HWRS 561b: Physical Hydrogeology II

Pore scale fluids distribution

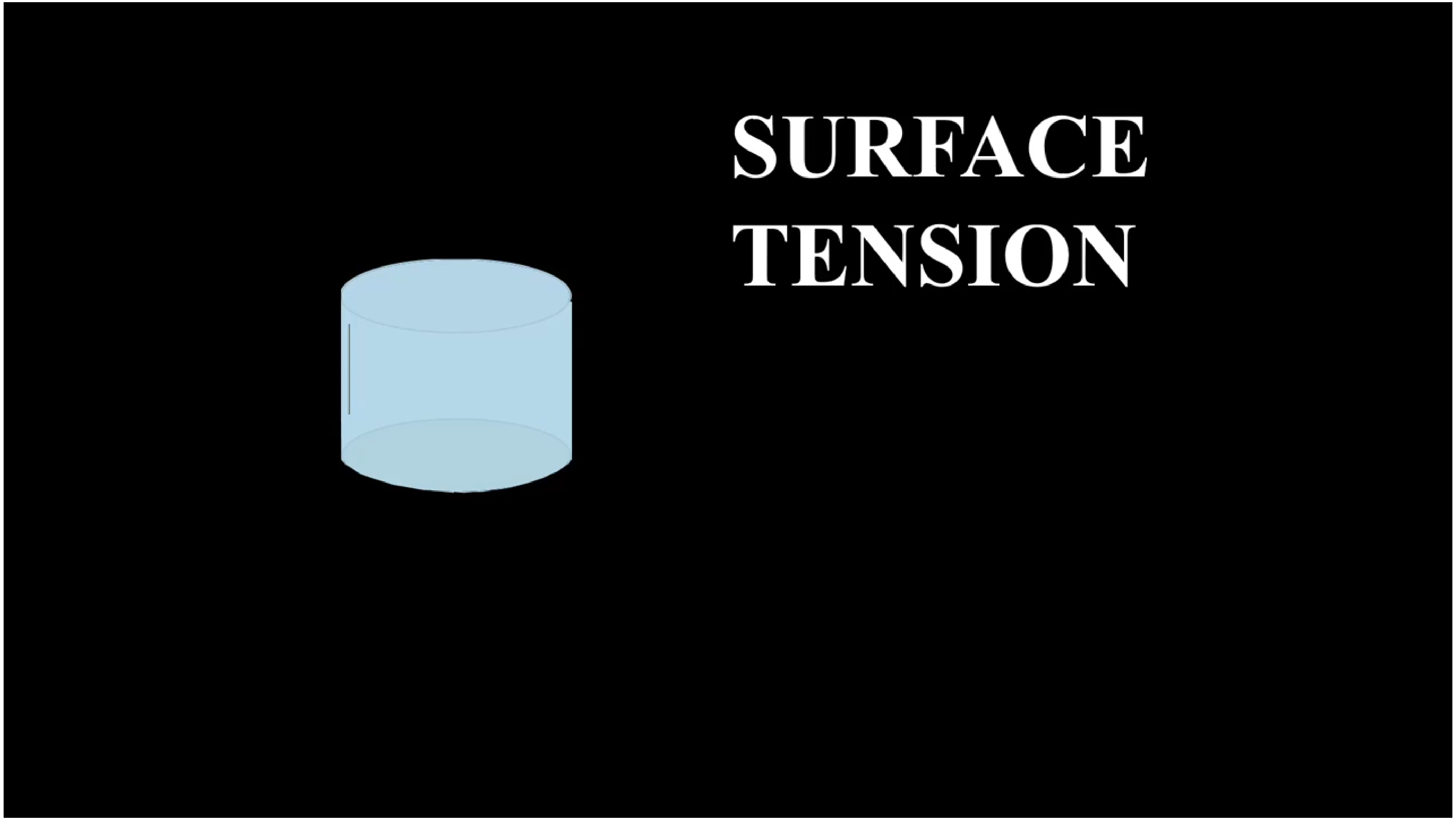
Agenda:

1. Air-water interface
2. Capillarity

Air-water system in capillary tubes

1. Why does the water try to hold together?
2. Why does the water not wet the surface?

Air-water system in capillary tubes

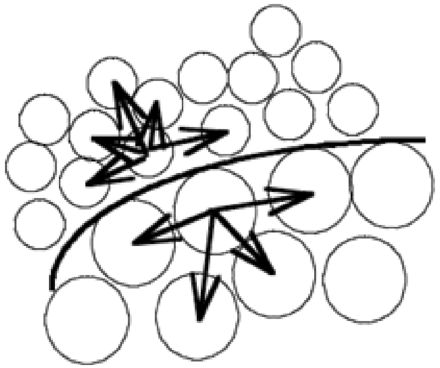


Link to the video: https://youtu.be/zMzqiAuOSz0

Air-water system in capillary tubes

- **Two and three phase systems: water, oil, air**

- *Interfacial tension (cohesive forces between fluid molecules)*

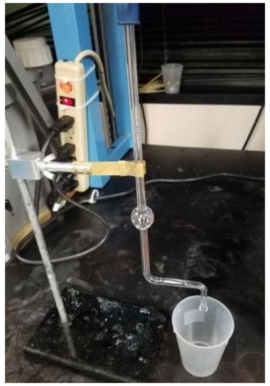


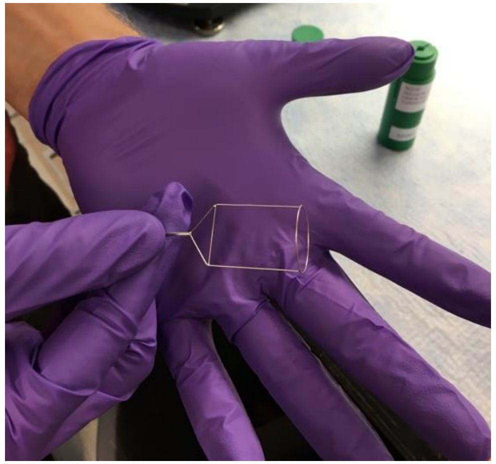
How to measure interfacial tension?

Typical values of surface tension:

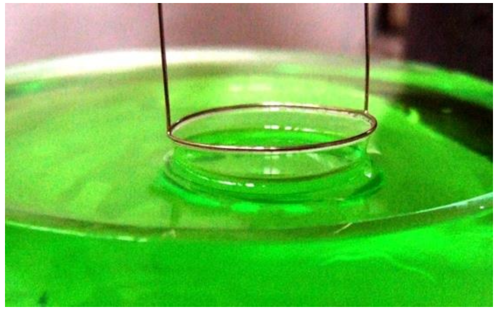
air-water	0.072 N/m
oil-water	0.20 N/m
oil-water w/ soap	0.0001 N/m

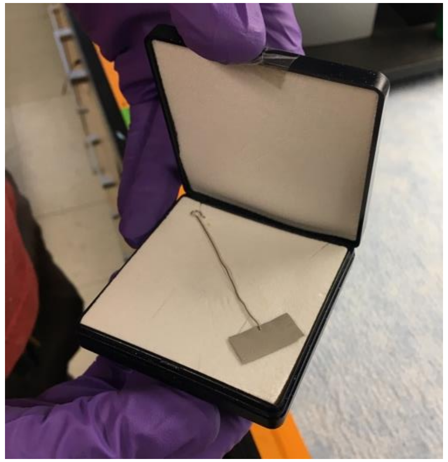
Drop weight method



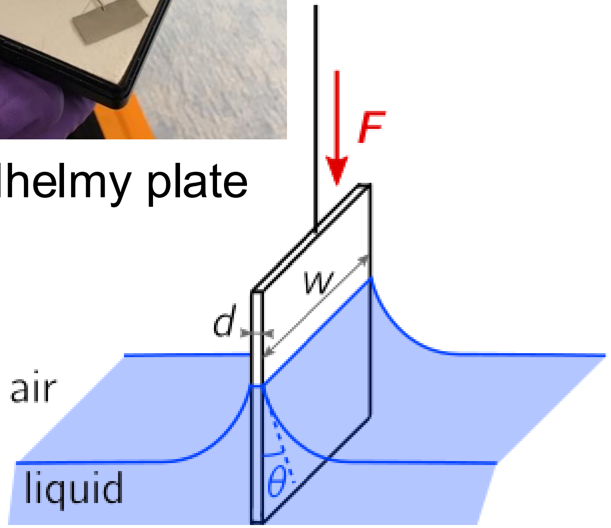


ring method



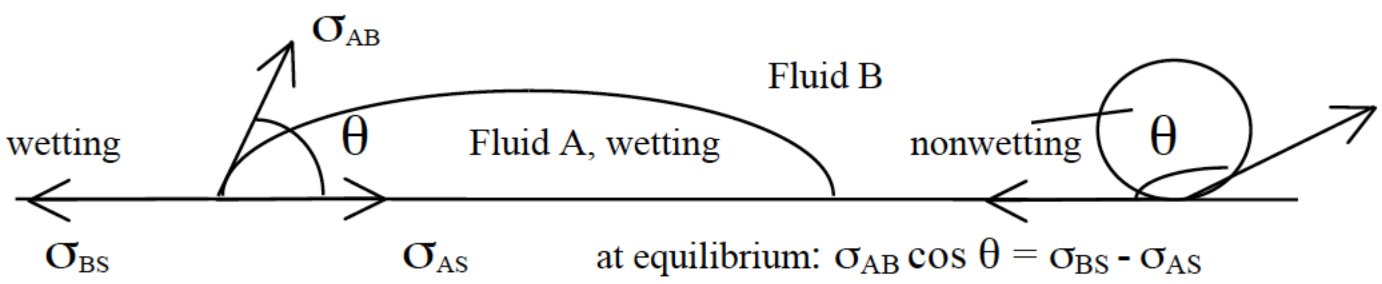


Wilhelmy plate



Air-water system in capillary tubes

- *Wettability (<u>adhesive</u> forces between the fluid and solid surface)*



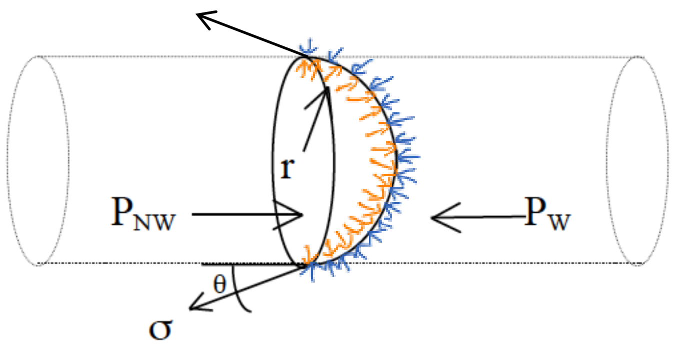
$\theta < 90^o$: fluid A is wetting with respect to fluid B on the solid S
$\theta > 90^o$: fluid A is nonwetting with respect to fluid B on the solid S

Wettability is a function of the fluid properties, soil properties, and history of contact. For most soils, the relative wettabilities are: water > oil > air

Recommended video for the concepts of *viscosity, cohesive and adhesive forces, surface tension, and capillary action* https://www.youtube.com/watch?v=P_jQ1B9UwpU

Air-water system in capillary tubes

Capillary pressure (difference between the nonwetting and wetting phase pressures)



Interface is static ⇒ Force balanced

$$\Rightarrow \quad 2\pi r \sigma \cos\theta = \pi r^2 P_{nw} - \pi r^2 P_w$$

$$\Rightarrow \quad P_{nw} - P_w = \frac{2\sigma\cos\theta}{r}$$

Define $P_c \equiv P_{nw} - P_w$

$$\Rightarrow \boxed{P_c = \frac{2\sigma\cos\theta}{r}}$$ Young-Laplace Eqn.

1°. More general equation for any nw-w interface: $P_c = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

For the capillary tube, $r_1 = r_2 = r/\cos\theta$

2°. For a perfectly wetting fluid $(\theta = 0)$, $P_c = \frac{2\sigma}{r}$

Air-water system in capillary tubes

Capillary pressure

Pressure jump across a fluid-fluid interface

Young-Laplace Equation

Pressure jump across a fluid-fluid interface is determined by interfacial tension + geometry of the interface (radii of the curvature)

Art of porous media flow

Optional, but strongly encouraged, Mini-project

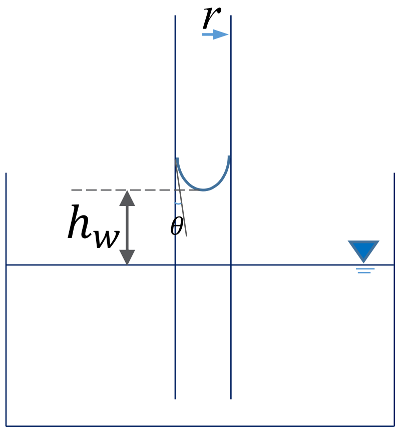
Take a photo or a video (< 2 min) in your day-to-day life that you think best illustrates some cool phenomena of porous media flow.

I will create a dropbox on D2L for you to upload the photo or video (due **April 26**).

Depending on the quality of your picture or video, you can receive up to 5 bonus points in your final grade (out of 100 points).

Air-water system in capillary tubes

Capillary rise in a Capillary tube



static ⇒ Force balanced

At the water table

$P_a = 0$

$$\left.\begin{aligned} P_w &= 0 \\ z &= 0 \end{aligned}\right\} \Rightarrow H = \frac{P_w}{\rho_w g} + z = 0$$

At the air-water interface in the tube

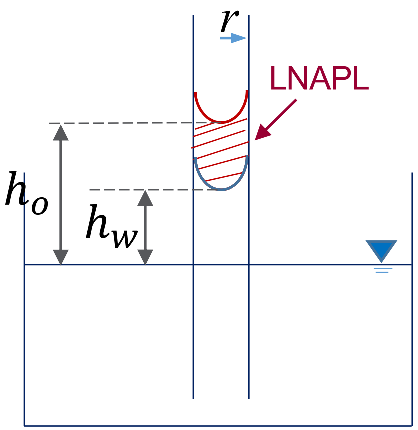
$P_a = 0$

$$\left.\begin{aligned} z &= h_w \\ H &= 0 \end{aligned}\right\} \Rightarrow \psi_w = H - z = -h_w \Rightarrow P_w = \rho_w g \psi_w = -\rho_w g h_w$$

$$\left.\begin{aligned} P^{cap} &= P_a - P_w = 0 - (-\rho_w g h_w) = \rho_w g h_w \\ &\text{Young-Laplace Eqn: } P^{cap} = \frac{2\sigma\cos\theta}{r} \end{aligned}\right\} \Rightarrow h_w = \frac{2\sigma\cos\theta}{\rho_w g r}$$

Air-water system in capillary tubes

Capillary rise in a Capillary tube in the presence of an LNAPL (Assuming zero contacts)



Note: w/o oil ($\theta = 0$)

$$h_w = \frac{2\sigma_{aw}}{\rho_w g r}$$

Static => Force balanced.

At the oil-water interface:

$$P_w = 0 - \rho_w g h_w = -\rho_w g h_w \quad \text{----} \quad (1)$$

$$P_o = P_{ow}^{cap} + P_w = \frac{2\sigma_{ow}}{r} - \rho_w g h_w \quad \text{---} \quad (2)$$

continuing through the oil to the oil-air interface, the oil pressure

$$P_o = \frac{2\sigma_{ow}}{r} - \rho_w g h_w - \rho_o g (h_o - h_w) \quad \text{--} \quad (3)$$

At the oil-air interface

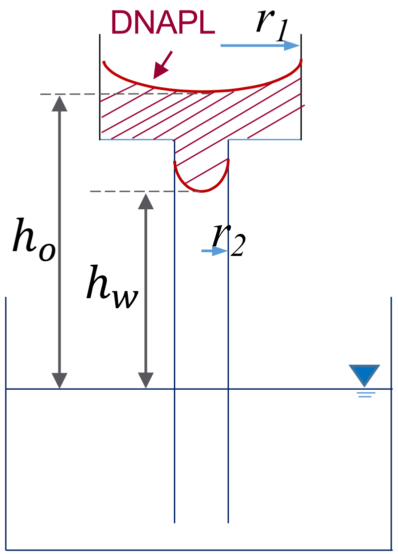
$$P_o = 0 - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r} \quad \text{----} \quad (4)$$

P_o in (3) and (4) are equal

$$\Rightarrow h_w = \left(\frac{2\sigma_{ow}}{r} + \frac{2\sigma_{oa}}{r} - \rho_o g h_o\right) \Big/ (\rho_w g - \rho_o g)$$

Air-water system in capillary tubes

Invasion of a nonwetting fluid into a pore (Assuming zero contacts)



Static ⇒ Force balanced

At the air-oil interface:

$$P_o = P_a - P_{oa}^{cap} = 0 - \frac{2\sigma_{oa}}{r_1}$$

continuing down to the oil-water interface, the pressure of oil is

$$P_o = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g$$

At the oil-water interface:

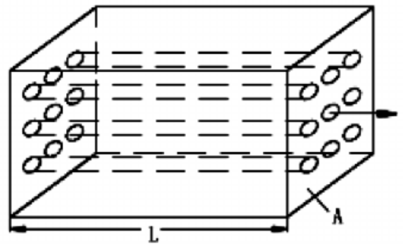
$$\begin{cases} P_{ow}^{cap} = P_o - P_w = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g - (-\rho_w g h_w) \\ P_{ow}^{cap} = \frac{2\sigma_{ow}}{r_2} \end{cases}$$

$$\Rightarrow h_o - h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_w g h_w\right] / (\rho_o g)$$

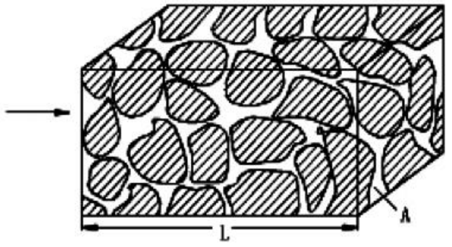
$$\Rightarrow h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_o g h_o\right) / \left[(\rho_w - \rho_o) g\right]$$

Air-water system in capillary tubes

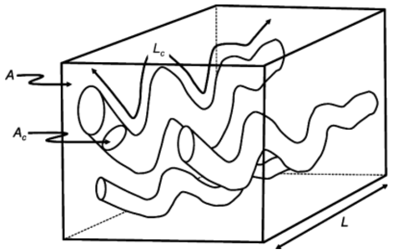
Model of a porous medium as a Bundle of Capillary Tubes



Bundle of capillary tubes



porous medium



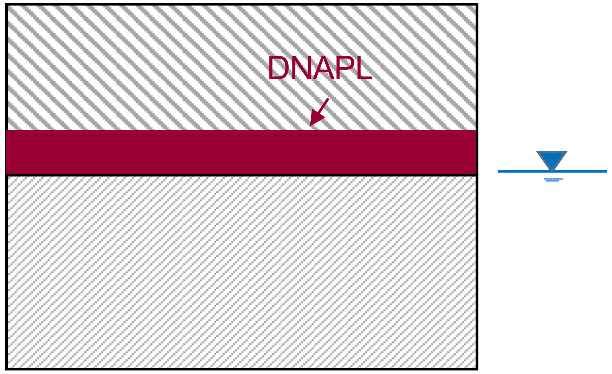
Bear (1972)

Bundle of <u>tortuous</u> capillary tubes

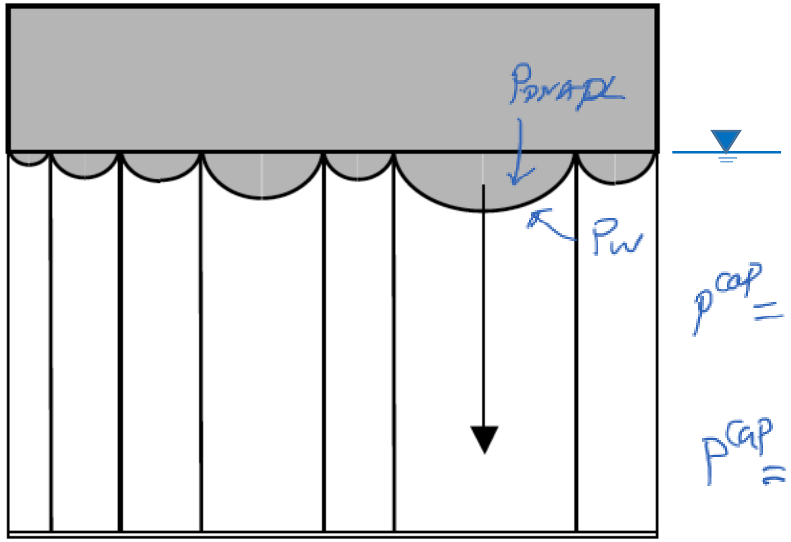
- Very simplified model, but its application has tremendously improved our understanding of fluid flow and transport phenomena in porous media. Some <u>examples</u>:
  - Permeability (already discussed)
  - Dispersion (already discussed)
  - Fluid invasion
  - Capillary transition zone
  - Soil water characteristic curve
  - Relative permeability

Air-water system in capillary tubes

Invasion of a nonwetting fluid into an aquifer



Representing the aquifer as a bundle of capillary tubes



$$p^{cap} = P_{DNAPL} - P_w$$

$$p^{cap} = \frac{2\sigma\cos\theta}{r}$$

1. Which is easier for DNAPL to invade?

   Coarse sand or fine-grained medium?

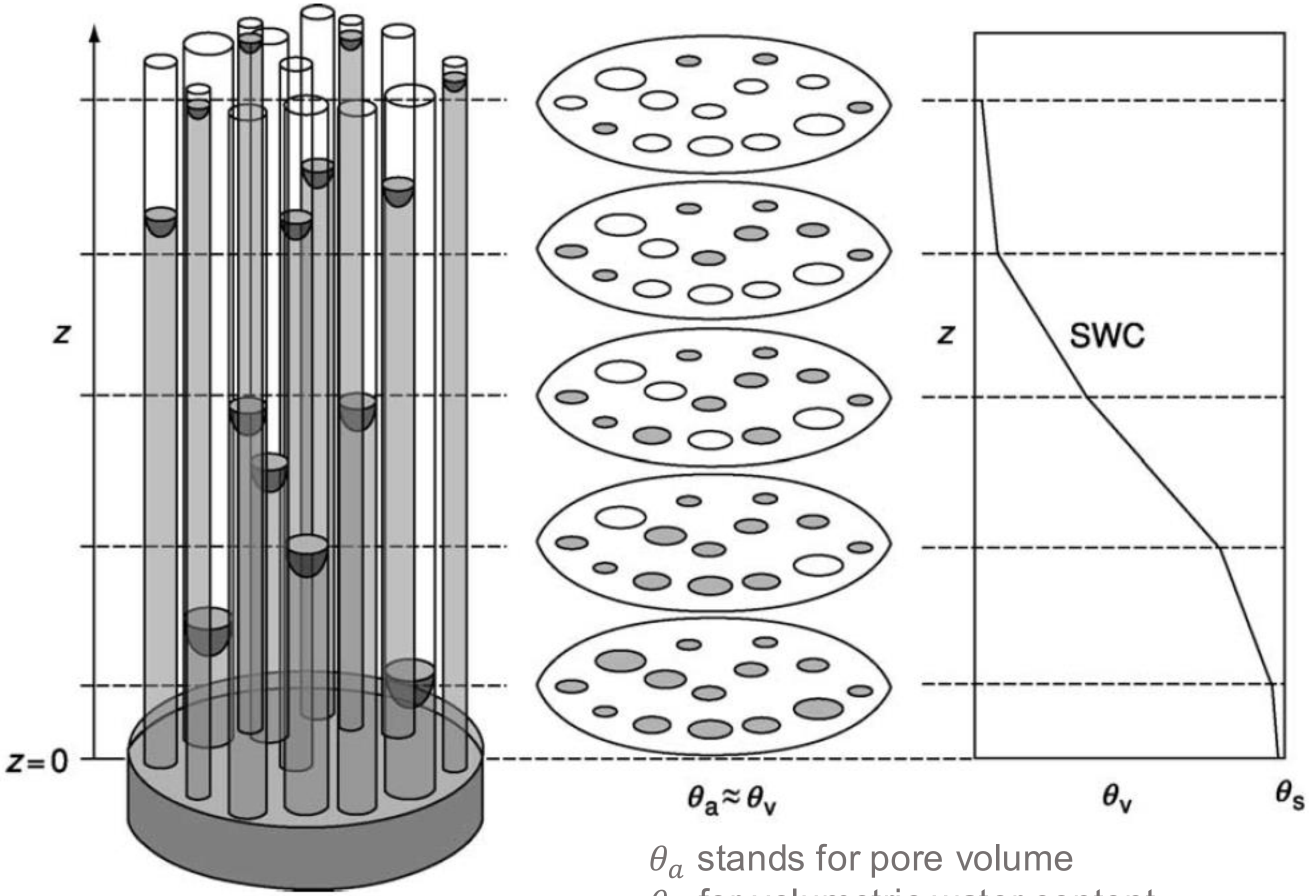
2. For some reason, if DNAPL modifies the wettability of the porous medium grain surfaces, e.g., the contact angle of water increases from 0º to something between 0º and 90º.

   What may happen to the DNAPL?

Air-water system in capillary tubes

Water retention (or capillary transition zone) in the vadose zone

Representing the soil as a
Bundle of Capillary Tubes

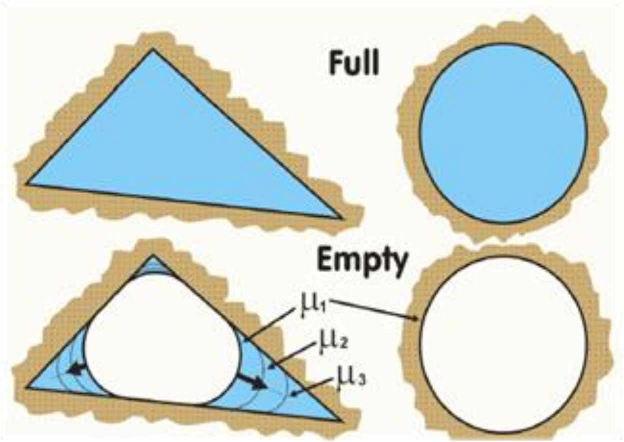


Tuller & Or (2004)

θ_a stands for pore volume
θ_V for volumetric water content
θ_s for volumetric water content at saturation

Air-water system in capillary tubes

Bundle of triangular capillary tubes vs. bundle of cylindrical capillary tubes



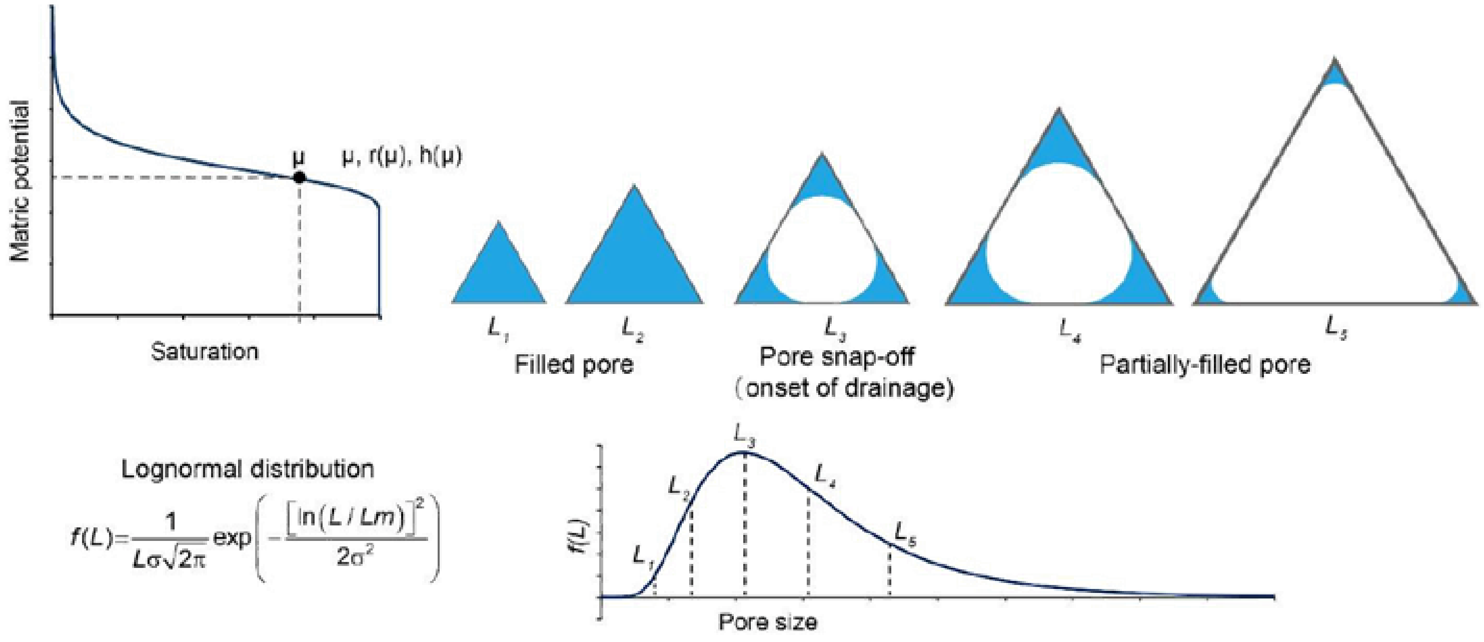
Tuller, Or, Dudley (1999)

Bundle of triangular capillary tubes model have several advantages:

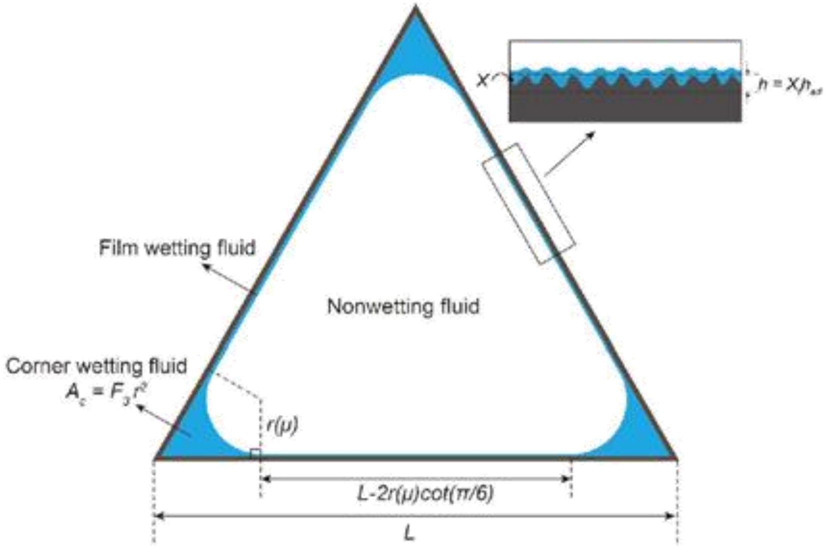
1. Can represent thin films and corner fluid
2. Saturation-dependent capillary pressure within a single-pore
3. More realistic representation of pore geometry
4. …

Air-water system in capillary tubes

An example study using the bundle of triangular capillary tubes model to examine the impact of surface roughness on fluid-fluid interfacial areas $A_{aw} = A_{aw}(S_w)$



Simulating the soil-water characteristics



Representing the surface roughness and films

Jiang, Guo, Brusseau (2020)