HWRS 561b: Physical Hydrogeology II

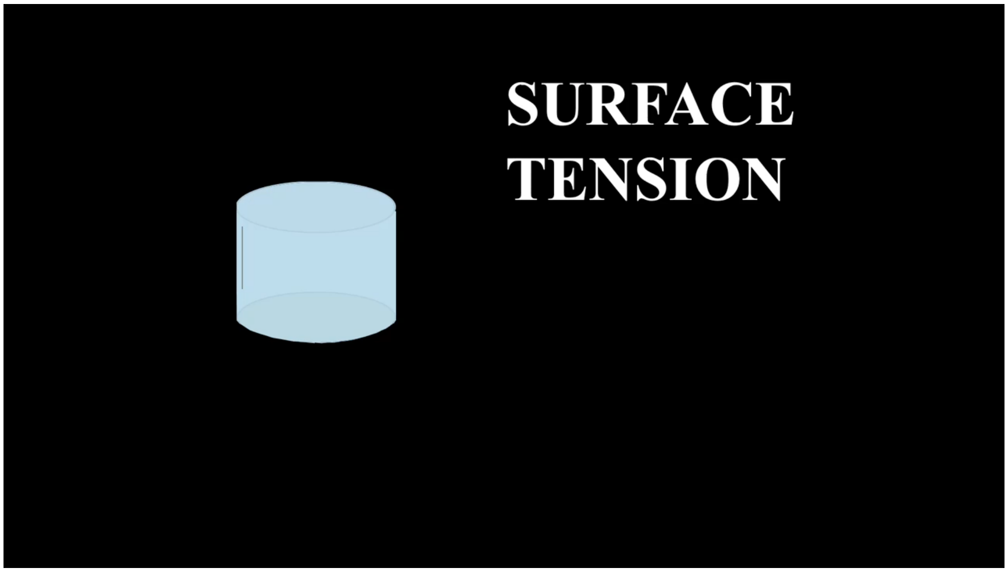
Pore scale fluids distribution

Agenda:
1. Air-water interface
2. Capillarity

Air-water system in capillary tubes

1. Why does the water try to hold together?
2. Why does the water not wet the surface?

Air-water system in capillary tubes



Link to the video: https://youtu.be/zMzqiAuOSz0

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Air-water system in capillary tubes

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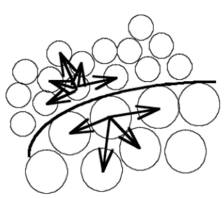
- **Two and three phase systems: water, oil, air**

- *Interfacial tension (<u>cohesive</u> forces between fluid molecules)*



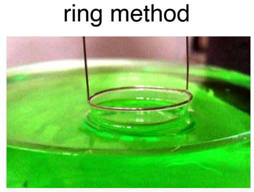
<u>Typical values of surface tension</u>:

air-water	0.072 N/m
oil-water	0.20 N/m
oil-water w/ soap	0.0001 N/m

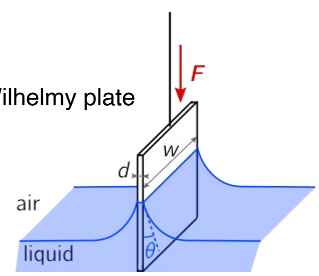
How to measure interfacial tension?

Drop weight method

ring method



Wilhelmy plate



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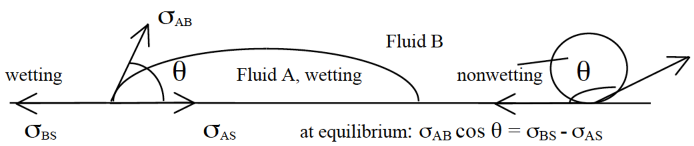
- *Wettability (<u>adhesive</u> forces between the fluid and solid surface)*



$\theta < 90^o$: fluid A is wetting with respect to fluid B on the solid S
$\theta > 90^o$: fluid A is nonwetting with respect to fluid B on the solid S

Wettability is a function of the fluid properties, soil properties, and history of contact. For most soils, the relative wettabilities are: water > oil > air

Recommended video for the concepts of *viscosity, cohesive and adhesive forces, surface tension, and capillary action* https://www.youtube.com/watch?v=P_jQ1B9UwpU

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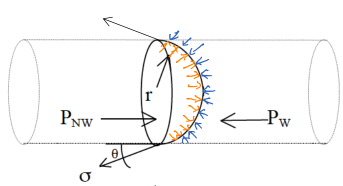
Capillary pressure (difference between the nonwetting and wetting phase pressures)



Force balance at equilibrium:

$$2\pi r \sigma \cos\theta = \pi r^2 P_{nw} - \pi r^2 P_w \Rightarrow P_{nw} - P_w = \frac{2\sigma\cos\theta}{r}$$

$$\Rightarrow \boxed{P_c = \frac{2\sigma\cos\theta}{r}}$$ Young - Laplace Equation.

1°. More general equation for any nw-w interface: $P_c = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

For the capillary tube: $r_1 = r_2 = r/\cos\theta$

2°. For a perfectly wetting fluid $(\theta = 0)$, $P_c = \frac{2\sigma}{r}$

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Air-water system in capillary tubes

Capillary pressure

Pressure jump across a fluid-fluid interface

Young-Laplace Equation

Pressure jump across a fluid-fluid interface is determined by interfacial tension + geometry of the interface (radii of the curvature)

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Art of porous media flow

Optional, but strongly encouraged, Mini-project

Take a photo or a video (< 2 min) in your day-to-day life that you think best illustrates some cool phenomena of porous media flow.

I will create a dropbox on D2L for you to upload the photo or video (due **April 26**).

Depending on the quality of your picture or video, you can receive up to 5 bonus points in your final grade (out of 100 points).

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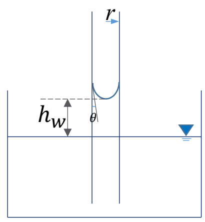
Capillary rise in a Capillary tube



At the water table:

$$\left.\begin{array}{l} p_a = 0 \\ p_w = 0 \\ z = 0 \end{array}\right\} \Rightarrow H = 0$$

At the air-water interface in the tube:

$$\left.\begin{array}{l} p_a = 0 \\ z = h_w \\ H = 0 \end{array}\right\} \Rightarrow \psi = -h_w \Rightarrow p_w = -\rho_w g h_w$$

$$p^{cap} = p_a - p_w = 0 - (-\rho_w g h_w) = \rho_w g h_w$$

Young-Laplace eqn: $p^{cap} = \frac{2\sigma\cos\theta}{r}$ $\Rightarrow$

$$h_w = \frac{2\sigma\cos\theta}{\rho_w g r}$$

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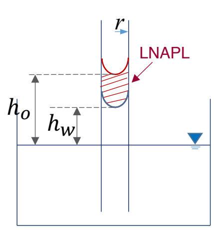
Capillary rise in a Capillary tube in the presence of an LNAPL (Assuming zero contacts)



Note: w/o oil ($\theta = 0$)

$$h_w = \frac{2\sigma_{aw}}{\rho_w g r}$$

At the oil-water interface:

$$p_w = 0 - \rho_w g h_w = -\rho_w g h_w \quad \text{------ (1)}$$

$$p_o = p^{cap}_{ow} + p_w = \frac{2\sigma_{ow}}{r} - \rho_w g h_w \quad \text{---- (2)}$$

continuing through the oil to the oil-air interface, the oil pressure

$$p_o = \frac{2\sigma_{ow}}{r} - \rho_w g h_w - \rho_o g (h_o - h_w) \quad \text{-- (3)}$$

At the oil-air interface:

$$p_o = 0 - p^{cap}_{oa} = -\frac{2\sigma_{oa}}{r} \quad \text{------ (4)}$$

p_o in (3) and (4) are equal ⇒

$$h_w = \left(\frac{2\sigma_{ow}}{r} + \frac{2\sigma_{oa}}{r} - \rho_o g h_o\right) / (\rho_w g - \rho_o g)$$

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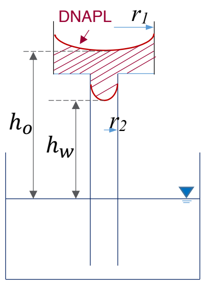
Invasion of a nonwetting fluid into a pore (Assuming zero contacts)



At the air-oil interface:

$$P_o = P_a - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r_1}$$

continuing down to the oil-water interface, the pressure of oil is:

$$P_o = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g$$

At the oil-water interface:

$$\begin{cases} P_{ow}^{cap} = P_o - P_w = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g + \rho_w g h_w \\ P_{ow}^{cap} = \frac{2\sigma_{ow}}{r_2} \end{cases}$$

$$\Rightarrow h_o - h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_w g h_o\right] / (\rho_o g)$$

$$\Rightarrow h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_o g h_o\right] / [(\rho_w - \rho_o) g]$$

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Air-water system in capillary tubes

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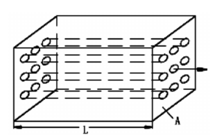
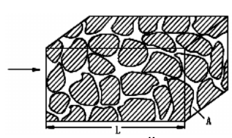
Model of a porous medium as a Bundle of Capillary Tubes

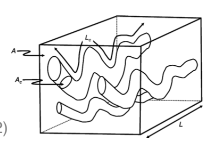


Bundle of capillary tubes



porous medium

Bear (1972)



Bundle of tortuous capillary tubes

❖ Very simplified model, but its application has tremendously improved our understanding of fluid flow and transport phenomena in porous media. Some examples:
- Permeability (already discussed)
- Dispersion (already discussed)
- Fluid invasion
- Capillary transition zone
- Soil water characteristic curve
- Relative permeability

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Air-water system in capillary tubes

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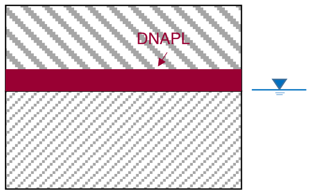
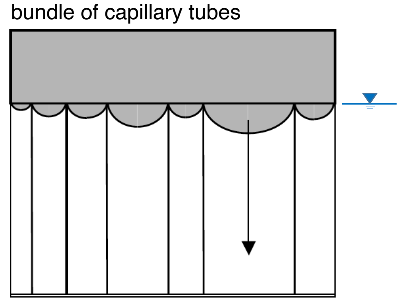
Invasion of a nonwetting fluid into an aquifer



Representing the aquifer as a bundle of capillary tubes



1. Which is easier for DNAPL to invade?

   Coarse sand or fine-grained medium?

2. For some reason, if DNAPL modifies the wettability of the porous medium grain surfaces, e.g., the contact angle of water increases from 0^{o} to something between 0^{o} and 90^{o}.

   What may happen to the DNAPL?

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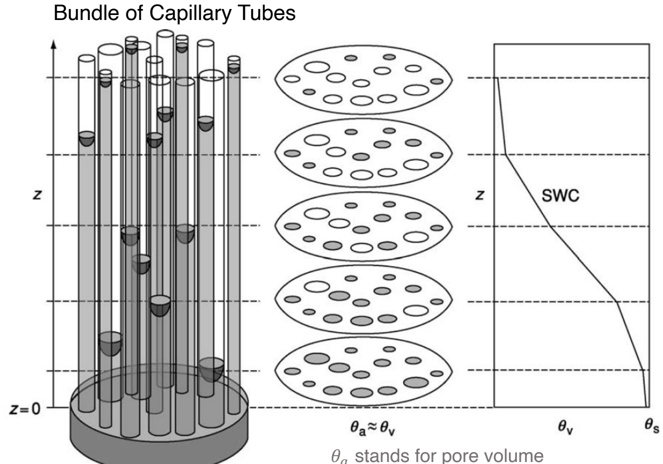
Water retention (or capillary transition zone) in the vadose zone

Representing the soil as a Bundle of Capillary Tubes



Tuller & Or (2004)

θ_a stands for pore volume
θ_v for volumetric water content
θ_s for volumetric water content at saturation

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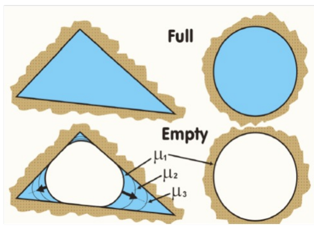
Bundle of triangular capillary tubes vs. bundle of cylindrical capillary tubes



Tuller, Or, Dudley (1999)

Bundle of triangular capillary tubes model have several advantages:

1. Can represent thin films and corner fluid
2. Saturation-dependent capillary pressure within a single-pore
3. More realistic representation of pore geometry
4. …

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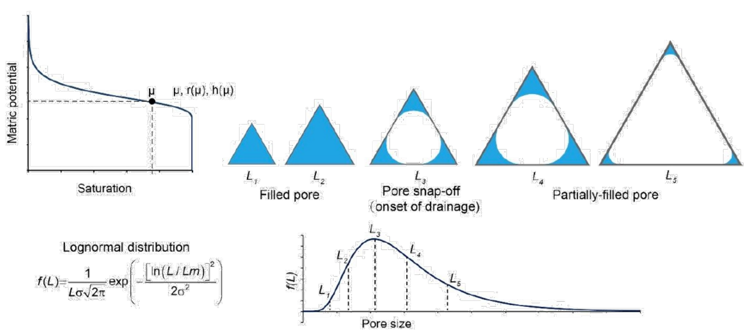
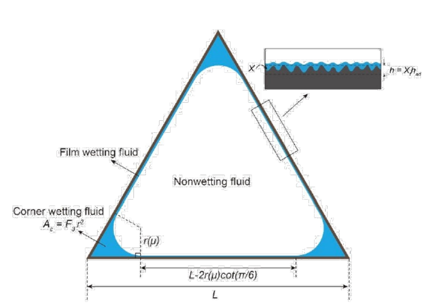
An example study using the bundle of triangular capillary tubes model to examine the impact of surface roughness on fluid-fluid interfacial areas $A_{aw} = A_{aw}(S_w)$



Simulating the soil-water characteristics



Representing the surface roughness and films

Jiang, Guo, Brusseau (2020)

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