

serie 5, Aufgabe 1

$$I = \int_1^2 \ln(x^2) dx$$

$$\text{error} = 10^{-5}$$

$$h = ?$$

Summierte Rechteckregel:

$$\left| \int_a^b f(x) dx - Rf(h) \right| \leq \frac{h^2}{24} (b-a) \cdot \max_{x \in [a,b]} |f''(x)|$$

error = 10^{-5}

$$f'(x) = \frac{2}{x}$$

$$f''(x) = -\frac{2}{x^2}$$

$$|f''(1)| = -\frac{2}{1} = -2 = 2 \quad |f''(2)| = -\frac{2}{4} = -0.5 = 0.5$$

(5. m2lab script)

$$\max_{x \in [2,5]} |f''(x)| = 2$$

$$10^{-5} \leq \frac{h^2}{24} \cdot 1 \cdot 2$$

$$10^{-5} \leq h^2 \cdot \frac{1}{12} \quad | \cdot 12$$

$$h^2 \geq 10^{-5} \cdot 12$$

$$h^2 \geq \sqrt{10^{-5} \cdot 12} = \sqrt{1.2 \cdot 10^{-6}} = \underline{0.0110}^{-3}$$

Summierte Trapezregel:

$$\left| \int_a^b f(x) dx - Tf(h) \right| \leq \frac{h^2}{12} (b-a) \cdot \max_{x \in [a,b]} |f''(x)|$$

$$10^{-5} \leq \frac{h^2}{12} \cdot 1 \cdot 2$$

$$h^2 \geq \sqrt{6 \cdot 10^{-5}} = \underline{0.0077}$$

$$\left| \int_a^b f(x) dx - S_f(h) \right| \leq \frac{h^4}{2880} (b-a) \cdot \max_{x \in [a,b]} |f^{(4)}(x)|$$

$$f^{(3)} = \frac{4}{x^3}$$

$$f^{(4)}(x) = -\frac{12}{x^4}$$

$$\underbrace{|f^{(4)}(1)| = \left| -\frac{12}{1} \right| = 12}_{\max} \quad f^{(4)}(2) = \left| -\frac{12}{16} \right| = \left| \frac{3}{4} \right| = 0.75$$

$$10^{-5} \leq \frac{h^4}{2880} \cdot 12 \quad / : 12$$

$$\frac{10^{-5}}{12} \leq \frac{h^4}{2880} \quad / \cdot 2880$$

$$h^4 \geq \frac{10^{-5} \cdot 2880}{12}$$

$$\underline{h \geq 0.2213}$$