Ukrainian Catholic University

Faculty of Applied Sciences

Computer Science Programme

Simultaneous Localization and Mapping

Linear Algebra final project report

Authors:
Bohdan Hlovatskyi, Mykhailo Pasichnyk, Stefan-Yuriy Malyk

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Abstract

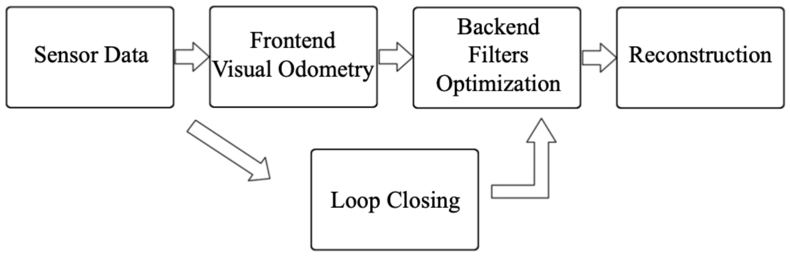
SLAM or Simultaneous Localization and Mapping - problem of constructing a map of unknown environment simultaneously tracking the object's location. Such algorithms utilize the data obtained from sensors, such as video camera, lidar etc to construct a 3-D model of the nearby objects. They are used in variety of fields: autonomous driving and robots, augmented reality and much more. The algorithms that rely on video data are referred to as Visual SLAM. They obtain the valuable information by means of Computer Vision and different techniques of Signal Processing. We implemented a variation of such algorithm - Monocular SLAM. It tracks position of object basing on input video stream from a single camera installed on the object. Source code: [1].

1 Introduction

Visual SLAM is a family of algorithms that solve the problem of Simultaneous Localization and Mapping by means of Computer Vision. Depending on the type of input video stream there are different types of visual SLAM:

- Monocular SLAM. Such SLAM takes as an input video stream from a single RGB camera. This is the simplest type of SLAM, though it has a lot of limitations, the most important of which is the fact that we can construct environment map only to some scale factor. In other words, we cannot obtain the exact depth of featured points in an environment from single image source.
- Stereo SLAM. Input video stream comes from two cameras. This allows to get the exact depth of the featured points.
- RGBD. Specific kind of SLAM that gets the input data from RGBD camera - the one that included depth of objects within an image. There are different technologies for such cameras though they are not particularly important for the algorithm itself.

The pipeline of SLAM algorithms will usually look in the following way:



In this work we are mainly interested in Frontend Visual Odometry, goal of which is to track the object that produces the source data and build non-optimized 3-D cloud map of environment. Backend optimization and loop closing are powerful techniques to optimize the SLAM and get rid of the noise that brings a lot of inaccuracy in the SLAM algorithm. Those algorithm, such as Kalman and Particle filters, are beyond the scope of Computer vision and are related mainly to signal processing, thus won't be focused on here.

2 Problem setting

2.1 Problem formulation

With the given setting, we are interested mainly in two parts:

1. Modeling the object's motion. In an abstract setting this can be modeled as $x_k = f(x_{k-1}, u_k, w_k)$, where x_k denotes a current position of an object, x_{k-1} - its previous position, u_k - change of its state and w_k - noise.

2. Collecting the landmarks of a nearby environment. The observed data $z_{k,j}$ will then be described by this equation: $z_{k,j} = f(y_j, x_k, v_{k,j})$. Here y_j - landmark point, x_k - its coordinates, $v_{k,j}$ - noise of the observation.

These two equation summarize the SLAM. For the Euclidean space the motion model can be expressed as the so-called special Euclidean Group:

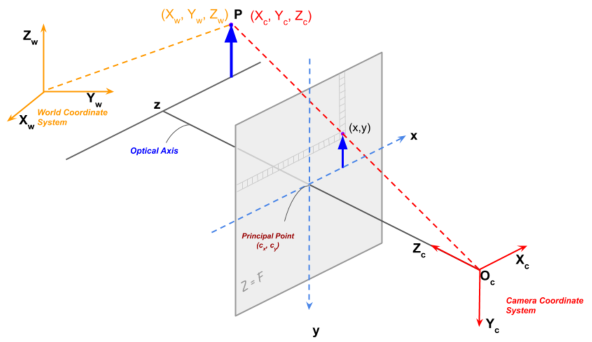
$$SE(3) = \{T = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \in \mathbf{R}^{4x4}\},$$

where R - rotation matrix, t - translation vector. It allows to express state a_1 as

$$a_1 = Ra_0 + t.$$

Important note here is that we associate the state of an object with the location of camera that produces some input data.

2.2 Camera model



Any point in space can be represented by three coordinate systems:

- The World Coordinate System - point $\mathbf{P} = (X_w, Y_w, Z_w)$. For example if our object is in a room we can threat floor as $x - y$ axis and a wall as z axis

- The Camera Coordinate System - (X_c, Y_c, Z_c) the same object in 3D space but from camera, where X_c points to the right, Y_w down and Z_w towards the camera

- The point projection to the plane (image itself)

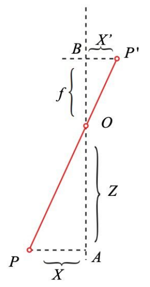
The world and camera coordinate systems are related by the rotation matrix $\mathbf{R}$ and translation vector $\mathbf{t}$. Thus

$$\begin{bmatrix} X_c \\ Y_c \\ Y_c \end{bmatrix} = \mathbf{R} \begin{bmatrix} X_w \\ Y_w \\ Y_w \end{bmatrix} + \mathbf{t}$$

We can rewrite statement above to make it more compact and call such $\begin{bmatrix}\mathbf{R}|\mathbf{t}\end{bmatrix}$ matrix the **Extrinsic matrix**

$$\begin{bmatrix} X_c \\ Y_c \\ Y_c \end{bmatrix} = \begin{bmatrix}\mathbf{R}|\mathbf{t}\end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Now that we have our point $\mathbf{P}$ in the camera coordinate system we can project it onto the plane.



Our projected point (x, y) can be found using following equations:

$$x = f\frac{X_c}{Z_c}, y = f\frac{Y_c}{Z_c}$$

where f is the focal length of the camera. One important thing to mention is that the image $\mathbf{P}$ going though camera's optical center would be inverted. However, since usage of such camera would be inconvenient camera's software performs some transformation. Thus, this step could be omitted and we can stick to the equations above. Next we construct such a matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \end{bmatrix} \begin{bmatrix} Y_c \\ X_c \\ Z_c \end{bmatrix}$$

It would be referred as the **Intrinsic Matrix**. Another important observation to make is that the optical center of the camera could be shifted from the origin of the camera coordinate system. Thus we need to modify the Intrinsic matrix as such:

$$\mathbf{K} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & f \end{bmatrix}$$

Combining all of the above we can get coordinates of the object on our image (u, v):

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & f \end{bmatrix} \begin{bmatrix} Y_c \\ X_c \\ Z_c \end{bmatrix}$$

where

$$u = \frac{u'}{w'}, v = \frac{u'}{w'}$$

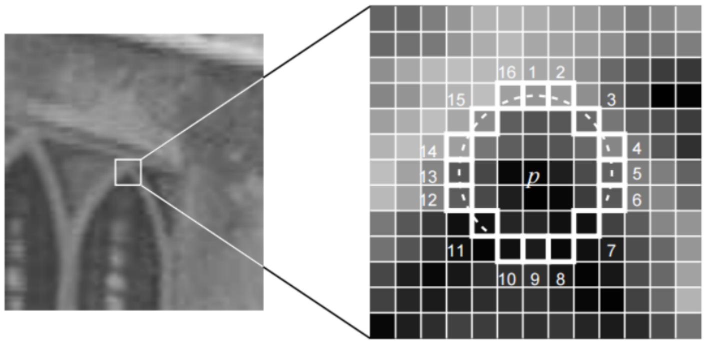
2.3 Feature detection

Firstly let define what a feature is. It is a some pattern in the image which has several proprieties:

1. **Repeatability**: The same feature can be found in different images.
2. **Distinctiveness**: Different features have different expressions.
3. **Efficiency**: In the same image, the number of feature points should be far smaller than the number of pixels.
4. **Locality**: The feature is only related to a small image area.

Historically there were 3 most popular approaches in algorithms of feature detection:

1. **SIFT** (Scale-Invariant Feature Transform) : is one of the most classic. It is fully consider the changes in illumination, scale, and rotation during the image transformation, but is so calculating expensive, so rarely used in SLAM.
2. **FAST** : is extremely fast to compute. Has following algorithm:
   (a) Select pixel p in the image, assuming its brightness as I_p
   (b) Set a threshold T (for example, 20% of I_p).
   (c) Take the pixel p as the center, and select the 16 pixels on a circle with a radius of 3.
   (d) If there are consecutive N points on the selected circle whose brightness is greater than $I_p + T$ or less than $I_p - T$, then the central pixel p can be considered a feature point. N usually takes 12, which is FAST-12. Also N could be 9 / 11 / etc, with respective names of FAST.
   (e) Iterate through the above four steps on each pixel.



[Fig 1] : Illustration of of FAST algorithm

This algorithm has an problem, that the detector does not have descriptors (fixed pattern, which try to find). Also, as it has fixed the radius of the circle = 3, there is a scaling problem. This two problems fix ORB algorithm.

3. **OBR** (Oriented FAST and Rotated BRIEF) : it solves both mentioned problem of FAST, by adding the description of scale and rotation. In addition, it uses the extremely fast binary descriptor BRIEF.

   (a) The scale invariance is achieved by the image pyramid and detect corner points on each layer

   (b) In terms of rotation, we calculate the gray centroid of the image near the feature point. The so-called centroid refers to the gray value of the image block as the center of weight. The specific steps are as follows:

      i. In a small image block B, define the moment of the image block as:

$$m_{pq} = \sum_{x,y \in B} x^p y^q I(x,y), p,q = \{0,1\}$$

      ii. Calculate the centroid of the image block by the moment:

$$C = (\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}})$$

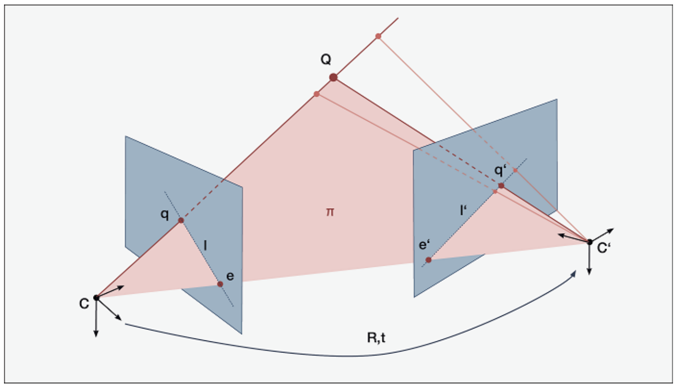
      iii. Connect the geometric center O and the centroid C of the image block to get a direction vector $\overrightarrow{OC}$, so the direction of the feature point can be defined as: $\theta = arctan(\frac{m_{01}}{m_{10}})$.

   (c) After extracting the Oriented FAST key points, we calculate the descriptor for each point. ORB uses an improved BRIEF feature description. BRIEF is a binary descriptor. Its description vector consists of many zeros and ones, which encode the size relationship between two random pixels near the key point (such as p and q): If p is greater than q, then take 1, otherwise take 0. If we take 128 such p, q pairs, we will finally get a 128-dimensional vector consisting of 0s and 1s. The BRIEF implements the comparison of randomly selected points, which is very fast. Since it expresses in binary, it is also very convenient to store and suitable for real-time image matching. The original BRIEF descriptor does not have rotation invariance, so it is easy to get lost when the image is rotated. The ORB calculates the direction of the key points in the FAST feature point extraction stage. The direction information can be used to calculate the Steer BRIEF feature after the rotation so that the ORB descriptor has better rotation invariance.

Due to the consideration of rotation and scaling, the ORB performs well under translation, rotation, and scaling. Meanwhile, the combination of FAST and BRIEF is very efficient, which makes ORB features very popular in real-time SLAM, so in our project we will use this approach.

2.4 Epipolar geometry

The object motion can be found from the correspondence between two consecutive video frames.



According to the camera model, each of the landmarks Q will be projected onto 2-D plane of pixels as q, q'. Motion, described by the rotation matrix R and transformation vector t can be reproduced than via Epipolar constraint.

Pixel points q, q' are described by the following equations:

$$\begin{cases} s_1 p_1 = KP \\ s_2 p_2 = K(RP + t) \end{cases} . \tag{1}$$

As was discussed in the setting, due to the limitations of the approach, we can represent the points only to some constant factor. Moreover, it is worth mentioning that p_1, p_2 will be the points matched by the feature matcher.

To find the coordinates on the normalized plane of two pixels, we can simply take the inverse of the matrix representing a camera: $x_{1,2} = K^{-1} p_{1,2}$. Then the equation above can be rewritten as $x_2 = Rx_1 + t$. This can be further transformed into

$$p_2^T K^{-T} t \times R K^{-1} p_1 = 0.$$

This equation is referred to as epipolar constraint. For the sake of simplicity, the notions of Fundamental and Essential matrices are introduced:

$$\begin{cases} E = t \times R \\ F = K^{-T} t \times R K^{-1} \\ x_2^T E x_1 = 0 \\ p_2^T F p_1 = 0 \end{cases} . \tag{2}$$

It is clear that usage of Essential matrix will make the calculations more accurate, as it uses precomputed calibration parameters for the camera. The process of camera pose estimation can then be summarized as:

1. Find E or F from the known pixel positions.

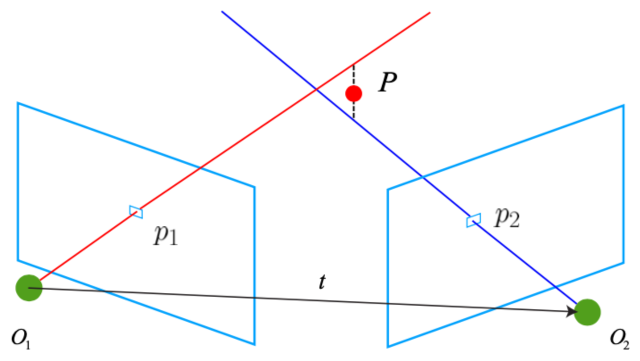
2. Find R, t based on those matrices.

In practice, the Essential matrix is found via Eight-Point algorithm and then R, t are found via SVD and triangulation to get the result that would satisfy the constraints given by the known projections of landmarks.

$$\begin{cases} R = U R_{Z(\pm pi/2)} V^T \\ t = U R_{Z(\pm pi/2)} \Sigma U^T \end{cases} . \tag{3}$$

2.5 Triangulation

The second goal of SLAM is to estimate the relative positions of the feature points. Knowing the relative position of the camera and the relative positions of those featuer points, we can reconstruct their global coordinates and construct the feature map of the environment.



From the epipolar constraint: $s_2x_2 = s_1Rx_1 + t$. Here $x_{1,2}$ - normalised coordinates (coordinates in the pixels' plane) of two feature points. Knowing R, t, we can now calculate $s_{1,2}$ that will represent the 'depth' of a pixel.

$$s_2x_2 \times x_2 = 0 = s_1x_2 \times Rx_1 + x_2 \times t, \tag{4}$$

which can be solved for s_1. Geometrically, we seek to find a 3D point on the ray O_1p_1 to make its projection close to p_2.

3 Overview of approaches

As was mentioned earlier, there exists different types of SLAM algorithms that depend on different type of input video stream. Though as we are focusing particularly on visual SLAM in this work, we would consider different approached to SLAM problem from the perspective of Computer Vision:

- The most popular implementation of visual SLAM is OpenVSLAM available open-source. It utilises notions of gloval and local key frame maps. For the local map it finds points via triangulation and for the global one uses pose-graph optimization. It also contain a global optimization module, which optimizes mathcing against global map. [7]

- Interesting to mention is the approach proposed by the ORB-SLAM. The authors use beg-of-words representation of encoded descriptors for each of the feature points in order to optimise the process of feature matching against the stored feature map. It has also a logical continuation as ORB-SLAM 2, which introduced ability to work with stereo input data. [3]

- Speaking of the camera calibration, the classical approach to it is using a chess board' photos from different perspective in order to estimate the intrinsic parameters. [4]. There are also some research conducted in the field of camera self-calibration. [5].

- It is worth mentioning that the research in the SLAM field is being actively maintained. The good example of it is creation of general frameworks such as expressed in the cited papeer. [8].

4 Implementation

The algorithm was implemented in Python mainly via pyopencv. The visualizer of the SLAM was written via OpegGL. For the feature extraction the Harris corner detector and BRIEF corner descriptor were used due to its computational efficiency. Feature matching was done in KNN manner because limitation on the number of features was only several hundreds. Because on the validation and test videos we had camera intrinsics, we used Essential matrix to recover rotation and transformation between two frames. The last step was to conduct triangulation so to get the 3-D coordinates of feature points. Demonstration of the algorithm can be found on Github or YouTube.

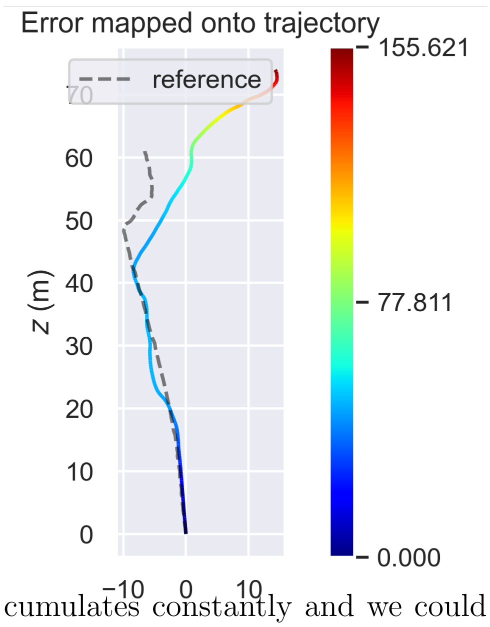
```
**Data:** stream of video frames
$init()$ /* make all the necessary initialization of visualizer and SLAM engine */ ;
$prev \leftarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$;
$currentFrame \leftarrow None$;
$prevKeyPoints \leftarrow None$;
while $true$ do
    $cur \leftarrow getFrame()$;
    $keyPoints \leftarrow getFeatures(currentFrame)$;
    if $prevKeyPoints$ $is$ $None$ then
        $prevKeyPoints \leftarrow keyPoints$;
        continue;
    else
        $matches \leftarrow getMatches(keyPoints, prevKeyPoints)$;
        $E \leftarrow recoverEssentialMatrix(method = RANSAC, matches)$;
        $R, t \leftarrow recoverPose(E)$;
        $points \leftarrow trinagulate(matches, prevFrame, cur)$;
        /* store all the necessary info */;
        $poses.append([R, t])$;
        $pointMap.extend(points)$;
        ;
        /* update all the values */;
        $tmp \leftarrow cur$;
        $cur \leftarrow prev@cur$;
        $prev \leftarrow tmp$;
        $prevKeyPoints \leftarrow keyPoints$;
    end
end
```

Algorithm 1: General algorithm of SLAM

5 Experiments

Experimentation was conducted on publicly available Kitty dataset [9]. It was collected via a standard station wagon with two high-resolution color and grayscale video cameras and is widely used in testing of different SLAM approaches. Accurate ground truth is provided by a Velodyne laser scanner and a GPS localization system. As we are working only with monocular SLAM we took only the video from the first camera to conduct testing. Moreover, the dataset also provided calibration data for cameras, which took out the need of researching of obraining camera calibration from a given video source, which is completely another topic.

Evaluation was conducted on a small subset of the provided dataset, due to lack of computational resources, to be more specific - **2011-09-26-drive-0095'**. To measure metrics, we used open-source librbary for SLAM researchers - **evo** [10]. It contains tools to measure absolute and relative errors, whether matrices that describe poses conform to SE(3) restriction and many more.



We mainly focused no **RPE** - relative pose error, which allows to investigate the local consistency of a SLAM trajectory, which is the most interesting metric for us as out SLAM is not stable on long distances because due to lack of backend optimizers, which result in a huge absolute error.

RPE compares poses along estimated and reference trajectory. It bases on delta pose difference [11]:

$$E_{i,j} = (P_{ref,i}^{-1} * P_{ref,j})^{-1} * (P_{est,i}^{-1} * P_{est,j})^{-1} \in SE(3); \tag{5}$$

$$RPE_{i,j} = ||E_{i,j} - I_{4x4}||_F \tag{6}$$

It is easy to notice the drift that the system suffers from - the reason for that is that the error accumulates constantly and we could not get rid of it without some non-linear optimizer, which was not in the focus of this project.

To see the drift ever more clearly, we would decompose the position into set of (x, y) coordinates:

Figure 1: Estimated y coordinate (in meters) against ground truth over iterations.

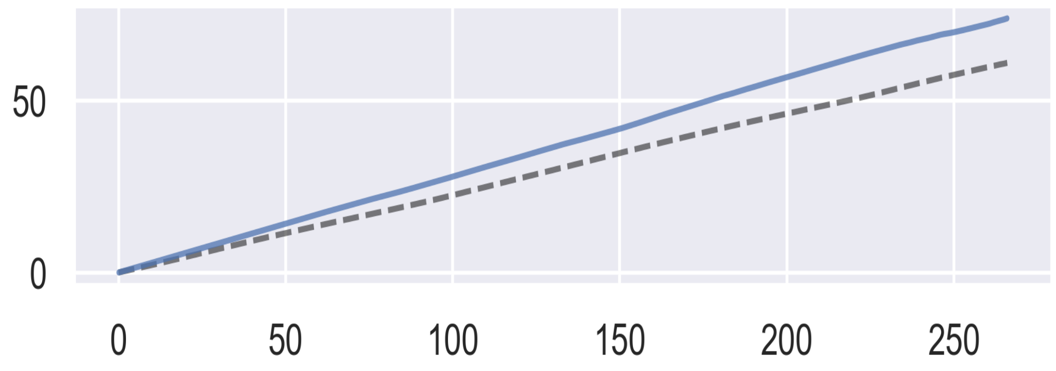
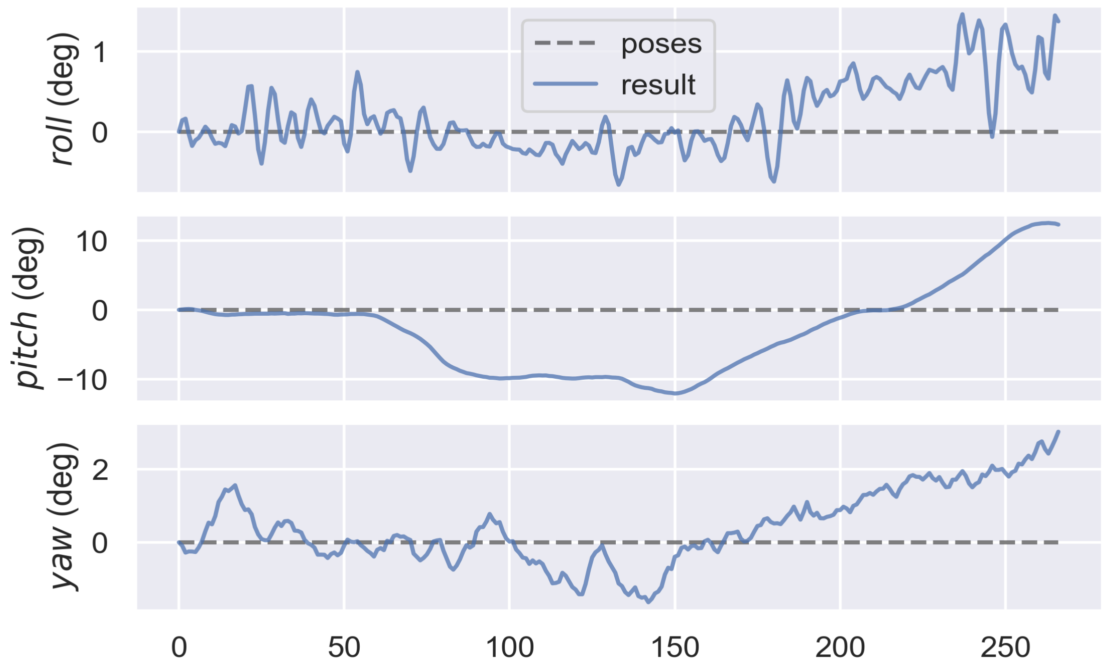
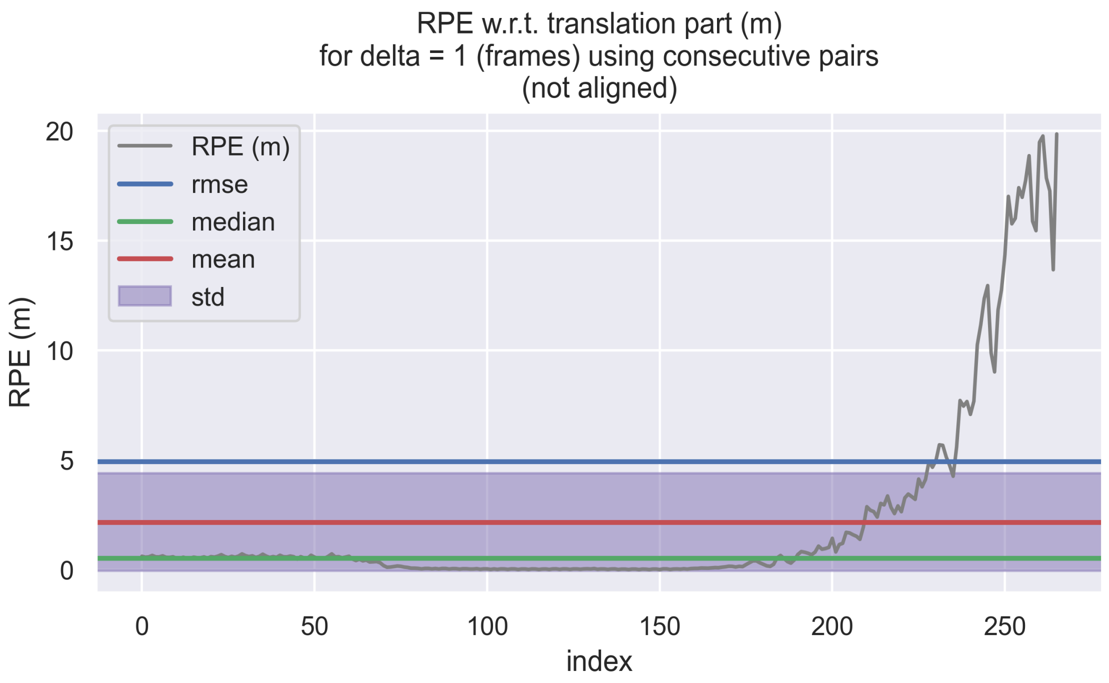


Figure 2: Relative error of estimated rotations.





6 Limitations

Monocular SLAM is the simplest SLAM possibles, thus it possesses many limitations. The most prominent from them are the following:

1. Main limitation of monocular SLAM is the fact that it can determine translation only to some constant factor, as while we projecting world onto image plane we lose the information about depth of objects that we see. To overcome this problem, one may use different type of camera - RGBD (measures depth as well) or different type of algorithm - Stereo SLAM - the one that takes images simultaneously captured from two or more cameras.

2. Triangulation is only possible if the enough amount of translation is present. If the amount of translation is small, pixel's change would result in big depth uncertainty. On the other hand, if translation is too big, the algorithm may fail to match the features. This problem will be especially seen in cases of so-called pure rotation, when there is no translation. Though in not so extreme cases, there are means to make this problem less visible - delayed triangulation etc.

3. Assumption about static sight of view brings a lot of limitations as well. For instance, the algorithm cannot be applied to autopilots as the moving object would make it think that the camera has moved.

7 Conclusions

In this work we developed a Monocular SLAM engine which allows us to track object given input video source from a single camera. It gives adequate results, speaking in terms of relative pose error, in short terms, though suffers from error accumulation over time. Basing on reviewed literature, back end optimizers, such as Extended Kalman Filter or Bundle Adjustement, which utilizes non-linear optimizers to make the error of projections of discovered features points in world coordinates as small as possible, could greatly improve the overall quality of SLAM and get rid of those accumulated errors. Sadly, this part if more connected with signal processing and non-linear optimization rather than with Linear Algebra thus we did not focus on that in this work. Speaking about the further work, we would propose to implement one of those optimizers to make SLAM more accurate. We also see this project as a great foundation for further research for us - combined with deep learning, 3-D map and poses obtained by SLAM could greatly influence the field of reinforcement learning for robots.

References

[1] https://github.com/bohdanhlovatskyi/OhISee

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[3] http://webdiis.unizar.es/ raulmur/MurMontielTardosTRO15.pdf

[4] https://www.microsoft.com/en-us/research/wp-content/uploads/2016/02/tr98-71.pdf

[5] https://users.aalto.fi/ kannalj1/publications/wacv2016.pdf

[6] https://github.com/gaoxiang12/slambook-en

[7] https://arxiv.org/pdf/1910.01122.pdf

[8] https://arxiv.org/pdf/1902.07995.pdf

[9] http://www.cvlibs.net/datasets/kitti/

[10] https://github.com/MichaelGrupp/evo/

[11] http://www2.informatik.uni-freiburg.de/ stachnis/pdf/kuemmerle09auro.pdf