Serie 9, Aufgabe 1

a) $J(a) = 2 \cdot \int_1^a x \cdot \ln(x^2) \cdot dx \quad \wedge \quad a = e$

Anzahl der Stützpunkte $= 4 \Rightarrow 4 = n+1 \Rightarrow n = 3$

Polynom höchstens 3. Grades

$P_n(x) = \sum_{i=0}^{n} l_i(x) y_i \qquad x = a \quad y = J(a)$

$\Rightarrow P_n(x) = \sum_{i=0}^{3} l_i(x) y_i = l_0(x) \cdot y_0 + y_1 l_1(x) + y_2 l_2(x) + + y_3 l_3(x)$

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

wobei $x_0 = e - \frac{1}{2}$, $x_1 = e - \frac{1}{4}$ $x_2 = e + \frac{1}{4}$ $x_3 = e + \frac{1}{2}$

$y_0 = 3.9203$, $y_1 = 5.9169$, $y_2 = 11.3611$, $y_3 = 14.8550$

$$l_0(x) = \frac{(x-e+\frac{1}{2})(x-e+\frac{1}{4})(x-e-\frac{1}{4})}{(e-\frac{1}{2}-e+\frac{1}{4})(e-\frac{1}{2}-e-\frac{1}{4})(e-\frac{1}{2}-e-\frac{1}{2})} =$$

$$= \frac{x^2 - x(e+\frac{1}{4}) + x(e+\frac{1}{2}) + (e+\frac{1}{2})(e+\frac{1}{4})}{-\frac{1}{4} \cdot (-\frac{3}{4})(-1)} =$$

$$= \frac{x^2 - x(e+\frac{1}{4}+e+\frac{1}{2}) + e^2 + \frac{1}{4}e + \frac{1}{2}e \cdot \frac{1}{8}}{-\frac{3}{16}}$$

$\Rightarrow$ in weiteren Schritten wurden ~~die~~ der konkrete Wert $x = e$ eingesetzt.

resp. $x = e$

$$l_0(e) = \frac{(\cancel{e}-e+\frac{1}{4})(\cancel{e}-e-\frac{1}{4})(\cancel{e}-e-\frac{1}{2})}{-\frac{3}{16}} = \frac{\frac{1}{32}}{-\frac{3}{4}} = \frac{1}{32_2} \cdot \left(-\frac{16^1}{3}\right) =$$

$= \cancel{\ } - \frac{1}{6}$

$$l_1(e) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} =$$

$$= \frac{(\cancel{e}-\cancel{e}+\frac{1}{2})(\cancel{e}-\cancel{e}-\frac{1}{4})(\cancel{e}-\cancel{e}-\frac{1}{2})}{(\cancel{e}-\frac{1}{4}-\cancel{e}+\frac{1}{2})(\cancel{e}-\frac{1}{4}-\cancel{e}-\frac{1}{4})(\cancel{e}-\frac{1}{4}-\cancel{e}-\frac{1}{2})} =$$

$$= \frac{\frac{1}{16}}{\frac{1}{4}\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{3}{4}\right)} = \frac{1}{16} \Big/ \left(+\frac{3}{32}\right) = \frac{1}{\cancel{16}_1}\cdot\left(+\frac{\cancel{32}^2}{3}\right) = +\frac{2}{3}$$

$$l_2(e) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} =$$

$$= \frac{(e-e+\frac{1}{2})(e-\cancel{e}+\frac{1}{4})(\cancel{e}-\cancel{e}-\frac{1}{2})}{(\cancel{e}+\frac{1}{4}-\cancel{e}+\frac{1}{2})(\cancel{e}+\frac{1}{4}-\cancel{e}+\frac{1}{4})(\cancel{e}+\frac{1}{4}-e-\frac{1}{2})} =$$

$$= \frac{-\frac{1}{16}}{\frac{3}{4}\cdot\frac{2}{4}\left(-\frac{1}{4}\right)} = -\frac{1}{16} \Big/ \left(-\frac{6}{64}\right) = -\frac{1}{\cancel{16}_1}\cdot\left(-\frac{\cancel{64}^4}{6}\right) = \frac{2}{3}$$

$$l_3(e) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_1)} =$$

$$= \frac{(\cancel{e}-\cancel{e}+\frac{1}{2})(\cancel{e}-\cancel{e}+\frac{1}{4})(\cancel{e}-\cancel{e}-\frac{1}{4})}{(\cancel{e}+\frac{1}{2}-\cancel{e}+\frac{1}{2})(\cancel{e}+\frac{1}{2}-\cancel{e}+\frac{1}{4})(\cancel{e}+\frac{1}{2}-e-\frac{1}{4})} =$$

$$= \frac{-\frac{1}{32}}{1\cdot\frac{3}{4}\cdot\frac{1}{4}} = -\frac{1}{\cancel{32}_2}\cdot\frac{\cancel{16}^1}{3} = -\frac{1}{6}$$

$$p_3(e) = 3.9203\cdot\left(-\frac{1}{6}\right) + 5.9169\cdot\frac{2}{3} + 11.3611\cdot\frac{2}{3} + 14.8550\cdot\left(-\frac{1}{6}\right)$$

$$= \underline{\underline{8.38945}}$$

b)

Exakter Wert:
berechnet mithilfe von Matlab s. script
"dubelbog_nureldeld_G6_Serie_9_Aufgabe1b"

$$2 \cdot \int_1^{e} x \cdot \ln(x^2) \cdot dx = 8.38905610$$

$$\text{error}_{abs} = \left| P_3(e) - \int_1^e J(e) \right| = \left| 8.38945 - 8.38905610 \right|$$

$$= \underline{0{,}0003939}$$

c) siehe Script => Polyberg schneidet besser ab

(Berechnung siehe script)

$\text{error}_{relativ} =$

Aufgabe 4

value	~~0~~	1	2 ~~000~~	3	4	5
m	0	1250	2500	3750	5'000	10'000
A-Druck	1013	? 872	747	? 638	540	226
	x_0		x_1		x_2	x_3

$$P_{i0} = y_i$$

$$P_{ij} = \frac{(x_i - x)\cdot P_{i-1,j-1} + (x - x_{i-j})\cdot P_{i,j-1}}{x_i - x_{i-j}}$$

Polynom höchstens 3. Grades.

Gesuchte Matrix

x	y			
x_0	$y_0 = P_{00}$			
x_1	$y_1 = P_{10}$	P_{11}		
x_2	$y_2 = P_{20}$	P_{21}	P_{22}	
x_3	$y_3 = P_{30}$	P_{31}	P_{32}	P_{33}

$$P_{11} = \frac{(x_1 - x)\cdot P_{00} + (x - x_0)\cdot P_{10}}{x_1 - x_0}$$

$$P_{21} = \frac{(x_2 - x)\cdot P_{10} + (x - x_1)\cdot P_{20}}{x_2 - x_1}$$

$$P_{31} = \frac{(x_3 - x)\cdot P_{20} + (x - x_2)\cdot P_{30}}{x_3 - x_2}$$

$$P_{22} = \frac{(x_2 - x)\cdot P_{11} + (x - x_0)\cdot P_{21}}{x_2 - x_0}$$

$$P_{32} = \frac{(x_3 - x)\cdot p_{21} + (x - x_1)\cdot p_{31}}{x_3 - x_1}$$

$$P_{33} = \frac{(x_3 - x)\cdot p_{22} + (x - x_0)\cdot p_{32}}{x_3 - x_0}$$

$P_{00} = 1013$ $P_{10} = 747$ $P_{20} = 540$ $P_{30} = 226$

$x_0 = 0$ $x_1 = 2500$ $x_2 = 5'000$ $x_3 = 10'000$

$x = 1250$

$$P_{11} = \frac{(2500 - 1250)\cdot 1013 + (1250 - 0)\cdot 747}{2500 - 0} = 880$$

$$P_{21} = \frac{(5'000 - 1250)\cdot 747 + (1250 - 2500)\cdot 540}{5'000 - 2500} = 850.5 \approx 851$$

$$P_{31} = \frac{(10'000 - 1'250)\cdot 540 + (1'250 - 5'000)\cdot 226}{10'000 - 5'000} = 775.5 \approx 776$$

$$P_{22} = \frac{(5'000 - 1'250)\cdot 880 + (1'250 - 0)\cdot 851}{5'000 - 0} = 872,75 \approx 873$$

$$P_{32} = \frac{(10'000 - 1'250)\cdot 851 + (1'250 - 2'500)\cdot 776}{10'000 - 2500} = 863.50 \approx 864$$

$$P_{33} = \frac{(10'000 - 1'250)\cdot 873 + (1'250 - 0)\cdot 864}{10'000 - 0} = 871.875 \approx 872$$

$x = 3'750$

$$P_{11} = \frac{(2'500 - 3'750)\cdot 1013 + (3'750 - 0)\cdot 747}{2'500 - 0} \approx 614$$

$$P_{21} = \frac{(5'000 - 3'750)\cdot 747 + (3'750 - 2'500)\cdot 540}{5'000 - 2'500} = 643.5 \approx 644$$

$$P_{31} = \frac{(10'000 - 3'750)\cdot 540 + (3'750 - 5'000)\cdot 226}{10'000 - 5'000} = 618.5 \approx 619$$

$$p_{22} = \frac{(5'000 - 3'750) \cdot 614 + (3'750 - 0) \cdot 644}{5'000 - 0} = 636.5 \approx 637$$

$$p_{32} = \frac{(10'000 - 3'750) \cdot 644 + (3'750 - 2'500) \cdot 613}{10'000 - 2'500} = 639.833... \approx 640$$

$$p_{33} = \frac{(10'000 - 3'750) \cdot \cancel{644}\, 637 + (3'750 - 0) \cdot 640}{10'000 - 0} = 638.125 \approx \underline{\underline{638}}$$