HWRS 561b
Bo Guo
Spring 2026

HWRS 561b: Physical Hydrogeology II

Solute transport (Part 3)

Agenda:

1. Analytical solutions for solute transport

Simplified 1D advection-dispersion equation

3D governing equation for solute transport in saturated porous media

$$\frac{\partial}{\partial t}(\phi C) + \nabla \cdot (\mathbf{q}C - \phi \mathbf{D} \nabla C) = 0$$

1D simplification (assuming constant porosity in space)

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}\left(VC - D\frac{\partial C}{\partial x}\right) = 0$$

Assuming constant porewater velocity and dispersion coefficient, we obtain

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial x} - D\frac{\partial^2 C}{\partial x^2} = 0$$

Simplified 1D advection-dispersion equation

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial x} - D\frac{\partial^2 C}{\partial x^2} = 0$$

Note:
1) This is a second-order partial differential equation.
2) It is linear, e.g., coefficients does not depend on *C*.
3) It can be solved analytically, i.e., we can obtain a close-form solution for *C* for any *x* and *t.*

To solve the 1D equation, we need to specify one initial condition and two boundary conditions.

Non-dimensional form and dimensionless number

1D step change in concentration

Let us first consider a semi-infinite domain.

Initial condition

$$C(x,0) = 0,\ x \geq 0$$

Boundary conditions (assuming semi-infinite domain)

$$C(0,t) = C_0,\ t \geq 0$$

$$C(\infty,t) = 0,\ t \to \infty$$

$$C = \frac{C_0}{2}\left[\operatorname{erfc}\left(\frac{x - Vt}{2\sqrt{Dt}}\right) + \exp\left(\frac{VL}{D}\right)\operatorname{erfc}\left(\frac{x + Vt}{2\sqrt{Dt}}\right)\right]$$

[Ogata and Banks, 1961]

1D step change in concentration

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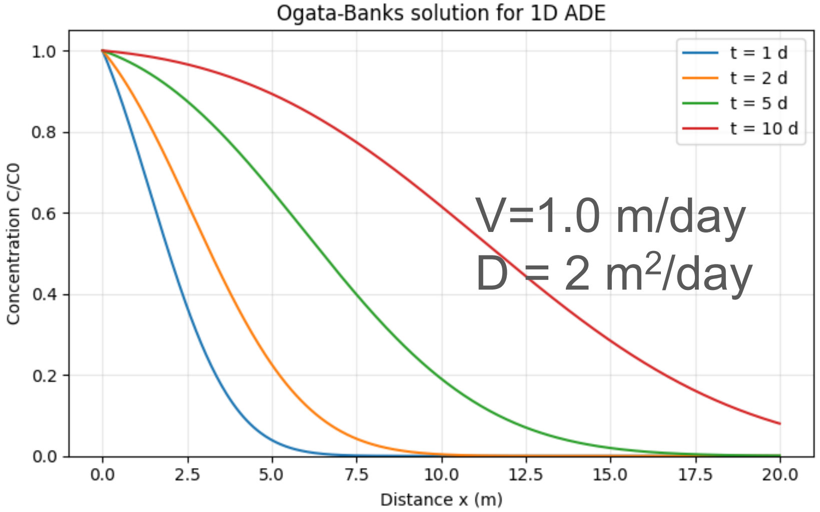

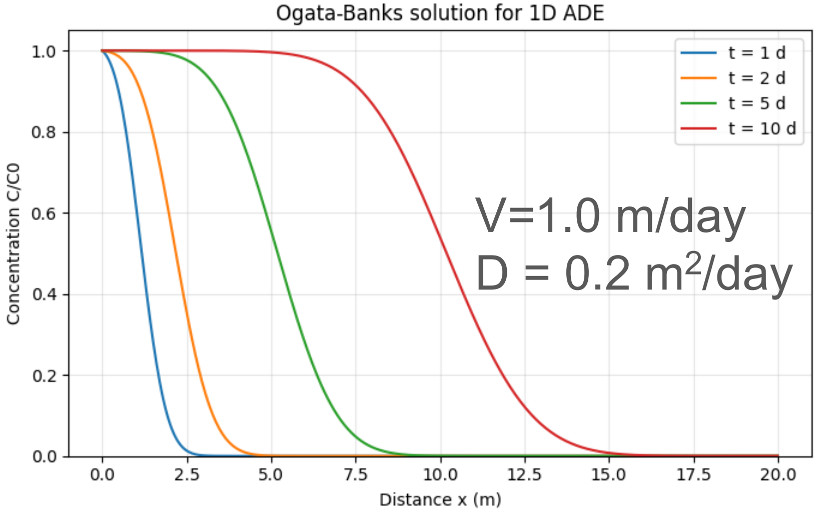

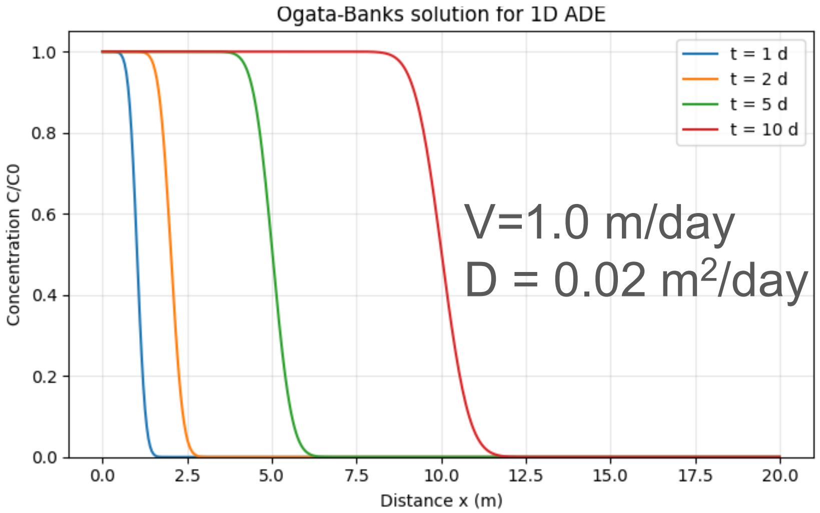

1D step change in concentration

Special case: *V=0* (only dispersion)

$$C = C_0 \,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$