Problem 1

Let X and Y be i.i.d. $\mathsf{Expo}(\lambda)$, and $T = \log(X/Y)$. Find the CDF and PDF of T.

Problem 2

Let X and Y be i.i.d. $\mathsf{Expo}(\lambda)$, and transform them to $T = X + Y$, $W = X/Y$.

(a) Find the joint PDF of T and W. Are they independent?

(b) Find the marginal PDFs of T and W.

Problem 3

Let $U \sim \mathsf{Unif}(0, 1)$ and $X \sim \mathsf{Expo}(\lambda)$, independently. Find the PDF of $U + X$.

Problem 4

Let X and Y be i.i.d. $\mathsf{Expo}(\lambda)$. Use a convolution integral to show that the PDF of $L = X - Y$ is

$$f_L(l) = \frac{\lambda}{2} e^{-\lambda|l|},$$

for all real l.

Problem 5

Use a convolution integral to show that if $X \sim \mathsf{N}(\mu_1, \sigma^2)$ and $Y \sim \mathsf{N}(\mu_2, \sigma^2)$ are independent, then

$$T = X + Y \sim \mathsf{N}(\mu_1 + \mu_2, 2\sigma^2).$$

You can use a standardization (location-scale) idea to reduce to the standard Normal case before setting up the integral. Hint: complete the square.

Problem 6

Let W_1 and W_2 be two random variables with the joint distribution:

$$\mathsf{P}(W_1 \le w_1, W_2 \le w_2) = \int_{-\infty}^{w_1} \int_{-\infty}^{w_2} \frac{1}{2\pi} \exp\left[-\frac{1}{2}(x^2 + y^2)\right] \mathrm{d}x\, \mathrm{d}y.$$

Consider two other random variables $Z_1 = |W_1|$ and $Z_2 = |W_2|$. In words, Z_1 is the absolute value of W_1, Z_2 is the absolute value of W_2.

(a) Show that Z_1 is independent of Z_2.

(b) Show that Z_1 and Z_2 have the same distribution, and find that distribution.