

Aufgabe 2, Seite 3

$$\int_0^{\pi} \cos(x^2) dx$$

Trapezregel:

$$T_f(h) = h \cdot \left(\frac{f(a) + f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right)$$

$$h = \frac{b-a}{n} \quad i \in \{0, 1, \dots, n\}$$

$$T_{10} = h_0 \cdot \frac{f(0) + f(\pi)}{2} = \pi \cdot \frac{\cos(0^2) + \cos(\pi^2)}{2} = 0.15286148$$

$$T_{10} \quad h_1 = \frac{\pi}{2} \quad n=2^1 \Rightarrow 2^1 = 2$$

$$T_{10} = \frac{\pi}{2} \cdot \left(\frac{\cos(0^2) + \cos(\pi^2)}{2} + \cos\left(0 + \frac{\pi^2}{2}\right) \right) =$$

$$= -1.150694033$$

$$T_{20} = \frac{\pi}{4} \cdot \left(\frac{\cos(0^2) + \cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{4}\right)^2\right) + \cos\left(\left(\frac{\pi}{2}\right)^2\right) + \cos\left(\left(\frac{3\pi}{4}\right)^2\right) \right)$$

$$= 0.64976090$$

$$T_{30} = \frac{\pi}{8} \cdot \left(\frac{\cos(0^2) + \cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{8}\right)^2\right) + \cos\left(\left(\frac{2\pi}{8}\right)^2\right) + \cos\left(\left(\frac{3\pi}{8}\right)^2\right) + \cos\left(\left(\frac{4\pi}{8}\right)^2\right) + \cos\left(\left(\frac{5\pi}{8}\right)^2\right) + \cos\left(\left(\frac{6\pi}{8}\right)^2\right) + \cos\left(\left(\frac{7\pi}{8}\right)^2\right) \right)$$

$$= 0.60262130$$

$$T_{40} = \frac{\pi}{16} \cdot \left(\frac{\cos(0^2) + \cos(\pi^2)}{2} + \cos\left(\left(\frac{\pi}{16}\right)^2\right) + \cos\left(\left(\frac{2\pi}{16}\right)^2\right) + \dots + \cos\left(\left(\frac{15\pi}{16}\right)^2\right) \right)$$

$$= 0.57452853$$

Romberg - Extrapolation:

$$T_{ik} = \frac{4^k \cdot T_{i+1, k-1} - T_{i, k-1}}{4^k - 1}$$

$$T_{01} = \frac{4 \cdot T_{10} - T_{00}}{3} = -1.58521254$$

$$T_{02} = \frac{4 \cdot T_{20} - T_{10}}{3} = 1.24991254$$

$$T_{03} = \frac{4 \cdot T_{30} - T_{20}}{3} = 0.58690810$$

$$T_{31} = \dots$$

$$T_{02} = \frac{16 \cdot T_{11} - T_{01}}{15} = \dots$$

Berechnungen gem. Script Aufgabe 3

| T_{i0} | T_{i1} | T_{i2} | T_{i3} | T_{i4} |
|-------------------------|-------------------------|------------------------|------------------------|-------------------------------|
| T_{00} 0.15286148 | T_{01} -1.58521254 | T_{02} 1.43892088 | T_{03} 0.52848220 | T_{04} <u>0.56418760</u> |
| T_{10} -1.15069403 | T_{11} 1.24991254 | T_{12} 0.54270781 | T_{13} 0.56404813 | T_{14} — |
| T_{20} 0.64976090 | T_{21} 0.58690810 | T_{22} 0.56371468 | T_{23} — | T_{24} — |
| T_{30} 0.60262130 | T_{31} 0.5651642 | T_{32} — | T_{33} — | T_{34} — |
| T_{40} 0.57452853 | T_{41} 0.000... | T_{42} — | T_{43} — | T_{44} — |