# Rocks from the Ground Up

## *A Firm Foundation in Geology*

*Sam Tenka* *2020*

Rocks are hard; their study, harder. These notes aim to survey rock theory not with the onomatopoeic impenetrability, weight, and dryness of museum glass, 3rd edition textbooks, and tasting sessions, but in basic terms that illustrate the unity of science. The ideal audience is Emmy Noether and Henri Poincaré. A brilliant high schooler might also enjoy these notes.

## *Scylla and Charybdis*

Science is marked by its emphasis on observation — above reason, aesthetics, intuition, and revelation from authority — as a means to know the world. Nevertheless, intuition plays a fundamental, unavoidable role in shaping our Bayesian priors. Kant, for instance, proposed that our ability to sense gives rise to synthetic propositions known prior to experience. To Kant, one's internal understanding of one's vision suggests before one sees any particular thing that space is three-dimensional. One thus expects to observe nature as a collection of objects of definite color, smell, and texture, located in a three-dimensional space and evolving though one-dimensional time. Shockingly, experiments in the 20th century revealed such coordinates of description to be far from independent, forcing a retreat from Kant's scene-setting intuitions.

Specifically, *relativity* denies us an absolute separation between space and time, for two may mix as easily as axes of space may rotate into each other. A 30 mph cow chasing a 20 mph calf perceives its kid as approaching at *more* than 10 mph.

Meanwhile, *quantum mechanics* denies us a parameterization of objects by their observable properties. Though via three separate experiments we may observe an apple's color, smell, or texture, to insist that the experiments could in principle be performed simultaneously — and thus that the apple in principle has a definite color, smell, and texture — leads to logical contradictions.

Relativity and quantum mechanics together suggest the (experimentally confirmed) possibility of particles of spin $1/2$ that necessarily obey *Pauli exclusion*. Such particles are the leading candidate for the building blocks of rocks.

## *Classifying particles*

An object's physical state inhabits the projective space $\mathcal{P}V$ associated to some Hilbert space $V$. The relativity group $SO(1;3)$ acts on Minkowski space $M$ and thus on $\mathcal{P}V$. We assume this action is smooth and linear, i.e. described by a Lie algebra action $\rho : \mathfrak{so}(1;3) \to \mathrm{End}(V)$. Whenever $\exp(g) = \mathrm{id}_M$ we have $\exp(\rho(g)) = c_g \cdot \mathrm{id}_V$ for some complex phase $c_g$. Strikingly, the value of $c_g$ has bearing on experiments, and it is not always 1.

## *Relativistic quantum fields*

## *Spin and statistics*

## *The solidity of rocks*