

HWRS 561b: Physical Hydrogeology II

Pore scale fluids distribution

Agenda:

1. Air-water interface
2. Capillarity

Air-water system in capillary tubes

HWRS 561b
Bo Guo
Spring 2026

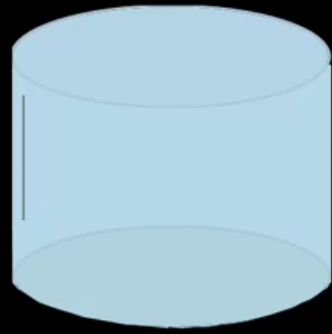


1. Why does the water try to hold together?
2. Why does the water not wet the surface?

Air-water system in capillary tubes

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**SURFACE
TENSION**



Link to the video: <https://youtu.be/zMzqiAuOSz0>

Air-water system in capillary tubes

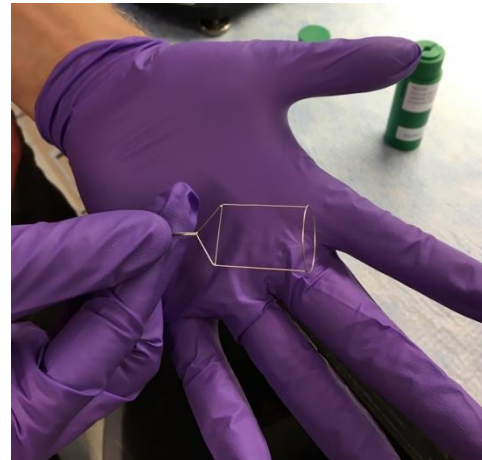
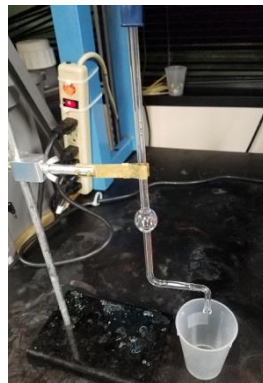
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- **Two and three phase systems: water, oil, air**
- *Interfacial tension (cohesive forces between fluid molecules)*

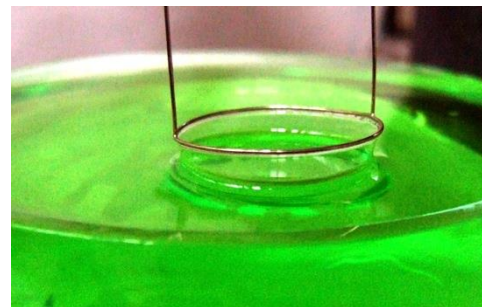


How to measure interfacial tension?

Drop weight method

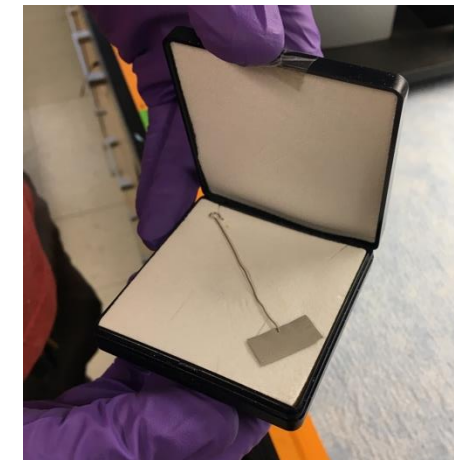


ring method

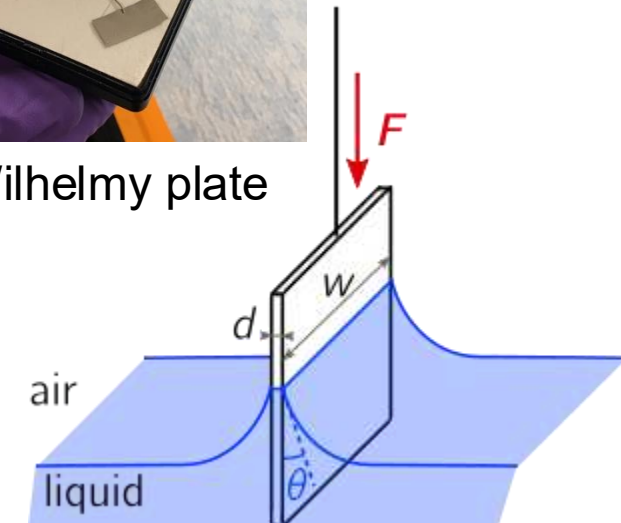


Typical values of surface tension:

air-water	0.072 N/m
oil-water	0.20 N/m
oil-water w/ soap	0.0001 N/m

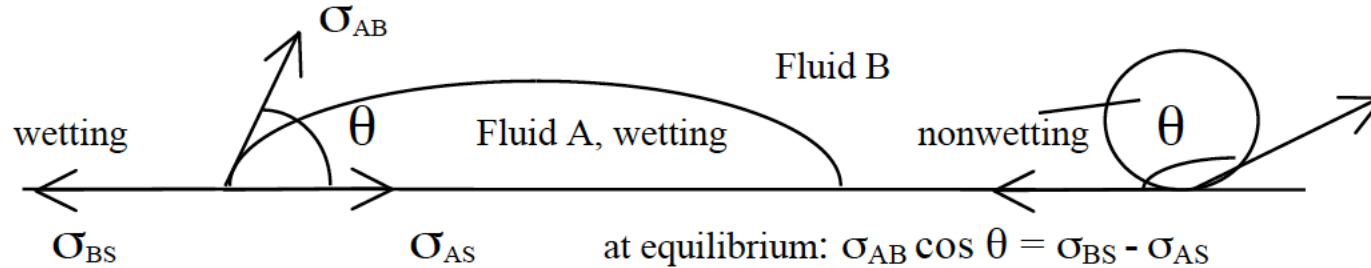


Wilhelmy plate



Air-water system in capillary tubes

- *Wettability (adhesive forces between the fluid and solid surface)*



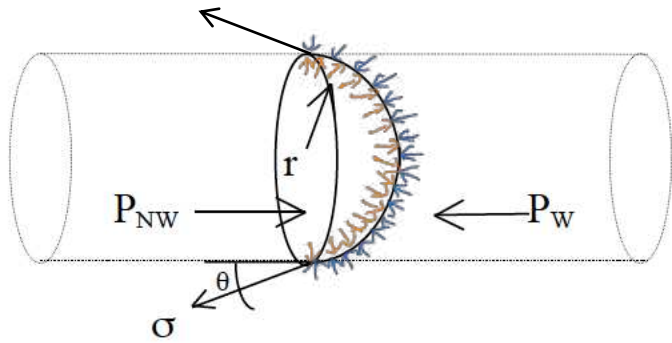
$\theta < 90^\circ$: fluid A is wetting with respect to fluid B on the solid S
 $\theta > 90^\circ$: fluid A is nonwetting with respect to fluid B on the solid S

Wettability is a function of the fluid properties, soil properties, and history of contact. For most soils, the relative wettabilities are: water > oil > air

Recommended video for the concepts of *viscosity, cohesive and adhesive forces, surface tension, and capillary action* https://www.youtube.com/watch?v=P_jQ1B9UwpU

Air-water system in capillary tubes

Capillary pressure (difference between the nonwetting and wetting phase pressures)



Interface is static \Rightarrow Force balanced

$$\Rightarrow 2\pi r \sigma \cos \theta = \pi r^2 P_{nw} - \pi r^2 P_w$$

$$\Rightarrow P_{nw} - P_w = \frac{2\sigma \cos \theta}{r} \quad \Rightarrow \quad P_c = \frac{2\sigma \cos \theta}{r} \quad \text{Young-Laplace Eqn.}$$

Define $P_c \equiv P_{nw} - P_w$


1°. More general equation for any mv-w interface: $P_c = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$
2°. For a perfectly wetting fluid, $P_c = \frac{2\sigma}{r}$ ($\theta = 0$)
For the capillary tube, $r_1 = r_2 = r / \cos \theta$

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Capillary pressure

Pressure jump across a
fluid-fluid interface



Young-Laplace Equation

Pressure jump across a fluid-fluid interface is
determined by interfacial tension + geometry
of the interface (radii of the curvature)



Optional, but strongly encouraged, Mini-project

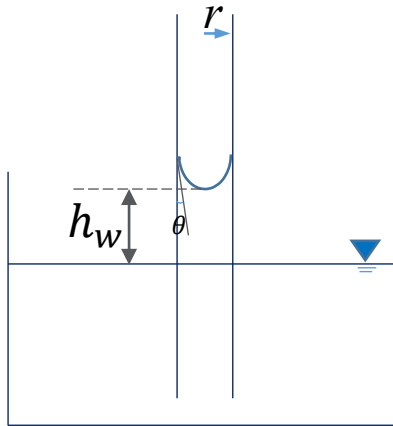
Take a photo or a video (< 2 min) in your day-to-day life that you think best illustrates some cool phenomena of porous media flow.

I will create a dropbox on D2L for you to upload the photo or video (due **April 26**).

Depending on the quality of your picture or video, you can receive up to 5 bonus points in your final grade (out of 100 points).

Air-water system in capillary tubes

Capillary rise in a Capillary tube



static \Rightarrow Force balanced

At the water table

$$P_a = 0$$

$$\left. \begin{array}{l} P_w = 0 \\ z = 0 \end{array} \right\} \Rightarrow H = \frac{P_w}{\rho_w g} + z = 0$$

At the air-water interface in the tube

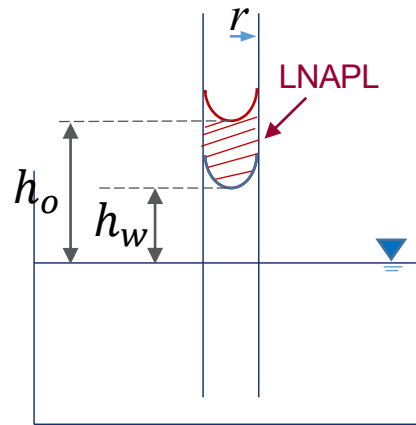
$$P_a = 0$$

$$\left. \begin{array}{l} z = h_w \\ H = 0 \end{array} \right\} \Rightarrow \psi_w = H - z = -h_w \Rightarrow P_w = \rho_w g \psi_w = -\rho_w g h_w$$

$$P^{cap} = P_a - P_w = 0 - (-\rho_w g h_w) = \rho_w g h_w \left\{ \begin{array}{l} \text{Young-Laplace Eqn: } P^{cap} = \frac{2\sigma \cos \theta}{r} \end{array} \right\} \Rightarrow h_w = \frac{2\sigma \cos \theta}{\rho_w g r}$$

Air-water system in capillary tubes

Capillary rise in a Capillary tube in the presence of an LNAPL (Assuming zero contacts)



static \Rightarrow Force balanced.

At the oil-water interface:

$$P_w = 0 - \rho_w g h_w = -\rho_w g h_w \quad \text{--- (1)}$$

$$P_o = P_{ow}^{cap} + P_w = \frac{2\sigma_{ow}}{r} - \rho_w g h_w \quad \text{--- (2)}$$

continuing through the oil to the oil-air interface, the oil pressure

$$P_o = \frac{2\sigma_{ow}}{r} - \rho_w g h_w - \rho_o g (h_o - h_w) \quad \text{--- (3)}$$

At the oil-air interface

$$P_o = 0 - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r} \quad \text{--- (4)}$$

P_o in (3) and (4) are equal

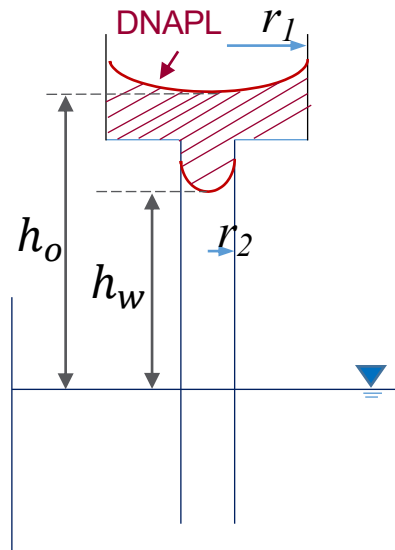
$$\Rightarrow h_w = \left(\frac{2\sigma_{ow}}{r} + \frac{2\sigma_{oa}}{r} - \rho_o g h_o \right) / (\rho_w g - \rho_o g)_{10}$$

Note: w/o oil ($\theta = 0$)

$$h_w = \frac{2\sigma_{aw}}{\rho_w g}$$

Air-water system in capillary tubes

Invasion of a nonwetting fluid into a pore (Assuming zero contacts)



Static \Rightarrow Force balanced

At the air-oil interface:

$$P_o = P_a - P_{oa}^{cap} = 0 - \frac{2\sigma_{oa}}{r_1}$$

Continuing down to the oil-water interface, the pressure of oil is

$$P_o = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g$$

At the oil-water interface:

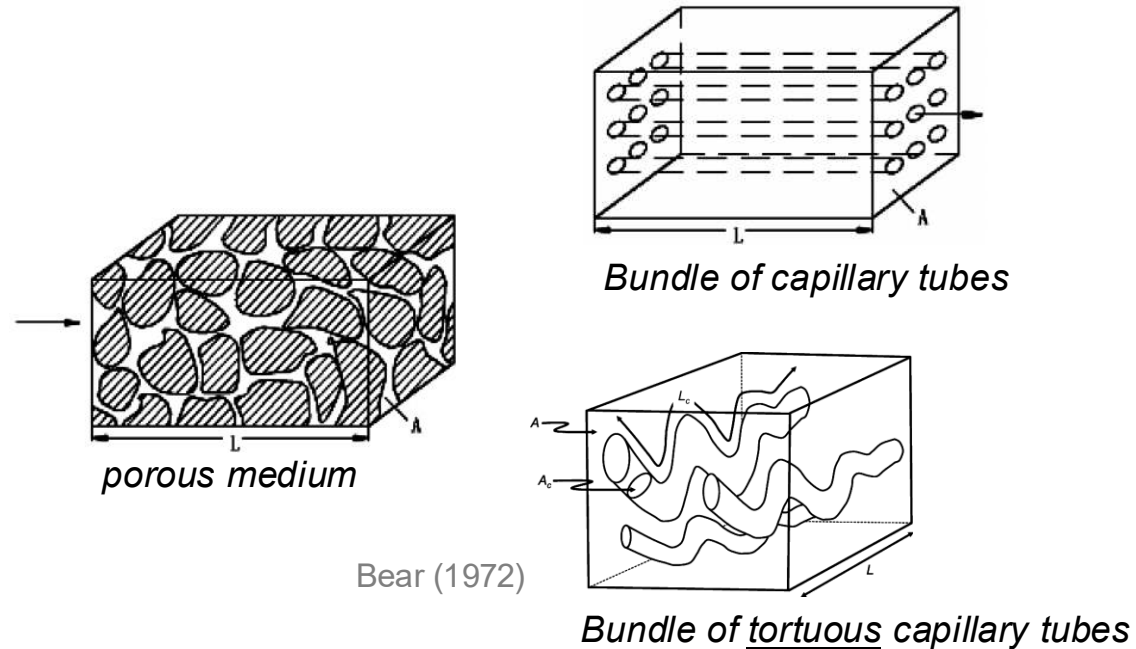
$$\begin{cases} P_{ow}^{cap} = P_o - P_w = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g - (-\rho_w g h_w) \\ P_{ow}^{cap} = \frac{2\sigma_{ow}}{r_2} \end{cases}$$

$$\Rightarrow h_o - h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_w g h_w \right] / (\rho_o g)$$

$$\Rightarrow h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_o g h_o \right] / [(\rho_w - \rho_o)g] \quad 11$$

Air-water system in capillary tubes

Model of a porous medium as a Bundle of Capillary Tubes



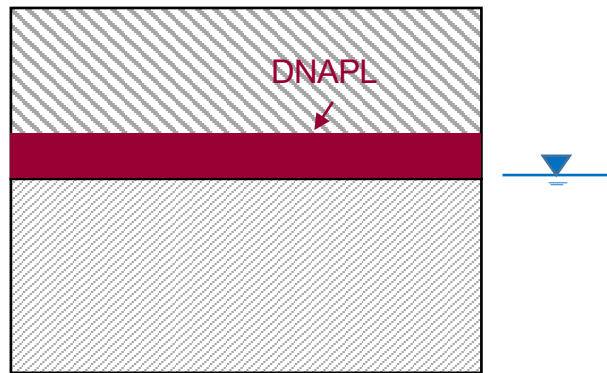
❖ Very simplified model, but its application has tremendously improved our understanding of fluid flow and transport phenomena in porous media.

Some examples:

- Permeability (already discussed)
- Dispersion (already discussed)
- Fluid invasion
- Capillary transition zone
- Soil water characteristic curve
- Relative permeability

Air-water system in capillary tubes

Invasion of a nonwetting fluid into an aquifer



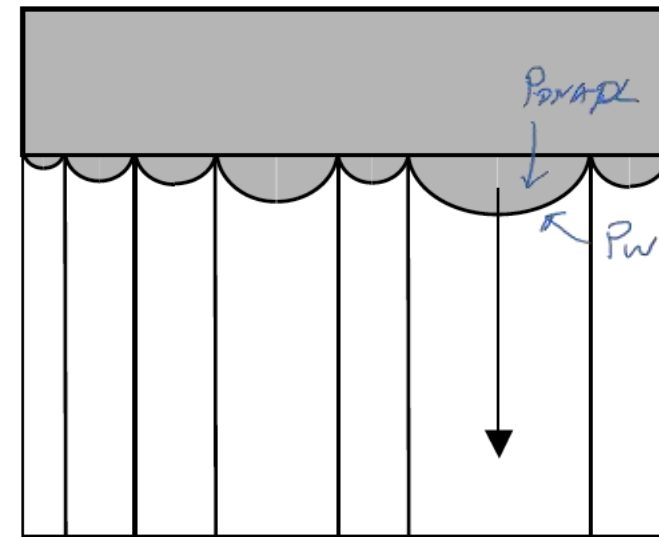
1. Which is easier for DNAPL to invade?

Coarse sand or fine-grained medium?

2. For some reason, if DNAPL modifies the wettability of the porous medium grain surfaces, e.g., the contact angle of water increases from 0° to something between 0° and 90° .

What may happen to the DNAPL?

Representing the aquifer as a bundle of capillary tubes

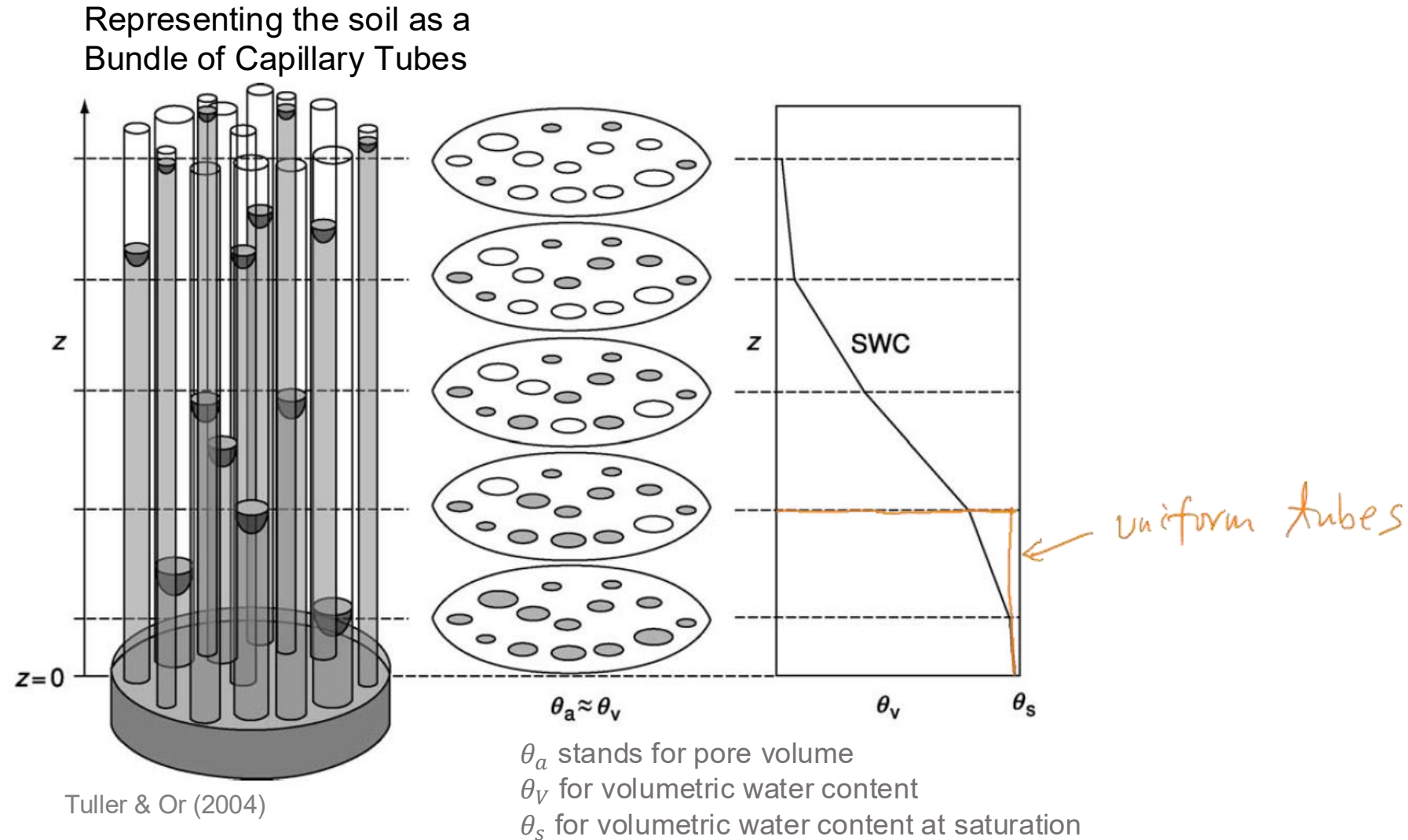


$$p^{cap} = P_{DNAPL} - P_w$$

$$p^{cap} = \frac{2\sigma \cos \theta}{r}$$

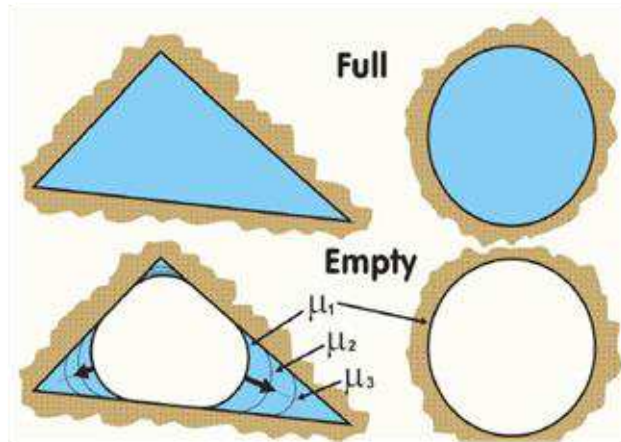
Air-water system in capillary tubes

Water retention (or capillary transition zone) in the vadose zone



Air-water system in capillary tubes

Bundle of triangular capillary tubes vs. bundle of cylindrical capillary tubes



Tuller, Or, Dudley (1999)

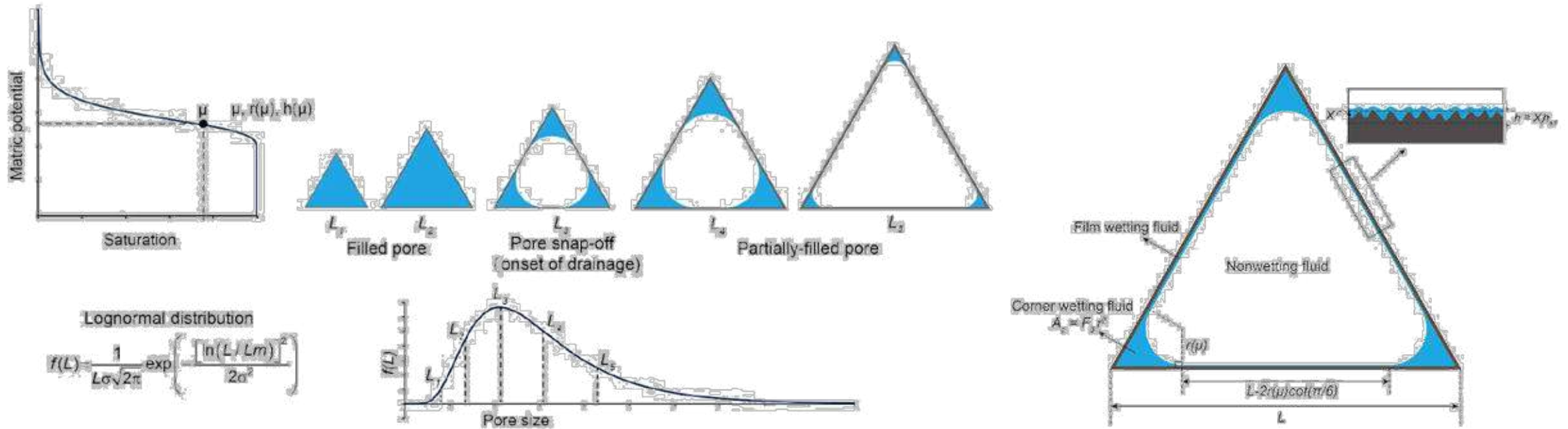
Bundle of triangular capillary tubes model have several advantages:

1. Can represent thin films and corner fluid
2. Saturation-dependent capillary pressure within a single-pore
3. More realistic representation of pore geometry
4. ...

Air-water system in capillary tubes

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An example study using the bundle of triangular capillary tubes model to examine the impact of surface roughness on fluid-fluid interfacial areas $A_{aw} = A_{aw}(S_w)$



Simulating the soil-water characteristics

Representing the surface roughness and films