

# HWRS 561b: Physical Hydrogeology II

## Solute transport (Part 2)

### Agenda:

1. Basic concepts
2. Flux law
3. Derivation of solute transport equation

# Solute transport under saturated flow

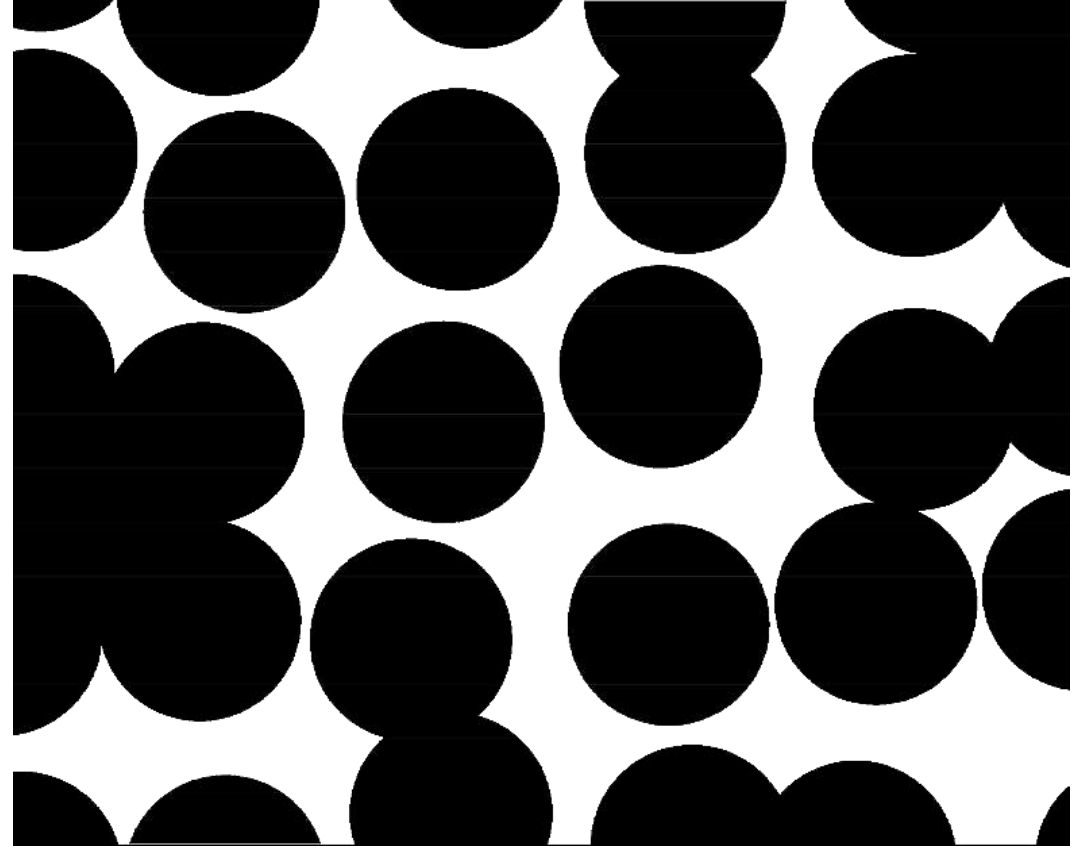
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HWRS 561b  
Bo Guo  
Spring 2026

Taylor-Aris Dispersion: Spatial variation of velocity leads to dispersion of solutes

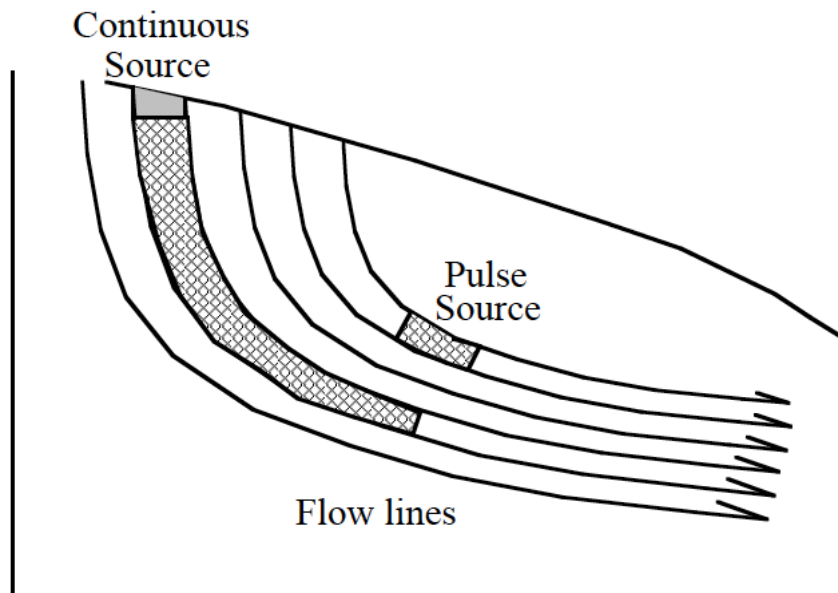
Spatial variation of velocity in a porous medium:

- (1) Velocity profile through a pore throat (related to T-A dispersion)
- (2) Velocity variation among pore throats
- (3) Tortuosity

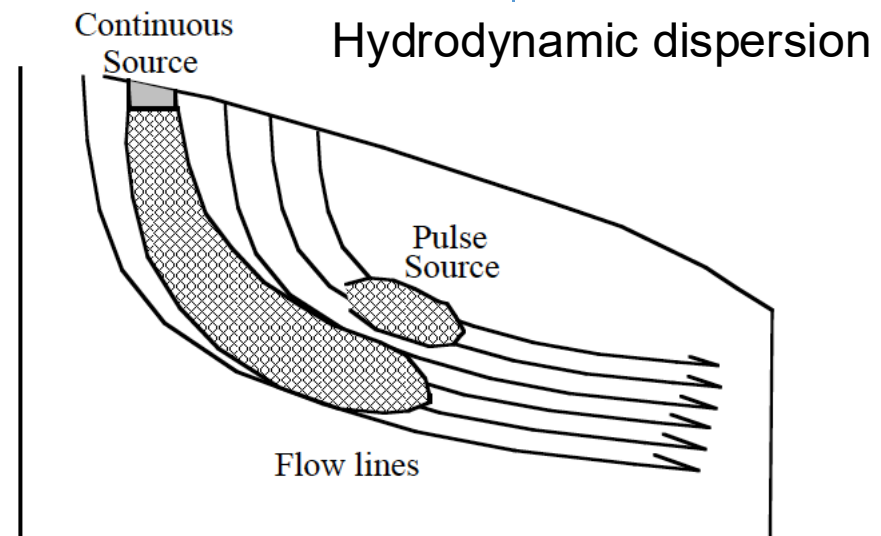


# Solute transport under saturated flow

Pure advection

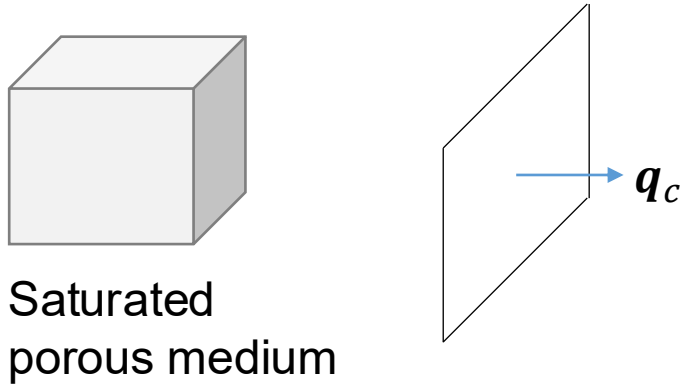


Advection + Mechanical dispersion + Molecular diffusion



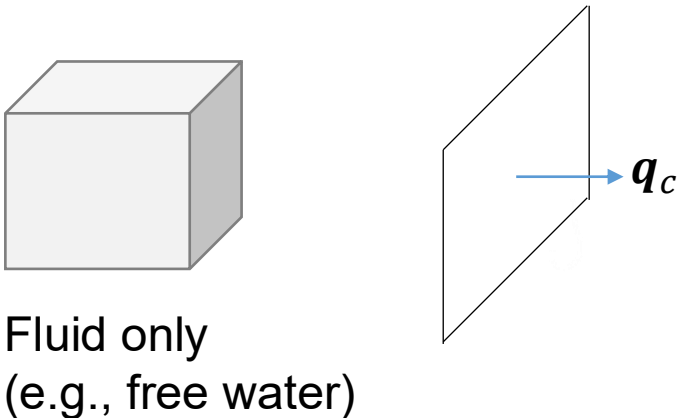
# Solute transport under saturated flow

A widely used theory for describing solute flux in saturated porous media [J. Bear (1961)]



$$\mathbf{q}_c = \phi(\mathbf{v}C - \mathbf{D}\nabla C),$$

- $\mathbf{q}_c$  is the solute flux vector.
- $\mathbf{v}$  is the velocity field.
- $\mathbf{D}$  is a tensor that depends on the velocity field and molecular diffusion in the porous medium.



$$\mathbf{q}_c = \mathbf{v}C - D_0\nabla C$$

Diffusion is isotropic,  
not a tensor.

# Solute transport under saturated flow

$$\mathbf{q}_c = \phi(\mathbf{v}C - \mathbf{D}\nabla C)$$

$$= \mathbf{q}C - \phi\mathbf{D}\nabla C$$

Outer product

$$\mathbf{D} = \alpha_T|\mathbf{v}|\mathbf{I} + (\alpha_L - \alpha_T)\frac{\mathbf{v}\mathbf{v}}{|\mathbf{v}|} + wD_0\mathbf{I}$$

$\alpha_L$  and  $\alpha_T$  are longitudinal and transverse dispersivities, respectively.

$$\mathbf{D} = \begin{pmatrix} \alpha_T|\mathbf{v}| + wD_0 + \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_x^2 & \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_xv_y & \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_xv_z \\ \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_yv_x & \alpha_T|\mathbf{v}| + wD_0 + \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_y^2 & \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_yv_z \\ \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_zv_x & \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_zv_y & \alpha_T|\mathbf{v}| + wD_0 + \frac{\alpha_L - \alpha_T}{|\mathbf{v}|}v_z^2 \end{pmatrix}$$

$$q_x = \left[ \alpha_T|\mathbf{v}| + (\alpha_L - \alpha_T)\frac{v_x^2}{|\mathbf{v}|} + wD_0 \right] \frac{\partial C}{\partial x} + \left[ (\alpha_L - \alpha_T)\frac{v_xv_y}{|\mathbf{v}|} \right] \frac{\partial C}{\partial y} + \left[ (\alpha_L - \alpha_T)\frac{v_xv_z}{|\mathbf{v}|} \right] \frac{\partial C}{\partial z}$$

$q_y$  and  $q_z$  can be written out in a similar way.

For a special case in 1D flow ( $v_y$  and  $v_z$  are zero)

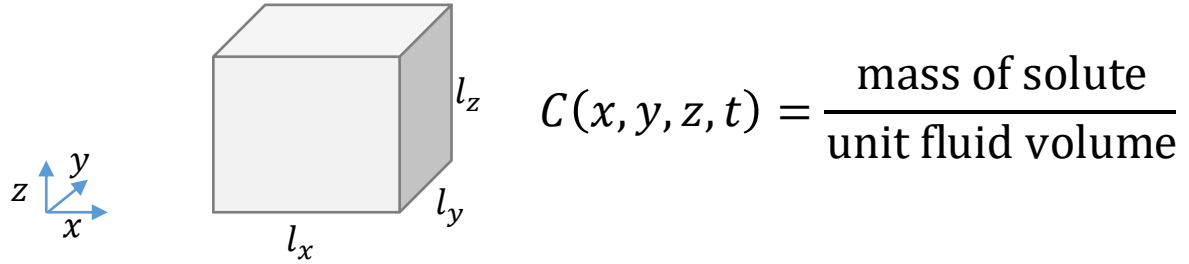
$$D_{xx} = \alpha_L v_x + D_d$$

$$D_{yy} = D_{zz} = \alpha_T v_x + D_d$$

$$\mathbf{D} = \begin{pmatrix} \alpha_L|\mathbf{v}| + wD_0 & 0 & 0 \\ 0 & \alpha_T|\mathbf{v}| + wD_0 & 0 \\ 0 & 0 & \alpha_T|\mathbf{v}| + wD_0 \end{pmatrix}$$

# Solute transport under saturated flow

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Mass conservation: Change of mass storage = mass in – mass out.

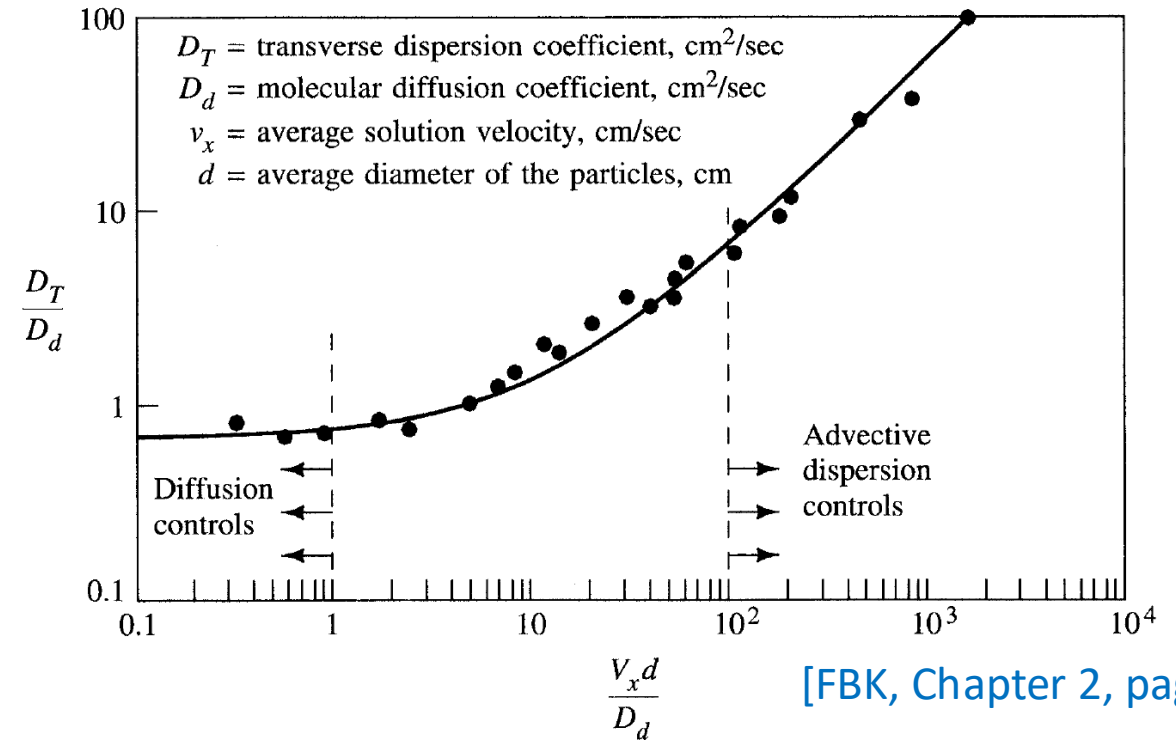
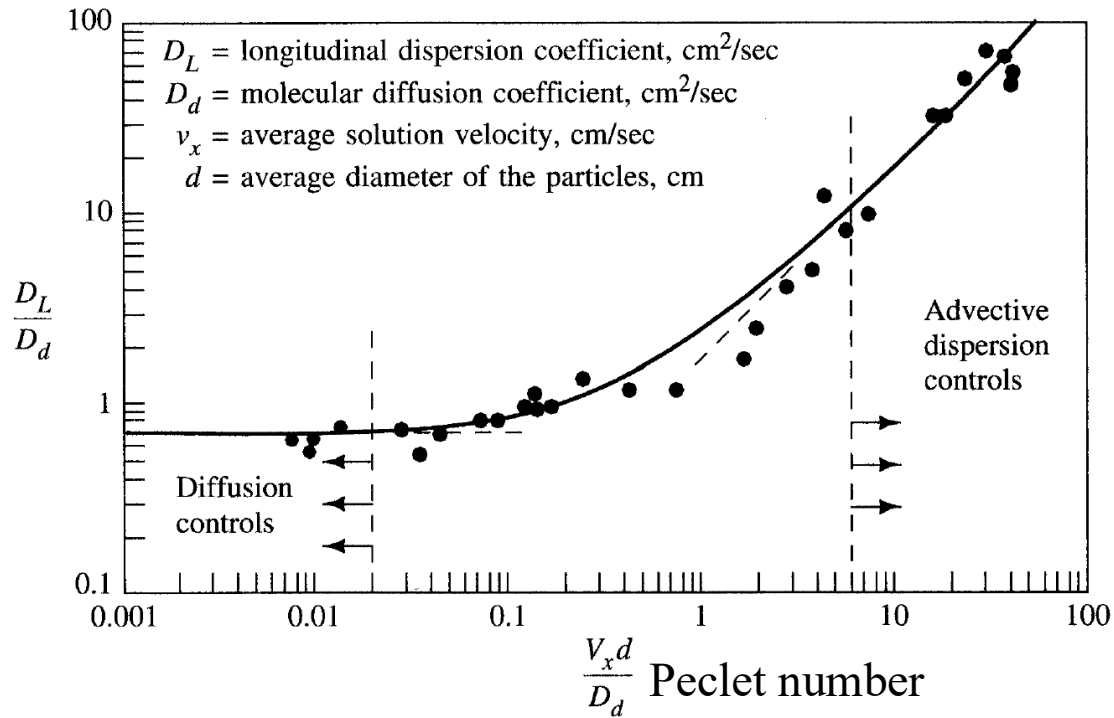
Rate of mass change:

Net fluxes:

# Solute transport under saturated flow

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# Solute transport under saturated flow



[FBK, Chapter 2, page 69]

- At lower Peclet numbers diffusion is the predominant force, and dispersion can be neglected
- At higher Peclet numbers mechanical dispersion is the predominant cause of mixing of the contaminant plume (Perkins and Johnson 1963; Bear 1972; Bear and Verruijt 1987) and the effects of diffusion can be ignored. Under these conditions  $D_i$  can be replaced with  $a_i v_i$  in the advection-dispersion equations.
- In between, there is a transition zone, where the effects of diffusion and longitudinal mechanical dispersion are comparable.