

Exercise 1

IA:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

IB:

$$n=1: \quad 1 = \frac{1 \cdot (1+1)}{2} \quad \checkmark$$

IS:

$$n = n+1: \quad \sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$$

$$\sum_{i=1}^n i + (n+1) = \frac{n^2 + 3n + 2}{2}$$

$$\frac{n(n+1)}{2} + (n+1) = \frac{1}{2}n^2 + \frac{3}{2}n + 1$$

$$\frac{1}{2}n^2 + \underbrace{\frac{1}{2}n + n + 1}_{\frac{3}{2}n} = \frac{1}{2}n^2 + \frac{3}{2}n + 1 \quad \checkmark$$

Exercise 2:

IA:

$$\sum_{i=1}^d (2^{d-i} \cdot i) \leq 2^{d+1} - d - 2 \quad \Rightarrow A(d) \leq B(d)$$

IB:

$$d=1: 2^{1-1} \cdot 1 = 1 \leq 2^{1+1} - 1 - 2 = 1 \quad \checkmark$$

IS:

$$\begin{aligned} d &= d+1 \\ \sum_{i=1}^{d+1} (2^{d+1-i} \cdot i) &\leq 2^{d+2} - (d+1) - 2 \\ &\leq 2 \cdot 2^{d+1} - d - 3 \\ &\leq 2 \cdot (2^{d+1} - d - 2) + d + 1 \\ 2 \cdot \sum_{i=1}^{d+1} (2^{d-i} \cdot i) &\leq \end{aligned}$$

$$2 \cdot \left(\sum_{i=1}^d (2^{d-i} \cdot i) + 2^{d-(d+1)} \cdot (d+1) \right) \leq$$

$$2 \cdot \left(\sum_{i=1}^d (2^{d-i} \cdot i) + 2^{-1} (d+1) \right) \leq$$

$$2 \cdot (A(d) + (d+1)) \leq 2 \cdot (B(d) + (d+1)) \quad \checkmark$$