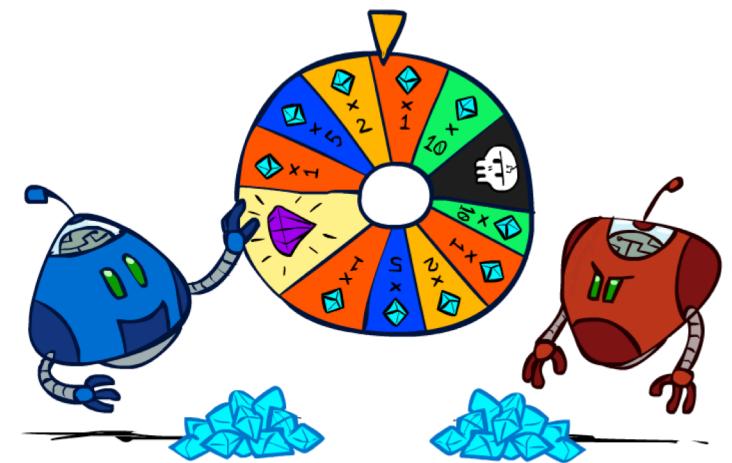
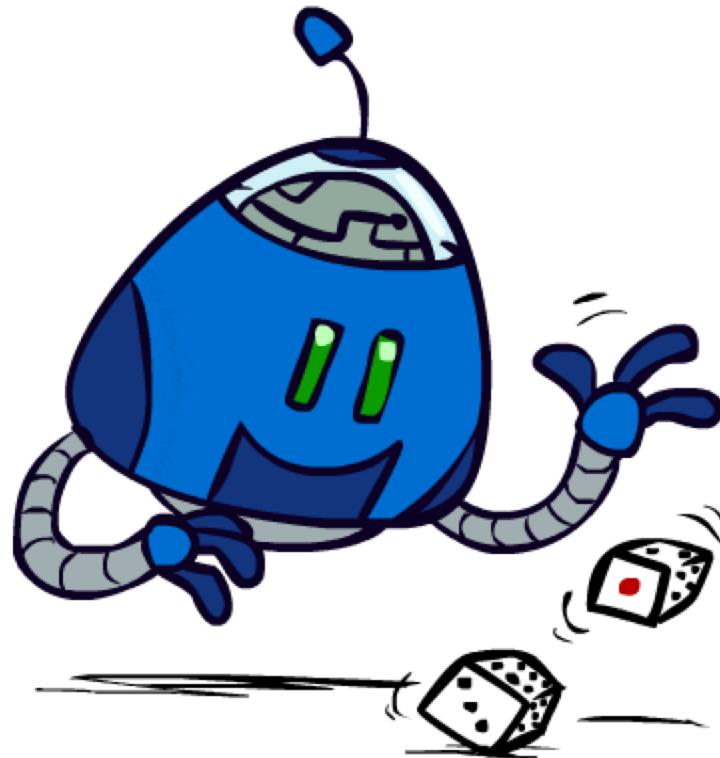

CSE 573: Artificial Intelligence

Hanna Hajishirzi
Expectimax – Complex Games

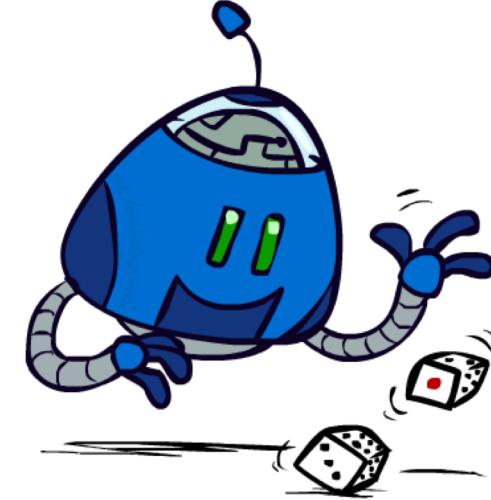
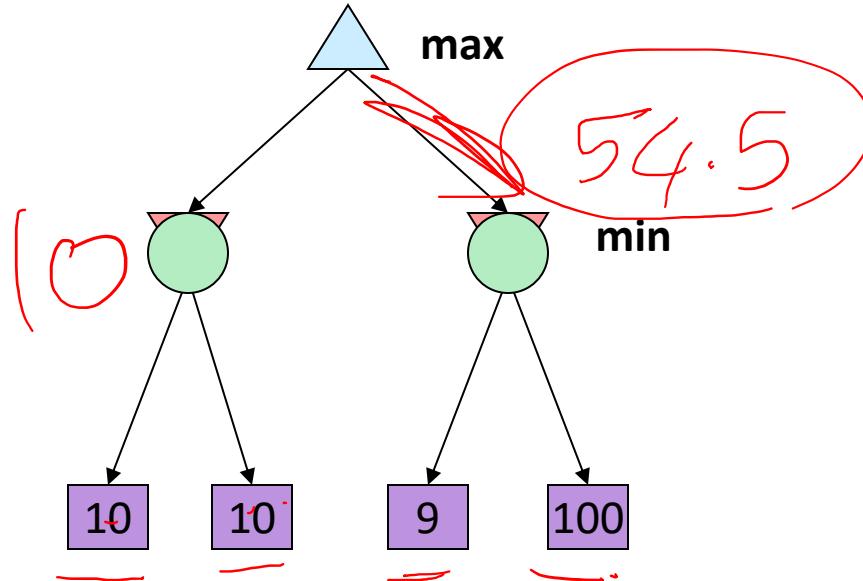
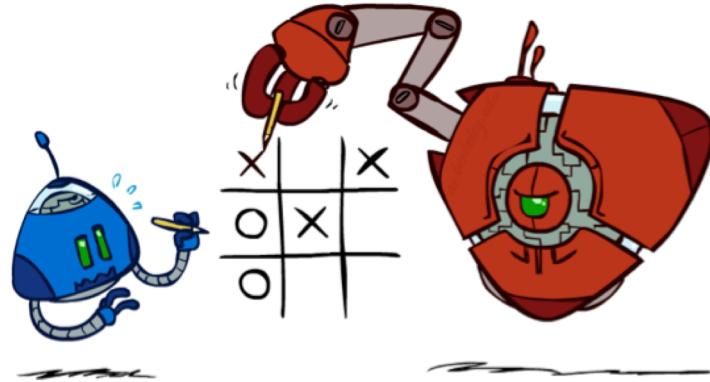
slides adapted from
Dan Klein, Pieter Abbeel ai.berkeley.edu
And Dan Weld, Luke Zettelmoyer



Uncertain Outcomes



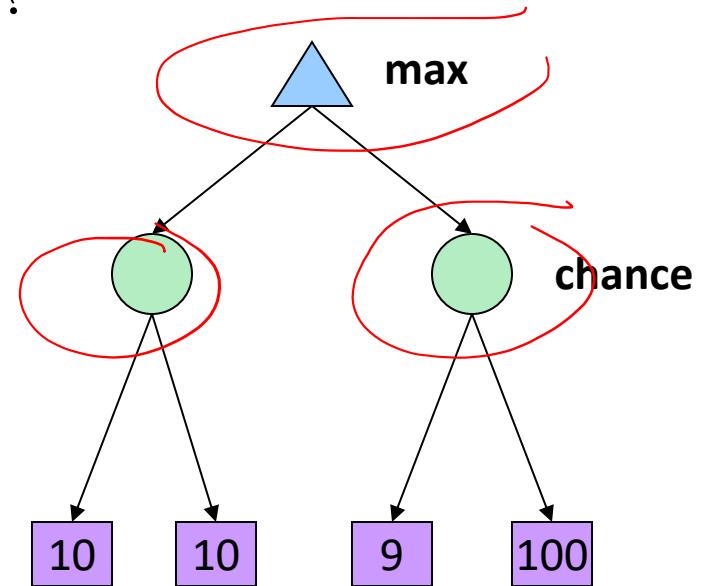
Worst-Case vs. Average Case



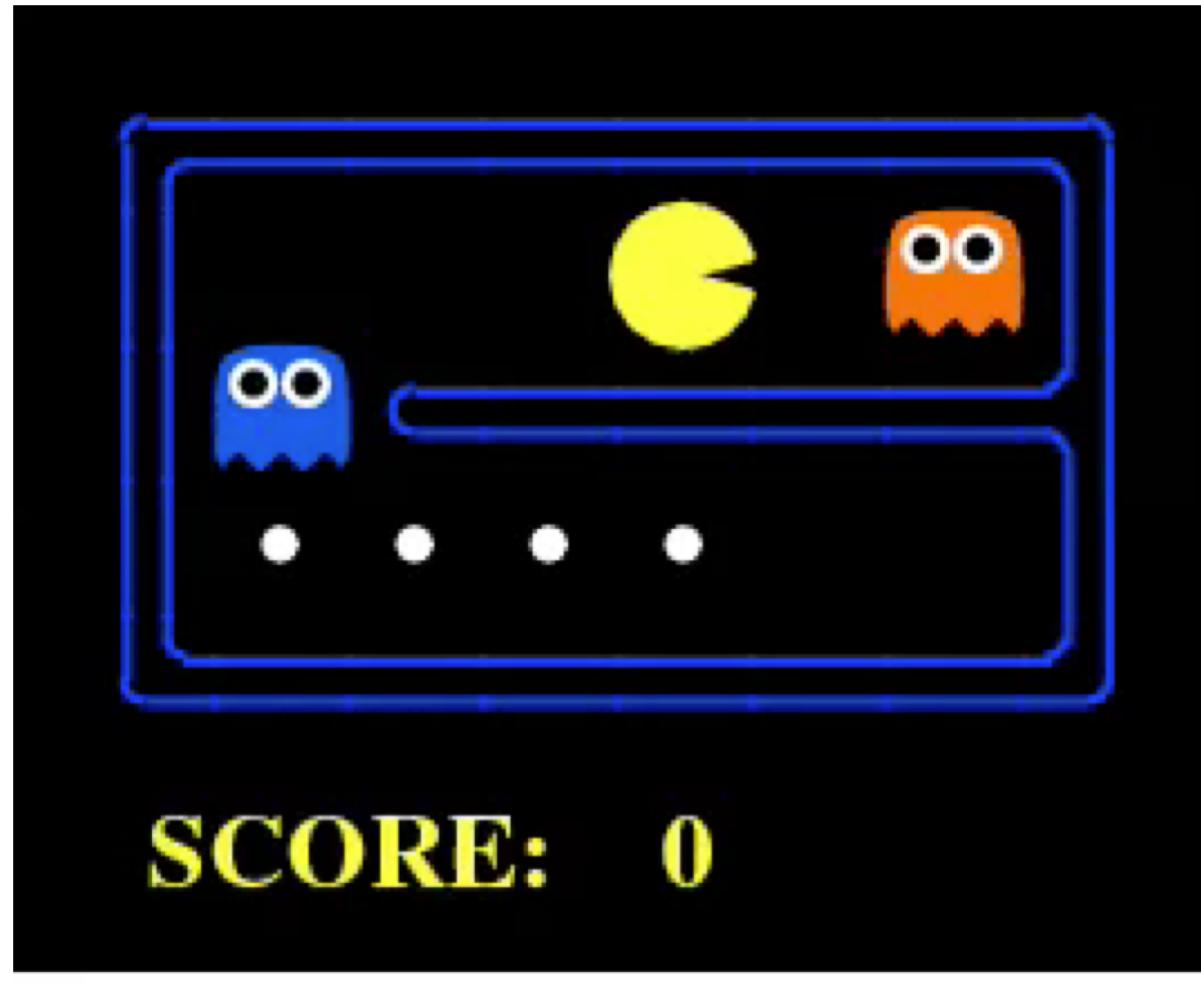
Idea: Uncertain outcomes controlled by chance, not an adversary!

Expectimax Search

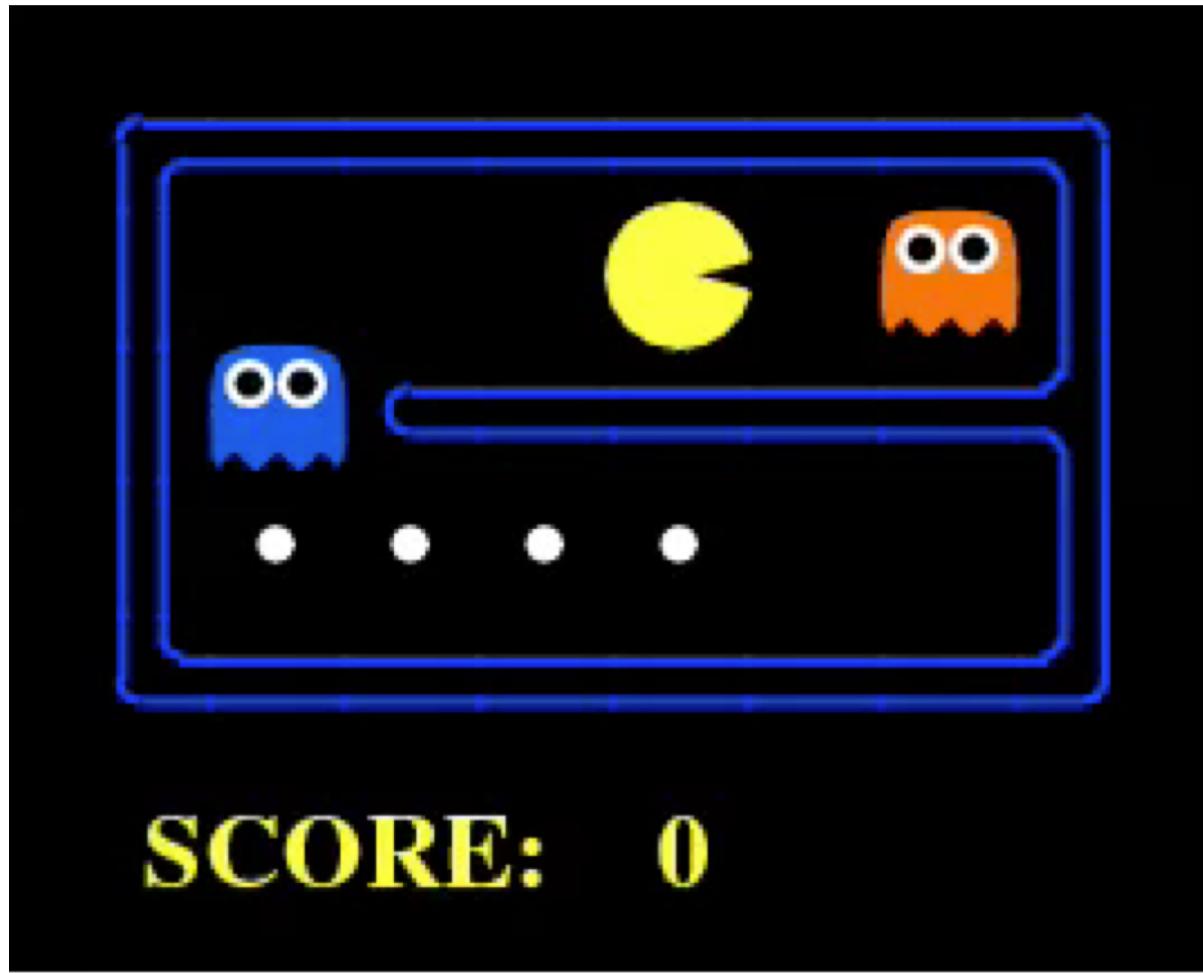
- Why wouldn't we know what the result of an action will be?
 - Explicit randomness: rolling dice
 - Unpredictable opponents: the ghosts respond randomly
 - Unpredictable humans: humans are not perfect
 - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- **Expectimax search:** compute the average score under optimal play
 - Max nodes as in minimax search
 - Chance nodes are like min nodes but the outcome is uncertain
 - Calculate their **expected utilities**
 - I.e. take weighted average (expectation) of children
- Later, we'll learn how to formalize the underlying uncertain-result problems as **Markov Decision Processes**



Video of Demo Min vs. Exp (Min)



Video of Demo Min vs. Exp (Exp)



Expectimax Pseudocode

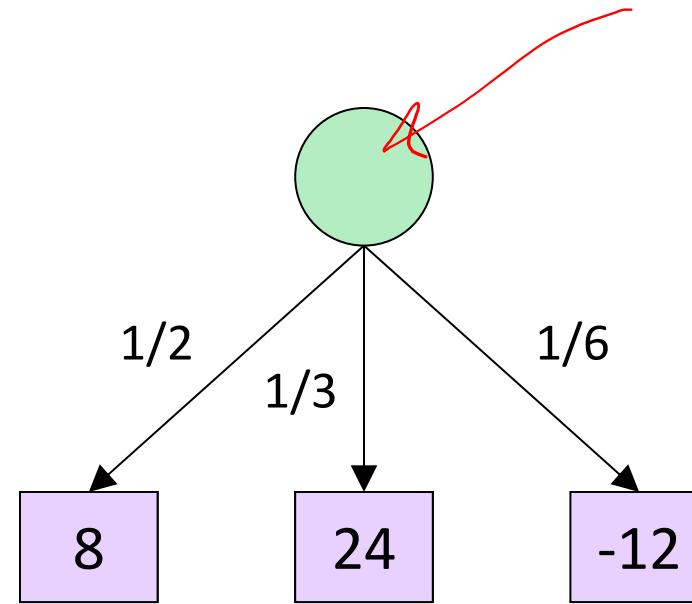
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

Expectimax Pseudocode

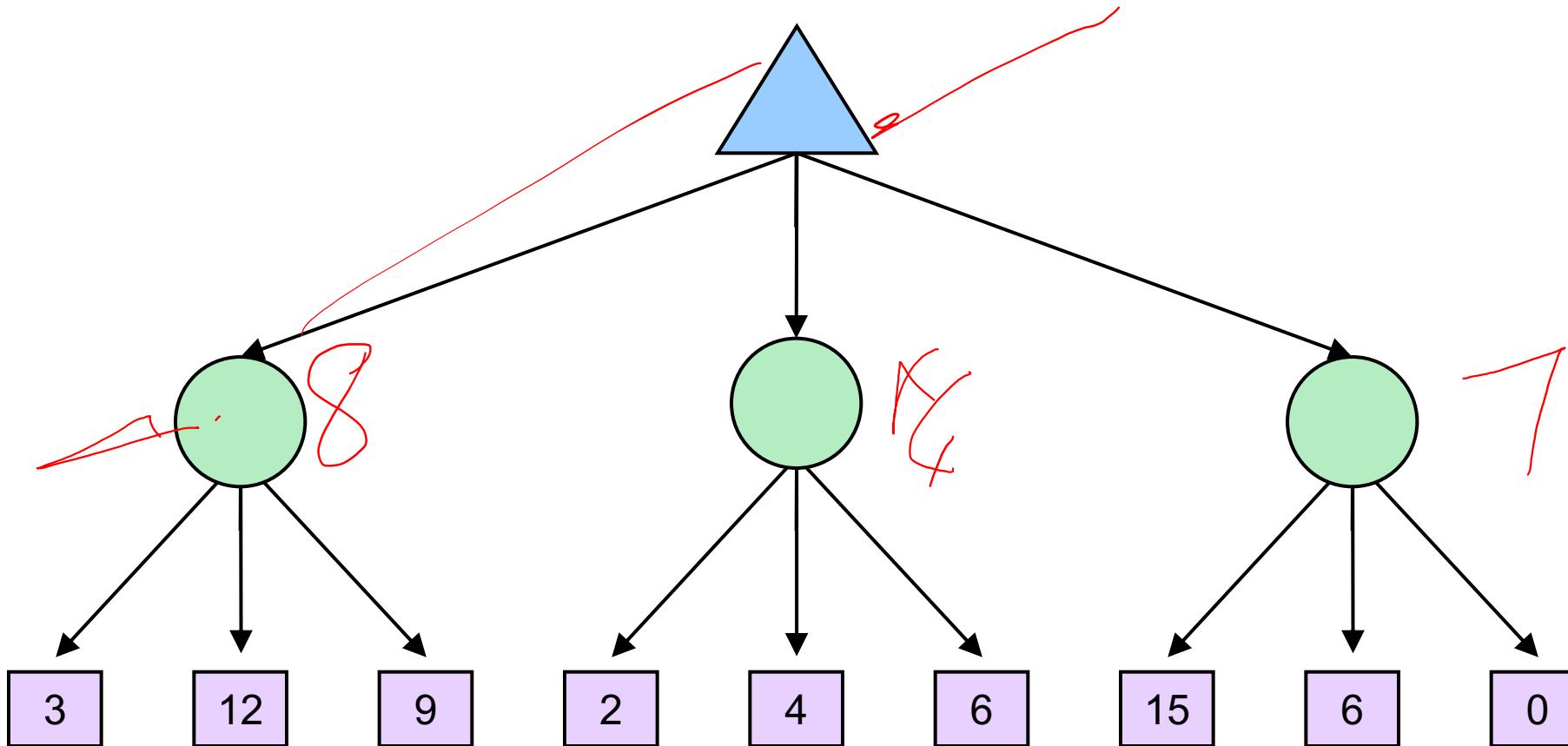
```
def exp-value(state):  
    initialize v = 0  
    for each successor of state:  
        p = probability(successor)  
        v += p * value(successor)  
    return v
```



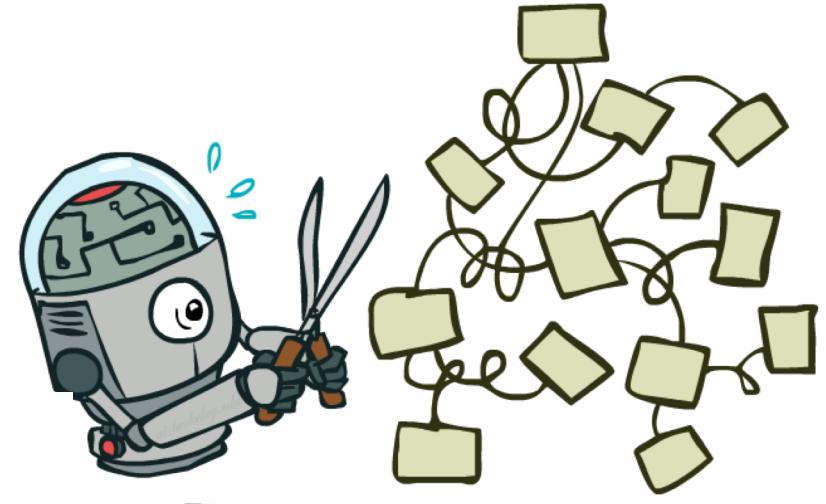
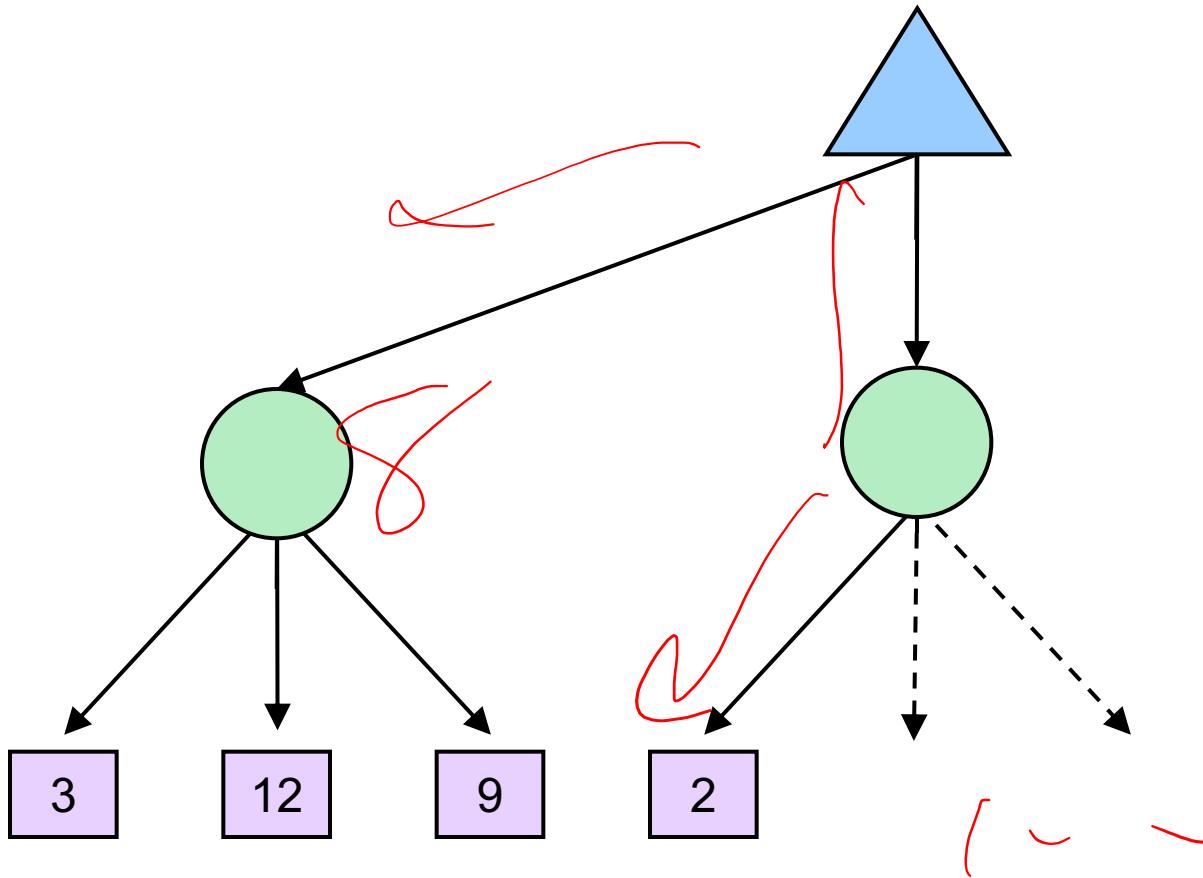
$$v = \left(\frac{1}{2} \right) (8) + \left(\frac{1}{3} \right) (24) + \left(\frac{1}{6} \right) (-12) = 10$$

Handwritten annotations in red show the calculation: $\cancel{8} \times \frac{1}{2}$, $\cancel{24} \times \frac{1}{3}$, and $\cancel{-12} \times \frac{1}{6}$.

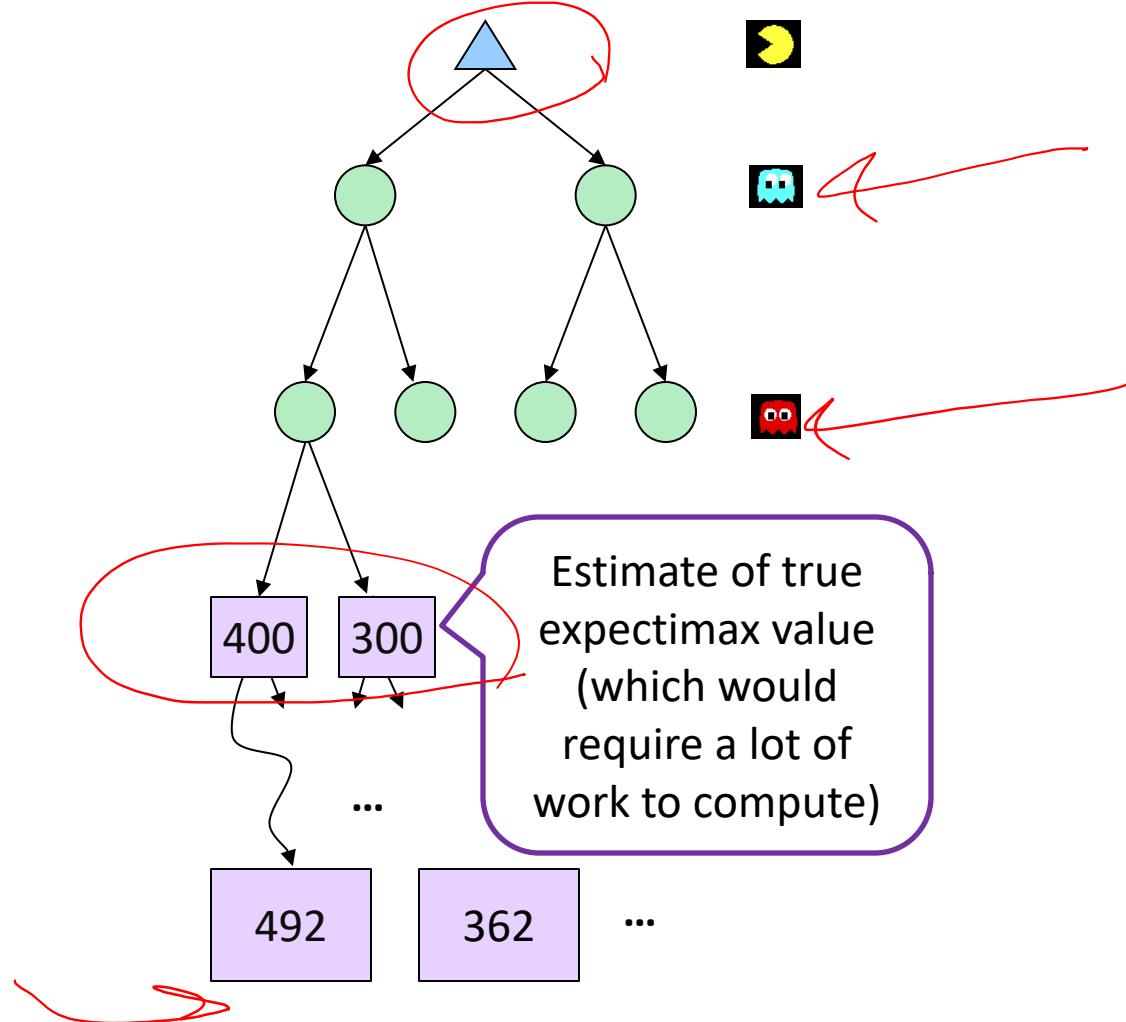
Expectimax Example



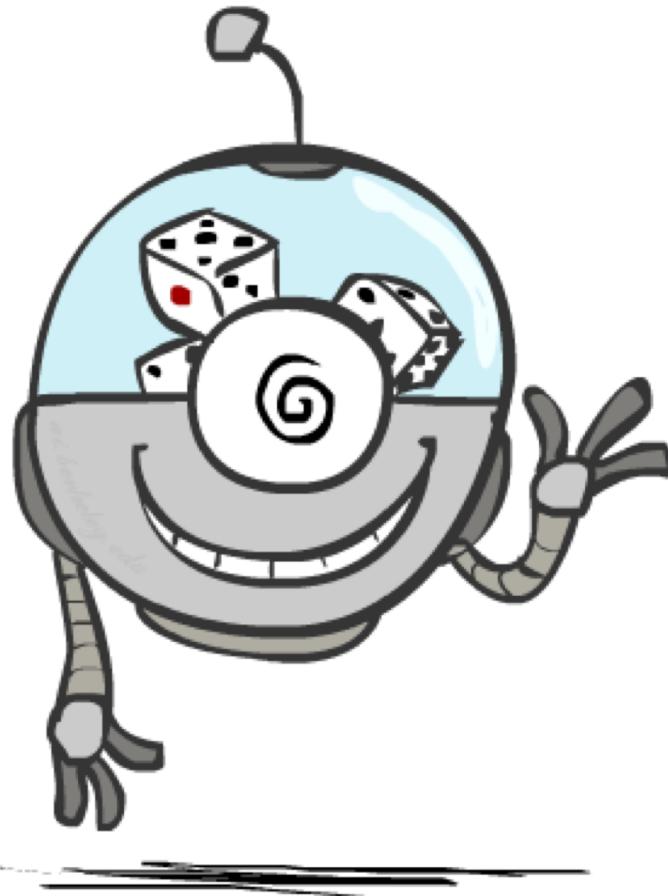
Expectimax Pruning?



Depth-Limited Expectimax

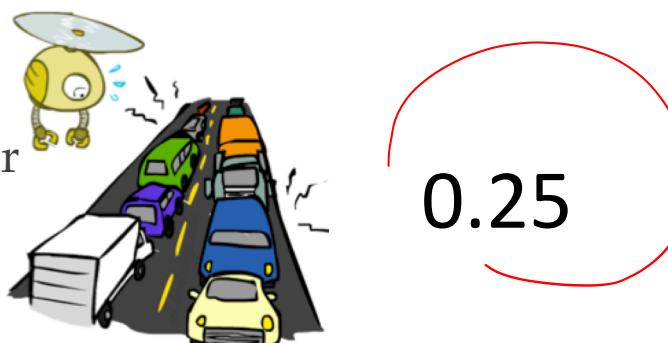
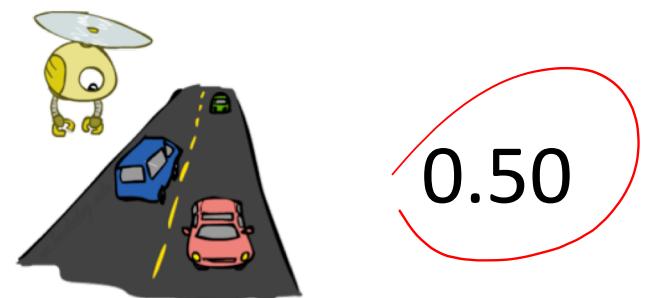
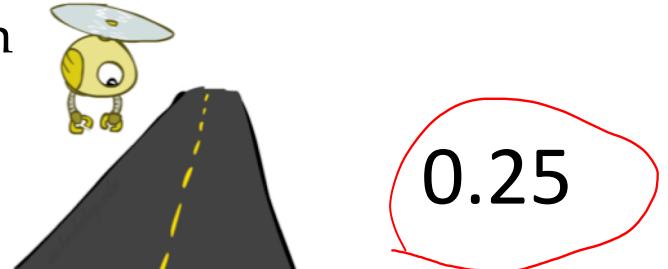


Probabilities



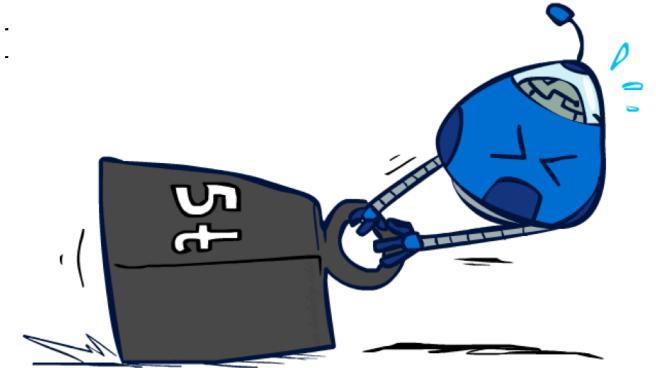
Reminder: Probabilities

- A **random variable** represents an event whose outcome is unknown
- A **probability distribution** is an assignment of weights to outcomes
- Example: Traffic on freeway
 - Random variable: $T =$ whether there's traffic
 - Outcomes: $T \in \{\text{none}, \text{light}, \text{heavy}\}$
 - Distribution: $P(T=\text{none}) = 0.25, P(T=\text{light}) = 0.50, P(T=\text{heavy}) = 0.25$
- Some laws of probability (more later):
 - Probabilities are always non-negative
 - Probabilities over all possible outcomes sum to one
- As we get more evidence, probabilities may change:
 - $P(T=\text{heavy}) = 0.25, P(T=\text{heavy} \mid \text{Hour}=8\text{am}) = 0.60$
 - We'll talk about methods for reasoning and updating probabilities later



Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes
- Example: How long to get to the airport?

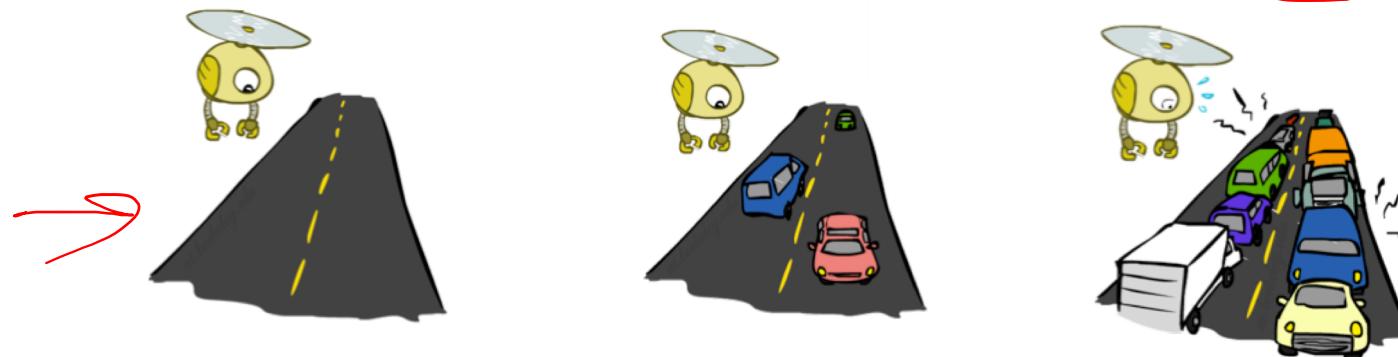


Time:

$$\begin{array}{ccc} \text{20 min} & + & \text{30 min} & + & \text{60 min} \\ \cancel{x} & & \cancel{x} & & \cancel{x} \\ \hline 0.25 & & 0.50 & & 0.25 \end{array}$$

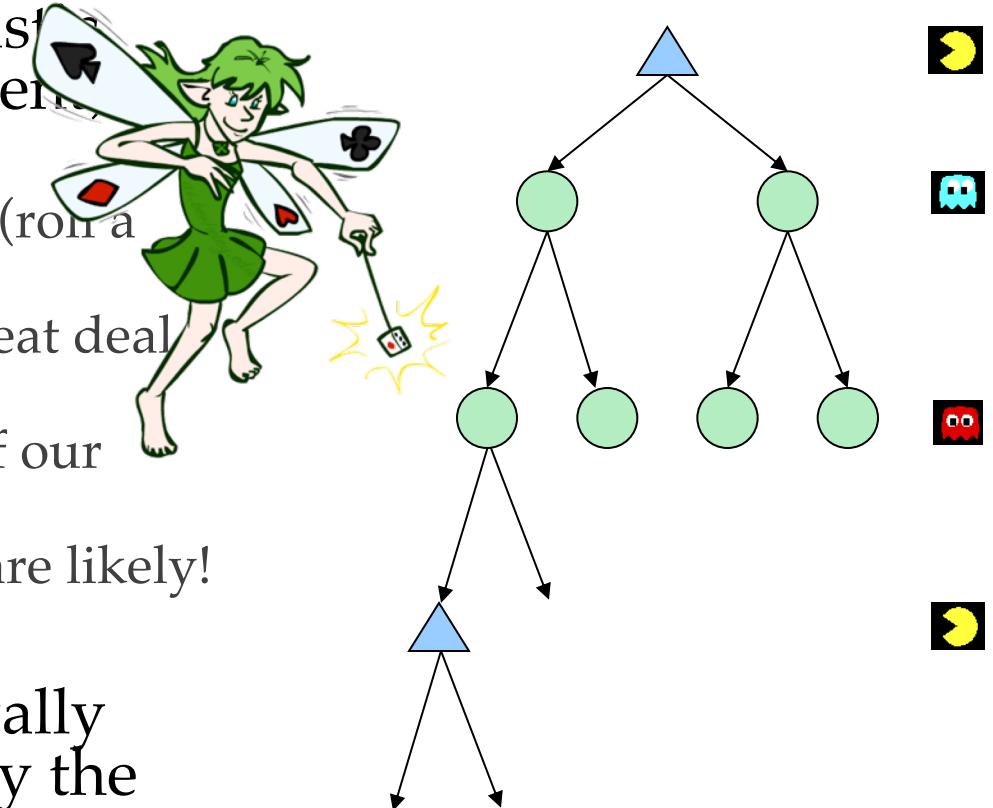
35 min

Probability:



What Probabilities to Use?

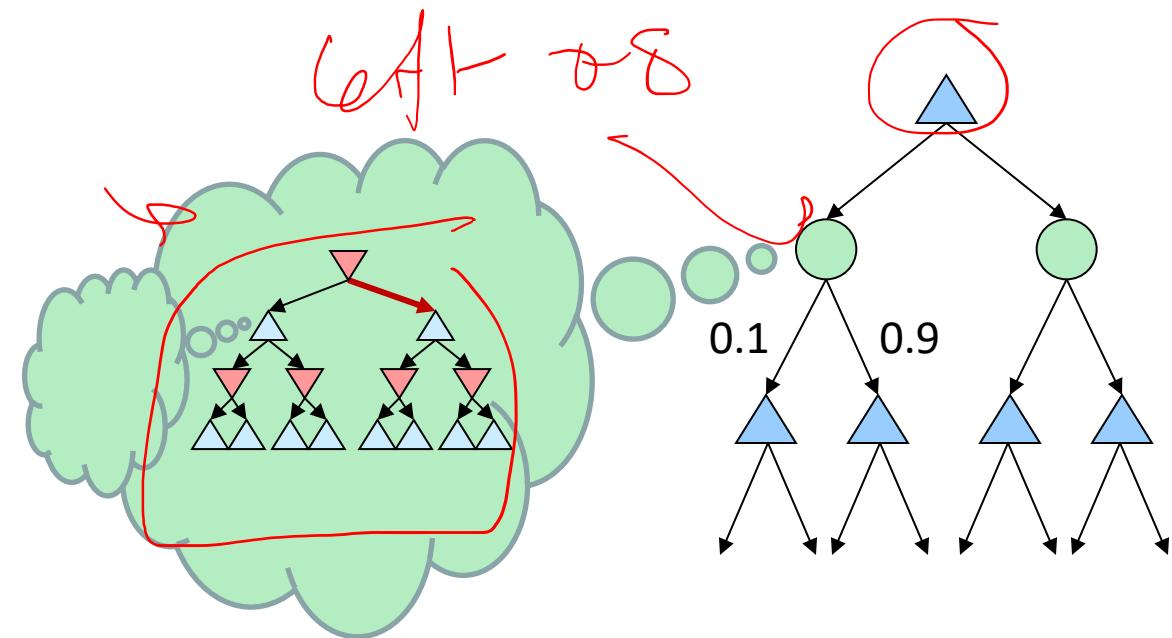
- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
 - Model could be a simple uniform distribution (roll a die)
 - Model could be sophisticated and require a great deal of computation
 - We have a chance node for any outcome out of our control: opponent or environment
 - The model might say that adversarial actions are likely!
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes



Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

Quiz: Informed Probabilities

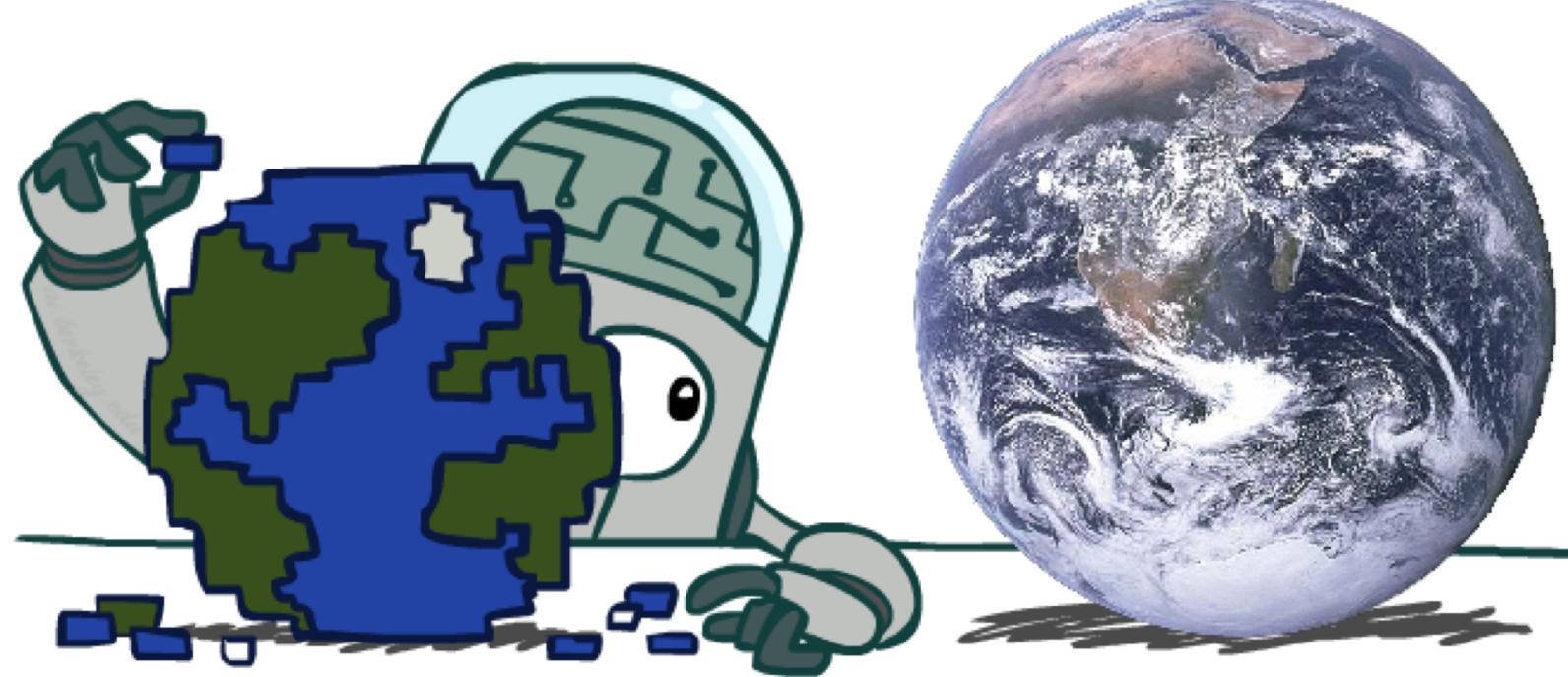
- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?



- Answer: Expectimax!

- To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
- This kind of thing gets very slow very quickly
- Even worse if you have to simulate your opponent simulating you...
- ... except for minimax and maximax, which have the nice property that it all collapses into one game tree

Modeling Assumptions



The Dangers of Optimism and Pessimism

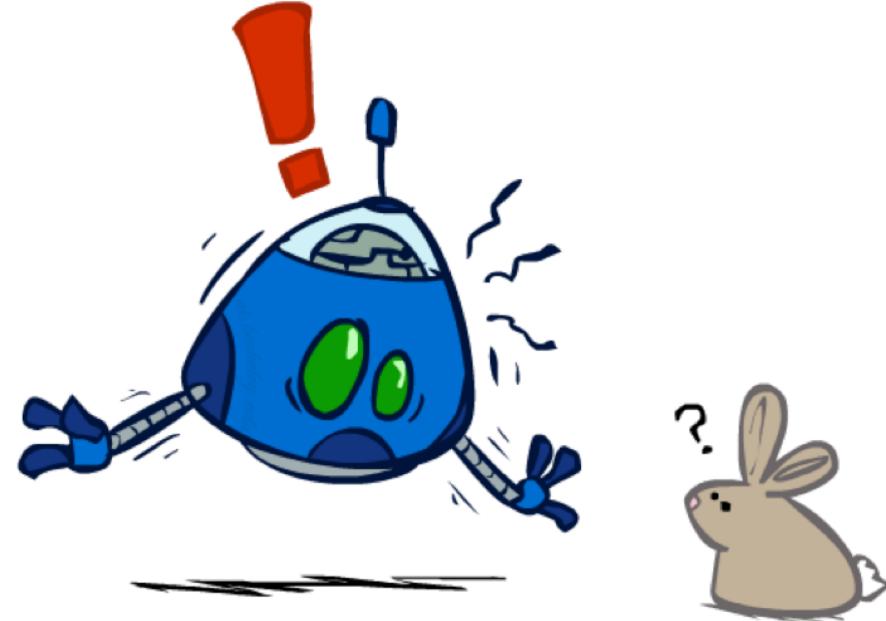
Dangerous Optimism

Assuming chance when the world is adversarial

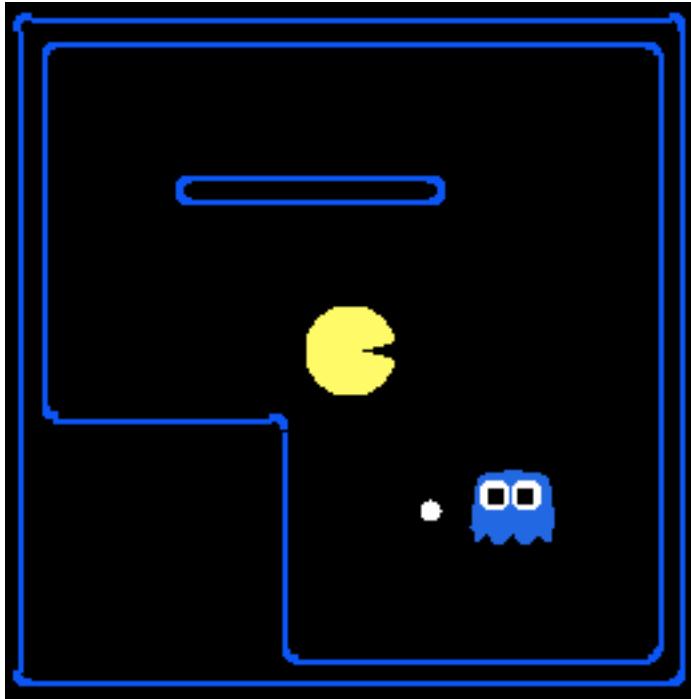


Dangerous Pessimism

Assuming the worst case when it's not likely



Assumptions vs. Reality



	Adversarial Ghost	Random Ghost
Minimax Pacman		
Expectimax Pacman		

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Video of Demo World Assumptions

Random Ghost – Expectimax Pacman

Video of Demo World Assumptions

Adversarial Ghost – Minimax Pacman



Video of Demo World Assumptions

Random Ghost – Minimax Pacman

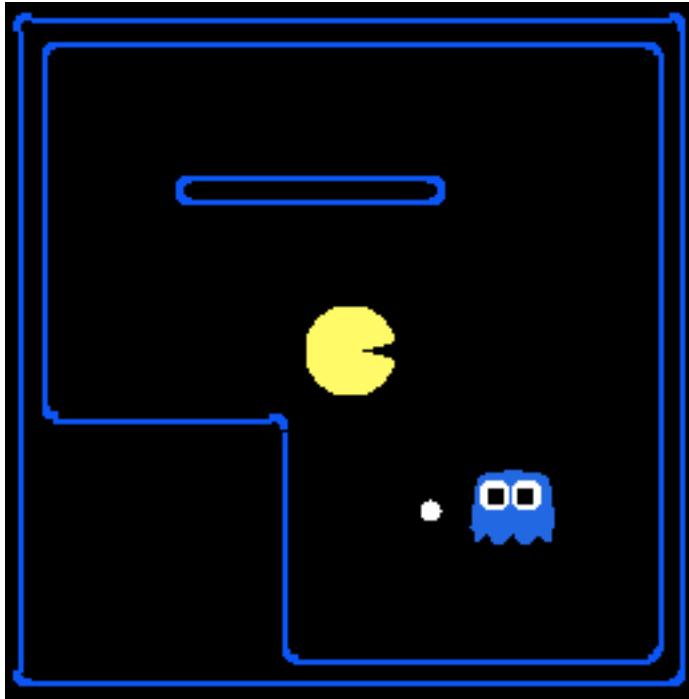


Video of Demo World Assumptions

Adversarial Ghost – Expectimax Pacman



Assumptions vs. Reality



	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 493
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Results from playing 5 games

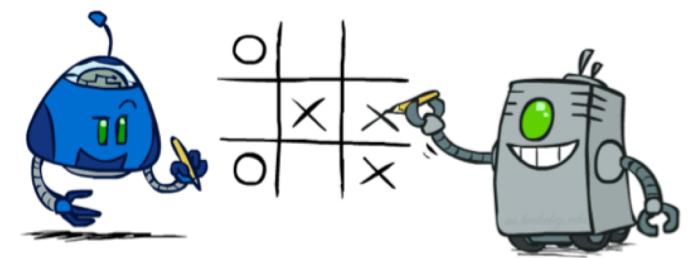
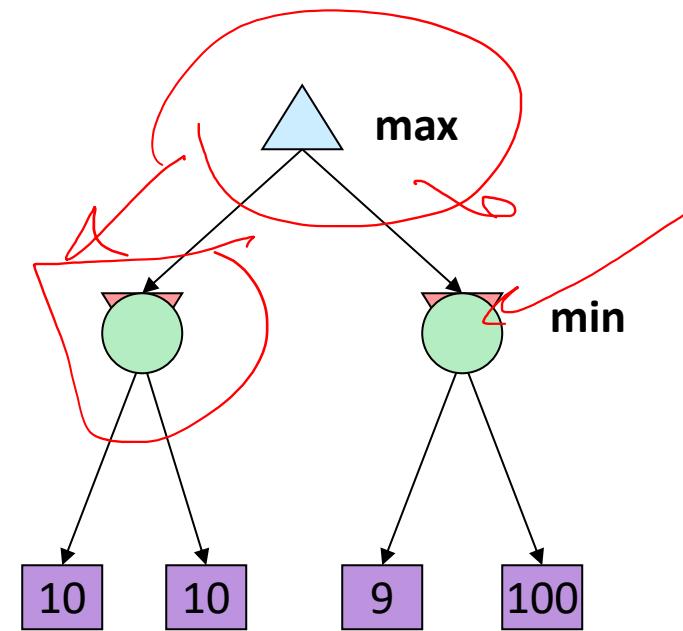
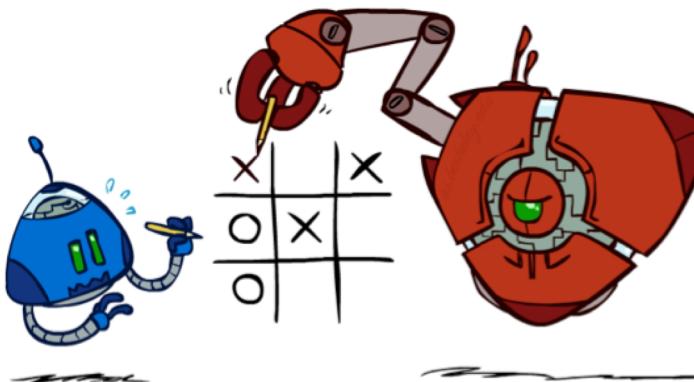
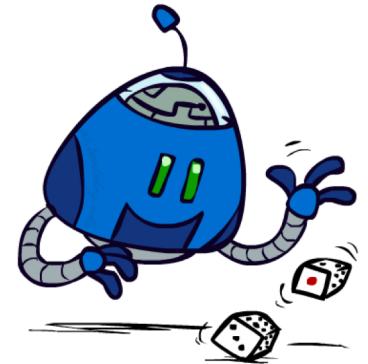
Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Announcements

- PS2 is released
 - Deadline: Feb. 5th
- HW1 is released
 - Deadline: Feb. 12th

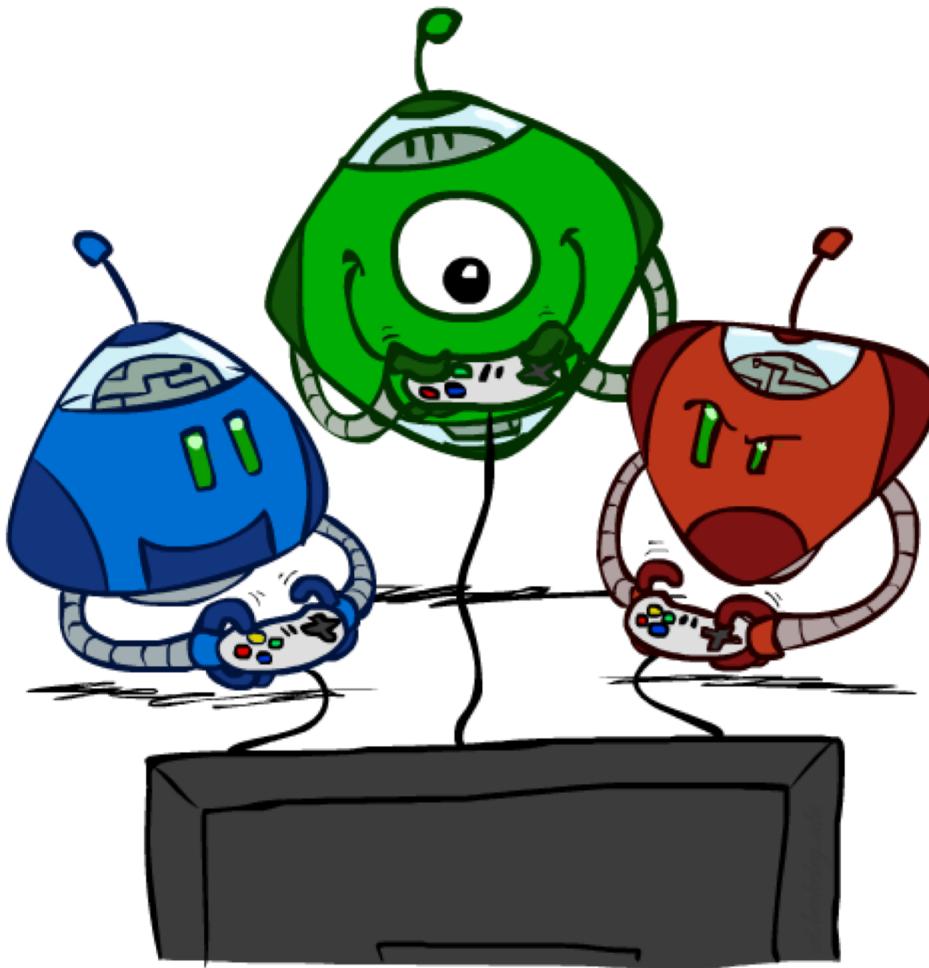
Recap: Worst-Case vs. Average Case

- Worst case reasoning is too conservative
- Need average case reasoning



Idea: Uncertain outcomes controlled by chance, not an adversary!

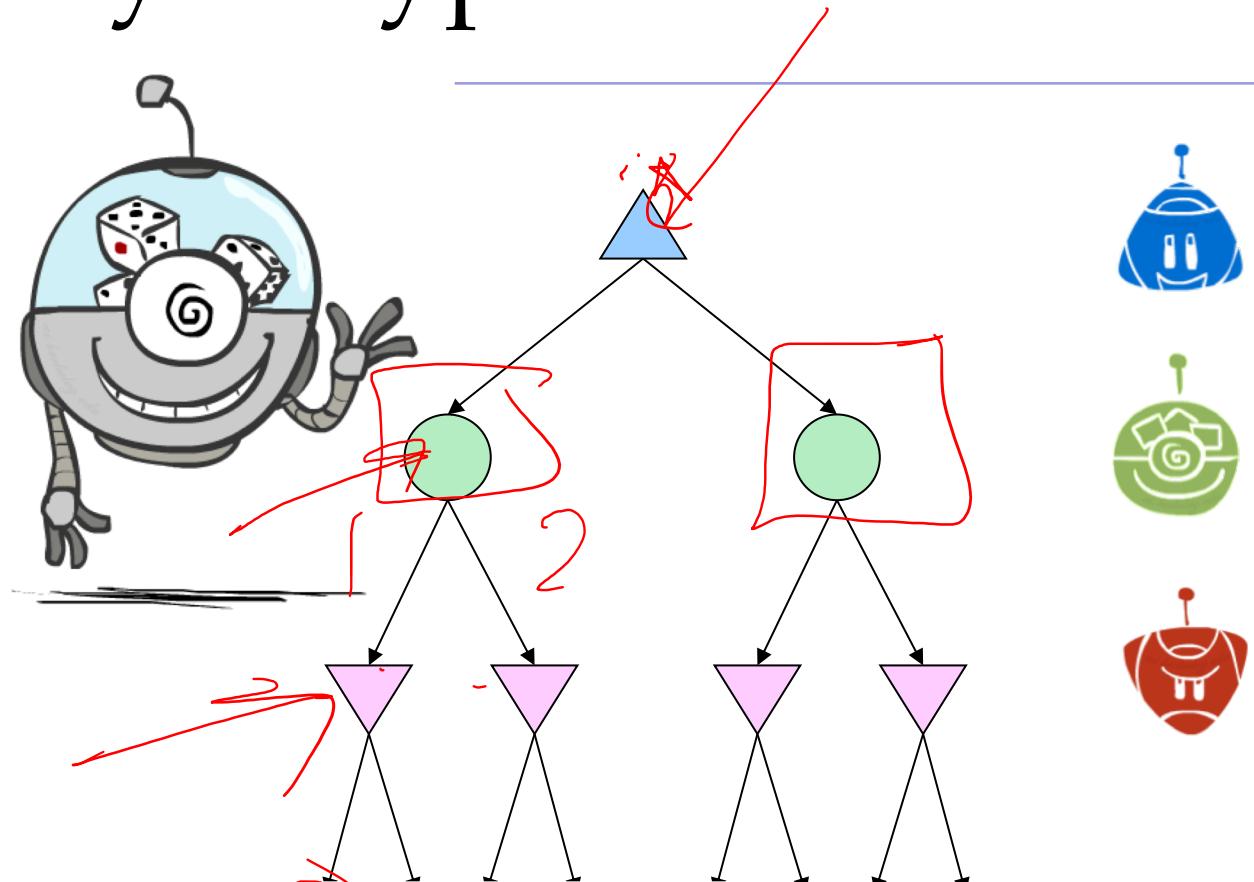
Other Game Types



Mixed Layer Types

- E.g. Backgammon
- Expecti-minimax
 - Environment is an extra “random agent” player that moves after each min/max agent
 - Each node computes the appropriate combination of its children

```
if state is a MAX node then  
    return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)  
if state is a MIN node then  
    return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)  
if state is a chance node then  
    return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
```



Example: Backgammon

- Dice rolls increase b : 21 possible rolls with 2 dice
 - Backgammon \approx 20 legal moves
 - Depth 2 = $20 \times (21 \times 20)^3 = 1.2 \times 10^9$
- As depth increases, probability of reaching a given search node shrinks
 - So usefulness of search is diminished
 - So limiting depth is less damaging
 - But pruning is trickier...
- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning:
world-champion level play
- 1st AI world champion in any game!

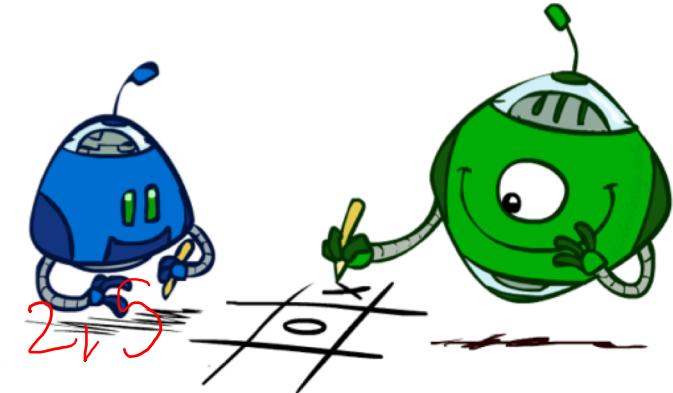
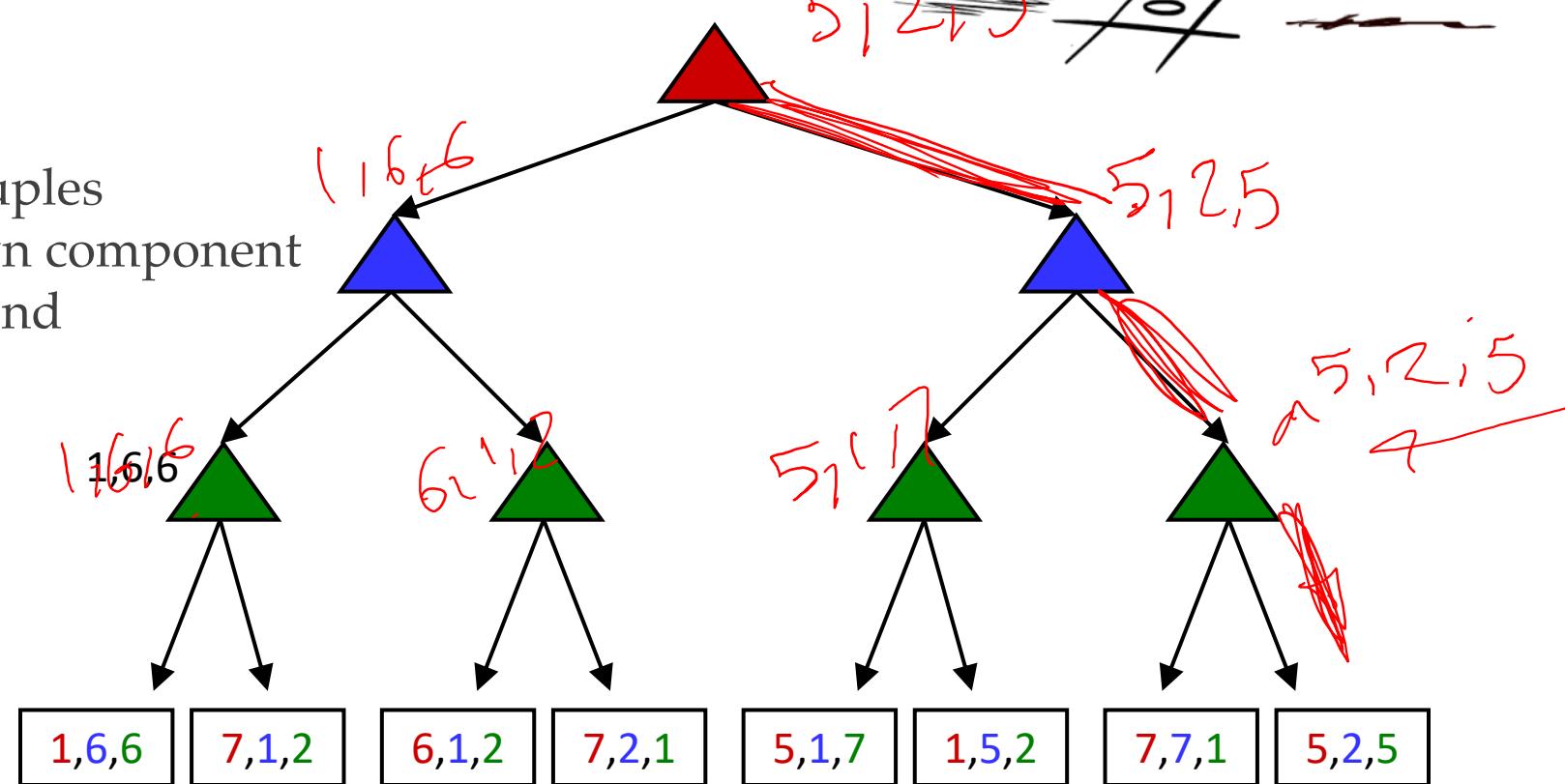
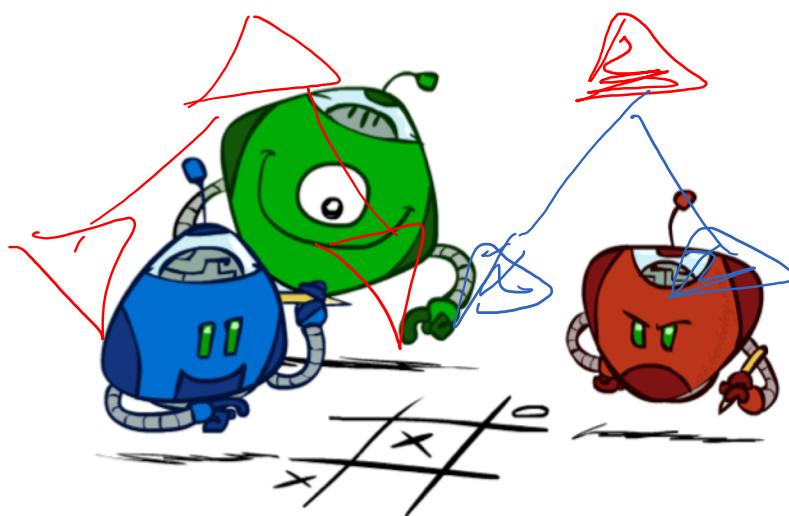


Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?

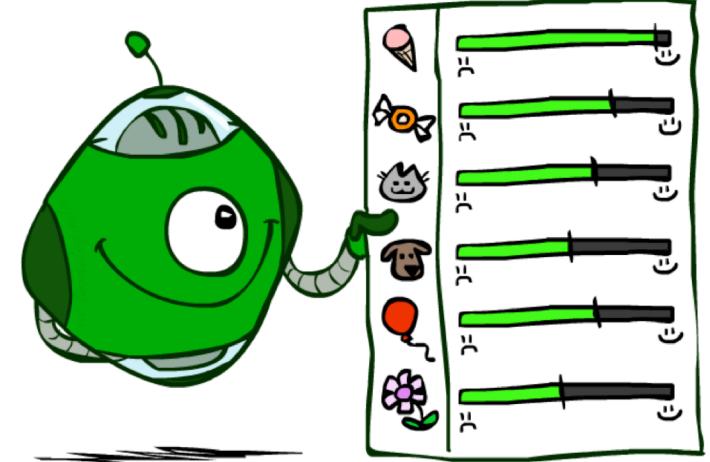
- Generalization of minimax:

- Terminals have utility tuples
- Node values are also utility tuples
- Each player maximizes its own component
- Can give rise to cooperation and competition dynamically...



Utilities

- Utilities: values that we assign to every state



- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
 - A rational agent should chose the action that **maximizes its expected utility, given its knowledge**

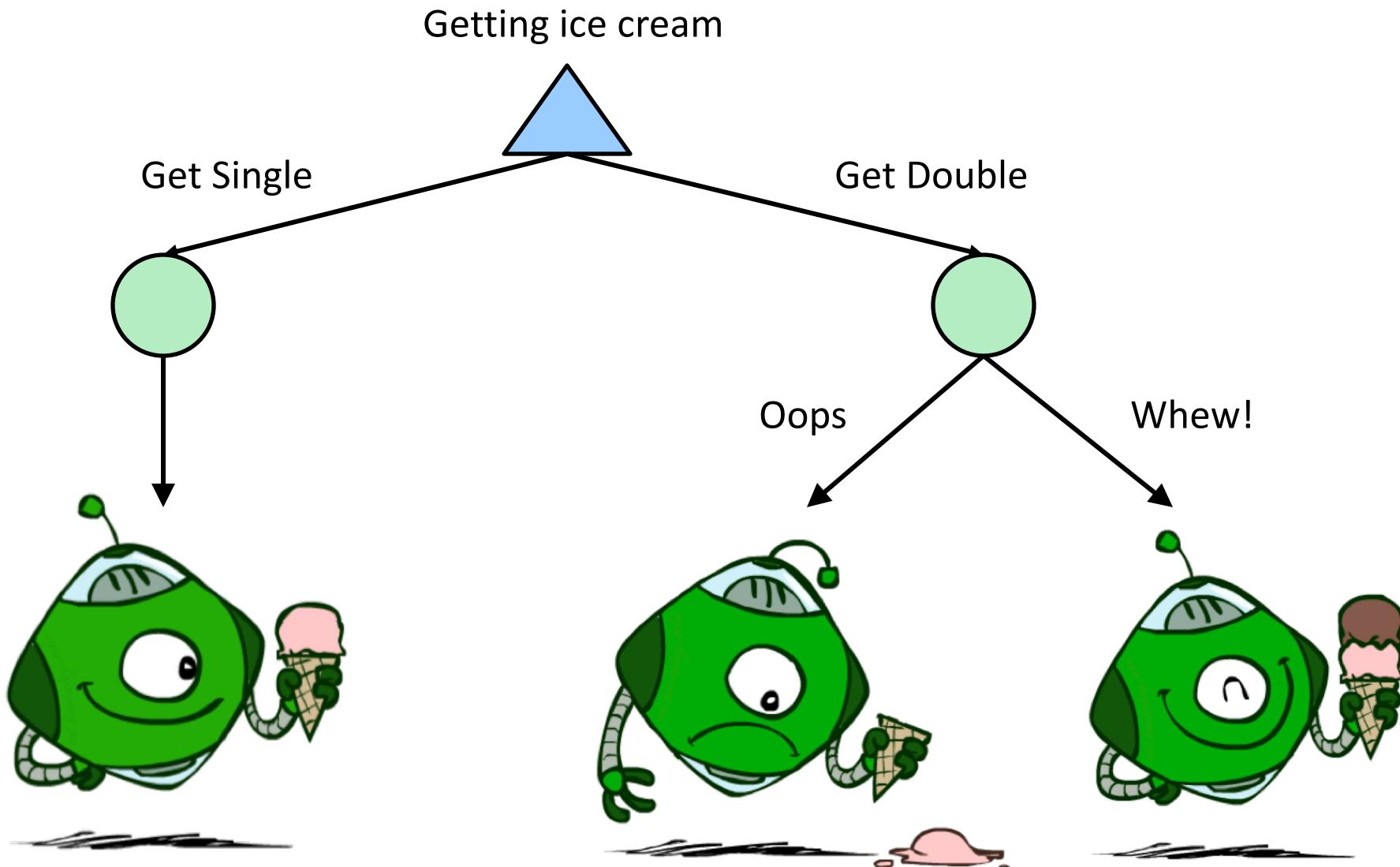


Utilities

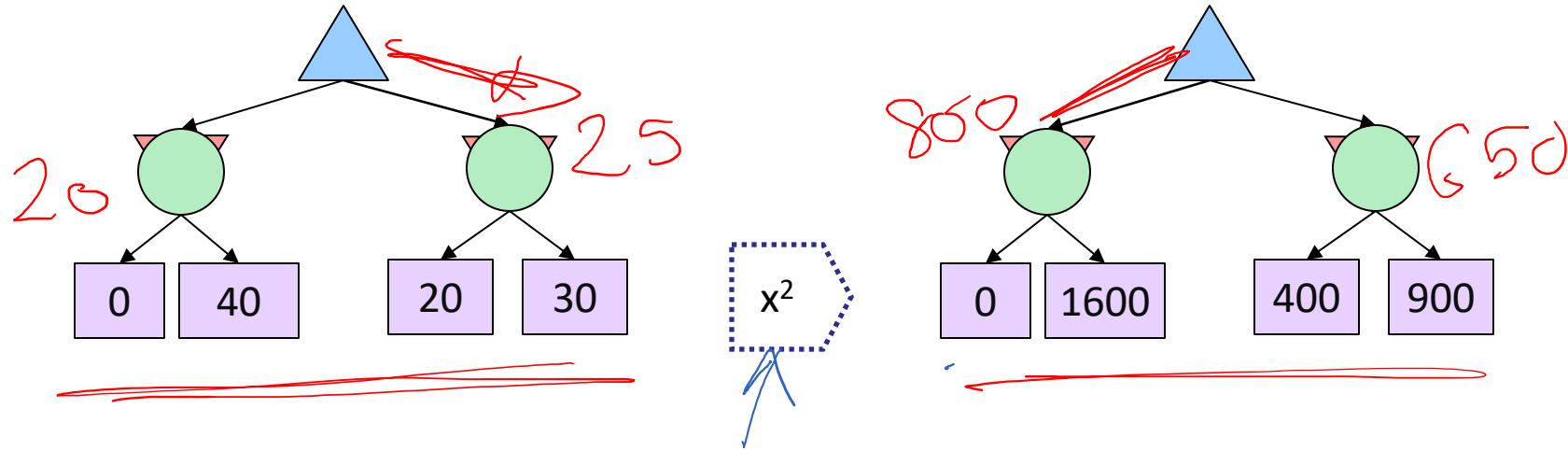
- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
 - In a game, may be simple (+1 / -1)
 - Utilities summarize the agent's goals
- We hard-wire utilities and let behaviors emerge
 - Why don't we let agents pick utilities?
 - Why don't we prescribe behaviors?



Utilities: Uncertain Outcomes



What Utilities to Use?



- For worst-case minimax reasoning, terminal function scale doesn't matter
 - We just want better states to have higher evaluations (get the ordering right)
 - We call this **insensitivity to monotonic transformations**
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

Next : MDPs!
