Aufgabe 2, Serie 5

$\frac{dx}{dy} = \frac{x^2}{y}$

exakte Lösung:

$y(x) = \sqrt{\frac{2x^3}{3} + 4}$

$0 \leq x \leq 2.1$

$y(0) = 2$

a) $h = 0.7$

$h = \frac{b-a}{n} \quad / \cdot n$

$hn = b - a \quad / : h$

$n = \frac{b-a}{h} = \frac{2.1-0}{0.7} = 3$

$x_{i+1} = x_i + h$

$y_{i+1} = y_i + h \cdot f(x_i, y_i)$

$x_0 = 0 \qquad y_0 = 2$

$x_1 = x_0 + h = 0 + 0.7 = 0.7$

$y_1 = 2 + 0.7 \cdot \left(\frac{0}{2}\right) = 2$

$x_2 = x_1 + h = 0.7 + 0.7 = 1.4$

~~$y_2 = 2 + 0.6 \left(\frac{0.7^2}{2}\right) = 2 + 0.6 \cdot 0.245 = 2.21$~~

$y_2 = y_1 + h\left(f(x_1, y_1)\right) = 2 + 0.7 \cdot \frac{0.7^2}{2} = 2.1715$

$x_3 = x_2 + h = 2.1$

$y_3 = 2.1715 + 0.7 \cdot \frac{1.4^2}{2.1715} = 2.803321$

$y(x_1) = y(0.7) = 2.0564 \qquad abs\_error = |2.0564 - 2.00| = 0.0564$

$y(x_2) = y(1.4) = 2.4144 \qquad abs\_error_{x_2} = |2.4144 - 2.1715| = 0.2429$

$y(x_3) = y(2.1) = 3.1897 \qquad abs\_error_{x_3} = |3.1897 - 2.803321| = 0.3863$

Aufgabe 2, Serie 5

b) Mittelpunkt-Verfahren

$n = 3$ $\quad h = 0.7$ (dito a) $\quad x_0 = 0 \quad y_0 = 2$

$x_{h/2} = x_i + \frac{h}{2}$ $\qquad y^{h/2} = y_i + \frac{h}{2} \cdot f(x_i, y_i)$

$x_{i+1} = x_i + h$ $\qquad y_{i+1} = y_i + h \cdot f(x_{h/2}, y^{h/2})$

Schritt 1 $\quad n = 1$

$x_{h/2} = 0 + 0.7/2 = 0.35$ $\qquad y_{h/2} = 2 + 0.35 \cdot \frac{0}{2} = 2$

$x_1 = 0 + 0.7 = 0.7$ $\qquad y_1 = 2 + 0.7 \cdot \frac{0.35^2}{2} = 2.0429$

Schritt 2 $\quad n = 2$

$x_{h/2} = 0.7 + 0.35 = 1.05$ $\qquad y_{h/2} = 2.0429 + \frac{0.35}{2.0429} \cdot 0.7^2 = 2.1268$

$x_2 = 0.7 + 0.7 = 1.4$ $\qquad y_2 = 2.0429 + 0.7 \cdot \frac{1.05^2}{2.1268} = 2.4057$

Schritt 3 $\quad n = 3$

$x_{h/2} = 1.4 + 0.35 = 1.75$ $\qquad y_{h/2} = 2.4057 + 0.35 \cdot \frac{1.4^2}{2.4057} = 2.6909$

$x_3 = 1.4 + 0.7 = 2.1$ $\qquad y_2 = 2.4057 + 0.35 \cdot \frac{1.75^2}{2.6909} = 3.2024$

Exakte Werte dito 2a)

$\text{abs\_error}_{x_1} = |2.0564 - 2.0429| = 0.0135$

$\text{abs\_error}_{y_2} = |2.41144 - 2.4057| = 0.0087$

$\text{abs\_error}_{y_3} = |3.1897 - 3.2024| = 0.0127$

c) Das modifizierte Euler-Verfahren

dito a und b

$n=3 \quad h=0.7 \quad a=0 \quad b=2.1 \quad x_0=0 \quad y_0=2$

$x_{i+1} = x_i + h \qquad y_{i+1}^{Euler} = y_i + h \cdot f(x_i, y_i)$

$k_1 = f(x_i, y_i) \qquad k_2 = f(x_{i+1}, y_{i+1}^{E})$

$x_{i+1} = x_i + h \qquad y_{i+1} = y_i + h \frac{(k_1 + k_2)}{2}$

Schritt 1 $n=1$

$x_1 = 0 + 0.7 = 0.7 \qquad y_{1+}^{E} = 2 + 0.7 \cdot \frac{0^2}{2} = 2$

$k_1 = \frac{0^2}{2} = 0 \qquad k_1 = \frac{0.7^2}{2} = 0.245$

$x_1 = 0.7 \qquad y_1 = 2 + 0.7 \frac{0 + 0.245}{2} = 2.0857$

Schritt 2 $n=2$

$x_2 = 0.7 + 0.7 = 1.4 \qquad y_2^{E} = 2.0857 + 0.7 \frac{0.7^2}{2.0857} = 2.2502$

$k_1 = \frac{0.7^2}{2.0857} = 0.2349 \qquad k_2 = \frac{1.4^2}{2.2502} = 0.8710$

$x_2 = 1.4 \qquad y_2 = 2.0857 + 0.7 \frac{0.2349 + 0.8710}{2} = 2.4728$

Schritt 3 $n=3$

$x_3 = 2.1 \qquad y_3^{E} = 2.4728 + 0.7 \frac{1.4^2}{2.4728} = 3.0277$

$k_1 = \frac{1.4^2}{2.4728} = 0.7926 \qquad k_2 = \frac{2.1^2}{3.0277} = 1.4566$

$x_3 = 2.1 \qquad y_3 = 2.4728 + 0.7 \cdot \frac{0.7926 + 1.4566}{2} = 3.26$

abs_error $y_1 = |2.0564 - 2.0857| = 0.0294$

abs_error $y_2 = |2.4144 - 2.4728| = 0.0584$

abs_error $y_3 = |3.1897 - 3.26| = 0.0704$