HWR 505 Homework #3 **Name _____**

Assigned: Tuesday, 1 October 2024
Due: Tuesday, 15 October 2024 (Upload answers in PDF to D2L)
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Semester: Fall 2024

1. (10 points) Propose two questions that you think can be used as midterm oral exam questions. The questions should focus on some of the most critical concepts that we have covered so far in this course. Keep in mind that these are **oral** exam questions, so they should not involve detailed calculations, rather they should focus on testing whether one can draw or develop the conceptual pictures and formulate physical arguments. Make sure to describe your questions concisely and accurately. Ask one of your classmates to answer your questions. Then, summarize her/his answers below. Please also write your own comments on the answers. You should also put down her/his name.
2. (10 points) Use the bundle of cylindrical capillary tubes to complete the following two problems.
   (1) Explain why the relative permeability coefficient is a function of fluid saturation. Then explain why the relative permeability coefficient always has values between 0 and 1.
   (2) The relative permeability function is almost always nonlinear. Examples are the Brooks-Corey and van Genuchten models. Explain why the relative permeability function is nonlinear. Additionally, explain how the relative permeability functions for wetting and nonwetting fluid phases are different and why.
3. (5 points) If the pressure in the air phase of an unsaturated soil remains essentially at atmospheric pressure, is it then reasonable to conclude that the air does not move? Why or why not? Does this type of air movement violate the Richards' assumptions?
4. (5 points) Suppose you wish to write a mathematical model for water flow in the unsaturated zone, which can also include zones of full saturation. Of the three equations: mixed, ψ-based, and θ-based, which is most appropriate? Which (if any) of the three is not appropriate? Explain your answers. Note: you can find these equations on slides 3&4 in Lecture 9 and on page 426 of Pinder & Celia (2006).
5. (20 points) Read the following article (see "Reading materials" on D2L): Celia, M.A., Bouloutas, E.T. and Zarba, R.L., 1990. A general mass-conservative numerical solution for the unsaturated flow equation. *Water resources research*, 26(7), pp.1483-1496. Answer the following questions. It would be best if you use bullet points rather than long paragraphs.
   (1) What were the major challenges for solving the transient Richards' equation at the time when this article was written? Why did the challenges arise?
   (2) How did Celia et al (1990) address those challenges? What was special about their new numerical scheme? Why is mass not conserved numerically when numerical schemes other than the mass-conservative one of Celia et al (1990) are used?
   (3) What is Picard iteration? How is it different from the Newton-Raphson iteration?
   (4) This article has been cited by 2000+ on Google Scholar, which indicates that this is a seminal contribution to the solution of Richards' equation. Why do you think it became a seminal article in this field?