Additional notes

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Please complete the following:

1. chapter 3: Expectation

   (a) **Proposition (Monotonicity of expectation).** Let X and Y be random variables such that X $\geq$ Y with probability 1. Then E(X)$\geq$E(Y), with equality holding if and only if X = Y with probability 1.

   (b) A hypergeometric can be considered a sum of bernoulli random variables, but with their probabilities conditioned on the previous iteration. However, when calculating hte expectation, each of the bernoullis are equally like to be picked first, so the expecation is n * p where p = w/w+b

   (c) **Theorem. Properties of Indicator Random Variables**

   1. $I_A^k = I_A$
   2. $I_{A^c} = 1 - I_A$
   3. $I_{A \cap B} = I_A I_B$
   4. $I_{A \Cup B} - I_A + I_B - I_{A \cap B}$

   (d) Inclusion Exclusion

   $P(A_1 \cup A_n) = \sum_i P(A_i) - \sum_{j>i} P(A_i \cap A_j) + ...(-1)^n P(A_1 ... A_n)$

2. Joint distributions

   (a) **Using 2d lotus to get Expected Value**

   $X, Y \sim N(0,1)$ find $E[|X-Y|] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x-y| \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dxdy$

   we know $X - Y \sim N(0,2) => X - Y = \sqrt{2}Z$ with $Z \sim N(0,1) => E[|X-Y|] = \sqrt{2}E[|Z|] = \sqrt{2}\int_{-\infty}^{\infty} |z| e^{-z^2/2} dz$

   $= 2\sqrt{2}\int_0^{\infty} z e^{-z^2/2} dz = \frac{2}{\sqrt{\pi}}$

   (b) **Properties of Covariance**// 4. Cov(aX,Y) = aCov(X,Y) for any constant a in R

   5. Cov(X + Y,Z) = Cov(X,Z) + Cov(Y,Z).

   6. Cov(X + Y,Z + W) = Cov(X,Z) + Cov(X,W) + Cov(Y,Z) + Cov(Y,W).

   7. Var(X + Y) = Var(X) + Var(Y) + 2Cov(X,Y).

   8. For any X1 X2 ... Xn $Var(X1 + X2 + ..Xn) = Var(X1) + ...Var(Xn) + 2\sum_{i<j} Cov(X_i, X_j)$

Have a great Holiday Season!