Exercice 1

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$n=1$: $\quad 1 = \frac{1\cdot(1+1)}{2} \quad \checkmark$

$n = n+1$: $\quad \sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$

$$\sum_{i=1}^{n} i + (n+1) = \frac{n^2+3n+2}{2}$$

$$\frac{n(n+1)}{2} + (n+1) = \frac{1}{2}n^2 + \frac{3}{2}n + 1$$

$$\frac{1}{2}n^2 + \underbrace{\frac{1}{2}n + n}_{\frac{3}{2}n} + 1 = \frac{1}{2}n^2 + \frac{3}{2}n + 1 \quad \checkmark$$

Exercice 2:

$$\sum_{i=1}^{d} (2^{d-i} \cdot i) \leq 2^{d+1} - d - 2 \qquad \Rightarrow A(d) \leq B(d)$$

$d=1$: $2^{1-1} \cdot 1 = 1 \leq 2^{1+1} - 1 - 2 = 1$ ✓

$d = d+1$

$$\sum_{i=1}^{d+1} (2^{d+1-i} \cdot i) \leq 2^{d+2} - (d+1) - 2$$

$$\leq 2 \cdot 2^{d+1} - d - 3$$

$$\leq 2 \cdot (2^{d+1} - d - 2) + d + 1$$

$$2 \cdot \sum_{i=1}^{d+1} (2^{d-i} \cdot i) \leq$$

$$2 \cdot \sum_{i=1}^{d} (2^{d-i} \cdot i) + 2^{d-(d+1)} \cdot (d+1) \leq$$

$$2 \cdot \sum_{i=1}^{d} (2^{d-i} \cdot i) + 2(d+1) \leq$$

$$2 \cdot \left(A(d) + (d+1)\right) \leq 2 \cdot \left(B(d) + (d+1)\right)$$ ✓