HWR 561b Homework #1

- Assigned: Sunday, 25 January 2026
- Due: Tuesday, 5 February 2026 (upload answers in PDF or Jupyter Notebook to D2L)
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- Semester: Spring 2026

1. Short-answer conceptual questions.

    - In the bundle-of-tubes model, permeability scales linearly with porosity. (a) Show why permeability scales linearly with porosity. (b) Based on the bundle-of-tubes model and this insight, explain why porosity alone is insufficient to characterize permeability. For example, is it possible that two soil media with the same porosity have different permeabilities? Similarly, is it possible that a soil medium with greater porosity has smaller permeability than a soil medium with smaller porosity? Explain why or why not.

    - Mechanical dispersion produces spreading of solute plumes at the Darcy scale. Explain why this spreading should not be interpreted as true mixing at the pore scale.

    - The advection–diffusion equation is written at the Darcy scale. (a) Which pore-scale processes are being averaged out to obtain this equation? (b) Give one example where this upscaling might fail or produce misleading results.

2. (10 points) Consider a semi-infinite horizontal soil column undergoing steady-state, saturated horizontal flow. The porosity of the soil is 0.3. The Darcy flux is 0.3 m/day. Now consider a solute transport problem. Initially, the domain is solute-free, so $C(x, t=0) = 0$. From $t = 0$ to $t = 1$ day, solute with concentration C_0 is injected into the domain from the left boundary at $x = 0$ cm.

    - Assuming that the water velocity and solute concentration at the REV scale are always uniform at any cross-section of the soil column, i.e., both flow and solute transport can be considered 1D, derive the governing equation that describes solute transport using the mass conservation and flux laws presented in class. Use the same methodology that we used in class, but do it for this specific 1D problem, i.e., do not derive or use the general 3D equation from class and simplify to 1D. Your control volume is an arbitrary 1D interval of the soil column, and your final derived governing equation should not contain any integrals. Be sure to define all parameters in your equation.

- In your derived equation, explain the physical meaning (i.e., what physical mechanisms they represent) and dimensions of the following parameters: dispersivity (α_L), tortuosity factor (ω), and molecular diffusion coefficient (D_d).
- Sketch the solutions of solute concentration $c(x,t)$ at three times, $t = 2$, 5, 10 days. You do not need to solve the equation to draw the solutions—just sketch what you conceptually think the solution would be. Explain why the shape of the concentration profile changes with time.

3. (10 points) Mechanical dispersion and Taylor–Aris dispersion.

- Explain, in your own words, the key insights from the analysis of Taylor–Aris dispersion.
- Use the insights from Taylor–Aris dispersion to conceptually defend the advection–dispersion model that has been widely used to describe solute transport processes in groundwater (while acknowledging the assumptions).
- If molecular diffusion were zero, would Taylor–Aris dispersion occur? Why or why not? Explain.

4. A clay liner of thickness $L = 2\,\mathrm{m}$ separates a landfill from the underlying aquifer. The liner is fully saturated and has an effective diffusion coefficient given by

$$D_{\text{eff}} = \omega D^*,$$

where $D^* = 2.03 \times 10^{-9}\,\mathrm{m}^2/\mathrm{s}$ is the molecular diffusion coefficient in free water and $\omega = 0.5$ is a tortuosity factor.

At time $t = 0$, the landfill side of the liner is suddenly exposed to landfill leachate with a constant chloride concentration of

$$C_0 = 2315\,\mathrm{mg/L}.$$

Initially, the chloride concentration everywhere in the liner is zero.

Ignore advection and reactions.

- Clearly state the governing equation and boundary conditions describing chloride transport in the liner.
- Explain physically why diffusion, rather than advection, is the dominant transport mechanism in this system.

- Estimate the characteristic diffusion time scale across the liner and use it to assess whether 50 years is a "short" or "long" time relative to that.
- Using an appropriate analytical solution, estimate the chloride concentration at the downgradient side of the liner after 50 years.
- If the clay liner were twice as thick but had the same material properties, how would the breakthrough time scale change? Explain without recalculating.

5. A canal leaks water into an underlying saturated aquifer. At time $t = 0$, the canal water becomes contaminated with a conservative, nonreactive organic compound (1,4-dioxane) at a constant concentration of

$$C_0 = 1.245\,\mathrm{mg/L}.$$

Groundwater flows horizontally away from the canal with a constant average porewater velocity of

$$v = 0.544\,\mathrm{m/day}.$$

Assume one-dimensional transport away from the canal, neglect transverse spreading, and ignore dispersion and diffusion.

- Write the governing transport equation for this problem and clearly state all assumptions.
- Describe in words how the concentration signal propagates through the aquifer under these assumptions.
- Calculate the travel time for groundwater to move 25 m away from the canal.
- Based on your result from above, determine the concentration of 1,4-dioxane at a location 25 m downgradient from the canal after 45 days.
- If longitudinal dispersion were included, how would the concentration at 25 m after 45 days change qualitatively? Discuss arrival time, peak concentration, and tailing.