5. Increasing Electromagnet Force

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Contents

1 Abstract

Since the concept of the electromagnet is present in both motors and generators we find it interesting to study the relationship between the electrical power the coil draws and the mechanical force with which the coil attracts a piece of ferromagnetic material or a magnet. The focus is usually on the

current and the number of turns but we include the voltage needed to deliver the current and then calculate the force to power ratio.

2 Clues

Go to any site with detailed information on electromagnets of various sizes, like Highcap[4], and collect the data, normalize it and calculate F/P:

d mm	**h** mm	**F** N	**P** W	**F/P**	**A** mm^2	**F/PA**
20	15	20	3	6.5	300	0.022
25	20	49	4	12.3	500	0.025
30	22	98	4	24.5	660	0.037
34	18	177	4	44.1	612	0.072
40	20	196	8	24.5	800	0.031
49	21	392	10	39.2	1029	0.038
60	34	686	13	52.8	2040	0.026
65	30	785	13	60.3	1950	0.031

As you can see we have also calculated A as a kind of cross-section area $A = dh$ and F/PA which seems as if it could be a constant. We plot F/P as a function of A:

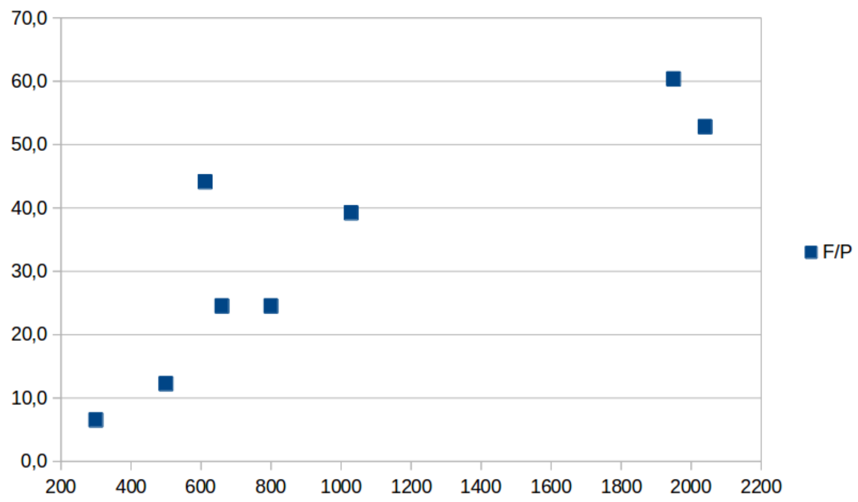


Figure 1: F/P(A)

The only outstanding data-point in this collection seems to be the fourth electromagnet.

3 Formula

Searching for a formula on this relationship we found[3, 1]:

$$F = \frac{B^2 A}{2\,\mu_0} \tag{1}$$

which can be rewritten using:

$$B = \frac{\mu N I}{l} \tag{2}$$

where l is the effective length of the magnetic flux path, as[3]:

$$F = \frac{\mu^2\,A}{2\,\mu_0}\,\frac{N_1 I_1\,N_2 I_2}{g^2} \tag{3}$$

where l now is g, the air-gap. It resembles Coulomb's law and Newton's law of gravitation. All three are inverse-square laws, where force is inversely proportional to the square of the distance.

Let us introduce A_F to make it easier to read:

$$A_F = \frac{\mu^2 A}{2\,\mu_0\,g^2} \tag{4}$$

and we get:

$$F = A_F\,N^2\,I^2 \tag{5}$$

Let us introduce a new concept; the relationship between the DC resistance and the number of turns in an inductor:

$$R = A_R\,N^2 \tag{6}$$

where A_R is the DC resistance factor of the inductor. Roughly we have:

$$A_R = 11\rho\,\frac{r}{d\,l} \tag{7}$$

The definition of A_R makes it possible to rewrite the formula:

$$F = \frac{A_F}{A_R}\,R\,I^2 \tag{8a}$$

$$= \frac{A_F}{A_R}\,P \tag{8b}$$

$$\frac{F}{P} = \frac{A_F}{A_R} \tag{8c}$$

It seems that the F/P ratio for an electromagnet is not a function of the number of turns, but apart from core-related parameters rather the amount of copper-wire; the more wire-mass, the bigger the form, the smaller the A_R the stronger force per input-power. We also see that the force is proportional to the power, when we increase the power the force increase.

4 Experiment

Eight coils a-h have been tested, five on a small form and three on a big form. They all had welding rods as their cores.

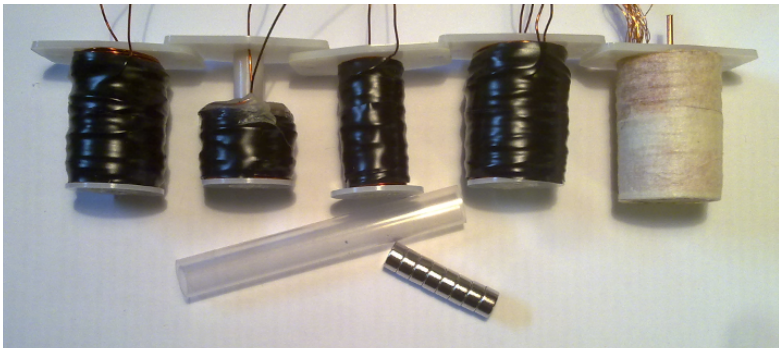


Figure 2: Coils a-e

Figure 3: Coils f-h

The testing was conducted using a small plastic tube and a stack of small magnets that could easily slide in the tube. The coil to be tested was placed on a table with its core in a vertical position, the tube with the magnets was placed on top of the core so the magnets would be lifted once the current started to flow. Voltage and current were measured when the magnets were lifted to the same height as for coil c. The current chosen for coil c was related to the limits of the power-supply. Both voltage and current were measured with meters external to the power-supply. [TODO: this needs to be redone]

coil	comment	r	d	l	ϱ	A_R	A	m_{tot}	m_{Cu}	W_d	N	R	A_R	L	A_L	V	I	P	N^2I^2A	Pm
	air core, no wire	8.4	8.7	34.5	1	5.2		4.2												
	welding rods core, no wire	8.4	8.7	34.5	1	5.2	30	11.0												
a	1 x 1, full	8.4	8.7	34.5	1	5.2	30	94.4	83.4	0.6	693	2.4	5.1	8.13	16.93	3.8	1.4	5.35	29.4	446
b	1 x 1, half length	8.4	8.7	17.3	1	10.4	30	54.9	43.8	0.6	350	1.2	10.1	2.57	20.98	3.3	2.1	7.01	16.5	307
c	1 x 1, half depth	6.2	4.3	34.5	1	7.7	30	44.3	33.3	0.6	350	1.0	8.5	2.04	16.65	4.0	2.9	11.67	31.8	388
d	1 x 2, (in series)	8.4	8.7	34.5	1	5.2	30	99.7	88.6	0.6	700	2.5	5.2	8.25	16.84	3.9	1.4	5.50	29.4	487
d	1 x 1, (half wires)	8.4	8.7	34.5	0.5	10.4	30	99.7	44.3	0.6	350	1.3	10.4	2.06	16.82	4.3	2.8	11.84	28.2	525
d	2 x 1, (in parallel)	8.4	8.7	34.5	1	5.2	30	99.7	88.6	0.6	350	0.7	6.0	2.07	16.90	2.6	2.8	7.49	29.9	664
e	1 x 1, full	8.4	8.7	34.5	1	5.2	30	107.3	96.3	0.2	6000	177.0	4.9	672.00	18.67	26.3	0.1	3.89	23.8	375
	air core, no wire	28.7	32.5	77.6	1	2.1		39												
	welding rods core, no wire	28.7	32.5	77.6	1	2.1	302	241												
f	4 x 1, not full	24.5	24.4	77.6	1	2.4	302	1438	1197	1	300	0.3	2.8	4.07	45.22	0.8	1.7	1.38	78.7	1650
g	5 x (3*2), overfull	29.2	33.7	77.6	1	2.1	302	2576	2335	0.6	1224	2.7	1.8	68.89	45.98	1.2	0.4	0.50	77.1	1171
g	5 x 3, overfull	29.2	33.7	77.6	0.5	4.1	302	2576	1168	0.6	612	1.5	4.1	17.14	45.77	1.2	0.8	0.98	70.3	1141
g	(2*5) x 3, overfull	29.2	33.7	77.6	1	2.1	302	2576	2335	0.6	612	0.7	2.0	17.15	45.79	0.8	0.8	0.62	74.3	1458
h	1 x 1, almost full	26.7	28.4	77.6	1	2.2	302	2322	2081	0.5	7200	92.6	1.8	2760.00	53.24	7.1	0.1	0.52	83.4	1082

header	unit	definition
coil		label
comment		p x s number of strands in parallel times number of strands in series
r	mm	mean radius for the windings
d	mm	depth for the windings
l	mm	core-length
ϱ		windings density, how much used, times 8/11
A_R	$\mu\Omega$	calculated from dimensions using formula
A	mm^2	core area
m_{tot}	g	mass
m_{Cu}	g	copper mass
W_d	mm	wire diameter
N		number of turns
R	Ω	resistance
A_R	$\mu\Omega$	calculated from resistance
L	mH	inductance
A_L	nH	calculated from inductance
V	V	voltage applied
I	A	current drawn
P	W	power drawn
N^2I^2A	A^2mm^2	part of the original expression for F
Pm	Wg	power times mass

The reading of the magnet lift, our force-measurement device, was not very exact.

The pole-area of the magnets was $50mm^2$, very well suited for coils a-e in the first set, but coil f-h in the second set has a much greater core-area than the magnets pole-area which makes an exact comparison between the two sets of coils difficult. The big coils in the experiment are producing a field of which most just pass the magnet by without affecting it.

During the experiment the coils got warm and the resistance changed.

We hoped for a smaller spread of the N^2I^2A values within each of the two coil-sets.

Coils that stick out are; d with a lower P, e with a lower P, f with a lot higher P and h with a lower P than expected.

Despite the spread in the results the difference between the power used in the two coil-sets is very clear; although the coil-core area A in the second set is much larger than the pole-area of the magnets, the second set with the bigger coils need less power to produce the same magnet-lift.

Watch a short, hasty and inexact version of the experiment with coil a,b and e at `https://www.youtube.com/watch?v=IIeRR6NjMPQ`.

5 Force to Power Ratio over Time

The above theories and experiment are dealing with the steady-state of the coil where the current is steady at its maximum and can be determined from the applied DC voltage and the internal DC resistance in the coil as $I = V/R$. This state is after 5τ, where τ is the time-constant of the coil defined as:

$$\tau = \frac{L}{R} = \frac{A_L N^2}{A_R N^2} = \frac{A_L}{A_R} \tag{9}$$

τ, measured in seconds, is constant for a given core and form regardless of the choice of wire gauge as long as the form is equally filled with wire. This means that when a bigger coil is used to increase the force to power ratio, which usually means that A_L increase and A_R decrease, τ increase.

Before 5τ the current is defined by the differential equation:

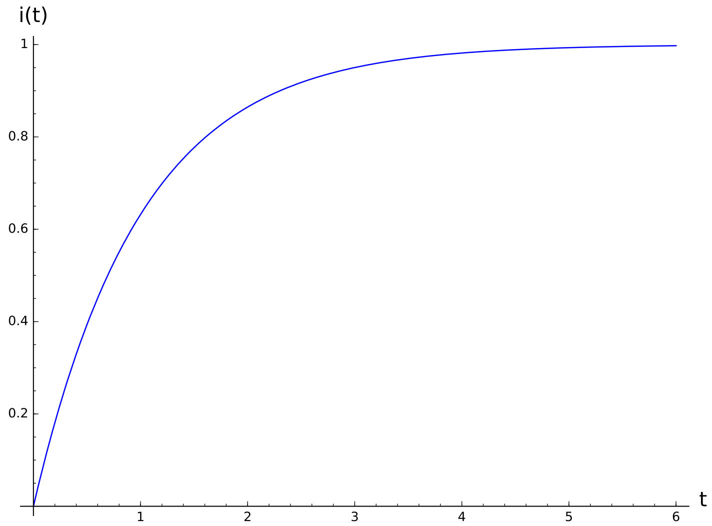
$$Li' + Ri = V \tag{10}$$

The solution to this equation is:

$$i(t) = \frac{V}{R}(1 - e^{-\frac{Rt}{L}}) \tag{11a}$$
$$= \frac{V}{R}(1 - e^{-\frac{t}{\tau}}) \tag{11b}$$

The main component here is the exponential decay function[2] $N(t) = N_0 e^{-\lambda t}$, which by going to zero makes the current go to V/R.

We set $L = R = V = 1$ and plot $i(t)$:



As t approach 5τ, in this case $\tau = L/R = 1$, $i(t)$ approaches V, in this case $V = 1$. We have:

$$F(t) = A_F\, N^2\, i^2(t) \tag{12a}$$
$$P(t) = V\, i(t) \tag{12b}$$

and so the force to power ratio is proportional to $i(t)$, or rather to $1-e^{-x}$:

$$F2P(t) = \frac{F(t)}{P(t)} \tag{13a}$$
$$= \frac{A_F\, N^2\, i^2(t)}{V\, i(t)} \tag{13b}$$
$$= \frac{A_F N^2}{V}\, i(t) \tag{13c}$$
$$= \frac{A_F N^2}{V}\, \frac{V}{R}(1 - e^{-\frac{t}{\tau}}) \tag{13d}$$
$$= \frac{A_F}{A_R}(1 - e^{-\frac{t}{\tau}}) \tag{13e}$$

and maximum force to power ratio is after $t = 5\tau$.

We have assumed that g is constant and not a function of time. If the force from the electromagnet is to be of any use it has to accelerate some mass. In that case g change and the force to power ratio no longer follows $i(t)$. If the mass to be accelerated is big enough the acceleration will be small enough to be discarded. But if the coil is part of a motor the mass is moving all the time so g is constantly changing. One solution might be to put the rotor magnet inside the motor coil, preferably inside a motor coil that is huge as a barrel[6]: `https://youtu.be/XryGfWn3ALk?t=25m10s`

6 Force to Power Ratio over A_R

We will now compare force to power ratio between two coils with the same A_L at a fixed time in seconds as a function of A_R. Let the force to power ratio now be the function:

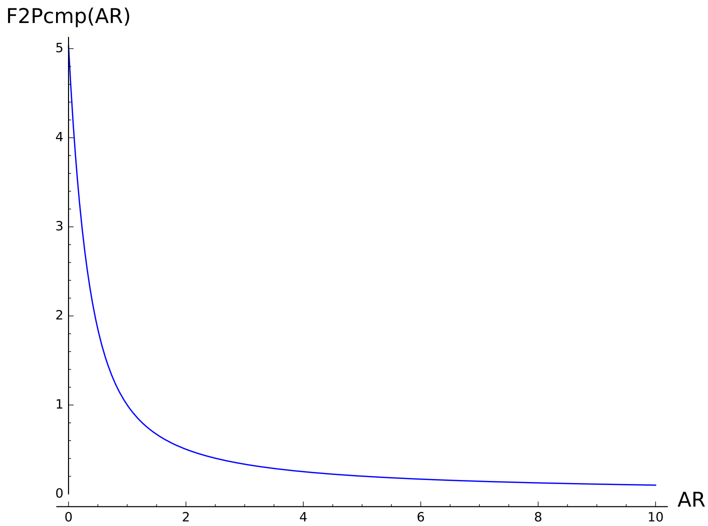
$$F2P(A_R, t) = \frac{A_F}{A_R}(1 - e^{-\frac{A_R t}{A_L}}) \tag{14}$$

Our reference coil will have $A_R = 1$ and we set $t = 5\tau = 5A_L/A_R$ for this coil so our comparison function is:

$$F2P_{cmp}(x) = \frac{F2P(x, 5A_L/1)}{F2P(1, 5A_L/1)} \tag{15a}$$

$$= \frac{e^5 - e^{5(1-x)}}{(e^5 - 1)x} \tag{15b}$$

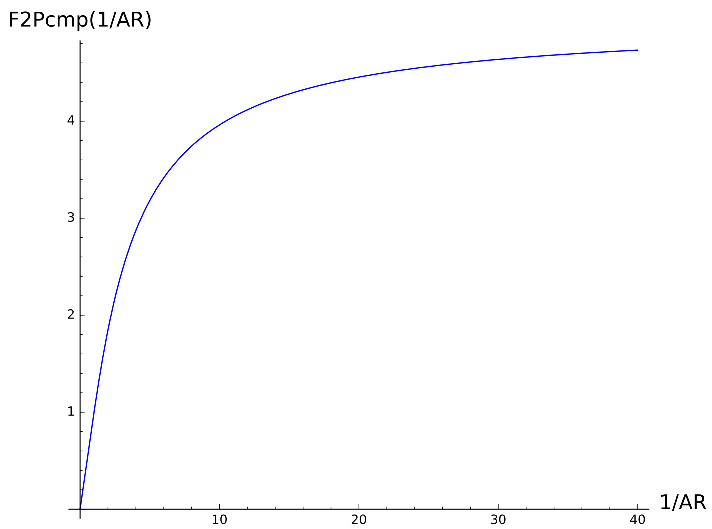
Let us plot $F2P_{cmp}(A_R)$:



It turns out that when time is fixed there is a maximum increase in force:

$$\lim_{x \to +0} F2P_{cmp}(x) = \frac{5e^5}{e^5 - 1} \approx 5.03 \tag{16}$$

Let us plot $F2P_{cmp}(1/A_R)$ to reflect the increase in copper instead of increase in resistance-factor:



For $1/A_R = 24$ the force is about 4.5 times greater and circa 90% of the maximum, and for $1/A_R = 11$ the force is about 4 times greater and circa 80% of the maximum. The time the coils are charged with respect to its own time-constant are $5/24\tau$ and $5/11\tau$ respectively.

7 Impulse to Energy Ratio over Time

The formulas for impulse and energy when force and power change over time are:

$$J = \int F(t)\mathrm{d}t \tag{17a}$$

$$E = \int P(t)\mathrm{d}t \tag{17b}$$

The impulse function over time:

$$\begin{aligned} J(t) &= \int_0^t F(x)\mathrm{d}x && (18a)\\ &= A_F\, N^2 \int_0^t i^2(x)\mathrm{d}x && (18b)\\ &= \frac{A_F\, N^2\, V^2}{R^3} \frac{2Rt - L(e^{-2\frac{Rt}{L}} - 4e^{-\frac{Rt}{L}} + 3)}{2} && (18c) \end{aligned}$$

If we set $L = A_L N^2$, $R = A_R N^2$, $P_{max} = V^2/R$ and $t = \tau x$:

$$J(x) = \frac{A_F A_L V^2}{A_R N^3} \frac{4e^{-x} - e^{-2x} + 2x - 3}{2} \tag{19a}$$
$$= \frac{A_F A_L V^2}{A_R^3 N^2} J_k(x) \tag{19b}$$
$$= \frac{A_F A_L V^2}{A_R A_R R} J_k(x) \tag{19c}$$
$$= \frac{A_F}{A_R} \tau P_{max} J_k(x) \tag{19d}$$

where:

$$J_k(x) = \frac{4e^{-x} - e^{-2x} + 2x - 3}{2} \tag{20}$$

The energy function over time:

$$E(t) = \int_0^t P(x)\mathrm{d}x \tag{21a}$$
$$= V \int_0^t i(x)\mathrm{d}x \tag{21b}$$
$$= \frac{V^2}{R^2}(Rt + L(e^{-\frac{Rt}{L}} - 1)) \tag{21c}$$

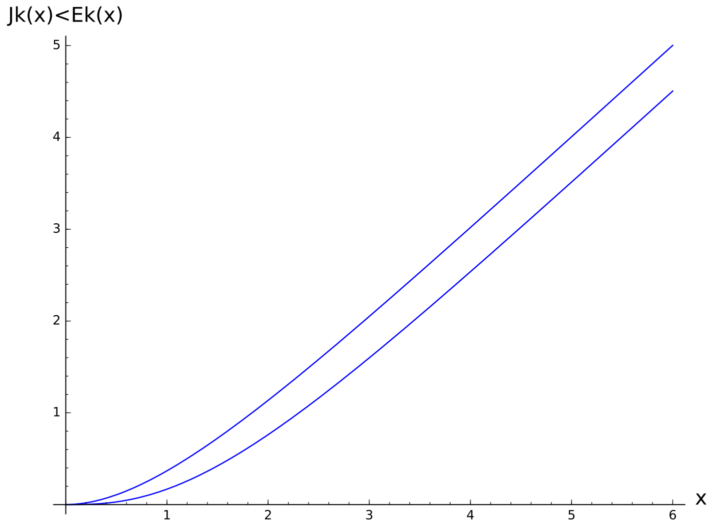
If we set $L = A_L N^2$, $R = A_R N^2$, $P_{max} = V^2/R$ and $t = \tau x$:

$$E(x) = \frac{A_L V^2}{A_R^2 N^2}(e^{-x} + x - 1) \tag{22a}$$
$$= \frac{A_L V^2}{A_R^2 N^2} E_k(x) \tag{22b}$$
$$= \frac{A_L V^2}{A_R R} E_k(x) \tag{22c}$$
$$= \tau P_{max} E_k(x) \tag{22d}$$

where:

$$E_k(x) = e^{-x} + x - 1 \tag{23}$$

We plot $J_k(x)$ and $E_k(x)$:



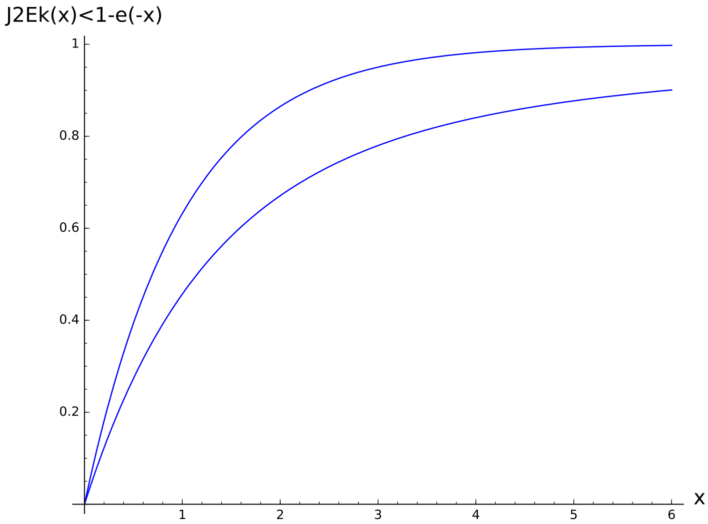
The asymptotes are $J_a(x) = x - 3/2$ and $E_a(x) = x - 1$.

The impulse to energy ratio function over time:

$$J2E(x) = \frac{J(x)}{E(x)} \tag{24a}$$
$$= \frac{\frac{A_F}{A_R}\tau P_{max} J_k(x)}{\tau P_{max} E_k(x)} \tag{24b}$$
$$= \frac{A_F}{A_R}\frac{J_k(x)}{E_k(x)} \tag{24c}$$
$$= \frac{A_F}{A_R} J2E_k(x) \tag{24d}$$

where:

$$J2E_k(x) = \frac{J_k(x)}{E_k(x)} \tag{25a}$$
$$= \frac{4 - e^{-x} + (2x-3)e^x}{2(1 + (x-1)e^x)} \tag{25b}$$



$1 - e^{-x}$ is always above $J2E_k(x)$. In fact, it is not until $x \approx 75$ that $J2E_k(x)$ reach the same level as $1 - e^{-x}$ was at $x = 5$, in other words $J2E_k(75) \approx 1 - e^{-5}$.

To benefit from the use of larger coils it is important to energize the coil long enough for the effect to appear. If a very large coil is used in a motor the RPM must be very low. Once again, look at: `https://youtu.be/r52VWrIZui8?t=11m26s`

8 Impulse to Energy Ratio over A_R

We will now compare impulse to energy ratio between two coils with the same A_L at a fixed time in seconds as a function of A_R. Let the impulse to energy ratio now be the function:

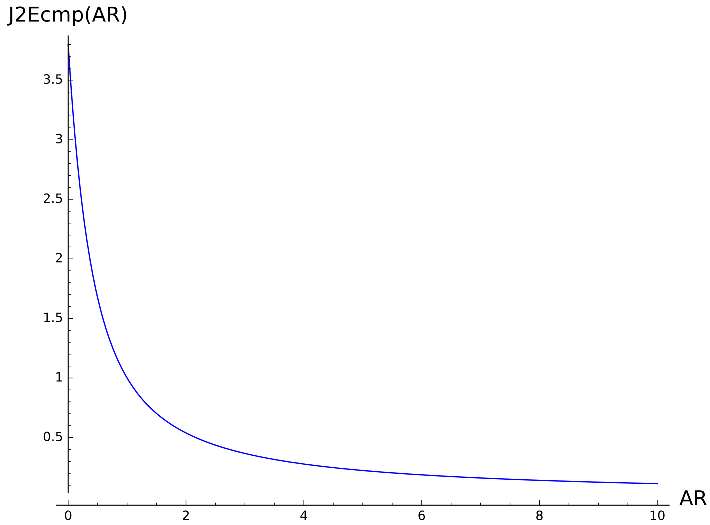
$$J2E(A_R, t) = \frac{A_F}{A_R}(1 - e^{-\frac{A_R t}{A_L}}) \tag{26}$$

Our reference coil will have $A_R = 1$ and we set $t = 5\tau = 5A_L/A_R$ for this coil so our comparison function is:

$$J2E_{cmp}(x) = \frac{J2E(x, 5A_L/1)}{J2E(1, 5A_L/1)} \tag{27a}$$

$$= \frac{(4e^5 + 1)((10x - 3)e^{10x} + 4e^{5x} - 1)e^{5-5x}}{(7e^{10} + 4e^5 - 1)((5x - 1)e^{5x} + 1)x} \tag{27b}$$

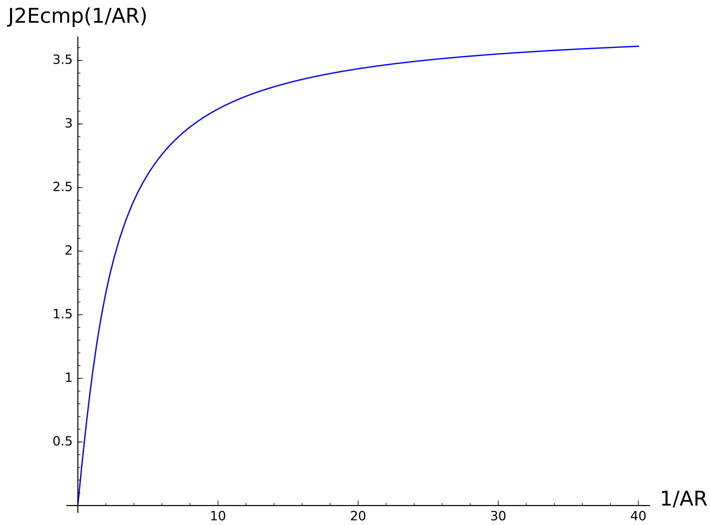
Let us plot $J2E_{cmp}(A_R)$:



It turns out that when time is fixed there is a maximum increase in force:

$$\lim_{x \to +0} J2E_{cmp}(x) = \frac{20e^5(4e^5+1)}{3(7e^{10}+4e^5-1)} \approx 3.80 \tag{28}$$

Let us plot $J2E_{cmp}(1/A_R)$ to reflect the increase in copper instead of increase in resistance-factor:



For $1/A_R = 19$ the impulse is about 3.42 times greater and circa 90% of the maximum, and for $1/A_R = 9$ the impulse is about 3.05 times greater and circa 80% of the maximum. The time the coils are charged with respect to its own time-constant are $5/19\tau$ and $5/9\tau$ respectively.

9 Conclusion

Although more can be explored in this area the general rule to get the best performance out of an electromagnet is to have the electromagnet as big as possible, which leads us to believe that motors should have as few and big coils as possible[6] to increase torque times speed per power.

A In Plain English

Whenever we spend something, be it time, money, energy or power, we want to get the most value back, so we calculate the performance per input ratio. In this paper we have proposed a way of doing the same with electromagnets. The amount of copper in an electromagnet is one important factor that affects the force per power ratio, not the number of turns or the wire thickness in the coil. The general rule is; the more copper the greater force per power.

B På Ren Svenska

När vi spenderar något, vare sig det är tid, pengar, energi eller kraft, vi vill få ut mesta möjliga tillbaka, så vi beräknar förhållandet mellan prestanda och input. I den här artikeln har vi föreslagit ett sätt att göra samma sak med elektromagneter. Mängden koppar i en elektromagnet är en viktig faktor som påverkar förhållandet mellan kraft och effekt, inte antalet varv eller trådtjocklek i spolen. Den generella regeln är; ju mer koppar desto större kraft per effekt.

C A_F, A_R and A_L

A_F was introduced to bundle together the parameters related to the core:

$$A_F = \frac{\mu^2 A}{2\,\mu_0\, g^2} \tag{29}$$

A_R is what we call the DC resistance factor and depends on the form of the windings of the inductor. If W is the wire then we have:

$$R = \rho \frac{W_{length}}{W_{area}} \tag{30a}$$

$$N = \frac{d}{W_{diam}} \frac{l}{W_{diam}} \tag{30b}$$

$$W_{length} = 2\pi r\, N \tag{30c}$$

$$W_{area} = \frac{\pi}{4} \frac{d\, l}{N} \tag{30d}$$

$$R = \frac{\rho}{\varrho} \frac{2\pi r\, N\, 4N}{\pi\, d\, l} \tag{30e}$$

$$R = 8\frac{\rho}{\varrho} \frac{r}{d\, l} N^2 \tag{30f}$$

$$A_R = 8\frac{\rho}{\varrho} \frac{r}{d\, l} \tag{30g}$$

$$R = A_R N^2 \tag{30h}$$

where ρ is the resistivity of the material in the wire, which for copper is $16.78\mathrm{n}\Omega\,\mathrm{m}$, ϱ is the tightness of the windings, r is the mean radius, d is the depth and l is the length. A realistic value for ϱ seems to be 8/11 which gives us:

$$A_R = 11\rho \frac{r}{d\, l} \tag{31}$$

which is what we use. A_R is similar in its definition to A_L, the inductance factor. The definition of A_L is $L = A_L\, N^2$. A_L is frequently specified by transformer-core manufacturers but can also be used for air-core coils. For a transformer-core we have:

$$A_L = \frac{\mu A}{l} \tag{32}$$

For a multilayer air-core coil we have[5]:

$$A_L = \frac{1}{31750} \frac{r^2}{6r + 9l + 10d} \tag{33}$$

where r, l, d are in m and A_L is in H,

D This Paper

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Figure 4: 1B79p75vQw4Rb1GQdmGYpDapFwEytFJDqw

We like to thank George Gray for proofreading v0.6 of this paper.

References

[1] U.A. Bakshi and A.V. Bakshi. *Electromagnet Theory*. 2009. URL: https://books.google.se/books?id=c369bbFJ1FYC&lpg=PP1&pg=SA7-PA121#v=onepage&q&f=false.

[2] *Exponential Decay*. URL: https://en.wikipedia.org/wiki/Exponential_decay.

[3] *Force exerted by magnetic field*. URL: https://en.wikipedia.org/wiki/Electromagnet#Force_exerted_by_magnetic_field.

[4] Highcap. *Elektromagneter*. URL: http://highcap.se/magneter/elektromagneter.

[5] *Inductance formulas*. URL: http://en.wikipedia.org/wiki/Inductor#Inductance_formulas.

[6] Joseph Westley Newman. *The Energy Machine of Joseph Newman*. 1984. URL: http://www.free-energy-info.tuks.nl.