HWR 505 Homework #2 **Name ___________________**

Assigned: Tuesday, 17 September 2024
Due: Thursday, 26 September 2024 (Upload answers in PDF to D2L)
Instructor: Bo Guo
Teaching Assistant: Jianwen Du
Semester: Fall 2024

NOTE: When submitting on D2L, convert your file to PDF and name it as "Last_name_HW1". For example, "Guo_HW1.pdf". Whether you type or handwrite, the text and figures need to be readable to an average human being. Nonreadable homework will be returned without grading.

1. (10 points) Consider a confined aquifer into which an LNAPL has been spilled. The LNAPL accumulates below the upper aquitard. The aquifer and aquitard materials are water wet with a contact angle $\theta = 0^{\circ}$. A representative pore radius for the aquitard is 0.05 mm. A representative pore diameter for the confined aquifer is 1 mm. The hydraulic head of the water in the aquitard, above the LNAPL, is identical to the hydraulic head of the water in the aquifer below the LNAPL, and water is hydrostatic everywhere. The densities of the water and LNAPL are 1.0 and 0.975 g cm^{-3}. The interfacial tension between the LNAPL and water is 0.02 N m^{-1}.
   a. Draw a system of capillary tubes that you can use to represent this system (*Hint:* You can approximate the aquifer and aquitard both as a bundle of tubes. For simplification, you can use one capillary tube to represent the aquifer and one capillary tube to represent the aquitard.)
   b. Write a general expression (in terms of $r_{aquitard}$, $r_{aquifer}$, ρ_{water}, ρ_{LNAPL}, $\sigma_{water\text{-}LNAPL}$, and h_{LNAPL}) that describes the maximum thickness of LNAPL that can exist below the aquitard before LNAPL first enters the aquitard. Use the values reported above to determine the value of the thickness for the example given.

2. (10 points) Calculate the capillary rise of water in air as a function of water table height above the base of a 100 cm long vertical tube for the following variable-diameter tube for both imbibition and drainage conditions. The diameter of the tube is 0.005 cm except for a section with a diameter of 0.075 cm. The larger diameter section is 10 cm in height. The lowest point of the larger diameter section is located 50 cm above the bottom of the tube. The air-water interfacial tension is 0.072 N/m. The static contact angle between water (the wetting fluid) and the wall of the tubes is zero.
   a. Plot the results with the y-axis showing the height of the air-water interface and the x-axis showing the height of the water table.
   b. Identify the area in your plot showing the hysteretic behavior.

3. (10 points) A loam has the van Genuchten parameters listed below. Make the Mualem assumption regarding the value of m ($m = 1 - 1/n$). From the form of the van Genuchten equation given in the lecture slides, write an expression for the water content (θ) as a function of water pressure head ($\psi = 0 - p_c$), α, n, θ_s, and θ_r. θ_s is the saturated water content, and θ_r is the residual water content. For the notations in the lecture slides, $s_{w,r} = \theta_r/\theta_s$ and $s_w = \theta/\theta_s$.

   a. Plot the volumetric water content (x-axis) vs. the water pressure head (y-axis) for pressure heads ranging from 0 to –200 cm.

b. From the form of the Brooks-Corey equation given in the lecture slides, write an expression for the water content as a function of the water pressure head, the displacement (entry) pressure head (p_d), λ, θ_s, and θ_r.

c. Using the "Solver" functionality in Excel (or any similar software), find values of the displacement (entry) pressure head and λ that give you the best fit of the Brooks-Corey model to the van Genuchten results (i.e., minimizing the root mean square of the difference between the water pressure head from the two models, $\Delta\psi$; when doing the fitting, divide $\theta_s - \theta_r$ into 100 uniform intervals). 1) Report the fitted values and show the plot of the van Genuchten results with your best fit Brooks-Corey fits. 2) Comment on the differences between the Brooks-Corey and van Genuchten curves. 3) Compare $1/\alpha$ and p_d. Are they equal? Why or why not?

	α (1/cm)	n	θ_r	θ_s
Loam	0.036	1.56	0.078	0.43

Hints:

(1) When plotting the Brooks-Corey curve, note that the volumetric water content is constant from 0 to p_d. You have to manually fix the water content to the saturated value in this range. The expression gives you water contents that are higher than the saturated water content, which is impossible. You can also use a conditional switch to fix this value in Excel.

(2) See this link for the instructions and examples for using the "Solver" functionality in Excel.