NMIT 2, Serie 1

Aufgabe 1

a) Taylorreihe ohne Restglied

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k$$

$f(x) = e^x \qquad x_0 = 0 \qquad n = 7$

$f'(x), f''(x), \dots f^{(7)}(x) = e^x$

$$p_7(x) = \sum_{k=0}^{7} \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k = f(x_0) + f'(x_0)(x-x_0) +$$

$$\frac{f''(x_0)}{2}(x-x_0)^2 + \frac{f^{(3)}(x_0)}{6}(x-x_0)^3 + \frac{f^{(4)}(x_0)}{24}(x-x_0)^4 +$$

$$\frac{f^{(5)}(x_0)}{120}(x-x_0)^5 + \frac{f^{(6)}(x_0)}{720}(x-x_0)^6 + \frac{f^{(7)}(x_0)}{5040}(x-x_0)^7$$

Einsetzen von $x_0 = 0$

$$p_7(x_0) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} \Rightarrow$$ [redacted]

b) $p_7(1) = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} = 2.7183$

$f(1) = e^1 = 2.7183$ (Eulerzahl)

error $= |p_7(1) - f(1)| = 0$

c) $$\sum_{k=0}^{n} \frac{1}{k!}$$