

Serie 10, Aufgabe 1

$$S_i(x) \quad \begin{array}{c|c|c|c|c} x_i & 0 & 1 & 2 & 3 \\ \hline q_i = y_i & 2 & 1 & 2 & 2 \end{array}$$

① $q_0 = 2 \quad q_1 = 1 \quad q_2 = 2 \quad q_3 = 2$

② $h_i = x_{i+1} - x_i$

$$h_0 = 1 - 0 = 1$$

$$h_1 = x_2 - x_1 = 2 - 1 = 1$$

$$h_2 = x_3 - x_2 = 3 - 2 = 1$$

③ $c_0 = 0, \quad c_n = 0 \Rightarrow n = i - 1 = 4 - 1 = 3$
 $\Rightarrow c_3 = 0$

④ ~~ebenfalls~~

• $i = 1:$

$$2(h_0 + h_1) \cdot c_1 + h_1 c_2 = 3 \frac{y_2 - y_1}{h_1} - 3 \frac{y_1 - y_0}{h_0}$$

$$2 \cdot 2 \cdot c_1 + 1 \cdot c_2 = \frac{3 \cdot 1}{1} - \frac{3 \cdot (-1)}{1}$$

$$4c_1 + c_2 = 3 + 3 = 6$$

• $i = 2, \dots, n-2$

$$h_{i-1} \cdot c_{i-1} + 2(h_{i-1} + h_i) c_i + h_i c_{i+1} = 3 \frac{y_{i+1} - y_i}{h_i} - 3 \frac{y_i - y_{i-1}}{h_{i-1}}$$

* der Schritt ist nicht gebraucht

• $i = n-1$

$$h_{n-2} \cdot c_{n-2} + 2(h_{n-2} + h_{n-1}) \cdot c_{n-1} = \frac{3y_n - y_{n-1}}{h_{n-1}} - 3 \frac{y_{n-1} - y_{n-2}}{h_{n-2}}$$

$$1 \cdot c_1 + 2(1+1) \cdot c_2 = 0 - 3$$

$$c_1 + 4c_2 = -3$$

Gleichungssystem;

$$\begin{cases} 4c_1 + c_2 = 6 \\ c_1 = -3 - 4c_2 \end{cases}$$

$$4(-3 - 4c_2) + c_2 = 6$$

$$-12 - 16c_2 + c_2 = 6$$

$$-15c_2 = 18 \quad | : (-15)$$

$$c_2 = -\frac{18}{15} = -\frac{6}{5} = -1.2$$

$$c_1 = -3 + 4.8 = 1.8$$

$$\underline{c_2 = -1.2}$$

$$\underline{c_1 = 1.8}$$

$$\textcircled{5} \quad b_i = \frac{y_{i+1} - y_i}{h_i} - \frac{h_i}{3} (c_{i+1} + 2c_i)$$

$$\underline{b_0} = \frac{y_1 - y_0}{h_0} - \frac{h_0}{3} (c_1 + 2c_0) = -1 - \frac{1}{3} \cdot 1.8 = \underline{-1.6}$$

$$\underline{b_1} = \frac{y_2 - y_1}{h_1} - \frac{h_1}{3} (c_2 + 2c_1) = 1 - \frac{1}{3} \cdot 2.4 = \underline{0.2}$$

$$\underline{b_2} = \frac{y_3 - y_2}{h_2} - \frac{h_2}{3} (c_3 + 2c_2) = 0 - \frac{1}{3} \cdot (-2.4) = \underline{0.8}$$

$$\textcircled{6} \quad d_i = \frac{1}{3h_i} (c_{i+1} - c_i)$$

$$\underline{d_0} = \frac{1}{3h_0} (c_1 - c_0) = \frac{1}{3} \cdot 1.8 = \underline{0.6}$$

$$\underline{d_1} = \frac{1}{3h_1} (c_2 - c_1) = \frac{1}{3} \cdot (-3) = \underline{-1}$$

$$\underline{d_2} = \frac{1}{3h_2} (c_3 - c_2) = \frac{1}{3} \cdot 1.2 = \underline{0.4}$$

⑦ Aufstellen der Polynome

$$S_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$$

$$S_0(x) = 2 - 1.6 \cdot x + 0.6(x-0)^3 = 2 - 1.6x + 0.6x^3 \quad x \in [x_0, x_1]$$

$$S_1(x) = 1 + 0.2(x-1) + 1.8(x-1)^2 - (x-1)^3 \quad x \in [x_1, x_2]$$

$$S_2(x) = 2 + 0.8(x-2) - 1.2(x-2)^2 + 0.4(x-2)^3 \quad x \in [x_2, x_3]$$