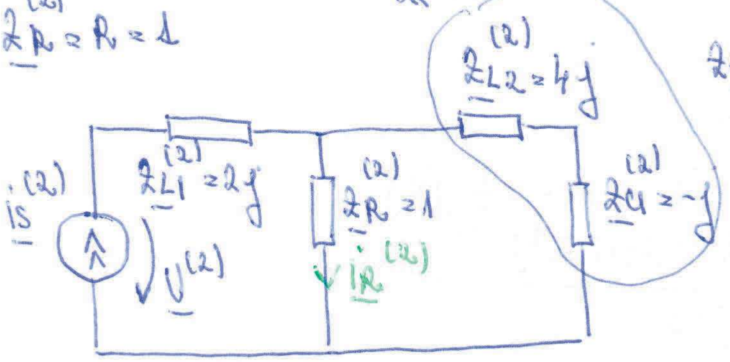
$$i_S^{(2)}(t) = 2\sqrt{2}\cos 2t = 2\sqrt{2}\sin\left(2t+\frac{\pi}{2}\right) \rightarrow \underline{\dot{I}_S}^{(2)} = 2e^{j\frac{\pi}{2}} = 2j.$$

$$\underline{Z_{L1}}^{(2)} = j\cdot 2\omega L_1 = 2j$$

$$\underline{Z_{L2}}^{(2)} = j\cdot 2\omega L_2 = 4j$$

$$\underline{Z_{C1}}^{(2)} = \frac{-j}{2\omega C_1} = \frac{-j}{2\cdot 1\cdot\frac{1}{2}} = -j.$$

$$\underline{Z_R}^{(2)} = R = 1$$



$$\underline{Z_{es}}^{(2)} = \underline{Z_{L2}}^{(2)} + \underline{Z_{C1}}^{(2)} = 4j - j = 3j$$

$$\underline{Z_{ep}}^{(2)} = \frac{\underline{Z_R}^{(2)}\cdot\underline{Z_{es}}^{(2)}}{\underline{Z_R}^{(2)}+\underline{Z_{es}}^{(2)}} = \frac{1\cdot 3j}{1+3j} = \frac{3j(1-3j)}{10}$$

$$= \frac{3j-9j^2}{10} = \frac{9+3j}{10}$$

$$\underline{Z_e}^{(2)} = \underline{Z_{L1}}^{(2)} + \underline{Z_{ep}}^{(2)} = 2j + \frac{3j-9j^2}{10} = \frac{20j+3j+9}{10} = \frac{23j+9}{10}$$

$$\underline{U}^{(2)} = \underline{\dot{I}_S}^{(2)}\cdot\underline{Z_e}^{(2)} = 2j\cdot\frac{9+23j}{10_{5}} = -\frac{23+9j}{5} = -\frac{23}{5}+\frac{9}{5}j$$

$$\boxed{u^{(2)}(t)} = \sqrt{\left(\frac{23}{5}\right)^2+\left(\frac{9}{5}\right)^2}\sqrt{2}\sin\left(2\omega t+\operatorname{arctg}\frac{9/5}{-23/5}\right)$$

$$= \boxed{\sqrt{21{,}6}\cdot\sqrt{2}\sin\left(2t+\operatorname{arctg}\frac{9}{-23}\right)}$$

$$u(t) = U^{(0)} + u^{(1)}(t) + u^{(2)}(t) = 2 + \sqrt{2}\sin\left(\omega t+\frac{\pi}{2}\right) + \sqrt{21{,}6}\cdot\sqrt{2}\sin\left(2t+\operatorname{arctg}\frac{9}{-23}\right)$$

$$U_{ef} = \sqrt{U_0^{(2)} + U^{(1)2} + U^{(2)2}} = \sqrt{2^2+1^2+21{,}6} \cong 5{,}3\,V = U$$

$$I_{ef} = I_s = \sqrt{I_0^{(2)} + I^{(1)2} + I^{(2)2}} = \sqrt{2^2+1^2+2^2} = 3\,A$$

$$S = U_{ef}\cdot I_{ef} = U\cdot I = 5{,}3\cdot 3 = 15{,}9\,VA.$$

$$P_g = U_0 I_0 + U_1 I_1\cos\varphi_1 + U_2 I_2\cos\varphi_2 = 2\cdot 2 + 1\cdot 1\cos\left(\frac{\pi}{2}-0\right) + 2\cdot 21{,}6\cos\left(\operatorname{arctg}\frac{9}{-23}-\frac{\pi}{2}\right)W$$

$$Q_g = U_1 I_1\sin\varphi_1 + U_2 I_2\sin\varphi_2 = 1\cdot 1\sin\frac{\pi}{2} + 2\sqrt{21{,}6}\sin\left(\operatorname{arctg}\frac{9}{-23}-\frac{\pi}{2}\right)VAR$$

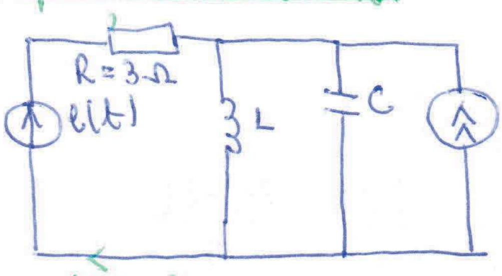
$$D = \sqrt{S^2-P_g^2-Q_g^2}\ [VAD].$$

$$P_C = R\left(I_R^{(0)^2} + I_R^{(1)^2} + I_R^{(2)^2}\right)$$

$$Q_C = 1 \cdot L_1 \cdot I_{L1}^{(1)^2} + 2 L_1 \cdot I_{L1}^{(2)^2} + 1 \cdot L_2 \cdot I_{L2}^{(2)^2} + 2 \cdot L_2 \cdot I_{L2}^{(2)^2} - \frac{1}{C} I_C^{(1)^2} - \frac{1}{2C} I_C^{(2)^2}$$

$$K_d = \frac{U_d}{U_a} = \frac{\sqrt{U^2 - U_1^2 - U_0^2}}{\sqrt{U^2 - U_0^2}} = \frac{\sqrt{U_2^{(2)} + U_3^{(2)}}}{\sqrt{U_1^{(2)} + U_2^{(2)} + \ldots}}$$

2) Aplicatie cu două surse



$R = 3\,\Omega$

$\omega L = 2\,\Omega$

$\frac{1}{\omega C} = 6\,\Omega$

? i(t) = ?

$e(t) = 3 + 12\sqrt{2}\sin(\omega t + \frac{\pi}{2})\,[V]$

$i_g(t) = 2\sqrt{2}\sin 3\omega t\,[A]$

$i_g(t) \to j(t)$

$e(t): E^{(0)};\ e^{(1)}(t)$

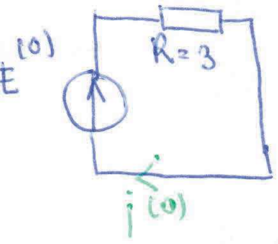
$i_g(t): i_g^{(3)}(t)$

c.c $(K=0)$ $E^{(0)} = 3$

! i_g nu are componentă pe armonica 0 se pasivizează —o o—

L →

C →



$$\boxed{I^{(0)}} = \frac{E^{(0)}}{R} = \frac{3}{3} = \boxed{1\,A}$$

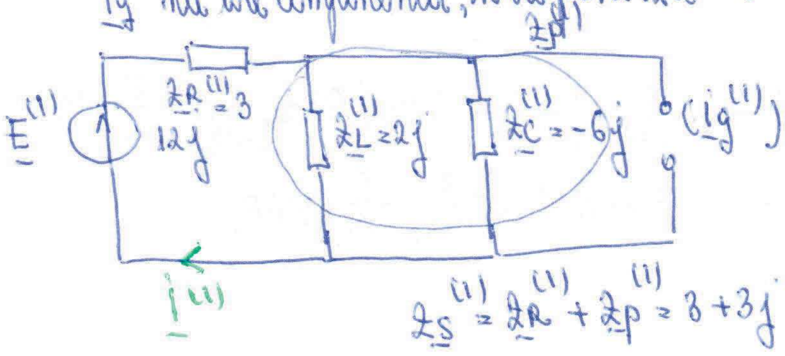
$K=1$ $e^{(1)}(t) = 12\sqrt{2}\sin(\omega t + \frac{\pi}{2}) \Rightarrow \underline{E}^{(1)} = 12 e^{j\frac{\pi}{2}} = 12j$

$\underline{Z}_R^{(1)} = R = 3$

$\underline{Z}_L^{(1)} = j\omega L = 2j$

$\underline{Z}_C^{(1)} = \frac{-j}{\omega C} = -6j$

$i_g^{(1)}$ nu are componentă, se va pasiviza —o o—



$$\underline{Z}_p^{(1)} = \frac{\underline{Z}_L^{(1)} \cdot \underline{Z}_C^{(1)}}{\underline{Z}_L^{(1)} + \underline{Z}_C^{(1)}} = \frac{2j(-6j)}{2j - 6j} = \frac{-12j^2}{-4j} = \frac{12}{-4j} = 3j$$

$$\underline{Z}_S^{(1)} = \underline{Z}_R^{(1)} + \underline{Z}_p^{(1)} = 3 + 3j$$

$$\underline{I}^{(1)} = \frac{\underline{E}^{(1)}}{\underline{Z}_S^{(1)}} = \frac{12j}{3(1+j)} = \frac{4j}{1+j}\Big\backslash^{1-j} = \frac{4j(1-j)}{2} = 2j(1-j) = 2(j-j^2) = 2(1+j) = 2\sqrt{2}e^{j\pi/4}$$

$$\boxed{i^{(1)}(t) = 2\sqrt{2}\cdot\sqrt{2}\sin\left(\omega t+\frac{\pi}{4}\right)}$$

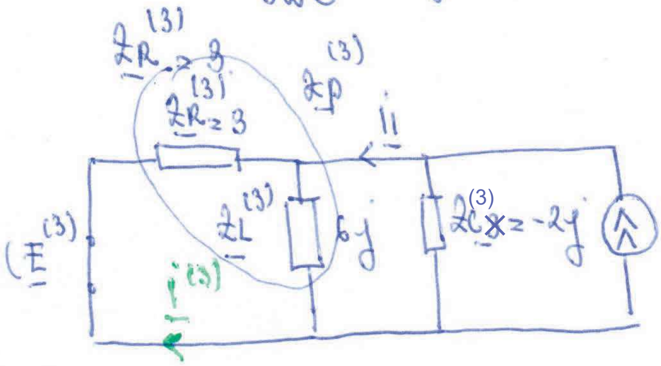
$\underline{k=3}$ $\underline{E}^{(3)}$ nu are componentă pe armonica 3 ⇒ se va pasiviza —∘

$$i_g^{(3)}(t) = 2\sqrt{2}\sin 3\omega t \to \underline{I_g}^{(3)} = 2e^{j0} = 2$$

$$\underline{Z_L}^{(3)} = 3j\omega L = 6j$$

$$\underline{Z_C}^{(3)} = \frac{-j}{3\omega C} = -2j$$

$\underline{Z_R}^{(3)} = 3$



$$\underline{Z_p}^{(3)} = \frac{\underline{Z_R}^{(3)}\cdot\underline{Z_L}^{(3)}}{\underline{Z_R}^{(3)}+\underline{Z_L}^{(3)}} = \frac{3\cdot 6j}{3(1+2j)} = \frac{6j}{1+2j}\Big\backslash^{1-2j} = \frac{6j(1-2j)}{5} = \frac{6j+12}{5}$$

$$\underline{I_1} = \underline{I_g}^{(3)}\cdot\frac{\underline{Z_C}^{(3)}}{\underline{Z_p}^{(3)}+\underline{Z_C}^{(3)}} = 2\cdot\frac{-2j}{-2j+\frac{6j+12}{5}} = \frac{(-4j)\cdot 5}{12+6j-10j} = \frac{-20j}{12-4j} = \frac{-10j}{6-2j} = \frac{-5j}{3-j} \to \frac{-5j(3+j)}{10}$$

$$= \frac{-15j-5j^2}{10} = \frac{1}{2}(1-3j)$$

$$\underline{I}^{(3)} = -\underline{I_1}\cdot\frac{\underline{Z_L}^{(3)}}{\underline{Z_R}^{(3)}+\underline{Z_L}^{(3)}} = -\frac{1}{\cancel{2}}(1-3j)\cdot\frac{\cancel{6}^{\,3}j}{\cancel{3+6j}_{\,2}} = \frac{(3j-1)j}{1+2j} = \frac{-j-3}{1+2j}\Big\backslash^{1-2j} = \frac{(-j-3)(1-2j)}{5}$$

$$= \frac{-j-2-3+6j}{5} = \frac{5j-5}{5} = -1+j = \sqrt{2}e^{j\frac{3\pi}{4}}$$

$$\boxed{i^{(3)}(t) = \sqrt{2}\cdot\sqrt{2}\sin\left(3\omega t+\frac{3\pi}{4}\right)}$$

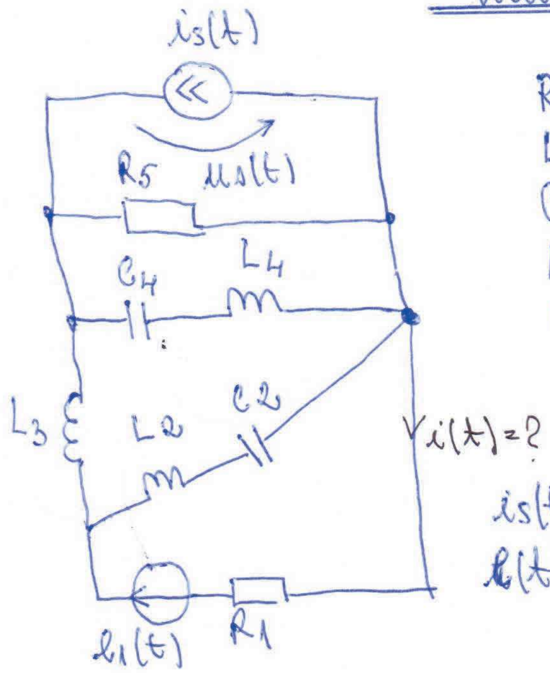
$$i(t) = I^{(0)} + i^{(1)}(t) + i^{(3)}(t) = 1 + 2\cdot 2\sin\left(\omega t+\frac{\pi}{4}\right) + \sqrt{2}\cdot\sqrt{2}\sin\left(3\omega t+\frac{3\pi}{4}\right)$$

$$k_d = \frac{I_d}{I_a} = \frac{\sqrt{2}}{\sqrt{2^2+2}} = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}}$$

$$I_{ef} = \sqrt{I_0^{(2)} + I^{(1)2} + I^{(3)2}} = \sqrt{1+4+2} = \sqrt{7}$$

$$P_{R_x} = R_x\cdot I_{ef}^2 = 3\cdot 7 = 21W$$

Circuit de regim deformant



$R_1 = R_5 = 3\,\Omega$

$L_2 = 4\,mH$

$C_2 = 250\,\mu F$

$L_3 = 3\,mH$

$L_4 = 2\,mH$

$C_4 = 125\,\mu F$

a) $i(t)$, $I_{ef} = ?$

b) puteri generate de $e(t)$

c) puterea activă consumată de R_5

d) $K_{di} = ?$, $K_{de} = ?$, $K_{dis} = ?$

$$i_s(t) = 8 + 2\sqrt{2}\sin\left(2\cdot 10^3 t - \frac{\pi}{4}\right)$$

$$e(t) = 12 + 12\sqrt{2}\sin\left(10^3 t + \frac{\pi}{4}\right) + 24\sin\left(2\cdot 10^3 t + \frac{\pi}{2}\right)$$

$k=0$, $i_s(t)$, $e(t)$

$k=1$, $e(t)$, $i_s(t)$ —o o—

$k=2$, $e(t)$, $i_s(t)$.

a)

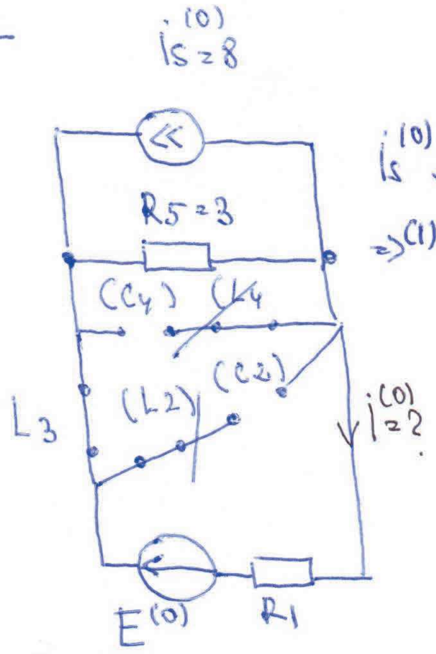
$\boxed{\text{I}\ k=0}$

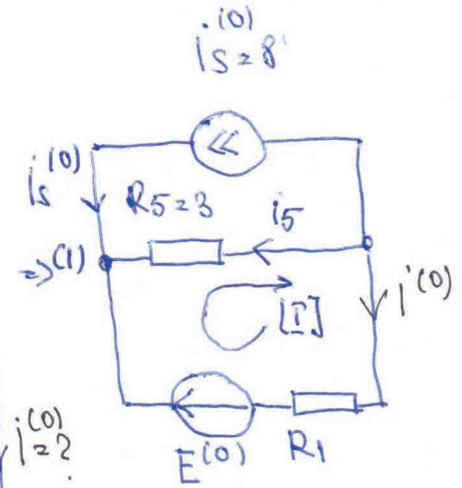
$E^{(0)} = 12$

$I_s^{(0)} = 8$

$L^{c.c.}$ —•—

$C^{c.c.}$ —o o—





TK1 (1) $I_s^{(0)} + I_5 + I^{(0)} = 0$

$8 + I_5 + I^{(0)} = 0$

TK2 [I] $I^{(0)} \cdot R_1 - I_5 R_5 = E^{(0)}$

$3 \cdot I^{(0)} - 3 I_5 = 12 \;|:3$

$I^{(0)} - I_5 = 4 \Rightarrow I_5 = I^{(0)} - 4$

$8 + I^{(0)} - 4 + I^{(0)} = 0 \Rightarrow 2 I^{(0)} + 4 = 0 \Rightarrow \boxed{I^{(0)} = -2}$

$\boxed{\text{II } K=1}$

$i_S^{(1)}(t)$ = nu are componentă pt $K=1 \xrightarrow[\text{sare}]{\text{pasivi-}}$ —o o—

$e^{(1)}(t) = 12\sqrt{2}\sin(10^3 t + \frac{\pi}{4}) \to \underline{E}^{(1)} = 12 e^{j\frac{\pi}{4}} = 12\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = 6\sqrt{2}(1+j)$

$\underline{Z}_{R1}^{(1)} = \underline{Z}_{R5}^{(1)} = 3$

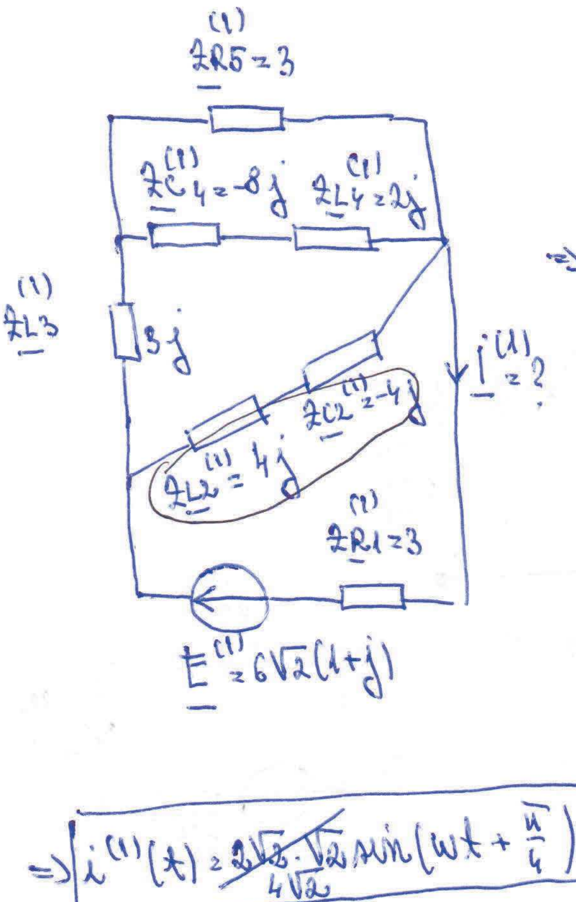
$\underline{Z}_{L2}^{(1)} = j\omega L_2 = j \cdot \cancel{1000} \cdot 4 \cdot \cancel{10^{-3}} = 4j$

$\underline{Z}_{L3}^{(1)} = j\omega L_3 = j \cdot \cancel{1000} \cdot 3 \cdot \cancel{10^{-3}} = 3j$

$\underline{Z}_{L4}^{(1)} = j\omega L_4 = j \cdot \cancel{1000} \cdot 2 \cdot \cancel{10^{-3}} = 2j$

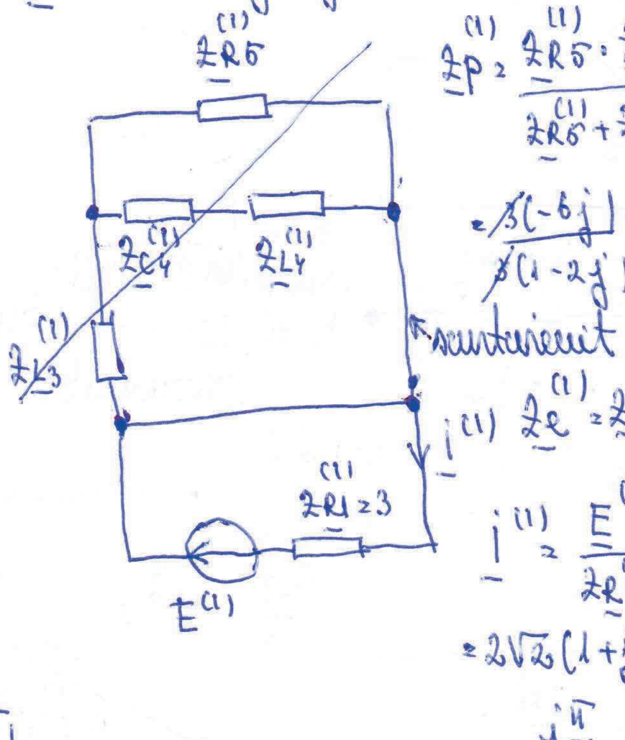
$\underline{Z}_{C2} = \frac{-j}{\omega C_2} = \frac{-j}{\cancel{1000} \cdot 25\cancel{0} \cdot 10^{-6}} = -4j$

$\underline{Z}_{C4} = \frac{-j}{\omega C_4} = \frac{-j}{1000 \cdot 125 \cdot 10^{-6}} = \frac{-1000j}{125} = -8j$



$\Rightarrow$

$\underline{Z}_{L2}^{(1)} + \underline{Z}_{C2}^{(1)} = 4j - 4j = 0$ rezonanță pt $K=1$ —o—o



$\underline{Z}_p^{(1)} = \frac{\underline{Z}_{R5}^{(1)} \cdot \underline{Z}_S^{(1)}}{\underline{Z}_{R5}^{(1)} + \underline{Z}_S^{(1)}} = \frac{3 \cdot (-6j)}{3 - 6j}$

$= \frac{\cancel{3}(-6j)}{\cancel{3}(1-2j)} = \frac{-6j(1+2j)}{5}$

$\underline{Z}_e^{(1)} = \underline{Z}_p^{(1)} + \underline{Z}_{L3}^{(1)} = 3j + \frac{12-6j}{5}$

$\underline{I}^{(1)} = \frac{\underline{E}^{(1)}}{\underline{Z}_{R1}^{(1)}} = \frac{6\sqrt{2}(1+j)}{3}$

$= 2\sqrt{2}(1+j) = 2\sqrt{2}\underset{\sqrt{2}}{\wedge} e^{j\frac{\pi}{4}}$

$= 4 e^{j\frac{\pi}{4}}$

$\Rightarrow \boxed{i^{(1)}(t) = \cancel{2\sqrt{2}\cdot\sqrt{2}}\,4\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)}$

$\boxed{\text{III } K=2}$

$$i_S^{(2)}(t) = 2\sqrt{2}\sin\left(2\cdot 10^3 t - \frac{\pi}{4}\right) \longrightarrow \underline{I}_S^{(2)} = 2e^{j\left(-\frac{\pi}{4}\right)} = 2\left(+\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = \sqrt{2}(1-j)$$

$$e^{(2)}(t) = \frac{24}{\sqrt{2}}\sqrt{2}\sin\left(2\cdot 10^3 t + \frac{\pi}{2}\right) \longrightarrow \underline{E}^{(2)} = \frac{24}{\sqrt{2}}e^{j\frac{\pi}{2}} = \frac{24}{\sqrt{2}}j$$

$$\underline{Z}_{R1}^{(2)} = \underline{Z}_{R5}^{(2)} = 3$$

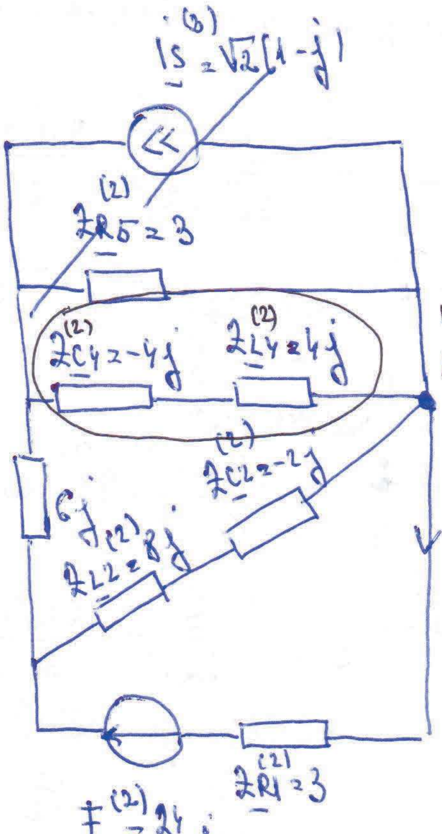
$$\underline{Z}_{L2}^{(2)} = j\cdot 2\cdot \omega L_2 = j\cdot 2\cdot \cancel{10^3}\cdot 4\cdot \cancel{10^{-3}} = 8j$$

$$\underline{Z}_{L3}^{(2)} = j\cdot 2\cdot \omega L_3 = j\cdot 2\cdot \cancel{10^3}\cdot 3\cdot \cancel{10^{-3}} = 6j$$

$$\underline{Z}_{L4}^{(2)} = j\cdot 2\cdot \omega L_4 = j\cdot 2\cdot \cancel{10^3}\cdot 2\cdot \cancel{10^{-3}} = 4j$$

$$\underline{Z}_{C2}^{(2)} = \frac{-j}{2\omega C_2} = \frac{-j}{2\cdot 1000\cdot 250\cdot 10^{-6}} = -2j$$

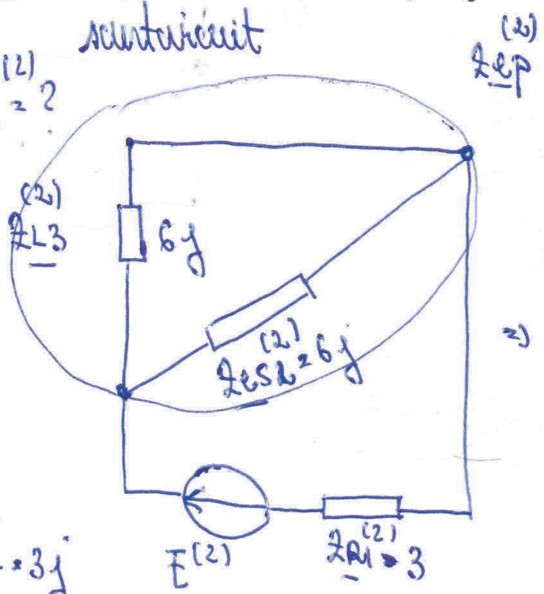
$$\underline{Z}_{C4}^{(2)} = \frac{-j}{2\omega C_4} = \frac{-j}{2\cdot 10^3\cdot 125\cdot 10^{-6}} = -4j$$



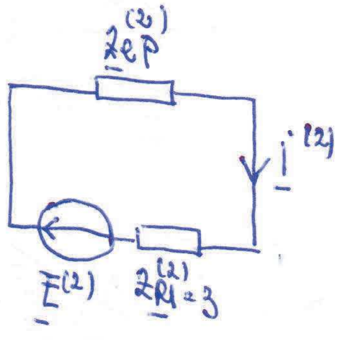
rezonanta serie pe armonica

$\underline{K=2}$ $\underline{Z}_{es}^{(2)} = \underline{Z}_{C4}^{(2)} + \underline{Z}_{L4}^{(2)} = -4j + 4j$

scurtcircuit



$\Rightarrow$



$$\underline{Z}_{ep}^{(2)} = \frac{\underline{Z}_{L3}^{(2)}\cdot \underline{Z}_{es2}^{(2)}}{\underline{Z}_{L3}^{(2)} + \underline{Z}_{es2}^{(2)}} = \frac{6j\cdot 6j}{6j+6j} = \frac{6j}{2} = 3j$$

$$\underline{I}^{(2)} = \frac{\underline{E}^{(2)}}{\underline{Z}_{ep}^{(2)} + \underline{Z}_{R1}^{(2)}} = \frac{24}{\sqrt{2}}j\cdot\frac{1}{3(1+j)} = \frac{8j\cdot\sqrt{2}}{(1+j)} = \frac{4j\sqrt{2}(1-j)}{\cancel{2}} = 2\sqrt{2}(j+1) = 2\cdot\sqrt{2}\cdot\sqrt{2}e^{j\frac{\pi}{4}} = 4e^{j\frac{\pi}{4}}$$

$$\Rightarrow \boxed{i^{(2)}(t) = 4\sqrt{2}\sin\left(2\omega t + \frac{\pi}{4}\right)}$$

$$i(t) = i^{(0)} + i^{(1)}(t) + i^{(2)}(t) = -2 + 2\sqrt{2}\cdot\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right) + 4\sqrt{2}\sin\left(2\omega t + \frac{\pi}{4}\right)$$

$$e(t) = 12 + 12\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right) + 24\sin\left(2\omega t + \frac{\pi}{2}\right)$$

$$I_{ef} = \sqrt{I^{(0)2} + I^{(1)2} + I^{(2)2}} = \sqrt{(-2)^2 + (2\sqrt{2})^2 + 4^2} = \sqrt{4+8+16} = \sqrt{28} = 2\sqrt{7}\,A$$

b) $$E_{ef} = \sqrt{12^2 + 12^2 + \left(\frac{24}{\sqrt{2}}\right)^2} = \sqrt{144+144+288} = \sqrt{576} = \sqrt{4\cdot 144} = \sqrt{(2\cdot 12)^2} = 24\,V$$

- $S = I_{ef}\cdot E_{ef} = 24\cdot 2\sqrt{7} = 48\sqrt{7}\,VA$
- $P_g = E^{(0)}\cdot I^{(0)} + E^{(1)}\cdot I^{(1)}\cos\varphi_1 + E^{(2)}\cdot I^{(2)}\cos\varphi_2$

$$\varphi_1 = \varphi_{E1} - \varphi_{i1} = \frac{\pi}{4} - \frac{\pi}{4} = 0$$

$$\varphi_2 = \varphi_{E2} - \varphi_{i2} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\Rightarrow P_g = 12(-2) + 12\cdot 2\sqrt{2}\cos 0 + 4\cdot\frac{24}{\sqrt{2}}\cos\frac{\pi}{4}$$

$$= -24 + 24\sqrt{2} + 4\cdot\frac{\overset{12}{\cancel{24}}}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{\cancel{2}}$$

$$= 24(-1+\sqrt{2}+2) = 24(1+\sqrt{2}) \simeq 57{,}94\,W$$

- $Q_g = E^{(1)}\cdot I^{(1)}\sin\varphi_1 + E^{(2)}\cdot I^{(2)}\sin\varphi_2 = 12\cdot 2\sqrt{2}\sin 0 + 4\cdot\frac{24}{\sqrt{2}}\sin\frac{\pi}{4}$

$$= 24\sqrt{2}\cdot 0 + 4\cdot\frac{\overset{12}{\cancel{24}}}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{\cancel{2}} = 48\,VAR$$

- $D = \sqrt{S^2 - P_g^2 - Q_g^2} = \sqrt{16128 - 3357{,}0 - 2304} = \sqrt{10467} \simeq 102{,}30\,VAD$

c) $P_{c(R_1)} = R_1\cdot I_{ef}^2 = 3\cdot 4\cdot 7 = 84\,W$

d) $$K_{di} = \frac{I_d}{I_a} = \frac{\sqrt{I^{(2)2}}}{\sqrt{I^{(1)2} + I^{(2)2}}} = \sqrt{\frac{4^2}{(2\sqrt{2})^2 + 4^2}} = \sqrt{\frac{16}{8+16}} = \frac{\sqrt{16}}{\sqrt{24}} = \frac{\cancel{4}}{\cancel{2}\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$K_{de} = \frac{E_d}{E_a} = \frac{\sqrt{24^2/2}}{\sqrt{12^2 + 24^2/2}} = \frac{\sqrt{288}}{\sqrt{292}} = \frac{\sqrt{4\cdot 4\cdot 18}}{\sqrt{4\cdot 73}} = \frac{\sqrt{2^4\cdot 2\cdot 3^2}}{\sqrt{2^2\cdot 73}} = \frac{2^{\cancel{2}}\,3\sqrt{2}}{\cancel{2}\sqrt{73}} = \frac{6\sqrt{2}}{\sqrt{73}}$$

$$K_{dis} = \frac{I_{sd}}{I_{sa}} = \frac{\sqrt{i_s^{(2)2}}}{\sqrt{i_s^{(1)2} + i_s^{(2)2}}} = \sqrt{\frac{2^2}{0^2 + 2^2}} = 1$$